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A new method for approximate solutions of some nonlinear equations: Residual power series method

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Abstract

In this work, a powerful iterative method called residual power series method is introduced to obtain approximate solutions of nonlinear time-dependent generalized Fitzhugh–Nagumo equation with time-dependent coefficients and Sharma–Tasso–Olver equation subjected to certain initial conditions. The consequences show that this method is efficient and convenient, and can be applied to a large sort of problems. The approximate solutions are compared with the known exact solutions.

Keywords

Residual power series method, nonlinear time-dependent generalized Fitzhugh–Nagumo equation, Sharma–Tasso–Olver equation

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Introduction

Partial differential equations can define a number of physical problems in different fields of science. These linear and nonlinear problems play important roles in applied science. There are many analytical approximate methods to solve problems in the literature such as the homotopy analysis method proposed by Liao,[1,2] the variational iteration method proposed by He,[3,4] and homotopy perturbation method.[5,6] Among these, residual power series method (RPSM) is a new algorithm.

The RPSM was developed as an efficient method for determining values of coefficients of the power series solution for fuzzy differential equations.[7] The RPSM is constituted with a repeated algorithm. This method is effective and easy to obtain power series solution for strongly linear and nonlinear equations without linearization, perturbation, or discretization. Unlike the classical power series method, the RPSM does not need to match the coefficients of the corresponding terms and a repeated relation is not required. This method calculates the coefficients of the power series by a chain of algebraic equations of one or more variables. Besides, the RPSM does not require any converting while changing from the higher order to the lower order; thus, the method can be applied directly to the given problem by choosing an appropriate initial guess approximation.

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It has been successfully put into practice to handle the approximate solution of the generalized Lane–Emden equation,[8] the solution of composite and non-composite fractional differential equations,[9] predicting and representing the multiplicity of solutions to boundary value problems of fractional order,[10] constructing and predicting the solitary pattern solutions for non-linear time-fractional dispersive partial differential equations,[11] the approximate solution of the nonlinear fractional KdV–Burgers equation,[12] the approximate solutions of fractional population diffusion model,[13] and the numerical solutions of linear non-homogeneous partial differential equations of fractional order.[14] The proposed method is an alternative process for getting analytic Maclaurin series solution of problems. This method has proved to be powerful and effective, and can easily handle a wide class of linear and nonlinear problems.

The purpose of this work is to employ RPSM to obtain the numerical solution for generalized Fitzhugh–Nagumo equation (FNE) with time-dependent coefficients[15] and Sharma–Tasso–Olver equation (STOE).[16] Nonlinear time-dependent generalized FNE is given by[15]

$$u_t + \cos(t)u_x - \cos(t)u_{xx} + 2\cos(t)(u(1-u)(\rho-u)) = 0$$
$$(x,t) \in [A,B]\times[0,T],\ 0 \le \rho \le 1 \tag{1.1}$$

subjected to the initial condition

$$u(x,0) = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right),\ x \in [A,B] \tag{1.2}$$

Using specific solitary wave ansatz and the tanh method (TanhM), new variety of soliton solutions are introduced in Triki and Wazwaz.[15] Bhrawy[17] applied the Jacobi–Gauss–Lobatto collocation method to solve the generalized FNE. In recent years, many physicists and mathematicians have paid much attention to the FNE on account of its importance in mathematical physics.[18–23]

The following nonlinear equation is obtained

$$u_t + \alpha\left(u^3\right)_x + \frac{3}{2}\alpha\left(u^2\right)_{xx} + \alpha u_{xxx} = 0 \tag{1.3}$$

where α is a real parameter and $u(x,t)$ is the unknown function depending on the variable t and x. Equation (1.3) be called STOE in literatures. The STOE appear in quantum field theory, relativistic physics, dispersive wave phenomena, plasma physics, nonlinear optics, and applied and physical sciences.[24–28] In addition, in Jafari et al.,[29] fractional sub-equation method is used to construct exact solution of the nonlinear fractional STOE.

The outline of the remainder of this article is as follows. In section "Numerical applications of the RPSM," we present some properties of RPSM and its numerical applications for generalized FNE with time-dependent coefficients and STOE. Section "Graphical results" shows formed graphics and drew tables for the reliability of obtained solutions. Finally, some concluding remarks are given and graphics are formed in section "Conclusion."

Numerical applications of the RPSM

In this section, we apply RPSM to solve the above-proposed equations.

Time-dependent generalized FNE

Consider generalized FNE with time-dependent coefficients (1.1) and (1.2).

The exact solution for equation (1.1) is[15]

$$u(x,t) = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho}{2}(x-(3-\rho)\sin(t))\right)$$

We apply the RPSM to find out series solution for this equation subjected to given initial conditions by replacing its power series expansion with its truncated residual function. From this equation, a repetition formula for the calculation of coefficients is supplied, while coefficients in power series expansion can be calculated repeatedly from the truncated residual function.[9,30]

Suppose that the solution takes the expansion form

$$u = \sum_{n=0}^{\infty} f_n(x)t^n,\ 0 \le t < R, x \in I \tag{2.1}$$

Next, we let u_k to denote kth, truncated series of u

$$u_k = \sum_{n=0}^{k} f_n(x)t^n, 0 \le t < R, x \in I \tag{2.2}$$

where $u_0 = f_0(x) = u(x,0) = f(x)$.

Equation (2.2) can be written as

$$u_k = f(x) + \sum_{n=1}^{k} f_n(x)t^n,\ 0 \le t < R,\ x \in I,\ k = \overline{1,\infty} \tag{2.3}$$

First, to find the value of coefficients $f_n(x)$, $n = 1,2,3,\ldots,k$ in series expansion of equation (2.3), we define residual function Res, for equation (1.1), as

$$Res = u_t + \cos(t)u_x - \cos(t)u_{xx} + 2\cos(t)(u(1-u)(\rho-u))$$

and the kth residual function, Res_k, as follows

$$Res_k = (u_k)_t + \cos(t)(u_k)_x - \cos(t)(u_k)_{xx} + 2\cos(t)(u_k(1-u_k)(\rho-u_k)) \quad (2.4)$$
$$k = 1,2,3,\ldots$$

As in Abu Arqub and colleagues,[7–10] it is clear that $Res = 0$ and $\lim_{k\to\infty} Res_k = Res$ for each $x \in I$ and $t \geq 0$. Then, $(\partial^r Res/\partial t^r) = 0$ when $t = 0$ for each $r = \overline{0,k}$. To determine $f_1(x)$, we write $k = 1$ in equation (2.4)

$$Res_1 = (u_1)_t + \cos(t)(u_1)_x - \cos(t)(u_1)_{xx} + 2\cos(t)(u_1(1-u_1)(\rho-u_1)) \quad (2.5)$$

where

$$u_1 = f(x) + tf_1(x)$$

for

$$u_0 = f_0(x) = f(x) = u(x,0) = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right)$$

From equation (2.5), we deduce that $Res_1 = 0$ $(t = 0)$ and thus

$$f_1(x) = \frac{1}{4}(-3+\rho)\rho^2 \sec h^2\left(\frac{\rho x}{2}\right) \quad (2.6)$$

Therefore, the 1st residual power series (RPS) approximate solutions are

$$u_1 = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right) + \frac{1}{4}(-3+\rho)\rho^2 \sec h^2\left(\frac{\rho x}{2}\right)t \quad (2.7)$$

Similarly, to find out the form of the second unknown coefficient, $f_2(x)$, we write

$$u_2 = f(x) + tf_1(x) + t^2 f_2(x)$$

in Res_2.

$(\partial Res_2/\partial t) = 0$ $(t = 0)$ and thus

$$f_2(x) = -\frac{1}{8}(-3+\rho)^2\rho^3 \sec h^2\left(\frac{\rho x}{2}\right)\tanh\left(\frac{\rho x}{2}\right) \quad (2.8)$$

Therefore, the 2nd RPS approximate solutions are

$$u_2 = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right) + \frac{1}{4}(-3+\rho)\rho^2 \sec h^2\left(\frac{\rho x}{2}\right)t - \frac{1}{8}(-3+\rho)^2\rho^3 \sec h^2\left(\frac{\rho x}{2}\right)\tanh\left(\frac{\rho x}{2}\right)t^2. \quad (2.9)$$

Similarly, we write

$$u_3 = f(x) + tf_1(x) + t^2 f_2(x) + t^3 f_3(x)$$

in Res_2.

$(\partial^2 Res/\partial t^2) = 0$ $(t = 0)$ and thus

$$f_3(x) = \frac{1}{48}\rho^2(3 - \rho + 54\rho^2 - 54\rho^3 + 18\rho^4 - 2\rho^5 + 3\cosh(\rho x) - \rho\cosh(\rho x) - 27\rho^2\cosh(\rho x) + 27\rho^3\cosh(\rho x) - 9\rho^4\cosh(\rho x) + \rho^5\cosh(\rho x))\sec h^4\left(\frac{\rho x}{2}\right) \quad (2.10)$$

Therefore, the 3rd RPS approximate solutions are

$$u_3 = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right) + \frac{1}{4}(-3+\rho)\rho^2 \sec h^2\left(\frac{\rho x}{2}\right)t - \frac{1}{8}(-3+\rho)^2\rho^3 \sec h^2\left(\frac{\rho x}{2}\right)\tanh\left(\frac{\rho x}{2}\right)t^2 + \frac{t^3}{48}\rho^2(3 - \rho + 54\rho^2 - 54\rho^3 + 18\rho^4 - 2\rho^5 + 3\cosh(\rho x) - \rho\cosh(\rho x) - 27\rho^2\cosh(\rho x) + 27\rho^3\cosh(\rho x) - 9\rho^4\cosh(\rho x) + \rho^5\cosh(\rho x))\sec h^4\left(\frac{\rho x}{2}\right) \quad (2.11)$$

STOE

Consider equation (1.3) with the initial condition[16]

$$u(x,0) = \frac{1}{1+e^{-x}}$$

The exact solution for equation (1.3) is[16]

$$u(x,t) = \frac{1}{1+e^{-(x-\alpha t)}}$$

We apply the RPSM to find out series solution for this equation. Suppose that the solution takes the expansion form

$$u = \sum_{n=0}^{\infty} f_n(x)t^n, 0 \leq t < R,\ x \in I \quad (2.12)$$

where u_k is the truncated series of u

$$u_k = \sum_{n=0}^{k} f_n(x)t^n, 0 \leq t < R, x \in I \quad (2.13)$$

where $u_0 = f_0(x) = u(x,0) = f(x)$.

To find the value of coefficients $f_n(x)$, $n = 1,2,3,\ldots,k$ in series expansion of equation (2.3), we define residual function Res, for equation (1.3), as

$$Res = u_t + \alpha\left(u^3\right)_x + \frac{3}{2}\alpha\left(u^2\right)_{xx} + \alpha u_{xxx}$$

and the kth residual function, Res_k, as follows

$$Res_k = (u_k)_t + \alpha\left(u_k^3\right)_x + \frac{3}{2}\alpha\left(u_k^2\right)_{xx} + \alpha(u_k)_{xxx},\ k = 1, 2, 3, \ldots \tag{2.14}$$

To determine $f_1(x)$, we write $k = 1$ in equation (2.14)

$$Res_1 = (u_1)_t + \alpha\left(u_1^3\right)_x + \frac{3}{2}\alpha\left(u_1^2\right)_{xx} + \alpha(u_1)_{xxx} \tag{2.15}$$

where

$$u_1 = f(x) + tf_1(x)$$

for

$$u_0 = f_0(x) = f(x) = u(x, 0) = \frac{1}{1 + e^{-x}}$$

From equation (2.15), we deduce that $Res_1 = 0$ $(t = 0)$ and thus

$$f_1(x) = -\frac{e^x\alpha}{(1 + e^x)^2} \tag{2.16}$$

The 1st RPS approximate solutions are

$$u_1 = \frac{1}{1 + e^{-x}} - \frac{e^x\alpha}{(1 + e^x)^2}t \tag{2.17}$$

Similarly, to find out the form of the second unknown coefficient, $f_2(x)$, we write

$$u_2 = f(x) + tf_1(x) + t^2f_2(x)$$

in Res_2.

$(\partial Res_2/\partial t) = 0$ $(t = 0)$ and thus

$$f_2(x) = -\frac{e^x(-1 + e^x)\alpha^2}{2(1 + e^x)^3} \tag{2.18}$$

Therefore, the 2nd RPS approximate solutions are

$$u_2 = \frac{1}{1 + e^{-x}} - \frac{e^x\alpha}{(1 + e^x)^2}t - \frac{e^x(-1 + e^x)\alpha^2}{2(1 + e^x)^3}t^2$$

Similarly, we write

$$u_3 = f(x) + tf_1(x) + t^2f_2(x) + t^3f_3(x)$$

in Res_2.

$(\partial^2 Res_2/\partial t^2) = 0$ $(t = 0)$ and thus

$$f_3(x) = -\frac{e^x(1 - 4e^x + e^{2x})\alpha^3}{6(1 + e^x)^4} \tag{2.19}$$

Therefore, the 3rd RPS approximate solutions are

$$u_3 = \frac{1}{1 + e^{-x}} - \frac{e^x\alpha}{(1 + e^x)^2}t - \frac{e^x(-1 + e^x)\alpha^2}{2(1 + e^x)^3}t^2 - \frac{e^x(1 - 4e^x + e^{2x})\alpha^3}{6(1 + e^x)^4}t^3 \tag{2.20}$$

and

$$f_4(x) = -\frac{e^x(-1 + 11e^x - 11e^{2x} + e^{3x})\alpha^4}{24(1 + e^x)^5} \tag{2.21}$$

$$u_4 = \frac{1}{1 + e^{-x}} - \frac{e^x\alpha}{(1 + e^x)^2}t - \frac{e^x(-1 + e^x)\alpha^2}{2(1 + e^x)^3}t^2 - \frac{e^x(1 - 4e^x + e^{2x})\alpha^3}{6(1 + e^x)^4}t^3 - \frac{e^x(-1 + 11e^x - 11e^{2x} + e^{3x})\alpha^4}{24(1 + e^x)^5}t^4 \tag{2.22}$$

Graphical results

In this section, we formed graphics and drew tables for the reliability of above-obtained solutions.

Figures 1–4 show that the exact error is smaller as the number of k increases. It is clear that the value of kth truncated series $u_k(x, t)$ affects the RPS approximate solutions. These figures clearly show that the convergence of the approximate solutions to the exact solution related to the order of the solution and the exact error is smaller as the order of the solution increases.

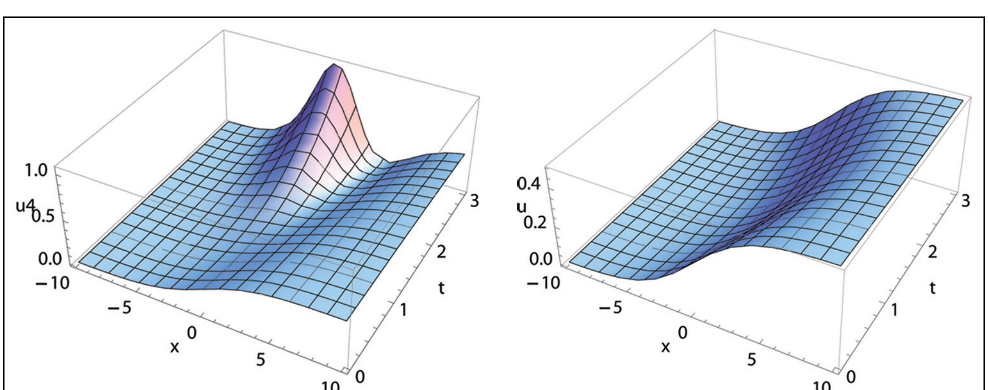

Figure 1. Surface graph of the RPS approximate solution and exact solution for equation (1.1) ($\rho = 0.5$).

Table 1. Comparison between RPS approximate solution $u_3(x,t)$ and exact solution of equation (1.1) ($\rho = 0.5$).

t	x				
	0.1	0.2	0.3	0.4	0.5
0.1	1.4606×10^{-9}	1.4285×10^{-7}	2.8588×10^{-7}	4.265×10^{-7}	5.6390×10^{-7}
0.2	2.3334×10^{-6}	3.5750×10^{-8}	2.2549×10^{-6}	4.5216×10^{-6}	6.7478×10^{-6}
0.3	2.3111×10^{-5}	1.1639×10^{-5}	1.3623×10^{-7}	1.1312×10^{-5}	2.2624×10^{-5}
0.4	1.0680×10^{-4}	7.1351×10^{-5}	3.5596×10^{-5}	1.9116×10^{-7}	3.5750×10^{-5}
0.5	3.3710×10^{-4}	2.5309×10^{-4}	1.6792×10^{-4}	8.2207×10^{-5}	3.424×10^{-6}

Table 2. Comparison between RPS approximate solution $u_4(x,t)$ and exact solution of equation (1.3) ($\alpha = 0.5$).

t	x				
	0.1	0.2	0.3	0.4	0.5
0.1	6.394×10^{-10}	6.012×10^{-10}	5.392×10^{-10}	4.574×10^{-10}	3.616×10^{-10}
0.2	2.0518×10^{-8}	1.9367×10^{-8}	1.7441×10^{-8}	1.4873×10^{-8}	1.1837×10^{-8}
0.3	1.5617×10^{-7}	1.4796×10^{-7}	1.3380×10^{-7}	1.1468×10^{-7}	9.188×10^{-8}
0.4	6.5927×10^{-7}	6.2694×10^{-7}	5.6929×10^{-7}	4.9035×10^{-7}	3.9545×10^{-7}
0.5	2.0145×10^{-6}	1.9228×10^{-6}	1.7531×10^{-6}	1.5174×10^{-6}	1.231×10^{-6}

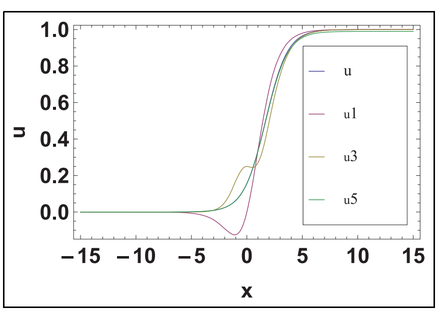

Figure 2. $u_k(x,t)$ solutions of (1.1) equation when $k = 1,3,5$ versus its exact solution ($\rho = 0.5, t = 1.5$).

Tables 1 and 2 clarify the convergence of the approximate solutions to the exact solution.

In Tables 3 and 4, comparison is made among approximate solutions with known results. These results are obtained using RPSM and TanhM.[15]

A comparison is made among approximate solutions with known results. These results are obtained using RPSM and the modified simple equation method (MSEM).[16]

Conclusion

The RPSM is applied successfully for solving the generalized FNE with time-dependent coefficients and STOE for certain initial conditions. The fundamental objective of this article is to introduce an algorithmic form and implement a new analytical repeated algorithm derived from the RPS to find numerical solutions for the FNE

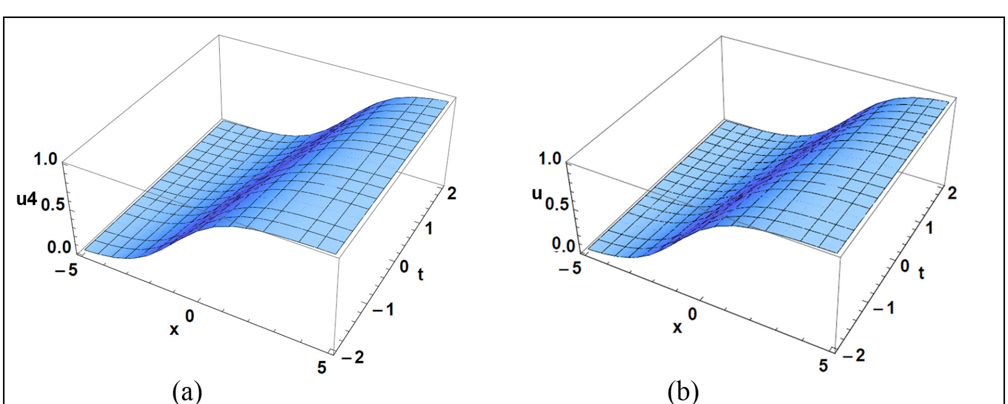

Figure 3. Surface graph of the RPS approximate solution and exact solution for equation (1.3) ($\alpha = 0.5$).

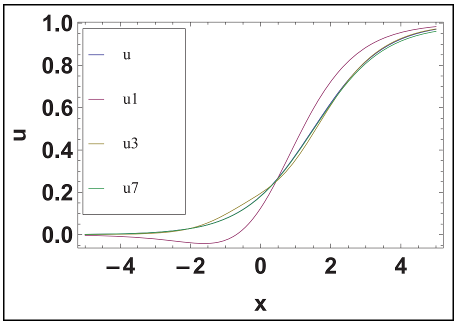

Figure 4. $u_k(x,t)$ solutions of (1.3) equation when $k = 1, 3, 7$ versus its exact solution ($\alpha = 1.5, t = 1$).

Table 4. Comparison between solutions u_{RPSM}, u_{MSEM}, and exact solution of equation (1.3) ($\alpha = 0.5, t = 0.5$).

x	u_{RPSM}	u_{MSEM}	u_{Exact}
0.1	0.462572	0.948998	0.46257
0.2	0.487505	0.90703	0.487503
0.3	0.512499	0.872409	0.512497
0.4	0.537431	0.843862	0.53743
0.5	0.562178	0.820404	0.562177

Table 3. Comparison between solutions u_{RPSM}, u_{Tanh}, and exact solution of equation (1.1) ($t = 1, q = 0.2$).

t	u_{RPSM}	u_{TanhM}	u_{Exact}
0.1	0.078322	0.102	0.077814
0.2	0.0792496	0.103998	0.0787668
0.3	0.0801809	0.105993	0.0797237
0.4	0.081116	0.107983	0.0806845
0.5	0.0820545	0.109967	0.0816491

and STOE. Graphical and numerical consequences are introduced to illustrate the solutions. Thus, it is concluded that the RPSM becomes powerful and efficient in finding numerical solutions for a wide class of nonlinear differential equations. The consequences emphasize the power of RPSM in handling a wide variety of nonlinear problems. The RPS does not require linearization, perturbation, or discretization of the variables, it is not impressed estimate of errors, and it is not confronted with necessity of large calculator memory and time. The main advantage of this method is the simplicity in calculating the coefficients of terms of the series solution using only the differential operators.

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