DEVELOPMENT OF LABORATORY EXPERIMENTS FOR CONTROL AND POWER SYSTEMS :THE WATER -TANK EXPERIMENT

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Title of the Thesis: **Development of Laboratory Experiments for Control and Power Systems: The Water -Tank Experiment**

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

DEVELOPMENT OF LABORATORY EXPERIMENTS FOR CONTROL AND POWER SYSTEMS :THE WATER -TANK EXPERIMENT

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Control system design is needed in many application areas such as manufacturing, automotive systems or power systems. Hence, a thorough education including practical experience is very beneficial for engineering students. In view of this, the thesis develops a water level control experiment for control laboratories. The system is designed to allow system modeling, set-point linearization and the application of various controller design algorithms both in continuous time and in discrete time. The thesis consists of two main parts. In the practical part, guidelines for the choice of system components such as pressure sensor, amplifiers, motor driver and water pump for the water tank system are given and supported by hardware experiment. In the theoretical part of the thesis, different variants of the water level control system are studied, and control methods such as pole placement, root locus, symmetrical optimum, Youlaparametrization and disturbance feedforward are applied. All these experiments are validated by simulations in Matlab/Simulink.

Keywords: Water level control, laboratory experiment, control system design, nonlinear modeling, linearization, control education

ÖZ

KONTROL VE GÜC SİSTEMLERİ İÇİN LABORATVAR DENEYLERİNİN GELİŞTİRİLMESİ: SU-TANK DENEYİ ÖRNEĞİ

ABAS ANWAR

Yüksek Lisans, Elektronik ve Haberleşme Mühendisliği Anabilim Dalı

TezYöneticisi: Doç. Dr. Klaus Werner SCHMIDT

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Kontrol sistem tasarımı sanayi, otomotiv ve güç sistemlerigibi birçok alanda uygulanmaktadır. Bunun için pratik deneyim içeren eksiksiz eğitim mühendislik öğrencileri için çok faydalıdır. Bu kapsamda, bu tez kontrol laboratuvarlar için su seviyesi kontrol deneyi geliştirmektedir. Bu sistemin tasarlanma nedeni, hem sürekli zaman hem de ayrık zaman içerisindesistem modelleştirmeye izin vermek, set-noktası doğrusallaştırma ve farklı kontrol tasarım algoritması uygulamalarını gerçekleştirmektir.

Bu tez iki ana bölümden uluşmaktadır. Pratik bölümde, su tankı sistemi için basınç algılayıcı,amplifikatörler, motor sürücü ve su pompası gibi sistem bileşenlerinin seçimi için kurallar verilmekte ve donanım deneyi ile desteklenmektedir. Tezin teorik bölümünde, su seviye kontrol sistemin farklı türevleri araştırılmaktadır, ve kutup yerleştirme, kök yer eğrisi, simetrik optimum, Youla parametrizasyonu ve ileri beslemeli karışıklık gibikontrol yöntemleri uygulanmaktadır. Tüm bu deneyler Matlab/Simulink simülasyonları ile doğrulanmaktadır.

Anahtar Kelimeler: Su seviyesi kontrol, laboratuvar deneyi, kontrol sistem tasarımı, doğrusal olmayan modelleme, doğrusallaştırma, kontrol eğitimi.

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Also special thanks are given to my parents. And my brothers Aytekin, levend, Ali and Mohammed . Their endless love and support are the motivation of my progress. They comprehend my career and suffer long time separation to me, Last, but by no means least, I thank my friends in Iraq and Turkey for their support and encouragement. For any errors or inadequacies that may remain in this work, of course, the responsibility is entirely my own.

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INTRODUCTION

Control systems are employed in many branches of industrial systems such as automation systems, production systems, power systems and power plants, automobiles, etc. [8, 19, 20]. Such control systems are designed most efficiently based on an analytical model of the system to be controlled. If such model is available, a large variety of methods .for this reason, control education is an important subject in Electronic Engineering, Mechatronics Engineering and Mechanical Engineering disciplines. The education is usually given on a theoretical level, but it is very important for students to apply their acquired knowledge in practical laboratory setups. Although control methods for varios types of systems such as production systems, power systems or automotive systems are studied, laboratory setups for education must fulfill certain requirements such as operational safety, reasonable cost, and applicability of many methods to the same system. for that reason, one of the most suitable laboratory setup is the water-tank system, that consists of different tanks that are connected with each other and fed by pumps. In line with the previous discussion, the topic of this In the theoretical part of the thesis, different versions of a three-tank system are modeled in the form of nonlinear differential equations. In addition, set-point linearization is applied in order to obtain linear time-invariant (LTI) models around the set-point. These LTI models are then used to perform various control designs with different control design methods that are relevant in practice. The methods employed in this thesis include the root locus method [20], pole placement [8], Youla parametrization [8], symmetric optimum [25] and disturbance feedforward [8]. All controller designs are supported by simulations of the linearized control system as well as the actual nonlinear control system using Matlab/Simulink. In addition, all controllers are converted to discrete-time for a convenient implementation on digital computers. It has to be noted that these techniques were applied to a water-tank system as application example. However, the same techniques can be used for other system types such as production systems, power systems or automotive systems.

The main task of the practical part of this thesis is the choice of low-cost equipment for a one-tank system in order to perform the developed control system designs using a real experimental setup. The chosen components include a DC water pump [22], a pressure sensor [17], a PIC microcontroller [18], a high-current motor driver [23] and several amplifiers [16]. The output of the thesis is the setup of a simple one-tank system that allows initial tests of control algorithms on the PIC microcontroller.

The organization of the thesis is as follows. In Chapter 1, the basic properties of the water-tank system are described. In addition, this chapter provides details of the hardware setup used in this thesis. Chapter 2 presents the nonlinear modeling of the water-tank system and Chapter 3 performs set-point linearization for different variants of the water-tank system. A large variety of controller design methods to be used for the water-tank system we explained in Chapter 4. These methods are then applied to the water-tank system in Chapter 5 and supported by simulations in Matlab/Simulink.In the last we will take summary for this thesis.

CHAPTER I

WATER-TANK SYSTEM DESCRIPTION

1.1 THE WATER-TANK SYSTEM AS CONTROL EXPERIMENT

The three tank system that is shown in Figure 1.1 represents an example of a nonlinear control system, whose operation principle is used in real life. The system is composed of three tanks, each of which has a given height, and there is a reservoir for supplying these tanks with water. It is assumed that all of these tanks have the same square area. In addition, the three tank system has three pumps that are driven by DC motors in order to both supply water from the reservoir (pump 1 and pump 3) are transport water between the tanks (pump 2). The tanks 1-2 and 2-3 are connected by pipes, whereby the connection can be manually opened or closed by valves that are attached to the pipes. Each tank has one more pipe for discharging water to the reservoir. Again, a valve allows to open or close this connection. Also, each tank has a pressure sensor that is located at its base. Here, it is intended to measure the water pressure in order to determine the water level in each tank. The described water-tank system is an interesting control system for educational purposes, since it offers a large variety of control problems, and allows to study and apply a large number of controller design techniques. In the following, we give a list of such control design problems.

One-tank System with Tank 1

- Keep the water level in the tank constant, assuming there is outflow into the reservoir

- Control the pump such that the tank is filled to a certain level as fast as possible
- Keep the water level in the tank constant, assuming that pump 2 removes water from tank 1

Two-tank System with Tank 1 and Tank 2

- Keep the water level in tank 2 constant, only using pump 1 and with possible discharge of water to the reservoir from tank 1 and/or tank 2
- Control pump 1 such that tank 2 is filled to a certain level as fast as possible
- Control pump 1 and 2 such that both tanks stay at a certain level

Three-tank System

- Control pump 1 and 3 such that tank 1 and tank 3 stay at a certain level
- Control pump 1 and 3 such that tank 2 stays at a certain level
- Control pump 1, 2, 3 such that all tanks stay at a certain level

In order to solve the described control design problems, there are various methods that can be applied, and that are taught in basic control courses. First, the systems allows to perform step response measurements for system identification. That is, it is possible to determine the parameters of a mathematical model based on measurements from the real system. Second, considering that the tank system is nonlinear, it is possible to use the technique of *set-point linearization*. As a result, a linear time-invariant state space model or transfer function model is obtained, that is suitable for linear control system design methods. Third, a large variety of linear controller design methods can be applied such as

- Root locus
- Symmetrical optimum
- Youla parametrization

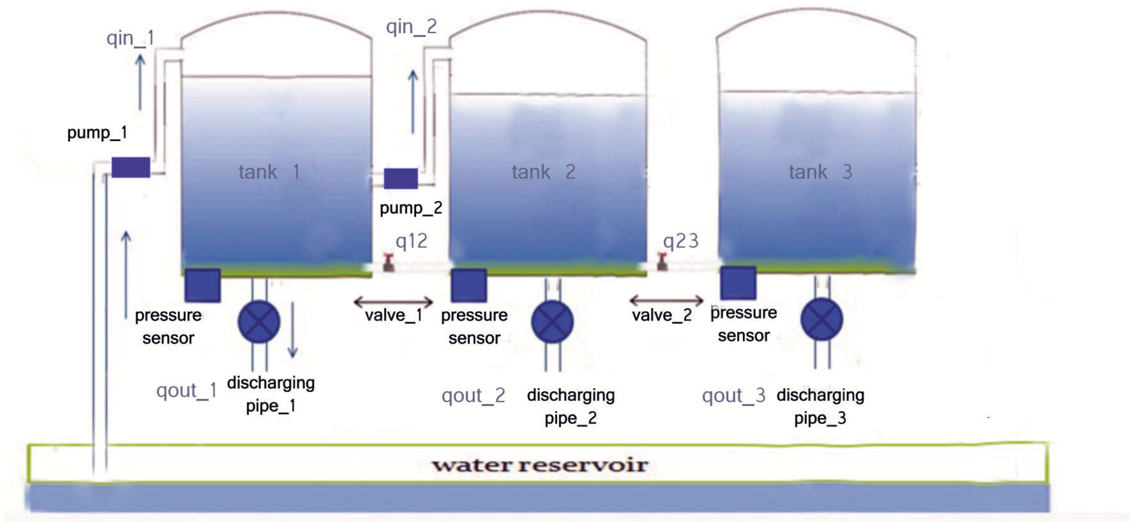


Figure 1.1: **Three tank fluid system**

- Pole placement
- Disturbance feedforward
- State feedback control

Fourth, it is possible to implement the designed algorithms on a digital controller (for example a PIC microcontroller). In that case, the technique of controller discretization with different integral approximations such as the Euler, Euler backward or trapezoidal method can be applied. Such realization also allows to study the effect of different sampling times in the control loop. Finally, since the system is nonlinear, it also enables the usage of more advanced nonlinear controller design methods. In addition, also intelligent control methods such as fuzzy control or neural network can be applied. Such study is for example done in [13]. In this thesis we do not consider intelligent control but focus on control method that are based on analytic model.

1.2 SETUP AND PARAMETERS

In this section, we describe the parameters of our three tank system laboratory experiment. The height of each tank is 100 cm and the (square) cross-section area of each tank is denoted as $A_T = 225$ cm^2. The connection pipes P12 and P23 between the tanks have the same (circular) cross-section area of $A_P = 6\,\pi$ cm^2. In addition, the pipes P1, P2 and P3 for discharging water to the reservoir have a cross-section area of $A_D = 16\,\pi$ cm^2. All pumps used in the system are identical. They are supplied by a DC voltage of up to 12 V and supply a maximum water flow of 20 l/min. The valves that are used in the system are operated manually. Generally, they are used either in the open position or in the closed position. For the valves V12 and V23, we write ($p_{V12} = 1$) ($p_{V23} = 1$) if the valve position is open and ($p_{V12} = 0$) ($p_{V23} = 0$) otherwise. Similarly, we use the valve position parameters p_{V1}, p_{V2}, p_{V3} for the valves V1, V2 and V3.

1.3 OUTLINE OF THE EXPERIMENTAL WATER-TANK SYSTEM

From the practical perspective, the water-tank system requires several hardware components. One important part of this thesis is the choice of these components and the test of their suitability. Main criteria for the component choice are component specifications according to the experiment requirements (as discussed below) and the availability in Turkey at low cost. The main components to be discussed are listed as follows.

- Pressure sensor: this component is needed to determine the water level in a tank.
- Pump: this component is used to supply water to a tank from the reservoir or from another tank.
- Microcontroller: this component is used to implement control algorithms. It receives measurement input from the pressure sensor and provides control input to the pump.

We now describe the different components chosen in this thesis together with additional required electronic components and measurements from the experimental setup.

1.3.1 Pressure Sensor

We choose the pressure sensor MPX7050 [17] for the water level measurement. This measurement is very important for the later control task, since it gives the actual feedback information about the state of the water-tank system. The relation between the water level and the pressure in the tank can be derived using basic physics and the following notation.

- p: is the pressure in a tank with the unit Pa (Pascal)
- g: is the gravitational acceleration with the value $9.81\,\mathrm{m/sec}^2$ or equivalently $981\,\mathrm{cm/sec}^2$
- ρ: is the density of water with the value $1\,\mathrm{kg/l}$ or equivalently $10^{-3}\,\mathrm{kg/cm}^3$
- h: is the water level in the tank with the unit cm
- A_i: is the area of the tank with the unit cm^2

Using these variables and parameters, the computation is as follows.

$$p = \frac{m\,g}{A_i} = \frac{A_i\,h\,\rho\,g}{A_i} = h\,\rho\,g \Rightarrow h = \frac{p}{\rho\,g} \tag{1.1}$$

We choose the sensor MPX7050 because of various reasons. First,our tank experiment is designed for water levels up to $h_{max} = 100\,\mathrm{cm}$. According to (1.1), this corresponds to a pressure of $p_{max} = 1\,\mathrm{m} \cdot 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9.81\,\frac{\mathrm{N}}{\mathrm{kg}} = 10\,000\,\frac{\mathrm{N}}{\mathrm{m}^2} = 10\,\mathrm{kPa}$. Since the MPX7050 can measure a pressure up to 50 kPa, it is suitable for the maximum pressure required in the water-tank system. In addition, it is important that the MPX7050 is a differential pressure sensor. That is, the sensor possesses two sides – a low-pressure side and a high-pressure side – and measures the pressure difference between both sides. In our experiment, the low-pressure side measures the atmospheric pressure, whereas the high-pressure side measures the water pressure in the tank (including the atmospheric pressure). As a result, changes in the atmospheric pressure will not influence the water level measurement. A picture of the MPX7050 with a special packaging (ending dp) is shown in Figure 1.2. This sensor is available in Turkey at a price of about 5 $.

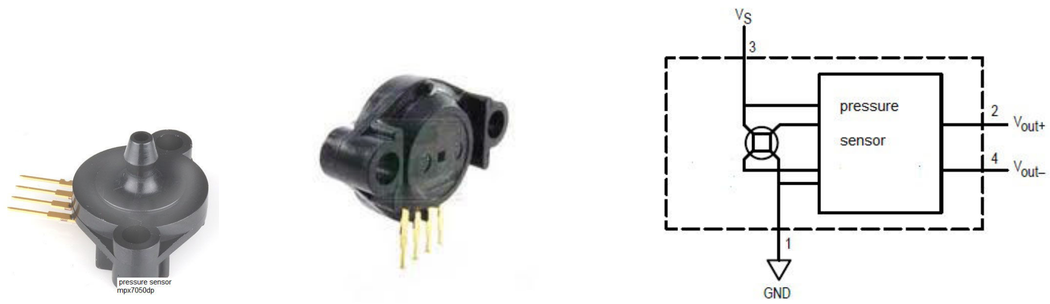


Figure 1.2: **Pressure sensor MPX7050dp: high pressure side (left), low pressure side (middle) and pin usage**

One further advantage of the MPX7050 is its linearity of the pressure measurement. The output voltage provided sensor increases linearly in the measured differential pressure. The following picture from the MPX7050 datasheet illustrates this fact. From this figure, it can be seen that the output voltage (denoted

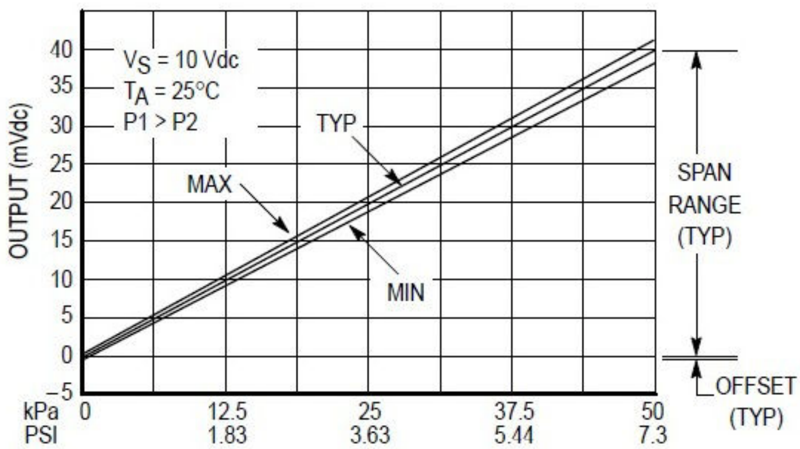


Figure 1.3: **Output voltage of the MPX7050 versus differential pressure.**

as v_{MPX}) is computed with the following formula.

$$v_{MPX} = v_{offset} + \frac{40\,\text{mV}}{50\,\text{kPa}} \cdot p = v_{offset} + 0.8\frac{\text{mV}}{\text{kPa}} \cdot p. \tag{1.2}$$

In this formula, v_{offset} is the so-called offset voltage that is provided by the MPX7050 at 0 differential pressure. The equation shows for example that an increase of the water level by 10 cm leads to a pressure increase of $\Delta p = 1\,\text{kPa}$, which leads to an output voltage increase of $\Delta v_{MPX} = 0.8\frac{\text{mV}}{\text{kPa}} \cdot 1\,\text{kPa} = 0.8\,\text{mV}$.

It can be seen from the previous computation that the voltage values are very small. Since they will be processed by the Analog to Digital converter of a PIC microcontroller, it is required to amplify v_{MPX}. Considering that the maximum differential pressure in the experiment is given by 10 kPa, the maximum output voltage provided by the MPX7050 is less than 10 mV (see Figure 1.3). The reason for this is that the pressure for a water level of 100 cm is 10 kpa, and the corresponding voltage is computed as 8 mV. In order to achieve a voltage range between 0 V and 5 V, we need an amplifier gain G_{MPX} of about $G_{MPX} = 500$. The most suitable amplifier Integrated circuit for this purpose is the instrumentational amplifier INA [11]. However, since this component is not available in Turkey, we use a different amplifier design based on the LM324 IC as is proposed in [9]. The schematic of the circuit is shown in Figure 1.4.

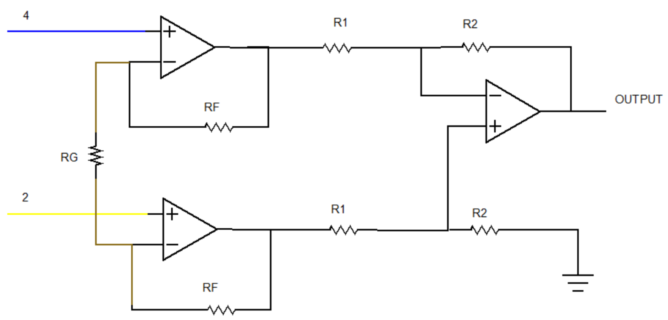


Figure 1.4: **Amplifier circuit for the pressure measurement.**

In our design, we choose the following resistance values.

- R1 = 1 kΩ
- R2 = 320 kΩ
- RF = 1 ´MΩ
- RG = 1 ´MΩ

and from these values we we find the amplifier gain with the following formula

$$G_{MPX} = G_1\, G_2 = \frac{Vo}{Vin}, \tag{1.3}$$

where

$$G_1 = 1 + 2\frac{RF}{RG} \text{ and } G_2 = \frac{R2}{R1} \tag{1.4}$$

As a result, we obtain an amplifier gain of $G_{MPX} = 960$.

Combining the results in (1.1) and (1.2) from before, it is now possible to determine the actual water level from the voltage measurement at the output of the amplifier v_{AMPL}:

$$p = \frac{v_{MPX} - v_{offset}}{0.8\,\frac{\mathrm{mV}}{\mathrm{kPa}}} = \frac{v_{AMPL}/G_{MPX} - v_{offset}}{0.8\,\frac{\mathrm{mV}}{\mathrm{kPa}}} \Rightarrow h = \frac{v_{AMPL}/G_{MPX} - v_{offset}}{0.8\,\frac{\mathrm{mV}}{\mathrm{kPa}}\,\rho\, g} \tag{1.5}$$

We test the combination of the pressure sensor and the amplifier circuit by two simple experiments. In the first experiment, we fill a water container with predefined water levels and measure the resulting voltage at the output of the amplifier. The result is shown in Table 1.1. From this measurement, the offset voltage of the pressure sensor (obtained for a water level of 0 cm) can be evaluated to $v_{offset} = 0.8\,\mathrm{V}/960 = 0.83\,\mathrm{mV}$. In addition, we can use (1.5) to compute the water level from the voltage measurement. The values are also given in the table.

Table 1.1: **Comparison of water level and pressure sensor measurement**.

water level	output voltage	estimated water level
0 cm	0.8 v	0 cm
5 cm	1.2 v	5.25 cm
10 cm	1.6 v	10.45 cm
15 cm	2 v	15.6 cm
20cm	2.4 v	20.12 cm
25 cm	2.8 v	26.08 cm
30 cm	3.2v	31.3 cm

Together, we can conclude that the values from the measurement fit the actual water level values precisely. Second, we perform an experiment with a constant inflow into a water container. In that case, it is expected that the water level – and equivalently the voltage output of the amplifier – increases linearly with time. The voltage measurement from an oscilloscope as shown in Figure 1.5 confirms the correct operation of the pressure sensor in combination with the amplifier circuit.

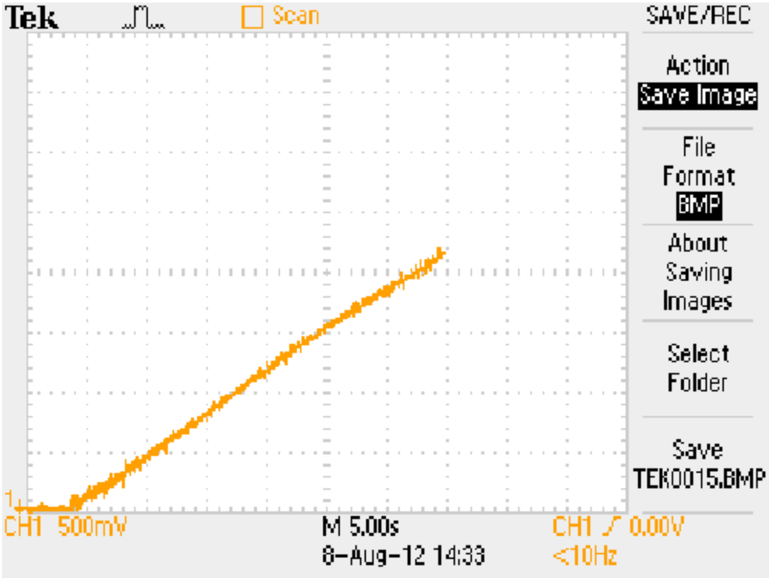


Figure 1.5: **Water level increase in the tank by using constant inflow.**

1.3.2 DC Pump

We use the DC pump that we see in Figure 1.6 to transport water from the reservoir to the water-tanks. It is operated by a DC motor, whose rotational

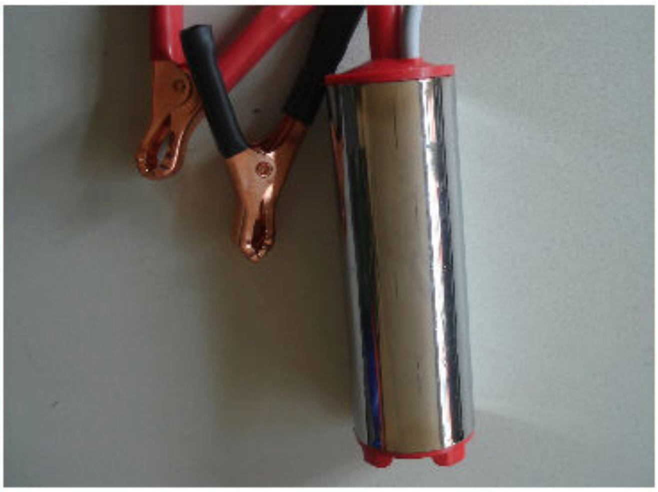
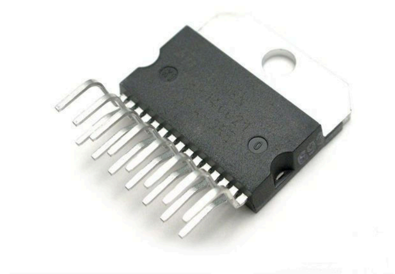


Figure 1.6: **DC water pump (Rich Multi Pump crom type)**

speed increases linearly with the provided input voltage v_{motor}. According to the specification, the maximum input voltage of the motor is 12 Volt, leading to a maximum water flow of 17 l/min at a maximum power of about 40 Watt. In our experiment, we use pulse-width modulation (PWM) in order to change the input voltage of the motor. That is, if we write u_{PWM} (in %) for the duty cycle of the PWM signal, we get

$$v_{motor} = u_{PWM} \cdot 12\,\mathrm{V} \tag{1.6}$$

for the input voltage of the DC motor. In our experiment, the PWM signal is supplied by a PIC microcontroller. Since the PWM signal cannot be used to supply power for the DC motor, a further driver circuit is needed to power the DC motor depending on the PWM signal. We choose the driver circuit L298 [23] as shown in Figure 1.7.



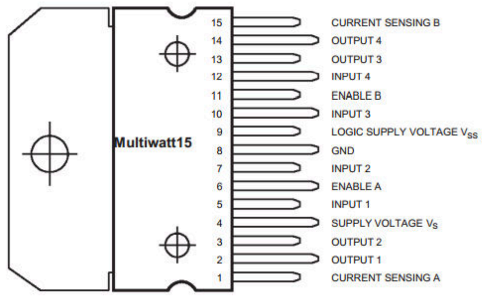


Figure 1.7: **L298n motor driver: packaging (left) and pin layout (right).**

It is suitable for our application, since it works for voltages up to 46 V and can supply current up to 4 A. In our application, we need currents up to 40 W/12 V = 3.33 A. The driver can actually supply power for two DC motors. In our experiment, we will only use the pins 1 to 9, which are enough to drive a single DC motor (our pump). It also has to be mentioned that this device has a low cost of about 17 Dollar.

We now illustrate the operation of the pump for different duty cycles of the PWM signal u_{PWM}. Since, we do not have a flow sensor, we supply water from the pump to a water-tank as can be seen in Figure 1.8. We measure the water level in the tank by using the pressure sensor as described in Section 1.3.1. As can be seen from the measurements in Figure 1.5, the water level increases linearly when a

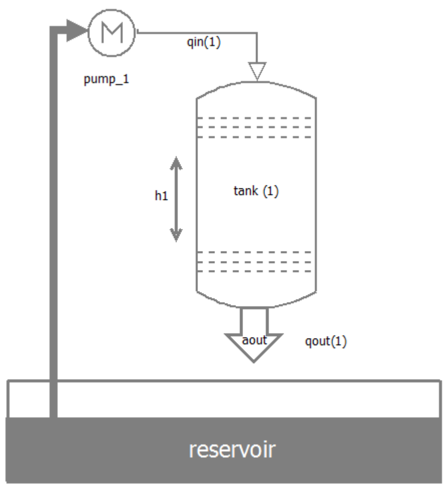


Figure 1.8: **One tank system with reservoir and DC pump.**

constant PWM value (PWM=170) is used.

1.3.3 Microcontroller (PIC16F877A)

In our current experimental setup, we use the PIC 16F877A in order to realize feedback control algorithms for the water level control. We have chose this PIC, since it can provide PWM output signals and also comes with ADC converters that are suitable for our pressure signal measurement. Furthermore, it provides the capability of transmitting data to a connected PC via the RS232 protocol or display data on an LCD screen, which will be important for data evaluation. The pin layout of the PIC 16F877A is shown in Figure 1.9.

In our experiment, we use pin 4 to digitalize the analog pressure sensor signal. Moreover, we use pin 16 to provide the PWM signal for the motor driver L298. The main control task is then to determine the control algorithm that changes the PWM duty cycle depending on the pressure sensor reading, which is the main task of the theoretical part of this thesis. These control algorithms are then implemented on the PIC16F877A in the form of a C-program using the sotware tool MicroC [5]. We use the flash-programmer (pickit 2) to write the machine code on the PIC.

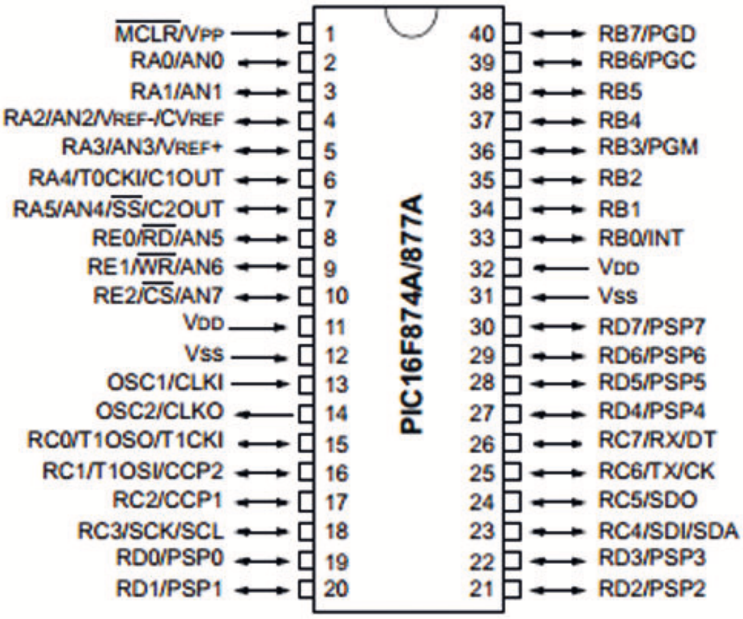


Figure 1.9: **Pic16f877a microcontroller chip Pin layout**

1.3.4 Pulse Width Modulation

The PWM is a technique used to control analog values by converting them to digital values in the form of a square wave as can be seen in Figure 1.10.

The PWM contains there relevant times periods ($T_{on}, T_{off}, T_{total}$=$(T_{on} + T_{off})$). By choosing these values, it is possible to achieve an analog voltage according to the following equation:

$$Vout = D \times Vin, \tag{1.7}$$

where D is called duty cycle and is equal to

$$D = \frac{T_{on}}{T_{total}} \tag{1.8}$$

PWM signals can be generated by many devices. In our experiment, we use the microcontroller PIC16F877A to generate the PWM. This PIC has two pins (ccp1,ccp2) for PWM output with voltage levels of 0 and 5 V. The duty cycle of the PWM signal can be set from our MicroC programming environment between values of 0 ($T_{on} = 0$) and 255 ($T_{off} = 0$). In the latter case, the PIC outputs a constant voltage of 5 V.

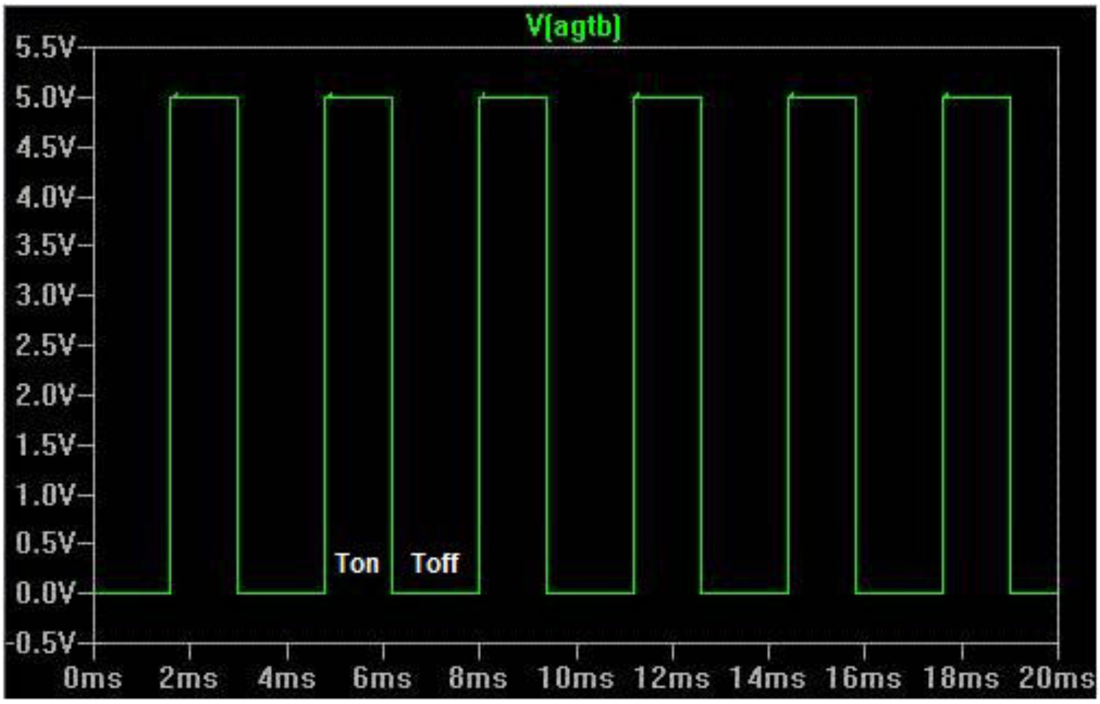


Figure 1.10: **The PWM signal from the Microcontroller**

1.3.5 Overall Experimental Setup

A schematic of the overall experimental setup is shown in Figure 1.11. In principle, we take the (low amplitude) voltage signal from the pressure sensor (pin 2 and 4) and amplify this signal by using our differential amplifier circuit. The output of the amplifier circuit is filtered by a RC (low-pass) filter in order to reduce high-frequency noise and then read by the ADC input of the PIC. The digitalized sensor reading is then processed by the PIC in order to determine an appropriate value of the PWM duty cycle. The PWM signal in turn regulates the output voltage of the motor driver so as to adjust the speed of the water pump. Note that the PWM signal is not directly fed to the motor driver, but the lower power PWM signal is separated from the higher power motor driver component by an additional amplifier that is only used as a buffer circuit.

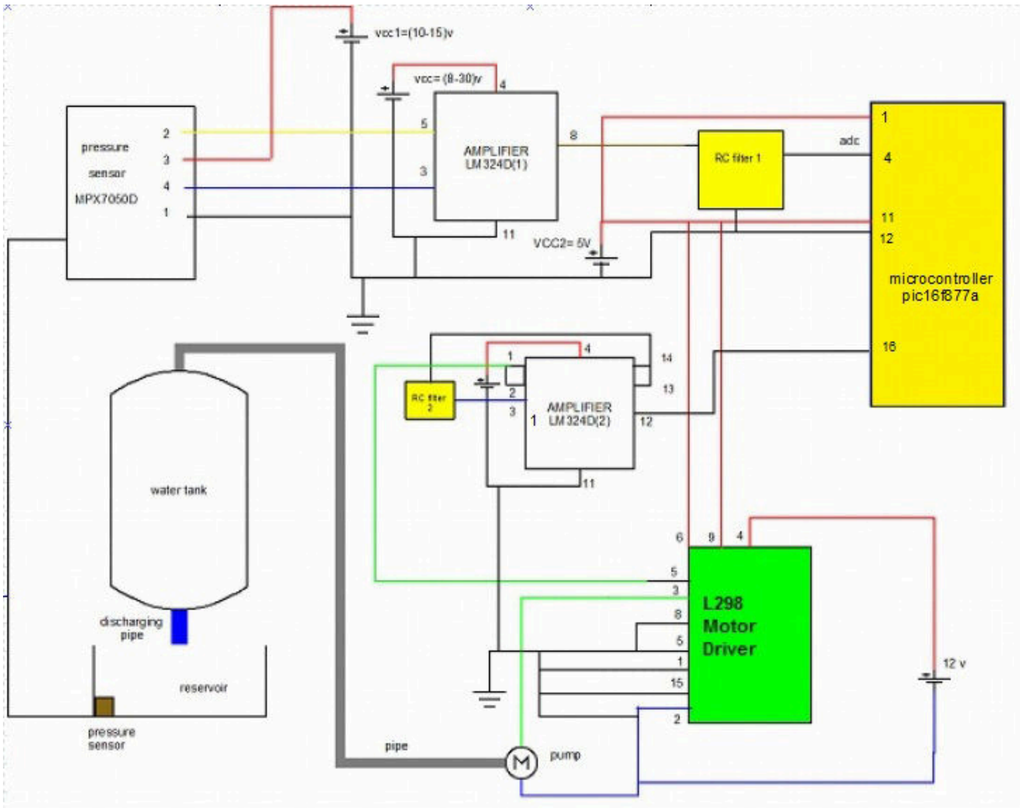


Figure 1.11: **Schematic Diagram of the water level control experiment.**

1.3.6 A Feedback Control Experiment

Now, we use a simple controller and test it with the one tank system. The controller that we use is PI controller and we convert it to a discrete-time representation such that we can realize it on the PIC microcontroller. The controller code is shown as follows.

```
int ek = 0; // error of the system
int dutycycle=190; // the initial duty cycle
unsigned ADC_Read(unsigned short channel);
unsigned  measurement;
int  eOld=0;
unsigned short uold=0;

void main()
{
TRISB=0;
PORTA=0xff; // ADC pin
  PORTC = 0xff; // PORTC is output
  PWM2_Init(20000); // Initialize PWM
PWM2_Start();

while(1){
measurement = ADC_Read(2);
  ek=(511-(int)measurement); // error signal
  dutycycle = (int)uold + (int)(.015*(float)ek - 0.005*(float)eOld);
if  (dutycycle > 255)
dutycycle=255;
if (dutycycle < 190)
dutycycle = 190;

uold = (unsigned short)dutycycle; // convert to unsigned int
eOld = ek; // memorize the old error for the next cycle
```

```
PWM2_Set_Duty((unsigned short)dutycycle);
Delay_ms(10); // wait for 10 ms
}
}
```

A measurement from the laboratory setup is shown in Figure 1.12. It shows the amplified voltage from the pressure sensor over time. It captures the return to the reference value of the water level after applying a disturbance (by switching of the pump). It has to be noted that, although the current experimental setup can be used in a feedback control experiment, the pump as described in Section 1.3.2 seems to be very insensitive to input changes. Hence, we suggest to find a different pump for a more precise experiment.

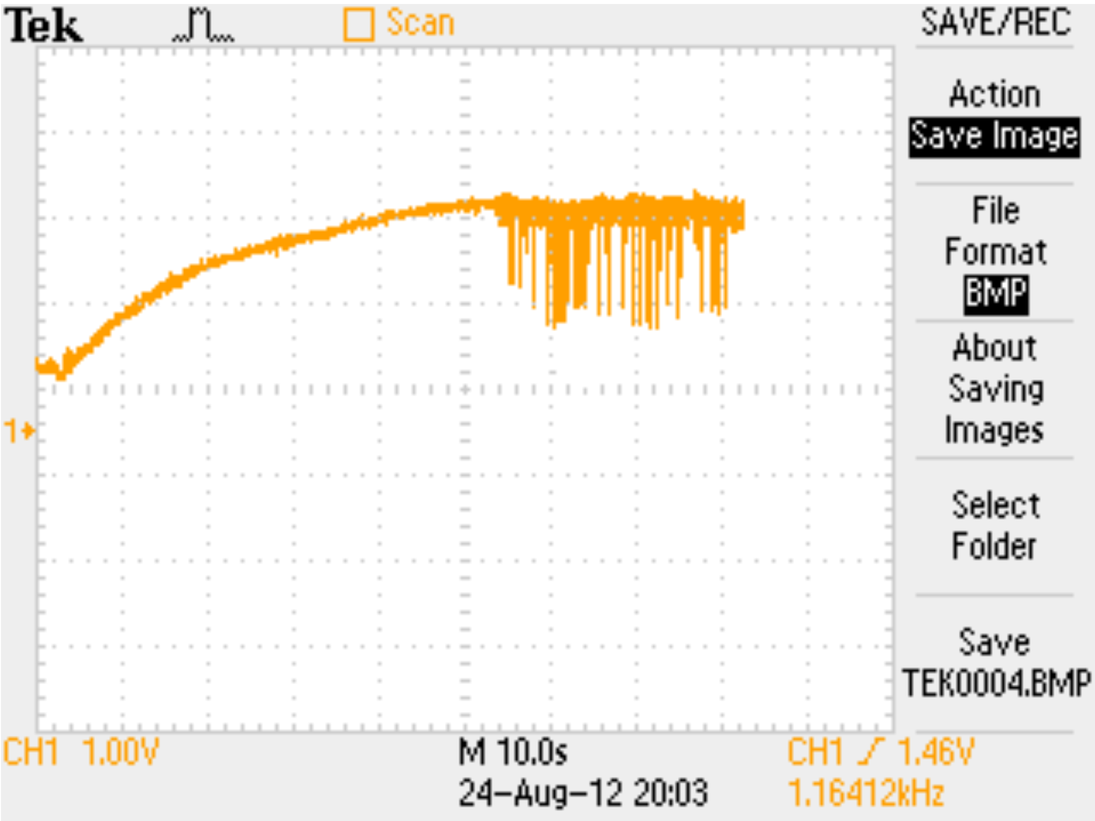


Figure 1.12: **Closed-loop experiment with PI controller on PIC micro-controller**

CHAPTER II

MODELING

The controller design methods to be used in this thesis are based on analytical models of the control system [8, 19]. Considering that the water-tank system turns out to be nonlinear, the nonlinear state equations are employed. They have the following form

$$\dot{x} = f(x, u) \tag{2.1}$$

$$y = h(x, u), \tag{2.2}$$

where x is the *state vector*, u is the control input vector, f is a nonlinear function in the state and input, and h is the nonlinear output function. In addition to the nonlinear state equations, we also use the linear state equations that are for example obtained after set-point linearization.

$$\dot{x} = A\,x + B\,u \tag{2.3}$$

$$y = C\,x + D\,u \tag{2.4}$$

Here, A is the *dynamic matrix*, B is the *input matrix*, C is the *output matrix* and D is the *feedthrough matrix*. Finally, we use transfer function models for both the plant and controllers. A plant transfer function model can for example be obtained from a linear state space model of the plant by evaluating

$$G(s) = C\,(s\,I - A)^{-1}\,B + D. \tag{2.5}$$

In this equation, s is the parameter of the Laplace transform and I is the identity matrix, whose dimension matches the dimension of A.

2.1 VARIABLES AND PARAMETERS

2.1.1 Pump

We use a DC pump as described in Section 1.3.2. The relevant parameters of the pump are given by

- Input voltage u_i for $i = 1, 2, 3$. The voltage will be between 0 V and 12 V
- Pump flow, which is input flow for the water-tank system: $q_{\text{in},i}$ for $i = 1, 2, 3$, the unit for the inflow is l/sec.

2.1.2 Tank

Water-tanks of the same kind with a maximum level of 100 cm will be used to store water. The exact specification of the water-tanks is

- Water level h_i for $i = 1, 2, 3$. The unit is cm and the maximum level of the water will be 100 cm.
- Surface area A_i for $i = 1, 2, 3$. The unit is cm^2.

2.1.3 Valves

There is one valve for each of the pipes, connecting different tanks. These valves can be manually closed (zero area) or opened (full area). In addition, there is on valve for each tank that can be manually opened for outflow of water. We use the following valve parameters.

- Connection valve area $a_{1,2}$ and $a_{2,3}$. The unit is cm^2.
- Outflow valve area $a_{\text{out},i}$ for $i = 1, 2, 3$. The unit is cm^2.

In our experiment, the connection valves have a diameter of about 6 cm and the outflow valves have a diameter of about 8 cm.

2.2 COMPONENT MODELS

In this section, we develop analytical models for the different components of the water-tank system.

2.2.1 Pump

For the pump, we propose to use a first-order lag model, since the pump is driven by a DC motor. We introduce the pump gain K_i and the pump time constant T_i for $i = 1, 2, 3$. The first-order differential equation is then

$$\dot{q}_{\text{in},i} = \frac{1}{T_i}(-q_{\text{in},i} + K_i\, u_i) \tag{2.6}$$

when$Ti = 1, Ki = 200$

2.2.2 Water-Tank

For the tank, we compute the water level change depending on inflow and outflow. The situation is shown in Figure 2.1. We call the inflow q_{in} and the outflow q_{out}. Then, we can write the water level change as follows.

$$A_i\, \dot{h} = q_{\text{in}} - q_{\text{out}} \Rightarrow \dot{h} = \frac{1}{A_i}(q_{\text{in}} - q_{\text{out}}) \tag{2.7}$$

2.2.3 Valves and Pipes

We model the pipes with valves using the law of Torricelli:

- The discharging pipe is connected to the water-tank with a water level h and discharges water to the reservoir if the valve is open (area a_{out}. The modeling equation for the outflow q_{out} from the discharging pipe is

$$q_{out} = a_{out}\, \sqrt{2\, g\, h}. \tag{2.8}$$

- The connection pipe between two tanks has a water level h_1 in one tank and a water level of h_2 in the other tank. If the valve on the pipe is open,

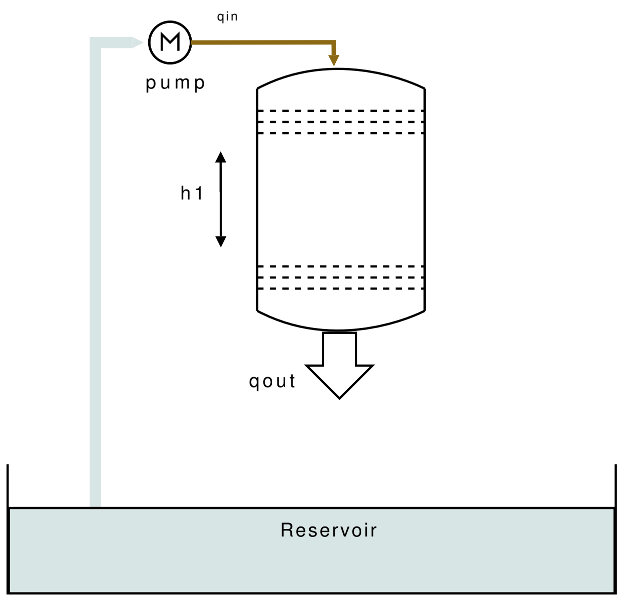


Figure 2.1: **Water-tank with inflow and outflow**

the surface for water flow between tanks is a. Then, the flow $q_{1,2}$ from tank 1 to tank 2 can be computed as follows.

$$q_{1,2} = a\sqrt{2\,g\,(h_1 - h_2)} \tag{2.9}$$

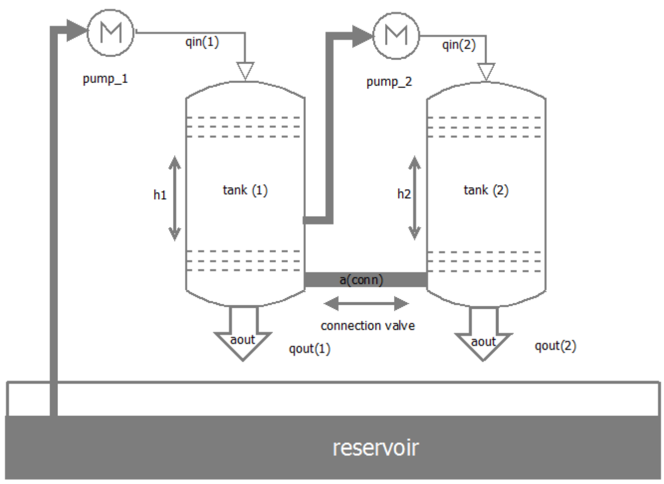


Figure 2.2: **Modeling a connection pipe between two tanks**

2.3 OVERALL NONLINEAR WATER-TANK MODEL

We now put together the component models from the previous sections in order to obtain an overall model of the water-tank system. Considering that the equations for the flow between tanks and the outflow are nonlinear, the overall model will be nonlinear. Since we consider three tanks, we use the water levels h_1, h_2, h_3, the pump voltages u_1, u_2, u_3, the pump flows $q_{in,1}, q_{in,2}, q_{in,3}$ and the outflows and connection flows as described before. We find one first-order state equation for each pump flow and one first-order state equation for each water level.

$$\dot{q}_{\mathrm{in},1} = \frac{1}{T_1}(-q_{\mathrm{in},1} + K_1\, u_1) \tag{2.10}$$

$$\dot{h}_1 = \frac{1}{A_1}(q_{\mathrm{in},1} - q_{\mathrm{out},1} - q_{1,2} - q_{\mathrm{in},2}) \tag{2.11}$$

$$\dot{q}_{\mathrm{in},2} = \frac{1}{T_2}(-q_{\mathrm{in},2} + K_2\, u_2) \tag{2.12}$$

$$\dot{h}_2 = \frac{1}{A_2}(q_{\mathrm{in},2} - q_{\mathrm{out},2} - q_{2,3}) \tag{2.13}$$

$$\dot{q}_{\mathrm{in},3} = \frac{1}{T_3}(-q_{\mathrm{in},3} + K_3\, u_3) \tag{2.14}$$

$$\dot{h}_3 = \frac{1}{A_3}(q_{\mathrm{in},3} - q_{\mathrm{out},3} + q_{2,3}) \tag{2.15}$$

In these state equations, we use the output flow

$$q_{\mathrm{out},i} = a_{\mathrm{out},i}\,\sqrt{2\,g\,h_i}, \quad i = 1, 2, 3 \tag{2.16}$$

and the flows in the connecting valves

$$q_{1,2} = \begin{cases} a_{1,2}\,\sqrt{2\,g\,(h_1 - h_2)} & \text{if } h_1 \geq h_2 \\ a_{1,2}\,\sqrt{2\,g\,(h_2 - h_1)} & \text{if } h_2 > h_1 \end{cases} \tag{2.17}$$

$$q_{2,3} = \begin{cases} a_{2,3}\,\sqrt{2\,g\,h_2 - h_3} & \text{if } h_2 \geq h_3 \\ a_{2,3}\,\sqrt{2\,g\,(h_3 - h_2)} & \text{if } h_3 > h_2 \end{cases} \tag{2.18}$$

Together, we obtain a 6-th order nonlinear model with three inputs u_1, u_2, u_3 and three outputs h_1, h_2, h_3, considering that the water level can be measured.

Since, we want to apply linear control system design methods, we next perform a linearization of the nonlinear system.

CHAPTER III

LINEARIZATION

In the study of dynamical systems, linearization is used for assessing the local stability of an equilibrium point of a system of nonlinear differential equations [4]. It is also employed in the process of approximating a nonlinear system model by a linear model, that is valid for small deviations from the equilibrium. The model of the three tank system is nonlinear. In this chapter, we present how we construct the linearized model of different subsystems of the three tank system.

We employ *set-point linearization*, where the set-point is defined as a stationary (non-changing) state of the system such that the system state maintains a constant set-point value. We compute a small signal approximation of the nonlinear model, that is valid close to the set-point. In principle, the system state and output equations are approximated using Taylor expansion, and the difference variables, that represent the deviation of state, input and output from the set point. An important restriction for the set point linearization is that the linear model is only valid in the vicinity of the set-point.

In our system, the water level is selected to be the state variable for the set point. In the following section, we study a one-tank system, and two different two-tank systems as subsystems of the three-tank system. After performing the set-point linearization, we compare the original non-linear model and the linearized model by doing simulations in Matlab.

3.1 LINEARIZATION OF THE ONE-TANK SYSTEM

The three tank system becomes a one-tank system when we close the valve that will connect between tank 1 and tank 2, and do not run the pump between tank 1 and tank 2 as shown in Figure 3.1.

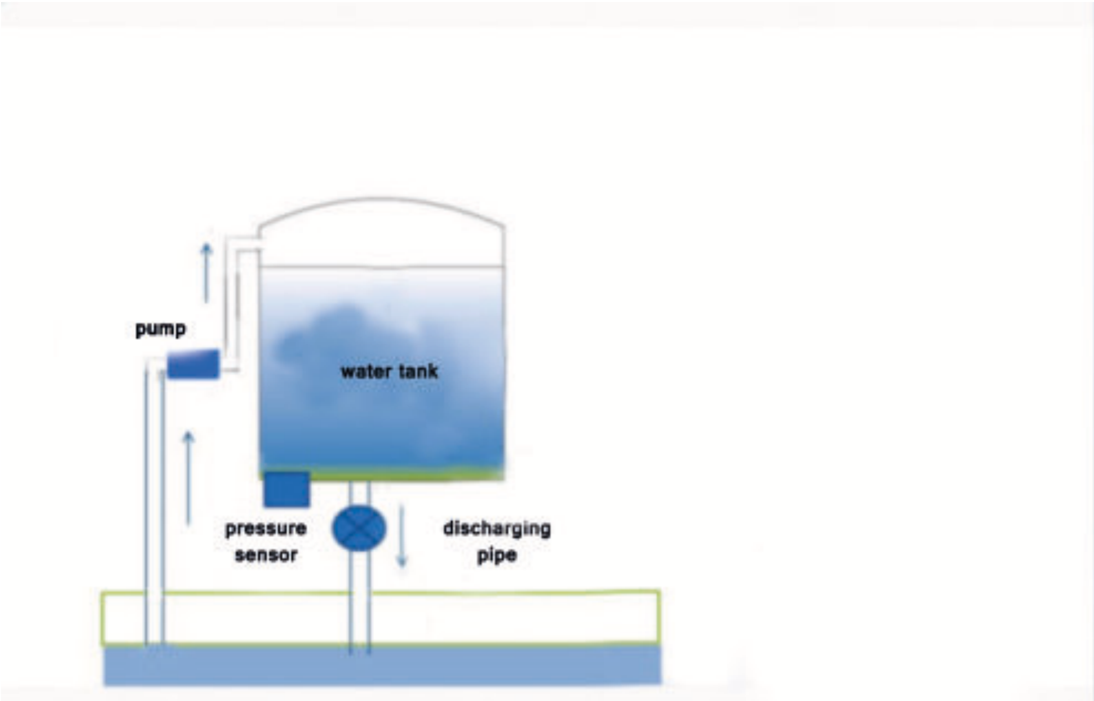


Figure 3.1: **One tank system.**

The system model before linearization is represented with the following differential equations which describe the dynamics of water flow, $q_{\text{in},1}$ and water level h_1 in tank 1.

$$\dot{q}_{\text{in},1} = \frac{1}{T_1}(-q_{\text{in},1} + K_1\, u_1) \tag{3.1}$$

$$\dot{h}_1 = \frac{1}{A_1}(q_{\text{in},1} - a_{\text{out},1}\sqrt{2\, g\, h_1}) \tag{3.2}$$

Let u_{1SP}, h_{1SP} and q_{in1SP} denote the set points for u_1, h_1 and $q_{\text{in},1}$ respectively. There is no deviation in the system state at the set point hence, by equating (3.1) and (3.2) to zero where we get:

$$q_{in1SP} = a_{out,1}\sqrt{2 \cdot g \cdot h_{1SP}} \tag{3.3}$$

$$u_{1SP} = \frac{q_{in1SP}}{K_1}. \tag{3.4}$$

We now apply the small signal analysis to find the matrices A, B, C, D in (2.3) and (2.4) by taking the partial derivatives with respect to $q_{in,1}$ and h_1. The resulting matrices are

$$A = \begin{bmatrix} -1/T_1 & 0 \\ 1/A_1 & \frac{a_{out} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot h_{1SP}}} \end{bmatrix} \tag{3.5}$$

$$B = \begin{bmatrix} \frac{k_1}{T_1} \\ 0 \end{bmatrix} \tag{3.6}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{3.7}$$

Note that $D = 0$ for our example.

Subsequently, the continuous time system is represented in the s-domain as shown in (2.5). We use the parameter values introduced in Section 2.1 and a set-point value for the water level as $h_{1SP} = 50$ (cm). Then, the transfer function of the one-tank system plant is

$$G(s) = \frac{H_1(s)}{U_1(s)} = \frac{0.4444}{s^2 + 0.5018\,s + 0.0008831}. \; . \tag{3.8}$$

Here, $H_1(s)$ and $U_1(s)$ denote the Laplace transforms of the water level h_1 and the input voltage u_1. We later use this transfer function in Chapter 5 for controller design.

If we want to change the set point for example to $h_{1SP} = 30$, we evaluate (3.4) and (3.4) with the new value of h_{1SP}. This leads to a modified transfer function

$$G(s) = \frac{H_1(s)}{U_1(s)} = \frac{0.4444}{s^2 + 0.5023s + 0.00114}. \; . \tag{3.9}$$

3.2 TWO-TANK SYSTEM WITH ONE PUMP

In this model, we close the valve that connects tank 2 and tank 3. In addition, we disable the second and third pumps as we see in Fig. 3.2.

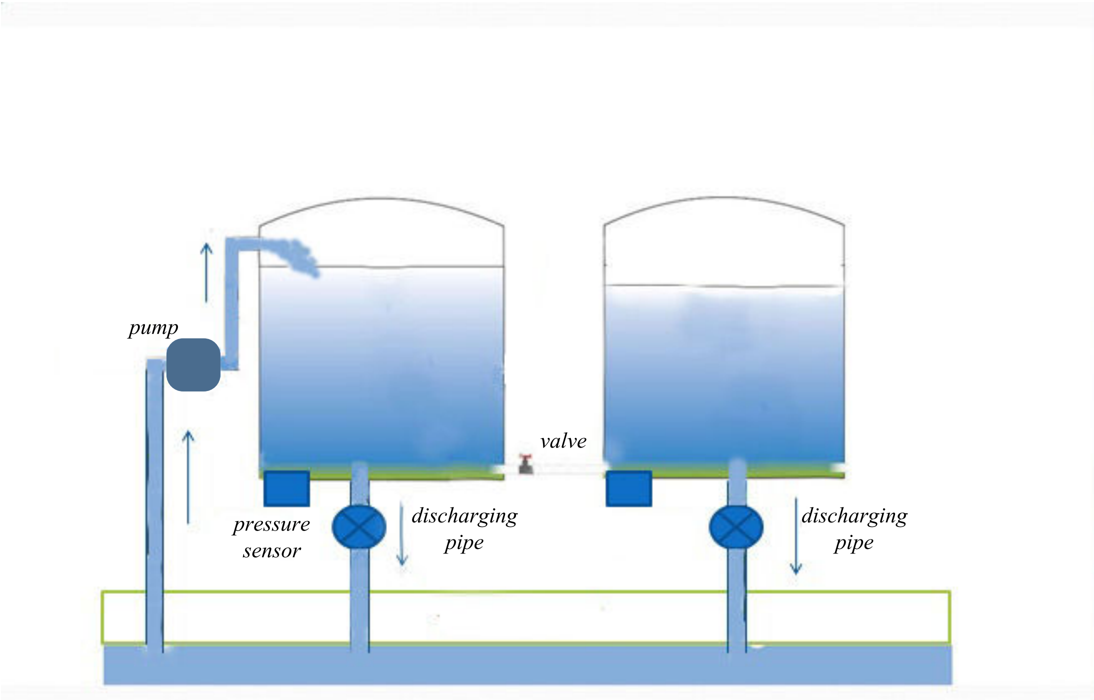


Figure 3.2: **Two tank system with one water pump.**

The differential equations for this system are as follows:

$$\dot{q}_{\text{in},1} = \frac{1}{T_1}(-q_{\text{in},1} + K_1\, u_1) \tag{3.10}$$

$$\dot{h}_1 = \frac{1}{A_1}(q_{\text{in},1} - q_{\text{out},1} - q_{1,2}) \tag{3.11}$$

$$\dot{h}_2 = \frac{1}{A_2}(q_{1,2} - q_{\text{out},2}) \tag{3.12}$$

$$\tag{3.13}$$

Let u_{1SP}, q_{in1SP}, h_{1SP} and h_{2SP} be the set-point values of the system variables.

Then, for the two-tank system:

$$a_{out}\sqrt{2 \cdot g \cdot h_1} - a_{conn}\sqrt{2 \cdot g \cdot (h_1 - h_2)} = 0 \tag{3.14}$$

$$\Rightarrow h_{1SP} = \frac{2 \cdot h_{2SP} \cdot a_{out}^2 + 2 \cdot g \cdot {a_{conn}}^2 \cdot h_{2SP}}{2 \cdot g \cdot a_{conn}^2} \tag{3.15}$$

From h_{1SP}, the values for q_{in1SP} and u_{1SP} can be computed in the same way as in the previous section.

Following the same method of small signal linearization as for the one tank system, we determine A, B, C, D as follows

$$A = \begin{bmatrix} -1/T_1 & 0 & 0 \\ 1/A & \frac{-a_{out1} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot h_{1SP}}} - \frac{a_{conn} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot h_{1SP} - h_{2SP}}} & \frac{a_{conn} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot (h_{1SP} - h_{2SP})}} \\ 0 & \frac{a_{conn} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot (h_{1SP} - h_{2SP})}} & \frac{-a_{out2} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot h_{2SP}}} - \frac{a_{conn} \cdot 2 \cdot g}{A \cdot 2 \cdot \sqrt{2 \cdot g \cdot (h_{1SP} - h_{2SP})}} \end{bmatrix} \tag{3.16}$$

$$B = \begin{bmatrix} 0 \\ \frac{k_1}{T_1} \\ \frac{k_2}{T_2} \end{bmatrix} \tag{3.17}$$

$$C = ([0 \quad 0 \quad 1]) \tag{3.18}$$

D is again zero. Assuming a set-point value of $h_{2SP} = 30$ (cm), we compute the plant transfer function of the two-tank system as

$$G(s) = \frac{H_2(s)}{U_1(s)} = \frac{0.01599}{s^3 + 0.5729\, s^2 + 0.03646\, s + 0.0000016} \tag{3.19}$$

3.3 TWO-TANK SYSTEM WITH TWO PUMPS

Lastly, we consider two tanks, each of which has one pump. In this system we close all the connection valves and run pump 1 and pump 2, as we see in Figure 3.3 and we make the outflow valves open. Pump 1 pumps water to tank 2 and pump 2 takes water from tank 1 and pumps it to tank 2.

Similar to the previous systems, we first construct the differential equations for the non-linear system as follows:

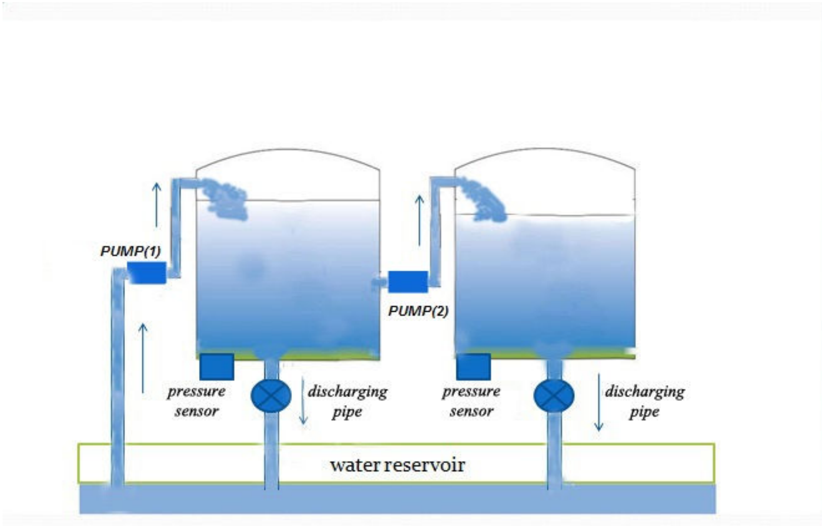


Figure 3.3: **Two-tank system without the connecting pipe.**

$$\dot{q}_{\text{in},1} = \frac{1}{T_1}(-q_{\text{in},1} + K_1\, u_1) \tag{3.20}$$

$$\dot{q}_{\text{in},2} = \frac{2}{T_2}(-q_{\text{in},2} + K_2\, u_2) \tag{3.21}$$

$$\dot{h}_1 = \frac{1}{A_1}(q_{\text{in},1} - q_{\text{out},1} - q_{\text{in},2}) \tag{3.22}$$

$$\dot{h}_2 = \frac{1}{A_2}(q_{\text{in},2} - q_{\text{out},2}) \tag{3.23}$$

$$\tag{3.24}$$

We now use the set-point values u_{1SP}, q_{in1SP}, h_{1SP}, h_{2SP}, q_{in2SP}. From the set-point computation, we obtain

$$q_{in(1)SP} = a_{out(2)} \cdot \sqrt{2 \cdot g \cdot h_{2SP}} \tag{3.25}$$

$$q_{in(2)SP} = q_{in(1)SP} \tag{3.26}$$

Using set-point linearization, we next compute the state space model, where $D = 0$.

$$A = \begin{bmatrix} \frac{-1}{T_1} & 0 & 0 & 0 \\ \frac{1}{A_1} & 0 & 0 & \frac{1}{A_2} \\ 0 & 0 & \frac{a_{out,1} \cdot 2 \cdot g}{A_2 \cdot 2 \cdot \sqrt{2 \cdot g \cdot h_{1SP}}} & \frac{1}{A_2} \\ 0 & 0 & 0 & \frac{-1}{T_2} \end{bmatrix} \tag{3.27}$$

$$B = \begin{bmatrix} \frac{K_1}{T_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_2}{T_2} \end{bmatrix} \tag{3.28}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.29}$$

Assuming that the set-point value of both water levels is 30 cm, we compute the transfer matrix between the inputs u_1, u_2 and the outputs h_1, h_2.

$$G(s) = \begin{bmatrix} \frac{0.4444}{s^2+0.5051s+0.002565} & -\frac{0.4444}{s^2+0.5051s+0.002565} \\ 0 & \frac{0.4444}{s^2+0.5051s+0.002565} \end{bmatrix}. \tag{3.30}$$

such that

$$\begin{bmatrix} H_1(s) \\ H_2(s) \end{bmatrix} = G(s) \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \tag{3.31}$$

CHAPTER IV

CONTROLLER DESIGN METHODS FOR LTI SYSTEMS

After completing the necessary steps of modeling and linearization in the previous sections, it is now possible to apply linear controller design methods for the control of the water-tank system. The basic feedback control loop for our setup is shown in Figure 4.1. Here, that plant represents the water-tank system to be controlled including the tank(s), the DC pump(s). The output signal of the plant is determined by the pressure sensor(s) in the tank(s). The controller is practically implemented on a PIC microcontroller. The main task of this section is to determine different control algorithms that are suitable for different control tasks of the water-tank system.

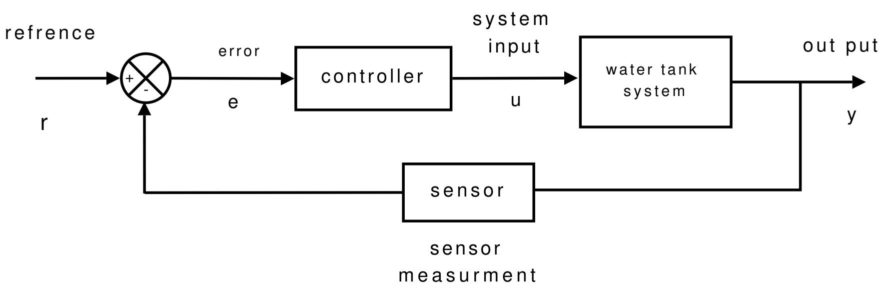


Figure 4.1: **Basic feedback loop for the three tank system**

The basic control tasks to be addressed in this thesis are related to the water-level control in one or two tank systems as modeled before. That is, we want to keep the water level in one or two tanks constant, while dealing with disturbances such as unknown outflow from one or two tanks. We will focus on design methods in the frequency domain. Methods to be used include the pole placement method, the root locus method, the symmetrical optimum method and the youla parametrization method.

The controller design experiments are chosen such that they can be applied in different laboratory sessions of the courses mentioned above. The controllers are designed in the frequency domain in continuous time, that is the resulting controller is represented by a transfer function. Then, the controllers are converted to a discrete-time representation for implementation on the microcontroller. All experiments are simulated in Matlab/Simulink and the simulation results are explained in reference to the different controller design methods.

4.1 POLE PLACEMENT

Pole placement method is one of the classical controller design methods for LTI systems in the basic feedback control loop that only work with state-space plant models [15] as shown in Figure 4.1. It can be applied to both continuous-time systems and discrete-time systems . The basic idea of the pole placement is to first choose the desired poles of the closed-loop transfer function of the system according to given performance specifications. That is, the denominator polynomial $Q(s)$ of the closed-loop system is pre-determined. Then, the controller transfer function $C(s)$ is directly computed. This method is studied in the courses ECE 441 (continuous-time systems) and ECE 438 (discrete-time systems).

Suppose that we have a closed-loop system described by the rational transfer function:

$$G(s) = \frac{B(s)}{A(s)}, \tag{4.1}$$

where $B(s)$ is the numerator polynomial

$$B(s) = b_n s^n + b_{n-1} s^{n-1} + ... + b_0 \tag{4.2}$$

and $A(s)$ is the denominator polynomial of the system:

$$A(s) = a_n s^n + a_{n-1} s^{n-1} + ... + a_0 \tag{4.3}$$

n is called the degree of the system. The controller transfer function is

$$C(s) = \frac{P(s)}{L(s)} \tag{4.4}$$

with the controller numerator

$$P(s) = P_m s^m + P_{m-1} s^{m-1} + ... + P_0 \tag{4.5}$$

and the controller denominator

$$L(s) = l_m s^m + l_{m-1} s^{m-1} + ... + l_0 \tag{4.6}$$

m is the controller degree. Now assume that $Q(s)$ represents the desired closed-loop polynomial. The task is to compute the coefficients of $P(s)$ and $L(s)$ such that the closed-loop polynomial $Q(s9$ is obtained. This is achieved by the design equation

$$Q(s) = A(s)\, L(s) + B(s)\, P(s). \tag{4.7}$$

In the lecture, basically two versions of the pole placement design are discussed: pole placement without additional requirements and pole placement with integral action.

- For the pole placement method without additional requirements, the controller degree is usually chosen as $m = n - 1$. This means, if the degree of the system is $n = 1$, the controller type will be proportional controller $m = 0$ but when the system order is larger $(n > 1)$, then the controller type will be a lead/lag compensator in the following form

  $$C(s) = \frac{p_0 + ... + p_{n-1}\, s^{n-1}}{l_0 + ... + l_{n-1}\, s^{n-1}} \tag{4.8}$$

  with $l_0 \neq 0$. The only disadvantage of this method is that it does not lead to an integral controller. That is, the feedback loop will generally show a non-zero steady-state error for reference and disturbance steps.

- Pole placement with integral action solves this problem by increasing the controller degree. The choice is now $m = n$ and the controller transfer function :

$$C(s) = \frac{p_0 + \cdots + p_n\, s^n}{(l_0 + \cdots + l_{n-1}\, s^{n-1})\, s} \tag{4.9}$$

  is used. Evaluating the design equation leads to integral control. Depending on the system degree n, different controller types are achieved. For example, $n = 1$ leads to PI-control and $n = 2$ leads to PID-control.

Although pole placement directly assigns the denominator polynomial of the closed-loop system transfer function, there is no direct influence on the numerator polynomial. As a result, pole placement design can lead to large overshoot in the reference or disturbance step response. This problem can be solved by adding a pre-filter to the feedback control loop as shown in Figure 4.2.

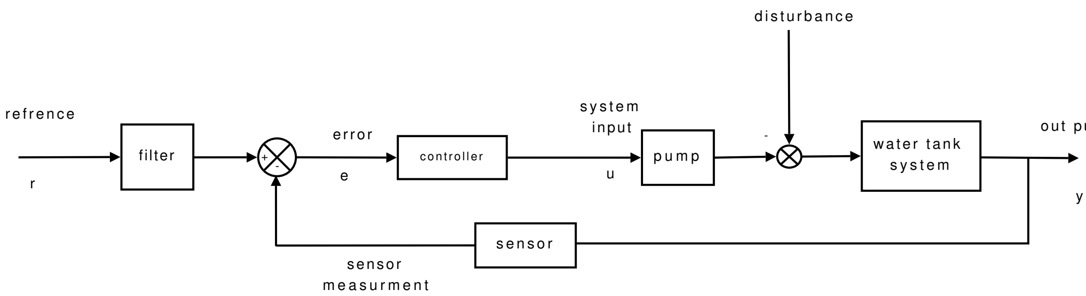


Figure 4.2: **Water-tank control system with filter**

Consider that the closed-loop system transfer function is given by the complementary sensitivity $T(s) = \dfrac{V^+(s) \cdot V^-(s)}{Q(s)}$. Here, we let $V^+(s)$ contain the zeros of the numerator polynomial in the right half plane and $V^-(s)$ contain the zeros of the numerator polynomial in the open left half plane. Then, the pre-filter transfer function is chosen as

$$F(s) = \frac{Q(0)}{V^-(s)}. \tag{4.10}$$

The idea of the pre-filter is to cancel out the numerator polynomial of the closed-loop transfer function $T(s)$, which removes the overshoot.

4.2 ROOT LOCUS DESIGN

Root locus design is a graphical technique for the closed-loop design in the basic feedback control loop. The root locus allows us to find the poles of the closed-loop system by starting from the open-loop system's poles and zeros. Using this information it is possible to perform controller design based on the root locus for both stability and transient system response. The root locus method can be applied to systems of arbitrary order, however, the root locus can become very complex for systems of degree $n = 4$. The root locus design is explained in ECE 388, ECE 441 and ECE 438.

The basic steps in applying the root locus method are as follows

- Determine the open loop transfer function with a free gain parameter K: $G_o(s) = K\,C(s)\,G(s)$
- Sketch the root locus plot of $G_o(s)$ (this can be done manually or using Matlab)
- Move the closed-loop poles to desired locations according to the closed-loop specification
- Determine the controller gain K
- Simulate the feedback loop with the designed controller and verify if the closed-loop behavior is as desired

4.3 SYMMETRICAL OPTIMUM METHOD

The (Kessler's) symmetrical optimum method is used for controller designs that lead to reasonable responses both for reference steps and disturbance steps. Usually, responses are fast with zero steady-state error but with overshoot. The symmetrical optimum method requires plant models that can be represented in the following form.

$$G(s) = \frac{K}{(1 + s\,T_1)\cdots(1 + s\,T_n)(1 + s\,\tau)}. \tag{4.11}$$

In this model, $T_1, \dots, T_n$ are time constants that are large compared to the single time constant τ. If the plant model is accordingly, a controller of the following form is used.

$$C(s) = K_p \frac{(1 + s\, T_p)^n}{s\,(1 + s\,\tau_P)^{n-1}}, \tag{4.12}$$

where

- $n + 1$ is the order of the plant
- K_P is the controller gain
- T_P is the numerator time constant
- τ_P is the denominator time constant that is usually chosen as $\tau_P < 0.1\, T_P$

Using the plant parameters n, $T_1, \dots, T_n$, τ and K, the design equation for the symmetric optimum controller is given by

$$K_p = \frac{1}{2\,K\,\tau} \frac{T_1 \cdots T_n}{(4\,n\,\tau)^n}$$

$$T_p = 4\,n\,\tau$$

In summary, the symmetrical optimum enables the design of PID-type controllers for systems with several large time constants and one small time constant. It has a straightforward design equation, and leads to closed loops with fast responses to reference and disturbance steps . The symmetrical optimum is studied in ECE 441.

4.4 YOULA PARAMETERIZATION METHOD

This section presents the Youla parameterization method (or Q-Parameterization method), which is a modern control design method [14, 7]. The Youla parametrization actually allows to parametrize *all* stabilizing controllers for a control system design in the basic feedback control loop. The Youla parametrization requires that the plant is stable and has a positive relative degree. In order to perform the Youla parametrization, we look at the complementary sensitivity parametrized as $T(s) = Q(s)\,G(s)$. Here, $Q(s)$ is a desired open-loop controller that leads to the closed-loop transfer function $T(s)$. At the same time, the closed-loop transfer function is computed as

$$T(s) = \frac{G(s)\,C(s)}{1 + G(s)\,C(s)} \tag{4.13}$$

Solving for $C(s)$ gives

$$C(s) = \frac{Q(s)}{1 - Q(s)\,G(s)} = \frac{Q(s)}{1 - T(s)} \tag{4.14}$$

That is, if we know $Q(s)$, we can directly compute $C(s)$. In summary, the Youla-parametrization design works as follows. First a desired complementary sensitivity $T(s)$ is chosen. Then, the parameter $Q(s) = T(s)/G(s)$ is computed. Next, $C(s)$ is directly found from (4.14).

Finally, we list some properties of the Youla paraemtrization.

- Any complementary sensitivity with a relative degree larger or equal to the plant relative degree can be achieved. However, the controller order can be large and the controller might be non-standard
- Youla parametrization can lead to bad disturbance rejection
- Youla parametrization requires stable plants with positive relative degree

Youla parametrization is taught in the course ECE 441.

4.5 DISTURBANCE FEED-FORWARD CONTROL

Disturbance feedforward is a method that can be used if a disturbance, that acts on the plant, can be measured [1]. In that case, it is possible to directly react to the disturbance. We consider the block diagram in Figure 4.3, that has the structure of a one-tank system. The water-tank system is separated in the pump and the tank, and a disturbance acts between these two components. The disturbance in this case is a flow that removes water from the tank, and it is assumed that this flow can be measured by a flow sensor.

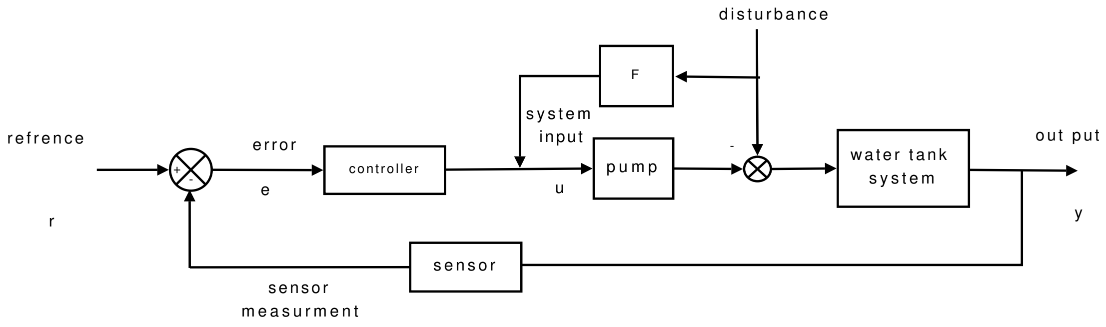


Figure 4.3: **Feed-forward controller architecture**

If we want to remove the disturbance before its effect to the output , we must fulfill the following equation according to the block diagram.

$$-1 + F(s)\,P_{1(s)} = 0 \tag{4.15}$$

Here, $F(s)$ is the disturbance feedforward transfer function and $P_{1(s)}$ is the pump transfer function. Solving for $F(s)$ leads to

$$F(s) = \frac{1}{P_{1(s)}} \tag{4.16}$$

If $F(s)$ in (4.16) is not proper, it is generally multiplied by a lag transfer function. An example of this is shown in Section 5.3.2. Disturbance feedforward is part of the lecture ECE 441.

4.6 CONTINUOUS AND DISCRETE-TIME CONTROL

In many cases, controllers are designed in the s-domain but realized on a digital computer such as a microcontroller or programmable logic controller (PLC). That is, the continuous-time controller transfer function has to be converted to a discrete-time (digital) representation. We use three classical approximation methods to perform this task.

- Euler method when:

$$s = \frac{z-1}{T} \tag{4.17}$$

- Euler backward method when:

$$s = \frac{z-1}{ZT} \tag{4.18}$$

- Trapeziodal method when:

$$s = \frac{2}{T}\frac{z-1}{z+1} \tag{4.19}$$

where T is the sampling period. This technique is studied in detail in the course ECE 438.

CHAPTER V

APPLICATION OF CONTROLLER DESIGN METHODS

In this chapter, we apply the controller design methods discussed in Chapter 4 to the different configurations of our water-tank system. We first discuss the one-tank system in Section 5.1. The different two-tank systems are explained in Section 5.2 and 5.3.

5.1 ONE-TANK SYSTEM

We apply the pole placement method with and without integral action to the one-tank system.

5.1.1 Pole placement method (without integral)

In order to apply pole placement, we first need a plant transfer function of the one-tank system as is shown in (4.1). Next, we have to decide on the desired pole locations of the closed-loop system in the complex plane in order to determine the control system performance. Finally, we will use the design equation 4.7 to find the controller transfer function.

From the linearization in Section 3.1, we get the plant transfer function

$$G(s) = \frac{0.4444}{s^2 + 0.5018s + 0.0008831} \tag{5.1}$$

That is, we find the numerator polynomial

$$B(s) = 0.4444 \tag{5.2}$$

and the denominator polynomial

$$A(s) = s^2 + 0.5018s + 0.0008831 \tag{5.3}$$

The plant is of degree $n = 2$ with poles at $s_1 = -0.5$ and $s_2 = -0.0018$. In order to speed up the behavior of the closed loop in comparison to the plant, we choose closed-loop poles at $s = -0.2$. Moreover, we get $m = n-1$, which leads to degree 3 of the closed-loop polynomial:

$$Q(s) = (s + 0.2)^3 = s^3 + 0.6\, s^2 + 0.12\, s + 0.008. \tag{5.4}$$

It remains to evaluate the design equation

$$Q(s) = A(s)\, L(s) + B(s)\, P(s), \tag{5.5}$$

and we find the controller transfer function

$$C(s) = \frac{0.133\, s + 0.0178}{s + 0.1}. \tag{5.6}$$

As discussed before, the controller is of degree $m = 1$ and is of the so-called lead/lag type. In order to realize this controller transfer function on a digital controller, the next step is to determine the discrete-time approximation of the controller transfer function. We use the formulas (4.17), (4.18) and (4.19) as presented in Section 4.6. We get the following discrete-time transfer functions for the different methods:

- Euler method:
$$C_{Euler} = \frac{0.133\, z - 0.133 + 0.0178\, T}{z - 1 + 0.1\, T} \tag{5.7}$$

- Euler backward method:
$$C_{Eulerbackward} = \frac{(0.133 + 0.0178\, T)\, z - 0.133}{(1 + 0.1\, T)\, z - 1} \tag{5.8}$$

- Trapezoidal method:

$$C_{rapezoidal} = \frac{(2\,0.133 + 0.0178\,T)\,z + (-2\,0.133 + 0.0178\,T)}{(2 + 0.1\,T)\,z + (-2 + 0.1\,T)} \tag{5.9}$$

As the final step of this controller design, we verify the correctness of the computation by simulation in Matlab/Simulink. We perform a step of the water-level reference value from the value 0 to 5. The block diagram for the simulation is shown in Figure 5.1. For comparison, we show both the continuous-time (s-domain) and discrete-time (z-domain) case. In discrete-time, we use different values of the sampling time T in order to find a good sampling time for the one-tank system.

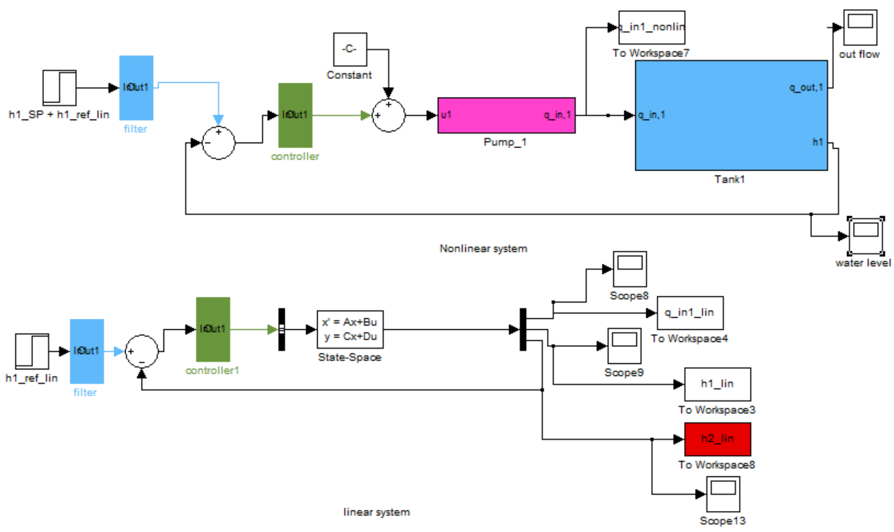


Figure 5.1: **Simulink model of the one-tank system control**

The following figures illustrate the simulation results for sampling times $T = 0.01$, $T = 0.1$ and $T = 1$ (according to the modeling in Chapter 2, the unit is seconds). The left hand plot shows the time evolution of the water level, whereas the right hand figure shows the flow of the pump.

Looking at the previous figures, the feedback loop is stable for all values of T. However, it can be observed that for example the Euler backward method at $T = 1$ leads to oscillations around the reference value of the water level. Hence, we conclude that the sampling time should be chosen sufficiently smaller, for example in the order of $T = 0.1$. We also observe that all experiments lead to

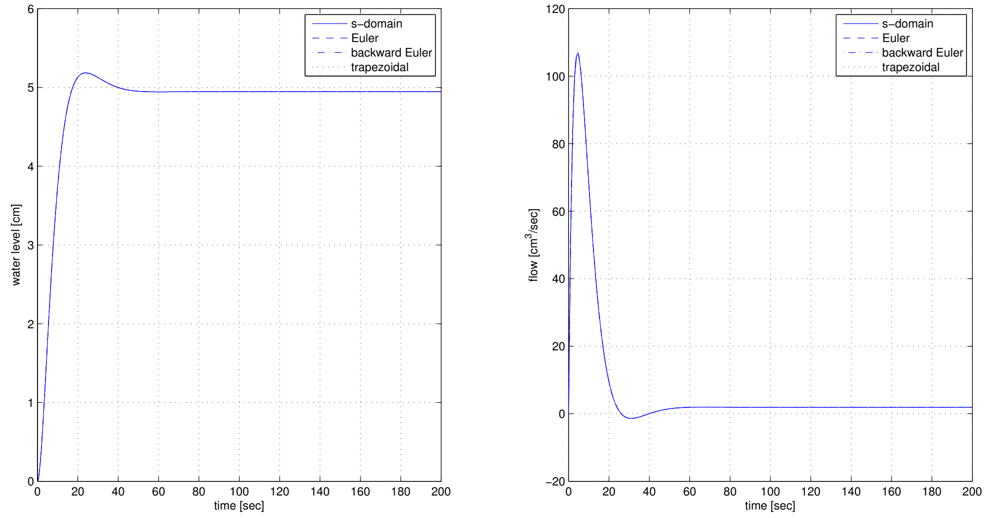


Figure 5.2: **Water level control for the one-tank system by using pole placement without integral and without pre-filter for T = 0.01.**

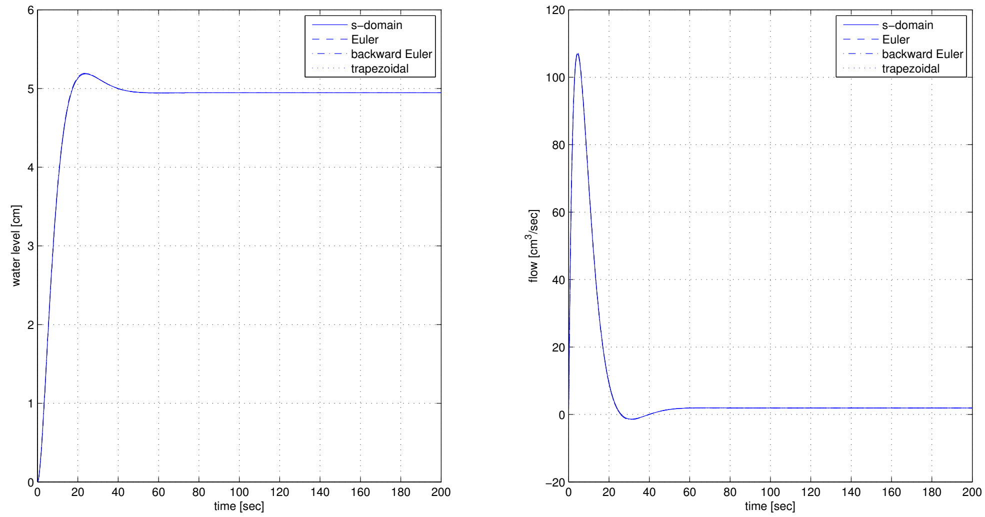


Figure 5.3: **Water level control for the one-tank system by using pole placement without integral and without pre-filter for T = 0.1 .**

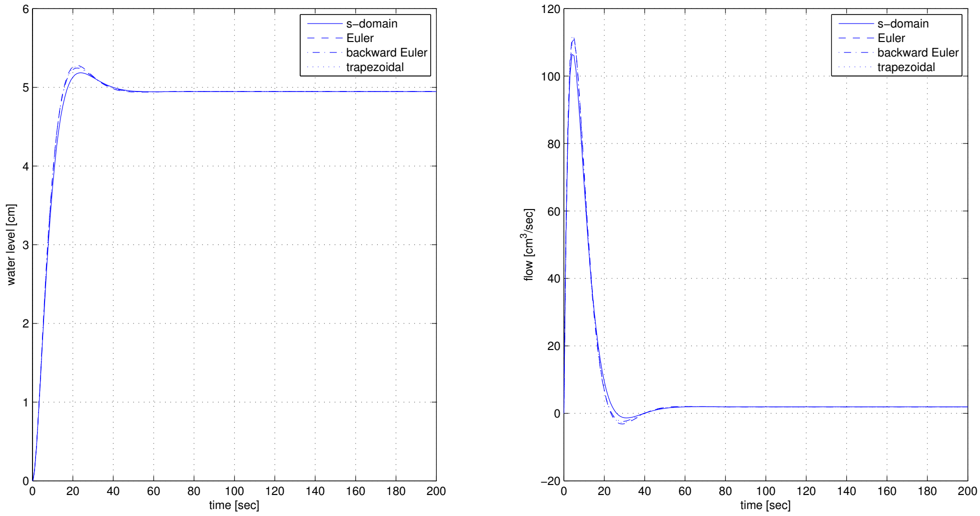


Figure 5.4: **Water level control for the one-tank system by using pole placement without integral and without pre-filter for T = 1.**

significant overshoot. This is a basic property of the pole placement design. By assigning the poles of the closed-loop arbitrarily, the closed-loop transfer function obtains zeros, that cause this overshoot. A remedy is to use a pre-filter, that filters the reference signal before it is applied to the feedback loop. We compute the filter as described in Section 4.1.

We now perform the same simulations as before, only using the additional pre-filter. It can be seen that the previously observed overshoot is almost entirely removed from the step response. This can also be explained when looking at the flow of the pump. In comparison to the case without pre-filter, the initial flow of the pump is much reduced. Hence, a steady increase of the water level without overshoot is achieved.

$$F(s) = \frac{0.008}{0.05912\,s + 0.007912} \tag{5.10}$$

The simulation result in Figure 5.5 to 5.7 now shows that there is no more overshoot and that the final value is achieved without any steady-state error.

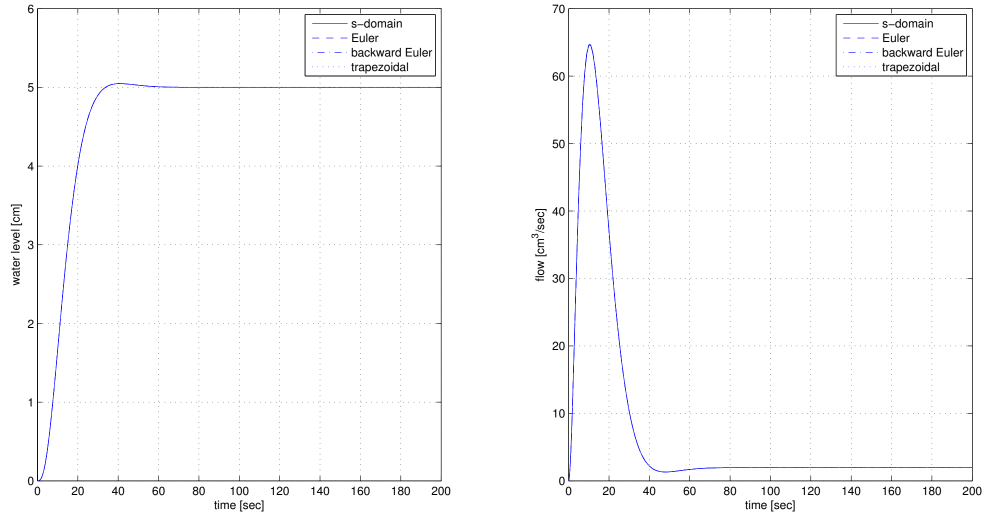


Figure 5.5: **Water level control for the one-tank system by using pole placement without integral but with pre-filter for T = 0.01.**

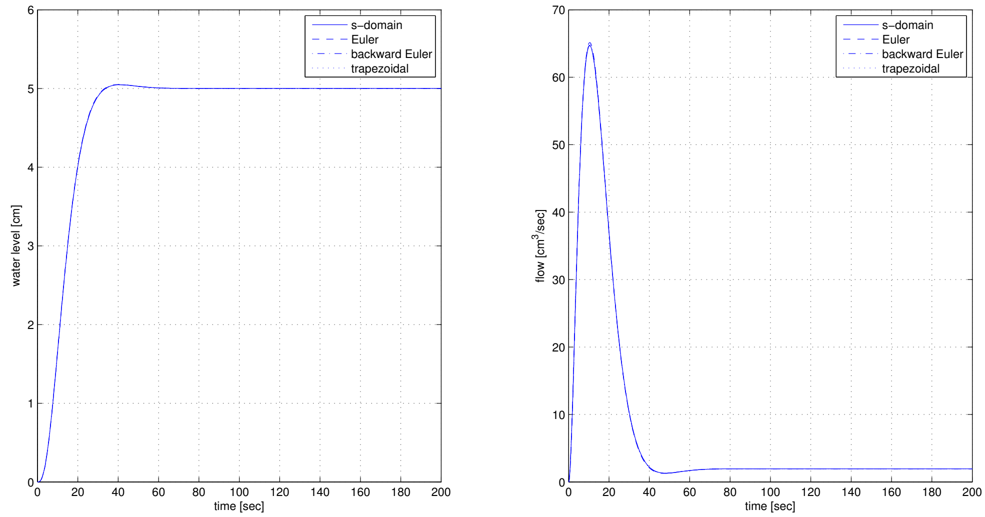
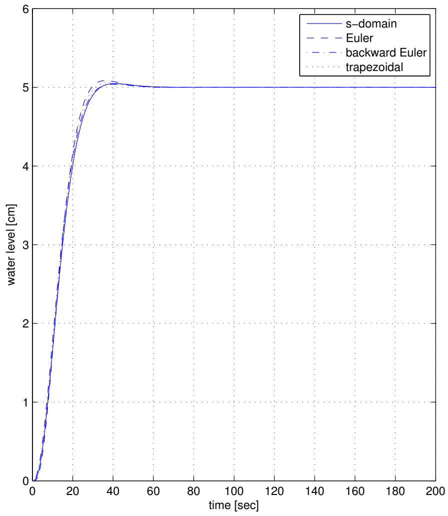


Figure 5.6: **Water level control for the one-tank system by using pole placement without integral but with pre-filter for T = 0.1.**



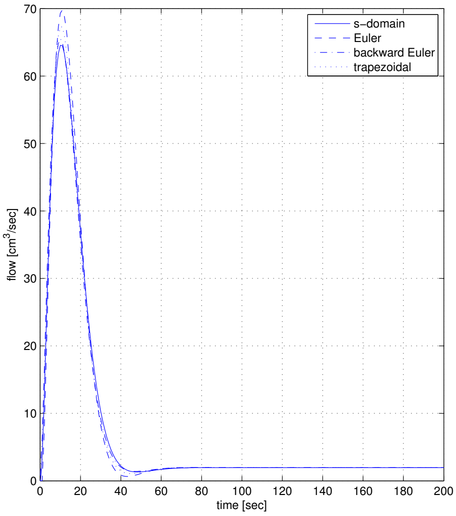


Figure 5.7: **Water level control for the one-tank system by using pole placement without integral but with pre-filter for T = 1.**

5.1.2 Pole Placement Method (with integral)

The controller derived in the previous section is of lead/lag type. Hence, it does not allow a steady-state error of zero for reference or disturbance steps. If we want to achieve this, we would like to design PI- or PID-controllers, that is, we need to add an integral action to the controller. We proceed as described in Section 4.1 for the pole placement design with integral action. As discussed before, the plant is of degree $n = 2$ and $A(s)$ and $B(s)$ can be found from (5.2), (5.3). We again choose closed-loop poles at $s = -0.2$. Moreover, we get $m = n - 1 + 1$, which leads to degree 4 of the closed-loop polynomial:

$$Q(s) = (s + 0.2)^4 = s^4 + 0.8\,s^3 + 0.24\,s^2 + 0.032\,s + 0.0016 \qquad (5.11)$$

The resulting controller transfer function is given as

$$C(s) = \frac{0.2075\,s^2 + 0.07052\,s + 0.0036,\,s + 0.01751}{s^2 + 0.3\,s}. \qquad (5.12)$$

Looking at $C(s)$, it can be seen that the controller is of second order with an integral part. Similar to the previous section, we finally transform the continuous-time controller transfer function to discrete time. We apply the different approximation methods as described before. Again, T denotes the sampling time.

- Euler method:

$$C_{Euler}(s) = \frac{0.2075\, z^2 + (0.075052\, T - 2\, 0.2075)\, z + (0.2075 - 0.075052\, T + 0.0036\, T^2)}{z^2 + (0.3\, T - 2)\, z + 1 - 0.3\, T} \tag{5.13}$$

- Euler backward method:

$$C_{Eulerbackward}(s) = \frac{(0.207 + 0.07052\, T + 0.0036\, T^2)\, z^2 + (-0.07052\, T - 2\, 0.207)\, z + 0.207}{(1 + 0.3\, T) z^2 + (-2 - 0.3\, T)\, z + 1} \tag{5.14}$$

- Trapezoidal method:

$$C_{rapezoidal}(s) = \frac{(0.8292 + 0.14104\, T + 0.0036\, T^2)\, z^2 + (-1.65 + 0.0072\, T^2)\, z + 0.8292 - 0.14104\, T + 0.0036\, T^2}{(4 + 0.6\, T)\, z^2 - 8\, z + 4 - 0.6\, T} \tag{5.15}$$

The simulation result of a reference step from 0 to 5 for the closed feedback loop is investigated for the three sampling times $T = 0.01$, $T = 0.1$ and $T = 1$. The result is shown in the following figures.

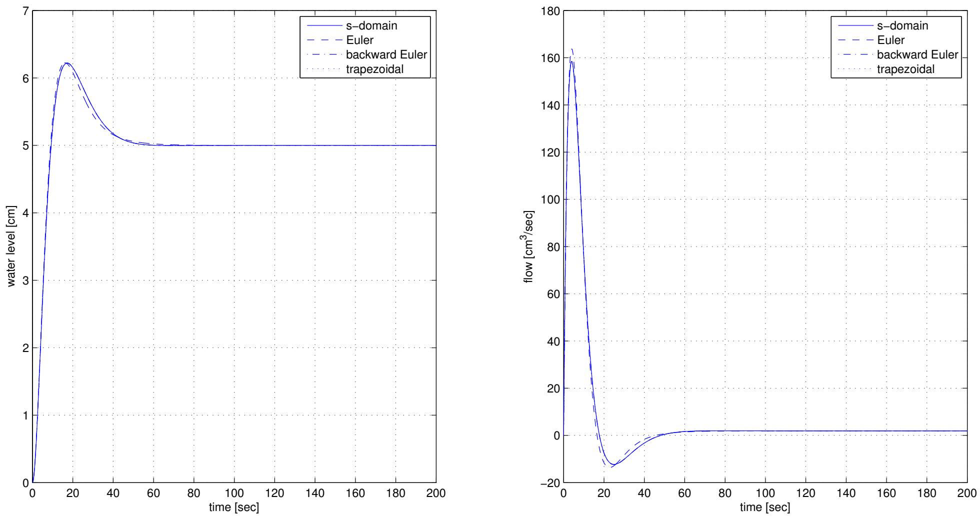


Figure 5.8: **Water level control for the one-tank system by using pole placement with integral and without pre-filter for T = 0.01.**

We see from the plots that all approximation algorithms and all selected sampling times lead to a stable closed-loop system. However, it has to be observed that the deviation from the continuous-time simulation increases for larger sampling times. Hence, the most suitable sampling time is again $T = 0.1$ (which corresponds to 100 ms). In addition, there is overshoot for all experiments, since no pre-filter is

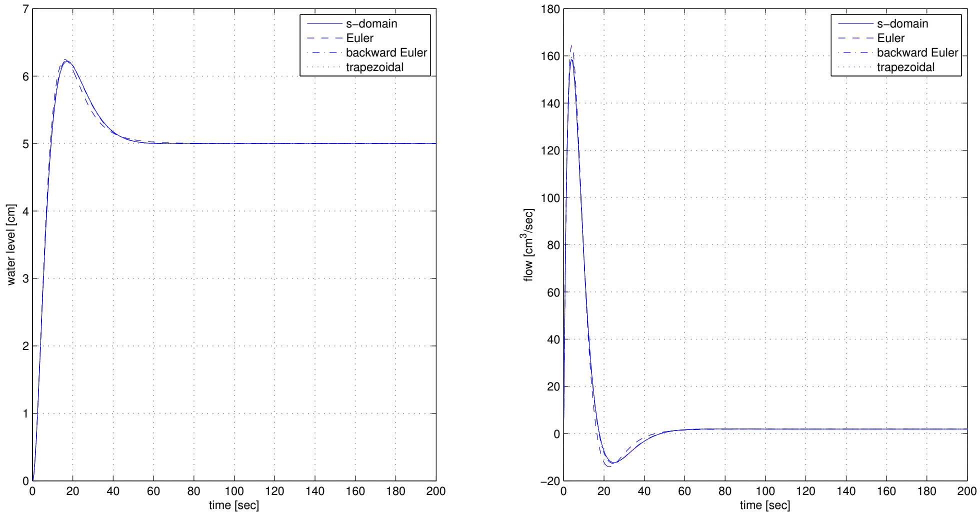


Figure 5.9: **Water level control for the one-tank system by using pole placement with integral and without pre-filter for T = 0.1 .**

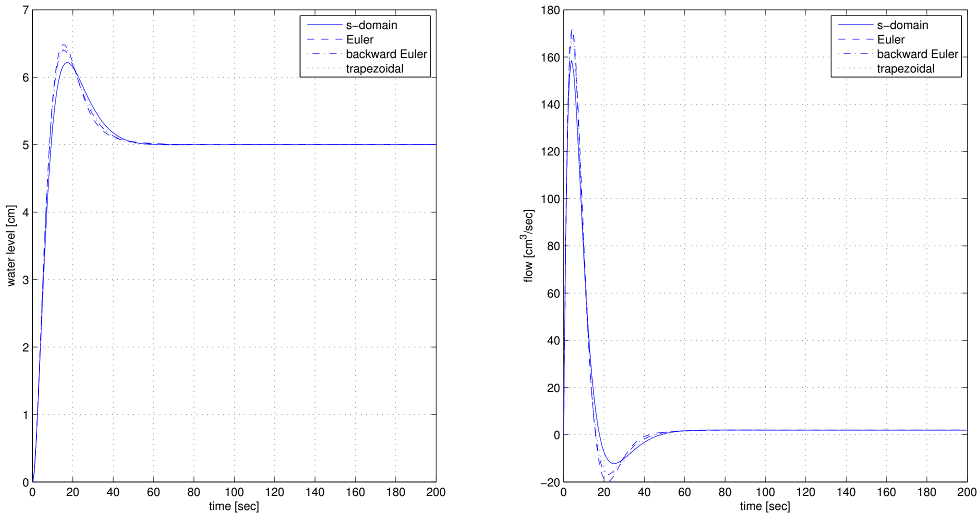


Figure 5.10: **Water level control for the one-tank system by using pole placement with integral and without pre-filter for T = 1 .**

used up to now. We now design a suitable pre-filter for the water level control of the one-tank system with pole placement and integral action. We now design a pre-filter in order to reduce the overshoot. The filter transfer function is as follows in the s-domain

$$F(s) = \frac{0.0016}{0.09221\, s^2 + 0.03134\, s + 0.0016} \tag{5.16}$$

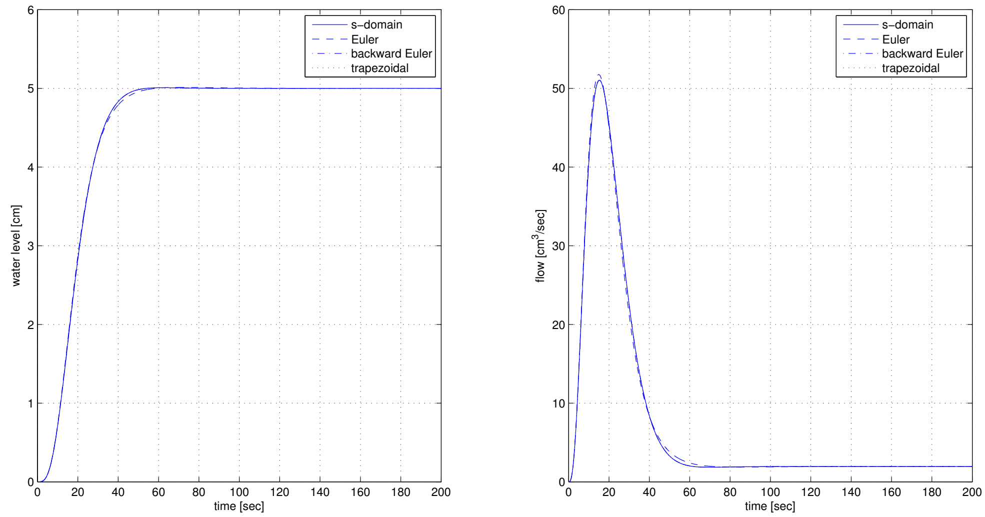


Figure 5.11: **Water level control for one tank system by using pole placement with integral (with filter) when T=0.01**

The simulation result in Figure 5.11 to 5.13 now shows that there is no more overshoot and that the final value is achieved without any steady-state error.

5.2 TWO-TANK SYSTEM (WITH ONE PUMP)

The one-tank system from the previous section is now extended by a second tank. This tank is connected to the first tank by a pipe as described in Section 3.2. We apply different controller design techniques, and want to control the water level

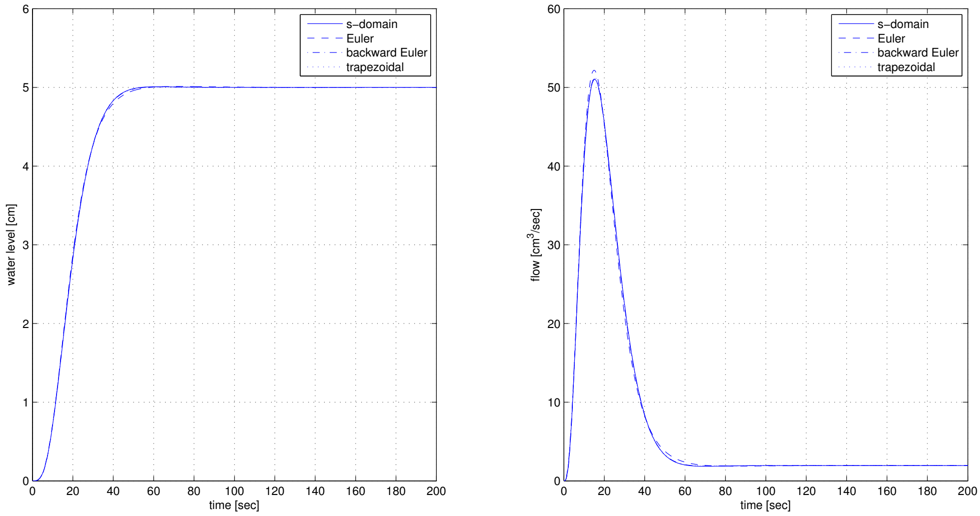


Figure 5.12: **Water level control for one tank system by using pole placement with integral (with filter) when T=0.1**

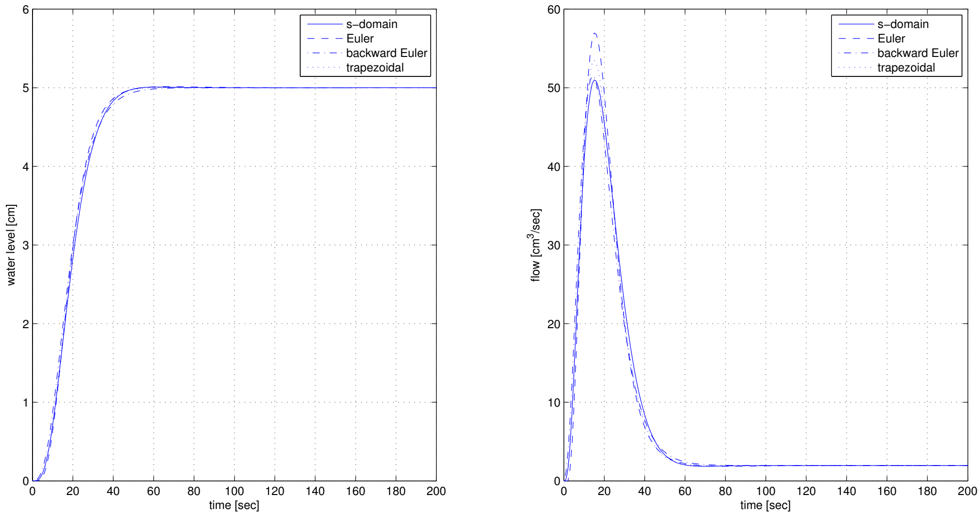


Figure 5.13: **Water level control for one tank system by using pole placement with integral (with filter) when T=1**

in the second tank by manipulating the voltage of the inflow pump for the first tank. The results are simulated in Matlab/Simulink. As computed in Section 3.2, the linearized model of this two-tank system is given by the transfer function in (3.19)

$$G(s) = \frac{0.01599}{s^3 + 0.5729\, s^2 + 0.03646\, s + 0.0000016}$$

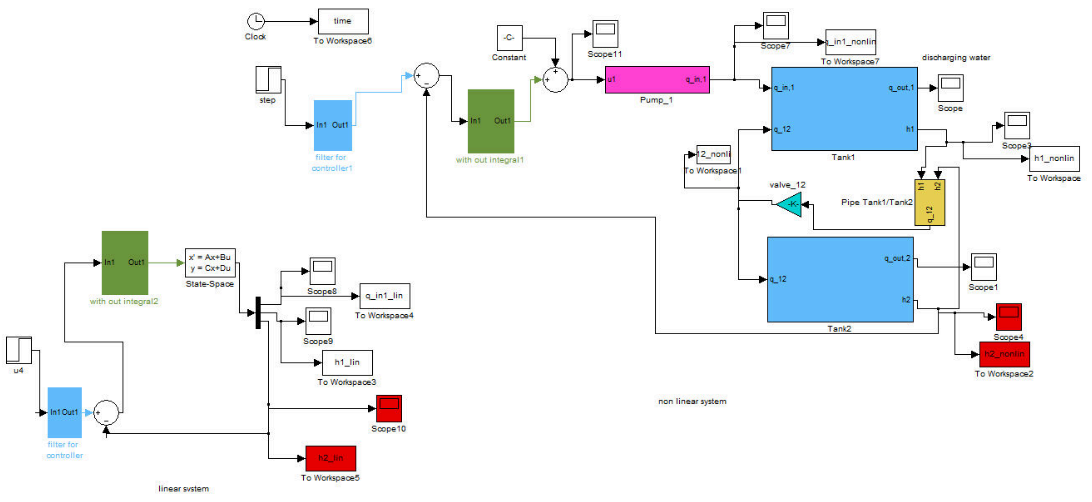


Figure 5.14: **Simulink control loop for the two-tank system with one pump**

5.2.1 Pole placement (without integral)

We apply the pole placement method as described in Section 4.1 with the desired closed-loop polynomial

$$Q(s) = (s + 0.2)^5 = s^5 + s^4 + 0.4s^3 + 0.08s^2 + 0.008s + 0.00032 \qquad (5.17)$$

Using $G(s)$ and $Q(s)$, we find the following controller transfer function in the s-domain.

$$C(s) = \frac{0.3764s^2 + 2.889s + 0.1512}{s^2 + 0.4694s + 0.1354}. \qquad (5.18)$$

By design, this controller does not have an integrator. We finally transfer this controller to the z-domain as described in Section 4.6. Note that we already discussed the influence of the sampling time T on the closed-loop system. In Section 5.1, we found that a sampling time of $T = 0.1\,(\text{sec})$ is suitable for the water-tank system. In order to keep the presentation short, we now continue using this sampling time $T = 0.1$. We find the controller transfer functions for the different integral approximations.

- Euler method:

$$C_{Euler}(z) = \frac{0.3764z^2 - 0.4639z + 0.08901}{z^2 - 1.953z + 0.9544} \tag{5.19}$$

- Euler backward method:

$$C_{Eulerbackward}(z) = \frac{0.6361z^2 - 0.9937z + 0.3591}{z^2 - 1.953z + 0.9539} \tag{5.20}$$

- Trapezoidal method:

$$C_{rapezoidal}(z) = \frac{0.5091z^2 - 0.7346z + 0.2269}{z^2 - 1.953z + 0.9542} \tag{5.21}$$

The resulting step response for a reference step (water level in tank 2) of 5 (cm) is shown in Figure 5.15. Since there is no integrator in the controller, it can be seen that there is a steady-state error.

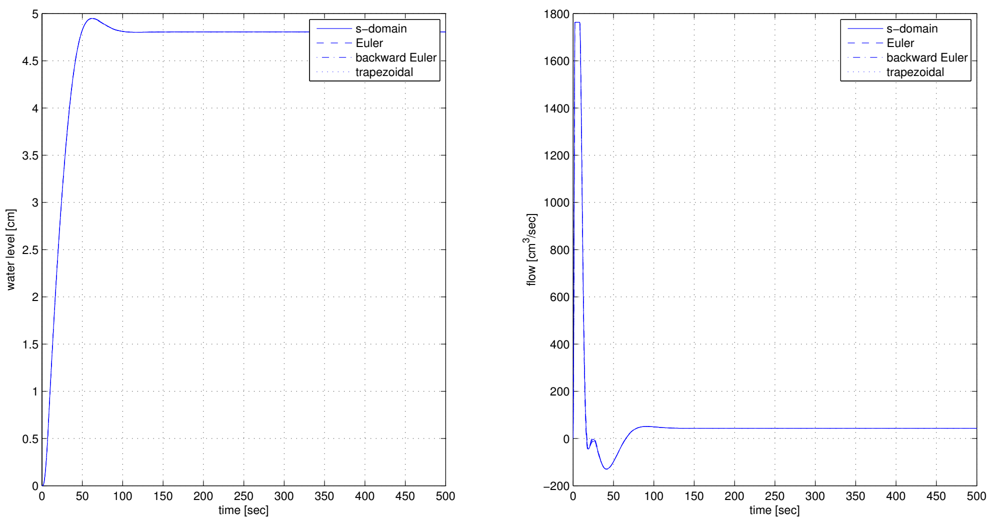


Figure 5.15: **Pole placement for the two-tank system with one pump (no filter) with T = 0.1 .**

Note that the flow on the right hand side of Figure 5.15 is negative, because it represents the difference from the set-point. We next design the pre-filter as described in Section 4.1. The filter transfer function in the s-domain is

$$F(s) = \frac{0.00032}{0.0007627s^2 + 0.005854s + 0.0003065}. \tag{5.22}$$

In the z-domain (for $T = 0.1$), we get

- Euler method:

$$F_{Euler}(z) = \frac{0.004196}{z^2 - 1.232z + 0.2365} \tag{5.23}$$

- Euler backward method:

$$F_{Eulerbackward}(z) = \frac{0.002368z^2}{z^2 - 1.562z + 0.5645} \tag{5.24}$$

- Trapezoidal method:

$$F_{rapezoidal}(z) = \frac{0.0007575z^2 + 0.001515z + 0.0007575}{z^2 - 1.443z + 0.4457} \tag{5.25}$$

The step response for the closed loop including the pre-filter is shown in Figure 5.16. It can be observed that now the steady-state error is removed. However there is still overshoot in the step response. This can be explained as follows. The controller design uses the linearized model of the two-tank system. However the real system is nonlinear. This leads to small deviations from the designed closed-loop behavior.

5.2.2 Pole placement (with integral)

We now add integral action to the controller using pole placement. The desired closed-loop polynomial is

$$Q(s) = (s + 0.2)^6 = s^6 + 1.2\,s^5 + 0.6\,s^4 + 0.16\,s^3 + 0.024\,s^2 + 0.00192\,s + 0.000064 \tag{5.26}$$

We get the controller in the s-domain

$$C(s) = \frac{13.74s^3 + 10.06s^2 + 0.9362s + 0.03158}{s^3 + 0.6694s^2 + 0.2293s} \tag{5.27}$$

and convert the $C(s)$ to the z-domain for a sampling time of $T = 0.1$.

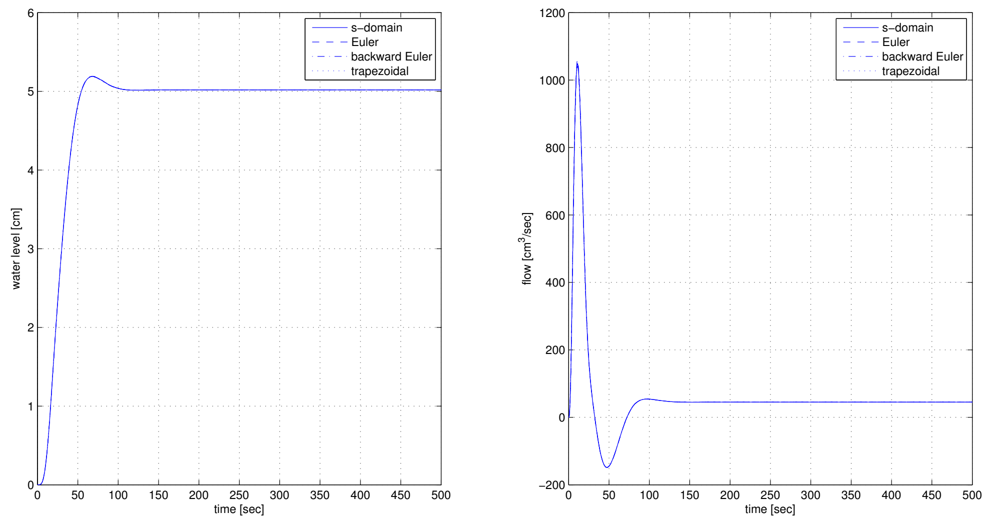


Figure 5.16: **Pole placement for the two-tank system with one pump (filter) with T = 0.1 .**

- Euler method:

$$C_{Euler} = \frac{13.74z^3 - 40.21z^2 + 39.22z - 12.74}{z^3 - 2.933z^2 + 2.868z - 0.9354} \tag{5.28}$$

- Euler backward method:

$$C_{Eulerbackward} = \frac{13.8z^3 - 40.44z^2 + 39.49z - 12.85}{z^3 - 2.933z^2 + 2.868z - 0.9352} \tag{5.29}$$

- Trapezoidal method:

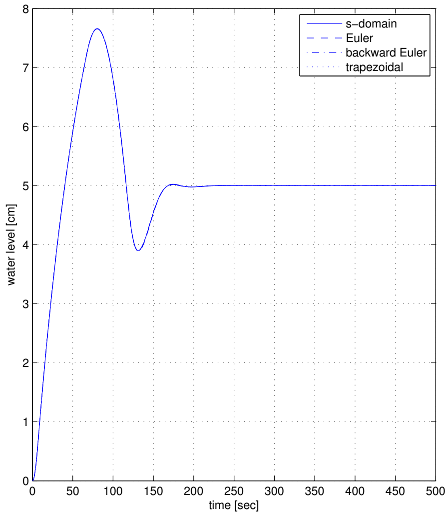
$$C_{rapezoidal} = \frac{13.78z^3 - 40.35z^2 + 39.37z - 12.8}{z^3 - 2.933z^2 + 2.868z - 0.9353} \tag{5.30}$$

The step response in the closed loop is shown in Figure 5.17. Due to the integral action of the controller, the steady-state error is zero. However, there is large overshoot because of zeros that appear in the closed-loop transfer function.

We now design a pre-filter in order to reduce the overshoot. The filter transfer function is as follows in the s-domain

$$F(s) = \frac{6.4\,10^{-5}}{0.02785s^3 + 0.02038s^2 + 0.001897s + 6.4\,10^{-5}} \tag{5.31}$$

and in the z-domain (for $T = 0.1$)



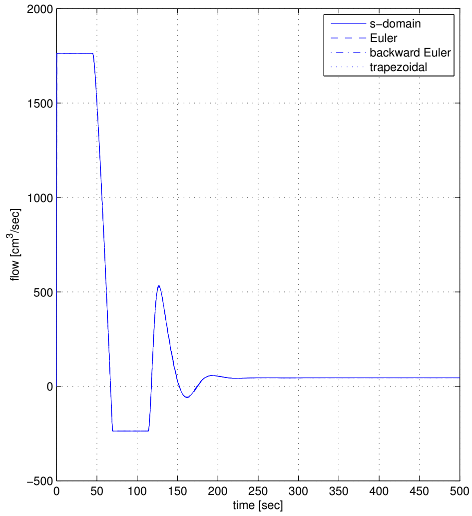


Figure 5.17: **Pole placement method with integral method for two tank system with connection pipe when the T=0.01**

- Euler method:

$$C_{Euler} = \frac{2.298\,10^{-6}}{z^3 - 2.927z^2 + 2.854z - 0.9275} \tag{5.32}$$

- Euler backward method:

$$C_{Eulerbackward} = \frac{2.14\,10^{-6}z^3}{z^3 - 2.931z^2 + 2.862z - 0.9312} \tag{5.33}$$

- Trapezoidal method:

$$C_{rapezoidal} = \frac{2.771\,10^{-7}z^3 + 8.312\,10^{-7}z^2 + 8.312\,10^{-7}z + 2.771\,10^{-7}}{z^3 - 2.929z^2 + 2.858z - 0.9294} \tag{5.34}$$

The step response simulation is shown in Figure 5.18. It can be seen that the overshoot is significantly reduced compared to the case without filter. However, there is still some overshoot because of the nonlinearity of the water-tank system and the saturation of the control input.

5.2.3 Root Locus Method

We next use the root locus method as described in Section 4.2 in order to find a controller for the two-tank system with one pump. This method requires the poles

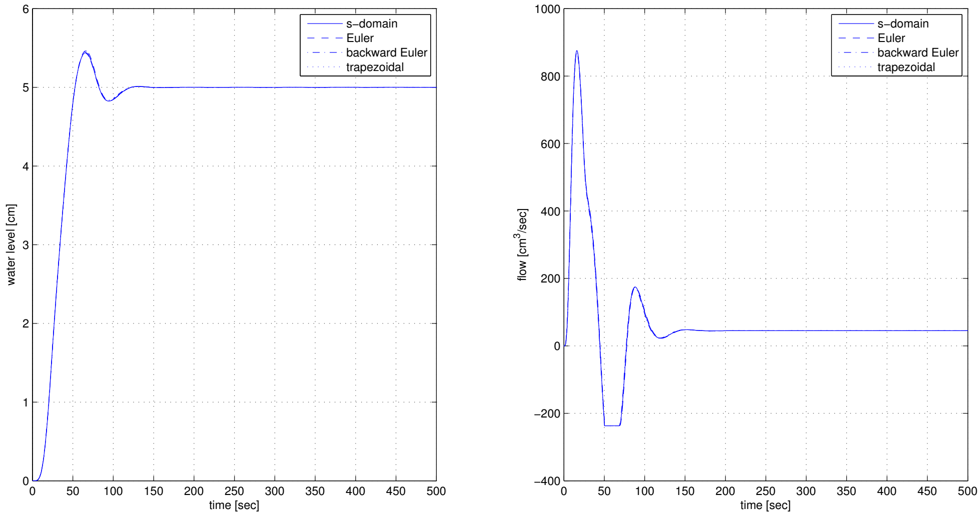


Figure 5.18: **Filter for pole placement method (with integral) when T=0.1**

and zeros of the open-loop transfer function. Hence, we first choose a suitable structure of the controller transfer function as follows.

$$C(s) = K_p \, \frac{(1 + T2\, s)((1 + T1\, s))}{s(1 + s\frac{\tau}{10})} \tag{5.35}$$

In order to choose $T1$ and $T2$, we look at the poles of the plant transfer function $G(s)$

$$P1 = -0.5000, P2 = -0.0212, P3 = -0.0094 \tag{5.36}$$

We suggest to compensate the two slow poles of $G(s)$ at $P2 = -0.0212$ and $P3 = -0.0094$ by the choice of $T1 = -1/P2$ and $T2 = -1/P3$. In addition, we introduce the fast time constant $\tau = -0.1/P1$ such that the controller transfer function is proper. Together, we obtain the open-loop transfer function

$$G(s)\, C(s) = \frac{50.73}{s^3 + 5.5\, s^2 + 2.5\, s}. \tag{5.37}$$

The root locus plot of $G(s)\, C(s)$ is shown in Figure 5.19. We see that the root locus has three branches, whereby two branches move to the instability region for large values of K_p. In order to make the closed loop as fast as possible, we choose the closed-loop poles at the intersection point of the root locus with the real axis. The corresponding value is $K_p = 0.00586$. The resulting controller transfer function is

$$C(s) = \frac{29.34s^2 + 0.8977s + 0.00586}{0.2s^2 + s} \tag{5.38}$$

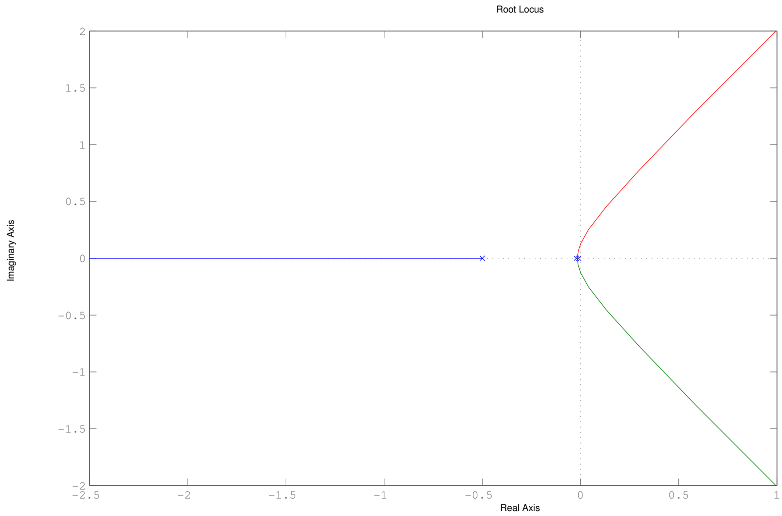


Figure 5.19: **Pole locations for the two tank system with one pump**

The controller transfer functions in discrete-time (for $T = 0.1$) are computed as follows.

- Euler method:

$$C_{Euler} = \frac{146.7z^2 - 293z + 146.3}{z^2 - 1.5z + 0.5} \tag{5.39}$$

- Euler backward method:

$$C_{Eulerbackward} = \frac{98.1z^2 - 195.9z + 97.8}{z^2 - 1.667z + 0.6667} \tag{5.40}$$

- Trapezoidal method:

$$C_{rapezoidal} = \frac{117.5z^2 - 234.7z + 117.2}{z^2 - 1.6z + 0.6} \tag{5.41}$$

It can be seen from the closed-loop simulation in Figure 5.20 that the response reaches the reference value very fast. However, it has to be remarked that a very large pump flow is required at the beginning of the experiment. This leads to overshoot and a slow return to the reference value.

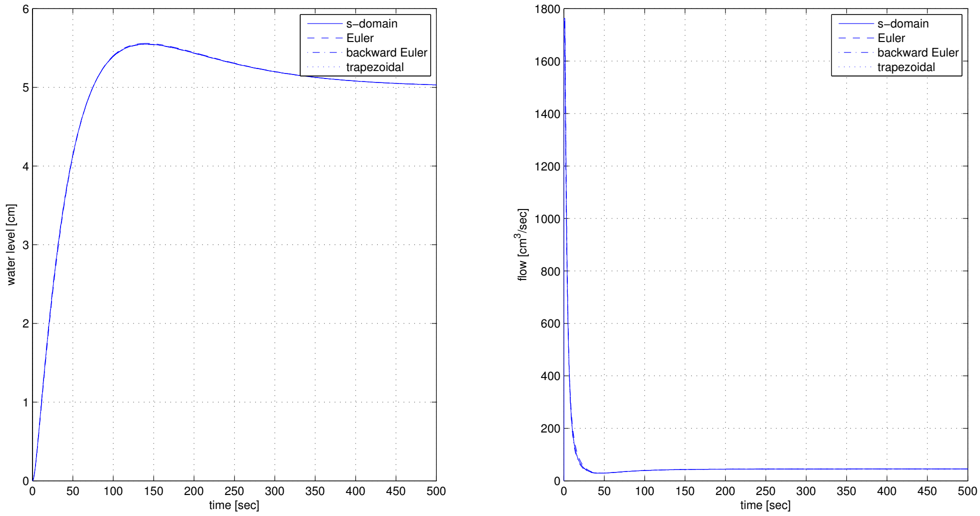


Figure 5.20: **Step response for the root locus design**

5.2.4 Symmetrical Optimum

In order to apply the symmetrical optimum method as described in Section 4.3, we need to look at the poles/time constants of the plant transfer function $G(s)$. The poles are

$$P1 = -0.5000, P2 = -0.0212, P3 = -0.0094 \tag{5.42}$$

as already discussed in Figure 5.19. Accordingly, we can write the plant transfer function in the time constant form as

$$G(s) = \frac{9993}{(1 + 47\,s)\,(1 + 106\,s)\,(1 + 2\,s)}. \tag{5.43}$$

It can be observed that $G(s)$ is stable and contains one small time constant $\tau = -1/P1 = 2$ and two larger time constants $T1 = -1/P2 = 47$ and $T3 = -1/P3 = 106$. Then, we can apply the formula in Section 4.3 to find the controller transfer function

$$C(s) = \frac{431.8s^3 + 53.98s^2 + 2.249s + 0.03124}{1.44s^3 + 2.4s^2 + s} \tag{5.44}$$

We again convert the controller to discrete time for $T = 0.1$.

- Euler method:

$$C_{Euler}(z) = \frac{8104z^3 - 2.421\,10^4z^2 + 2.411\,10^4z - 8003}{z^3 - 2.833z^2 + 2.674z - 0.8403} \tag{5.45}$$

- Euler backward method:

$$C_{Eulerbackward}(z) = \frac{6992z^3 - 2.089\,10^4z^2 + 2.08\,10^4z - 6905}{z^3 - 2.846z^2 + 2.698z - 0.8521} \tag{5.46}$$

- Trapezoidal method:

$$C_{rapezoidal}(z) = \frac{7516z^3 - 2.245\,10^4z^2 + 2.236\,10^4z - 7422}{z^3 - 2.84z^2 + 2.686z - 0.8464} \tag{5.47}$$

The resulting step response simulation is shown in Figure 5.21. As is usual for the symmetrical optimum method, the response is fast, but leads to significant overshoot. and the flow will increasing and decreasing because the Sampling Time is too large then the controller needs Large changes of the output.

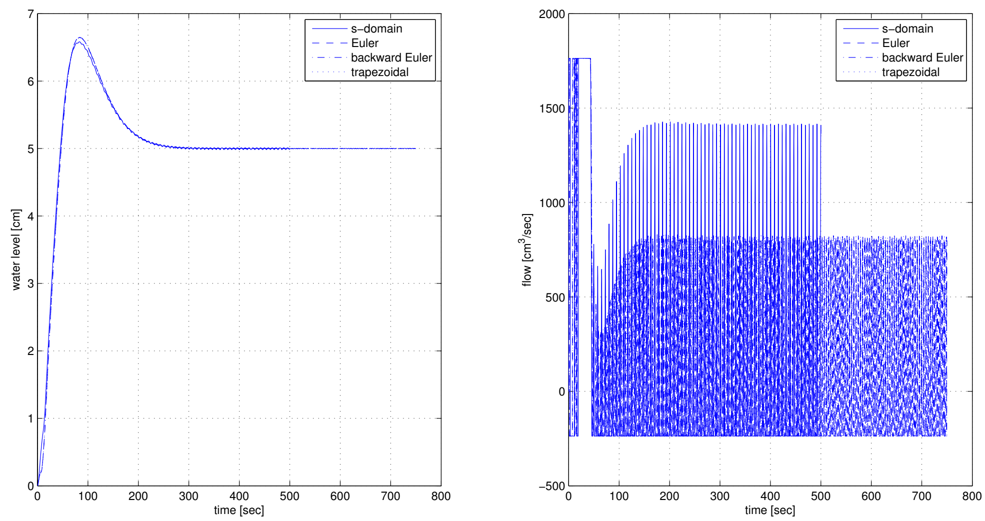


Figure 5.21: **Step response for the symmetrical optimum method**

5.2.5 Youla Parameterization

We finally apply the Youla parametrization method as explained in Section 4.4. We use the desired closed-loop transfer function

$$T(s) = \frac{1}{(1+s)^3} \tag{5.48}$$

and apply the formula in (4.14) to find out the controller transfer function

$$C(s) = \frac{493.5s^3 + 261.8s^2 + 7.647s + 0.04928}{s^3 + 3s^2 + 3s} \tag{5.49}$$

The conversion of $C(s)$ to the z-domain for $T = 0.1$ leads to

- Euler method:

$$C_{Euler} = \frac{493.5z^3 - 1454z^2 + 1428z - 467.4}{z^3 - 2.7z^2 + 2.43z - 0.73} \tag{5.50}$$

- Euler backward method:

$$C_{Eulerbackward} = \frac{390.8z^3 - 1153z^2 + 1133z - 371.1}{z^3 - 2.729z^2 + 2.481z - 0.7519} \tag{5.51}$$

- Trapezoidal method:

$$C_{rapezoidal} = \frac{437.7z^3 - 1290z^2 + 1268z - 415.1}{z^3 - 2.715z^2 + 2.456z - 0.7408} \tag{5.52}$$

The step response simulation result for this controller design is shown in Figure 5.22. It can be seen that, although fast poles are specified in $T(s)$, the response reaches the steady-state value slowly. The reason is again the nonlinearity of the real two-tank system. In the linear controller design, it is assumed that the pump can run in two directions (it can also take water from the tank). In the real nonlinear system, the only outflow is from the discharging pipe. This leads to the slow decrease of the water level.

5.3 TWO-TANK SYSTEM (WITH TWO PUMPS)

In this experiment, we consider the two-tank system that is equipped with two pumps as discussed in Section 3.3. The first pump pushes water to tank 1, while the second pump takes water from tank 1 and feeds it to tank 2. We now design controllers for both tanks and investigate the relationship between both control loops.

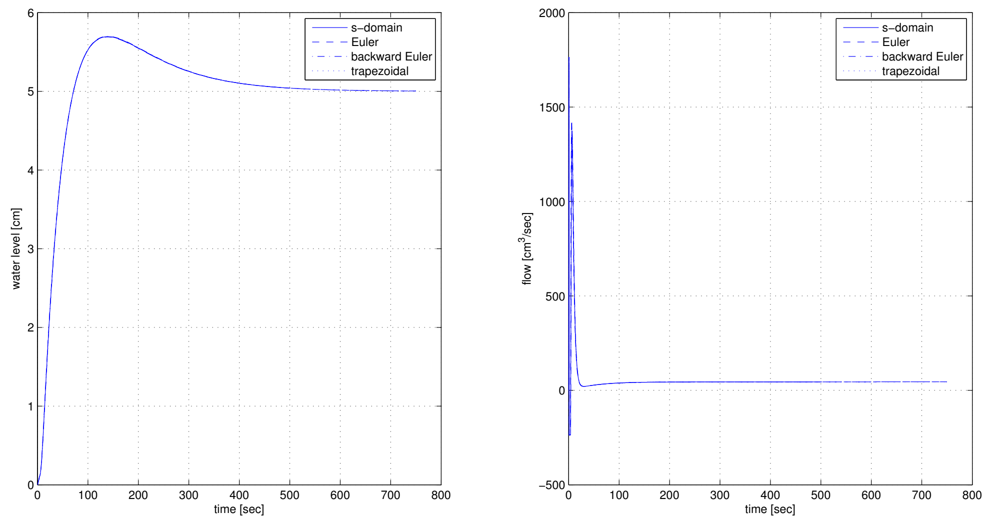


Figure 5.22: **Step response for the Youla parameterization method**

5.3.1 Pole Placement Mehtod

In this experiment, the plant consist of two one-tank systems as studied in Section 5.1. That is, we can apply the pole placement controller with integral and filter as used in Section 5.1.2´for both control loops. The controller transfer function is

$$C(s) = \frac{13.74s^3 + 10.06s^2 + 0.9362s + 0.03158}{s^3 + 0.6694s^2 + 0.2293s} \tag{5.53}$$

Figure 5.23 shows a step response simulation for a reference step at time 50 of the water level in the second tank by 10 (cm). The left hand plot shows the water level $h1$ of tank 1 and the right hand plot shows the water level $h2$ of tank 2. In order to increase the water level in tank 2, the flow of pump 2 is increased, which constitutes a disturbance for the water level in tank 1. Hence, the controller for tank 1 has to react in order to bring the water level back to the initial level.

5.3.2 Disturbance Feed-forward

We now use the same setup and controllers as in Section 5.3.1, but we assume that the flow of pump 2 is measurable. Then, we can apply disturbance feedforward

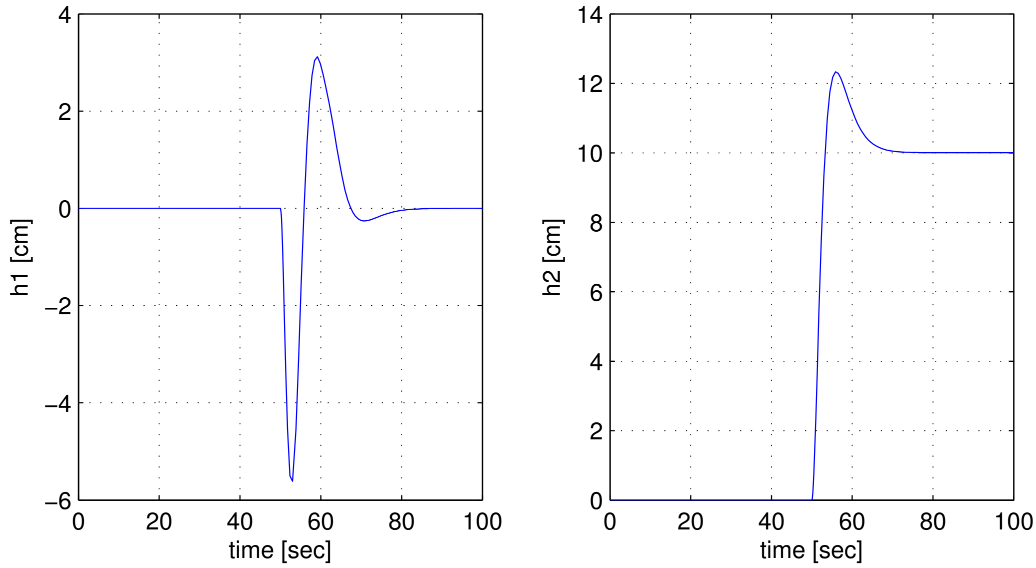


Figure 5.23: **pole placement method with filter for control on a water level in tank1 when T=0.1**

as described in Section 4.5. That is, we add a disturbance feedforward controller $F(s)$. Considering the pump transfer function

$$P1(s) = \frac{200}{1 + 2\,s}, \tag{5.54}$$

we compute with (4.16)

$$F(s) = \frac{1}{P1(s)} = \frac{1 + 2\,s}{200}. \tag{5.55}$$

Since $F(s)$ is not proper, we multiply with a first-order lag transfer function with a small time-constant (fast pole). We finally use

$$F(s) = \frac{1 + 2\,s}{200\,(1 + 0.2\,s)}. \tag{5.56}$$

Figure 5.24 shows the same step response experiment as in Section 5.3.1 with and without disturbance feedforward. It can be seen that the water level in tank 2 is not affected, since the disturbance feedforward is applied in the loop with tank 1. However, the disturbance response in the loop with tank 1 is dramatically improved if disturbance feedforward is used. This is due to the immediate reaction to the disturbance based on the disturbance measurement.

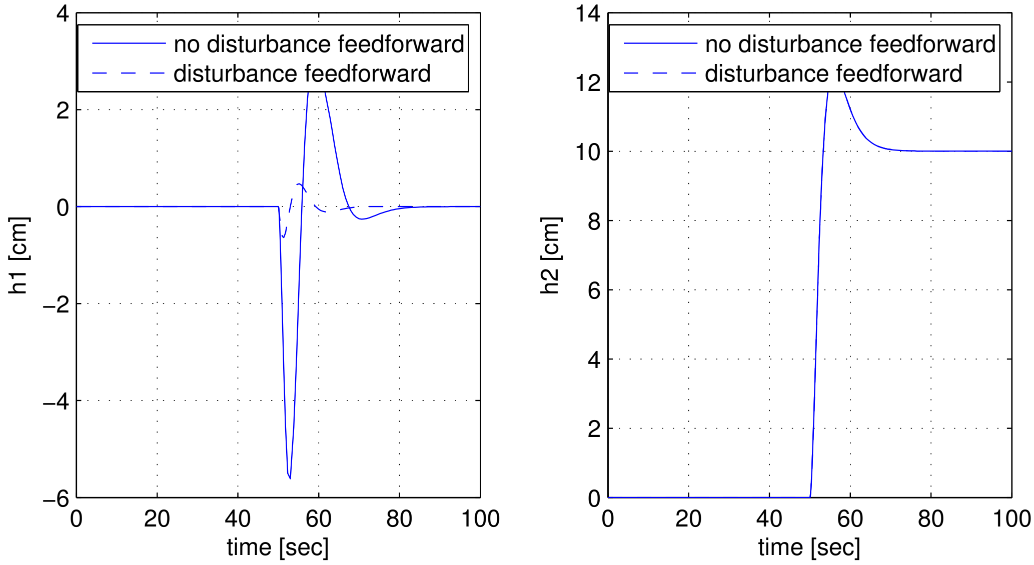


Figure 5.24: **Feed-forward control method when T=0.1**

CONCLUSIONS

As reported in the literature, the water level control is a very suitable experiment for control laboratories. It can be used to study many aspects of control systems, that can be easily transfered to other applications such as automation systems, power systems or automotive systems. First, it allows to study basic aspects of control system design such as nonlinear modeling and set-point linearization. Second it is suitable for demonstrating a large variety of controller design methods. Third, it makes students familiar with elementary equipment that is used in control systems such as pressure sensors and water pumps.

In this thesis, a three tank system for control laboratories is designed. In the theoretical part of the thesis, basic properties of the system are investigated and suitable control designs are evaluated. The works include

- Nonlinear modeling of the three tank system
- Set-point linearization for different variants of the three tank system
- Application of basic linear controller design methods
- Discretization of continuous-time control algorithms for the digital controller implementation
- Simulation and validation of control systems using Matlab/Simulink

In the practical part of the thesis, basic components of a one-tank system are selected and assembled for a feedback control experiment. The works include

- Microcontroller programming in the C language
- Usage of pressure sensors
- Design of high-gain instrumental amplifiers

- Usage of high-current motor drivers
- Usage of pulse width modulation (PWM)
- Usage of DC pumps
- Implementation of digital control algorithms on PIC microcontrollers

Using the above steps and equipment, it was possible to perform basic feedback control experiments. However, since the DC pump turned out to be very insensitive, more detailed experiments were not possible. As the main task for future work, it remains to obtain a more suitable DC pump. All other components proved to be suitable for the water-tank experiment. As an additional important result of the practical experiments, the connection and outflow pipes should be large enough to guarantee a practicable design of the motor tank experiment . The reason for this is that inflow of water into a tank is conveniently controlled by pumps. However, sufficient outflow is only possible if the pipes are large enough.

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APPENDIX

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