Special Issue Article

Advances in Mechanical Engineering

Advances in Mechanical Engineering
2017, Vol. 9(2) 1–8

DOI: 10.1177/1687814017690069
journals.sagepub.com/home/ade
SAGE

Analysis of logistic equation pertaining to a new fractional derivative with non-singular kernel

Devendra Kumar[1], Jagdev Singh[1], Maysaa Al Qurashi[2] and Dumitru Baleanu[3,4]

Abstract

In this work, we aim to analyze the logistic equation with a new derivative of fractional order termed in Caputo–Fabrizio sense. The logistic equation describes the population growth of species. The existence of the solution is shown with the help of the fixed-point theory. A deep analysis of the existence and uniqueness of the solution is discussed. The numerical simulation is conducted with the help of the iterative technique. Some numerical simulations are also given graphically to observe the effects of the fractional order derivative on the growth of population.

Keywords

Logistic equation, nonlinear equation, Caputo–Fabrizio fractional derivative, uniqueness, fixed-point theorem

Date received: 22 October 2016; accepted: 21 December 2016

Academic Editor: Xiao-Jun Yang

Introduction

The logistic equation describes the population growth. It was first proposed by Pierre Verhulst that is why it is also known as Verhulst model. The mathematical equation is a continuous function of time, but a modified version of the continuous model to a discrete quadratic recurrence model is said to be the logistic map which is also extensively used.

The continuous form of the logistic equation is expressed in the form of nonlinear ordinary differential equation as[1]

$$\frac{dN}{dt} = \lambda N\left(1 - \frac{N}{K}\right) \tag{1}$$

In the above equation (1), N indicates population at time t, $\lambda > 0$ represents Malthusian parameter expressing growth rate of species and K denotes carrying capacity. If we take $x = N/K$, then equation (1) reduces in the nonlinear differential equation written as

$$\frac{dx}{dt} = \lambda x(1 - x) \tag{2}$$

Equation (2) is said to be logistic equation.

Fractional calculus in mathematical modeling has been gaining great admiration and significance due largely to its manifest importance and uses in science, engineering, finance and social sciences. Due to its wide applications, many scientists and engineers investigated in this special branch and introduced various

[1]Department of Mathematics, JECRC University, Jaipur, India
[2]Department of Mathematics, College of Science, King Saud University, Riyadh, Saudi Arabia
[3]Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, Etimesgut, Turkey
[4]Institute of Space Sciences, Magurele-Bucharest, Romania

Corresponding author:
Devendra Kumar, Department of Mathematics, JECRC University, Jaipur 303905, Rajasthan, India.
Email: devendra.maths@gmail.com

denotations of fractional derivatives and integrals.[2–7] In this connection, a monograph by Baleanu et al.[8] presents applications of nanotechnology and fractional calculus. A monograph by Kilbas et al.[9] provides an excellent literature related to basic concepts and uses of fractional differential equations. In this sequel, Bulut et al.[10] analyzed differential equations of arbitrary order analytically. Atangana and Alkahtani[11] examined the fractional Keller–Segel model using iterative technique. Alkahtani and Atangana[12] analyzed a non-homogeneous heat model involving a new fractional order derivative. Atangana[13] studied a fractional generalization of nonlinear Fisher's reaction–diffusion equation using iterative scheme. Singh et al.[14] studied the Tricomi equation involving the local fractional derivative with the aid of local fractional homotopy perturbation sumudu transform technique. Kumar et al.[15] reported the numerical solution of fractional differential-difference equation using homotopy analysis Sumudu transform scheme. Choudhary et al.[16] examined the fractional model of temperature distribution and heat flux in the semi-infinite solid using integral transform technique. Yang et al.[17] obtained an exact traveling-wave solution for KdV equation associated with local fractional derivative. Yang et al.[18] investigated some novel uses for heat and fluid flows associated with fractional derivatives having non-singular kernel. Yang et al.[19] studied a new fractional derivative without singular kernel and showed its uses in the modeling of the steady heat flow. Hristov[20] examined Cattaneo concept of flux relaxation with a Jeffrey's exponential kernel in view of its association with heat diffusion pertaining to time derivative of fractional order termed in Caputo–Fabrizio sense. Golmankhaneh et al.[21] studied the synchronization in a non-identical fractional order of a modified system. The fractional generalization of logistic equation associated with Caputo fractional derivative is studied by many authors such as El-Sayed et al.,[22] Momani and Qaralleh[23] and many others.

Thus, the fractional modeling is very useful in description of natural phenomena. But the novel fractional derivative given by Caputo and Fabrizio is more suitable to describe the growth of population because its kernel is non-local and non-singular. Therefore, we replace the time derivative in equation (2) by a new fractional derivative discovered by Caputo and Fabrizio, and equation (2) converts to a time-fractional model of the logistic equation expressed in the following manner

$$ {}_{0}^{CF}D_t^{\beta}x(t) = \lambda x(t)(1 - x(t)) \tag{3} $$

subject to the initial condition

$$ x(0) = \alpha \tag{4} $$

The principal objective of this work is determining the novel fractional derivative to the nonlinear logistic model and imparting in detail the analysis of the solution of the nonlinear model with the aid of the fixed-point theory. The structure of this article is as follows: in section "Preliminaries," the fundamental concept of new fractional derivatives defined by the Caputo–Fabrizio is given. In section "Equilibrium and stability," the equilibrium stability of initial value problem (IVP) associated with new Caputo–Fabrizio fractional derivative is discussed. The fractional logistic equation and its stability analysis are examined in section "Fractional model of logistic equation associated with new fractional derivative." In section "Existence and uniqueness," the existence and uniqueness of the solution are examined. Section "Numerical results and discussions" contains the numerical simulation of fractional logistic equation. Finally, section "Conclusion" is dedicated to the conclusions.

Preliminaries

Definition 1. If $x \in H^1(a,b), b>a, \beta \in [0,1]$, then the new fractional derivative defined by Caputo and Fabrizio[5] is represented as

$$ D_t^{\beta}(x(t)) = \frac{M(\beta)}{1-\beta}\int_a^t x'(s)\exp\left[-\beta\frac{t-s}{1-\beta}\right]ds \tag{5} $$

In the above expression, $M(\beta)$ is a normalization of the function that satisfies the condition $M(0) = M(1) = 1$ presented by Losada and Nieto.[6]

But if $x \notin H^1(a,b)$, then the new derivative of arbitrary order can be defined as

$$ D_t^{\beta}(x(t)) = \frac{\beta M(\beta)}{1-\beta}\int_a^t (x(t)-x(s))\exp\left[-\beta\frac{t-s}{1-\beta}\right]ds \tag{6} $$

Remark 1. If $\sigma = \frac{1-\beta}{\beta} \in [0,\infty), \beta = \frac{1}{1+\sigma} \in [0,1]$, then equation (6) presume the form

$$ D_t^{\beta}(x(t)) = \frac{N(\sigma)}{\sigma}\int_a^t x'(s)\exp\left[-\frac{t-s}{\sigma}\right]ds, \quad N(0) = N(\infty) = 1 \tag{7} $$

Moreover

$$ \lim_{\sigma\to 0}\frac{1}{\sigma}\exp\left[-\frac{t-s}{\sigma}\right] = \delta(s-t) \tag{8} $$

The corresponding fractional integral resulted to be essential.[6]

Definition 2. Let $0<\beta<1$. If x be a function of t, then the fractional integral operator of order β is presented in the following form

$$I_\beta^t(x(t)) = \frac{2(1-\beta)}{(2-\beta)M(\beta)}x(t) + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t x(s)ds, \quad t \geq 0 \tag{9}$$

Definition 3. If $x(t)$ be a function of t, then the Laplace transform of the function ${}_0^{CF}D_t^\beta x(t)$ is written as (see Caputo and Fabrizio[5])

$$L\left[{}_0^{CF}D_t^\beta x(t)\right] = M(\beta)\frac{s\bar{x}(s) - x(0)}{s + \beta(1-s)} \tag{10}$$

In the above formula (10), $\bar{x}(s)$ stands for the Laplace transform of the function $x(t)$.

Equilibrium and stability

Let us take the following IVP associated with Caputo–Fabrizio fractional derivative

$${}_0^{CF}D_t^\beta x(t) = g(x(t)), \quad t>0, \ 0<\beta\leq 1 \tag{11}$$

and

$$x(0) = x_0 \tag{12}$$

To compute the equilibrium point for equation (11), put ${}_0^{CF}D_t^\beta x(t) = 0$, then it yields the following result

$$g(x_{eq}) = 0 \tag{13}$$

In order to find the asymptotic stability, take

$$x(t) = x_{eq} + \varepsilon(t) \tag{14}$$

Using equation (14) in (11), we get

$${}_0^{CF}D_t^\beta\left(x_{eq} + \varepsilon\right) = g\left(x_{eq} + \varepsilon\right) \tag{15}$$

which yields

$${}_0^{CF}D_t^\beta \varepsilon(t) = g\left(x_{eq} + \varepsilon\right) \tag{16}$$

As we know that

$$g\left(x_{eq} + \varepsilon\right) = g\left(x_{eq}\right) + g'\left(x_{eq}\right)\varepsilon + \cdots$$

which implies that

$$g\left(x_{eq} + \varepsilon\right) = g'\left(x_{eq}\right)\varepsilon \tag{17}$$

where $g(x_{eq}) = 0$, and then we have the following result

$${}_0^{CF}D_t^\beta \varepsilon(t) = g'\left(x_{eq}\right)\varepsilon(t), t>0, \text{ with } \varepsilon(0) = x_0 - x_{eq} \tag{18}$$

Further assume that the solution $\varepsilon(t)$ of equation (18) exists. Therefore, the equilibrium point x_{eq} is unstable if the function $\varepsilon(t)$ is increasing, and the equilibrium point x_{eq} is locally asymptotically stable if the function $\varepsilon(t)$ is decreasing.

Fractional model of logistic equation associated with new fractional derivative

Here, we examine the equilibrium and stability of the fractional generalization of logistic equation associated with the newly developed Caputo–Fabrizio fractional derivative.

Let us consider that $0<\beta\leq 1$, $\lambda>0$ and $x_0>0$; the fractional model of logistic equation is presented as

$${}_0^{CF}D_t^\beta x(t) = \lambda x(t)(1-x(t)), \quad t>0 \text{ and } x(0) = \alpha \tag{19}$$

To compute the equilibrium points, put

$${}_0^{CF}D_t^\beta x(t) = 0 \tag{20}$$

which gives the equilibrium points $x = 0, 1$.

Next, to investigate the stability of the equilibrium points, we find the following result

$$g'(x(t)) = \lambda(1 - 2x(t)) \tag{21}$$

which yields

$$g'(0) = \lambda \ \text{ and } \ g'(1) = -\lambda \tag{22}$$

Then, the solution of fractional order IVP

$${}_0^{CF}D_t^\beta \varepsilon(t) = g'\left(x_{eq} = 0\right)\varepsilon(t) = \lambda\varepsilon(t), t>0 \text{ with } \varepsilon(0) = x_0$$

is presented as

$$\varepsilon(t) = \frac{x_0}{(1-\lambda+\lambda\beta)}e^{\left(\frac{\lambda\beta}{1-\lambda+\lambda\beta}\right)t} \tag{23}$$

In this case, the point $x = 0$ is unstable.

In order to check the stability of the point $x = 1$, we consider the fractional order IVP

$${}_0^{CF}D_t^\beta \varepsilon(t) = g'\left(x_{eq} = 1\right)\varepsilon(t) = -\lambda\varepsilon(t), t>0 \text{ with } \varepsilon(0) = x_0 - 1 \tag{24}$$

which is (if $x_0>0$) the relaxation equation of arbitrary order, and its solution is presented as

$$\varepsilon(t) = \frac{x_0 - 1}{(1+\lambda-\lambda\beta)}e^{-\left(\frac{\lambda\beta}{1+\lambda-\lambda\beta}\right)t} \tag{25}$$

Therefore, the equilibrium point $x = 1$ is asymptotically stable.

Next, we present the existence and uniqueness for the solution of the logistic equation of fractional order (3).

Existence and uniqueness

Here, we present the analysis of the fractional model of logistic equation. Applying the Losada–Nieto fractional integral operator on equation (3) we get the following result

$$x(t)-x(0)=\frac{2(1-\beta)}{(2-\beta)M(\beta)}\{\lambda x(t)(1-x(t))\}+\frac{2\beta}{(2-\beta)M(\beta)}\int_0^t\{\lambda x(s)(1-x(s))\}ds \tag{26}$$

For simplicity, we interpret

$$x(t)=x(0)+\frac{2(1-\beta)}{(2-\beta)M(\beta)}K(t,x)+\frac{2\beta}{(2-\beta)M(\beta)}\int_0^t K(s,x)ds \tag{27}$$

The operator K has Lipschitz condition providing that the function x has an upper bound. So if the function x is upper bounded then

$$\|K(t,x)-K(t,y)\|=\left\|\lambda(x-y)-\lambda\left(x^2-y^2\right)\right\| \tag{28}$$

On using the inequality of triangle on equation (28), it yields

$$\begin{aligned}&\|K(t,x)-K(t,y)\|\le\lambda\|(x-y)\|+\lambda\left\|\left(x^2-y^2\right)\right\|\\&\le\lambda\|(x-y)\|+\lambda\|(x-y)(A+B)\|\\&\le\lambda(1+A+B)\|(x-y)\|\end{aligned} \tag{29}$$

Setting $\rho=\lambda(1+A+B)$, where $\|x\|\le A$ and $\|y\|\le B$ are bounded functions, we have

$$\|K(t,x)-K(t,y)\|\le\rho\|x-y\| \tag{30}$$

Therefore, the Lipschitz condition is fulfilled for K, and if additionally $0<\lambda(1+A+B)\le 1$, then it is also a counterstatement.

Theorem 1. Considering that the function x is bounded, then the operator presented below satisfies the Lipschitz condition

$$T(x)=x(0)+\frac{2(1-\beta)}{(2-\beta)M(\beta)}K(t,x)+\frac{2\beta}{(2-\beta)M(\beta)}\int_0^t K(s,x)ds \tag{31}$$

Proof. Suppose both the functions x and y are bounded with $x(0)=y(0)$, then we have

$$\begin{aligned}&\|T(x)-T(y)\|=\\&\left\|\frac{2(1-\beta)}{(2-\beta)M(\beta)}\{K(t,x)-K(t,y)\}+\frac{2\beta}{(2-\beta)M(\beta)}\int_0^t\{K(s,x)-K(s,y)\}ds\right\|\\&\le\frac{2(1-\beta)}{(2-\beta)M(\beta)}\|\{K(t,x)-K(t,y)\}\|\\&+\frac{2\beta}{(2-\beta)M(\beta)}\int_0^t\|\{K(s,x)-K(s,y)\}\|ds\\&\le\left(\frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho+\frac{2\beta}{(2-\beta)M(\beta)}\rho t_0\right)\|x-y\|\\&\le\eta\|x-y\|\end{aligned} \tag{32}$$

Hence, the theorem is proved.

Theorem 2. Considering that the function x is bounded, then the operator T_1 expressed as

$$T_1(x)=\lambda x(t)(1-x(t)) \tag{33}$$

satisfies the result

$$|\langle T_1(x)-T_1(y),x-y\rangle|\le\rho\|x-y\|^2 \tag{34}$$

In the above inequality (34), $\langle\cdot,\cdot\rangle$ indicates the inner product of function with the differentiation restricted in L^2.

Proof. Let us assume that x be bounded function, then we have

$$\begin{aligned}&|\langle T_1(x)-T_1(y),x-y\rangle|=\left|\left\langle\lambda(x-y)-\lambda\left(x^2-y^2\right),x-y\right\rangle\right|\\&\le\lambda|\langle(x-y),x-y\rangle|+\lambda\left|\left\langle\left(x^2-y^2\right),x-y\right\rangle\right|\\&\le\lambda\|(x-y)\|\|x-y\|+\lambda\left\|x^2-y^2\right\|\|x-y\|\\&\le\lambda(1+A+B)\|(x-y)\|^2\\&\le\rho\|(x-y)\|^2\end{aligned} \tag{35}$$

Hence, the theorem is proved.

Theorem 3. If it is assumed that the function x is bounded, then the operator T_1 satisfies the result

$$|\langle T_1(x)-T_1(y),w\rangle|\le\rho\|x-y\|\|w\|,\quad 0<\|w\|<\infty \tag{36}$$

Proof. Let $0<\|w\|<\infty$ and consider that the function x be bounded, then we have

$$\begin{aligned}&|\langle T_1(x)-T_1(y),w\rangle|=\left|\left\langle\lambda(x-y)-\lambda\left(x^2-y^2\right),w\right\rangle\right|\\&\le\lambda|\langle(x-y),w\rangle|+\lambda\left|\left\langle\left(x^2-y^2\right),w\right\rangle\right|\\&\le\lambda\|(x-y)\|\|w\|+\lambda\left\|x^2-y^2\right\|\|w\|\\&\le\lambda(1+A+B)\|(x-y)\|\|w\|\\&\le\rho\|(x-y)\|\|w\|\end{aligned} \tag{37}$$

Hence, the theorem is proved.

Existence of the solution

To show the existence of the solution, we employ the notion of iterative formula. In view of equation (27), we set up the following iterative formula

$$x_{n+1}(t) = \frac{2(1-\beta)}{(2-\beta)M(\beta)}K(t,x_n) + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t K(s,x_n)ds \tag{38}$$

and

$$x_0(t) = x(0) \tag{39}$$

The difference of the successive terms is represented as follows

$$\theta_n(t) = x_n(t) - x_{n-1}(t) = \frac{2(1-\beta)}{(2-\beta)M(\beta)}(K(t,x_{n-1}) - K(t,x_{n-2})) + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t (K(s,x_{n-1}) - K(y,x_{n-2}))ds \tag{40}$$

Its usefulness is to notice that

$$x_n(t) = \sum_{i=0}^{n} \theta_i(t) \tag{41}$$

Slowly but surely we assess

$$\|\theta_n(t)\| = \|x_n(t) - x_{n-1}(t)\| = \left\| \frac{2(1-\beta)}{(2-\beta)M(\beta)}(K(t,x_{n-1}) - K(t,x_{n-2})) + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t (K(s,x_{n-1}) - K(s,x_{n-2}))ds \right\| \tag{42}$$

Making use of the triangular inequality, equation (42) becomes

$$\|\theta_n(t)\| \le \frac{2(1-\beta)}{(2-\beta)M(\beta)}\|(K(t,x_{n-1}) - K(t,x_{n-2}))\| + \frac{2\beta}{(2-\beta)M(\beta)}\left\|\int_0^t (K(s,x_{n-1}) - K(s,x_{n-2}))\, ds\right\| \tag{43}$$

As the Lipschitz condition is fulfilled by the kernel, it yields

$$\|\theta_n(t)\| \le \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho\|x_{n-1} - x_{n-2}\| + \frac{2\beta}{(2-\beta)M(\beta)}\rho\int_0^t \|x_{n-1} - x_{n-2}\|ds \tag{44}$$

Then

$$\|\theta_n(t)\| \le \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho\|\theta_{n-1}(t)\| + \frac{2\beta}{(2-\beta)M(\beta)}\rho\int_0^t \|\theta_{n-1}(t)\|ds \tag{45}$$

Now taking the above result into consideration, we derive the following result expressed as the subsequent theorem.

Theorem 4. The fractional model of logistic equation associated with equation (3) has a solution under the condition that we can find t_0 satisfying the following inequality

$$\frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho + \frac{2\beta}{(2-\beta)M(\beta)}\rho t_0 < 1 \tag{46}$$

Proof. Here, we have the function $x(t)$ is bounded. Additionally, we have shown that the kernels fulfill the Lipschitz condition, hence on considering the result of equation (45) and by applying the recursive method, we get the inequality as follows

$$\|\theta_n(t)\| \le \left[\frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho + \frac{2\beta}{(2-\beta)M(\beta)}\rho t\right]^n x(0) \tag{47}$$

Therefore

$$x_n(t) = \sum_{i=0}^{n} \theta_i(t) \tag{48}$$

exists and is a smooth function. Next, we demonstrate that the function presented in equation (48) is the solution of equation (3). Now it is assumed that

$$x(t) - x(0) = x_n(t) - P_n(t) \tag{49}$$

Therefore, we have

$$\begin{aligned}\|P_n(t)\| &= \left\|\frac{2(1-\beta)}{(2-\beta)M(\beta)}(K(t,x) - K(t,x_{n-1})) + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t (K(s,x) - K(s,x_{n-1}))ds\right\| \\ &\le \frac{2(1-\beta)}{(2-\beta)M(\beta)}\|(K(t,x) - K(t,x_{n-1}))\| + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t \|(K(s,x) - K(s,x_{n-1}))\|ds \\ &\le \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho\|x - x_{n-1}\| + \frac{2\beta}{(2-\beta)M(\beta)}\rho\|x - x_{n-1}\|t\end{aligned} \tag{50}$$

On using this process recursively, it yields

$$\|P_n(t)\| \leq \left(\frac{2(1-\beta)}{(2-\beta)M(\beta)} + \frac{2\beta}{(2-\beta)M(\beta)}t\right)^{n+1} \rho^{n+1}A \tag{51}$$

Now taking the limit on equation (51) as n tends to infinity, we get

$$\|P_n(t)\| \to 0$$

Hence, proof of existence is verified.

Uniqueness of the solution

Here, we present the uniqueness of the solution of equation (3). Suppose, there exists an another solution for equation (3) be $y(t)$, then

$$x(t) - y(t) = \frac{2(1-\beta)}{(2-\beta)M(\beta)}(K(t,x) - K(t,y)) + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t (K(s,x) - K(s,y))ds \tag{52}$$

On taking the nom on both sides of equation (52), it yields

$$\|x(t) - y(t)\| \leq \frac{2(1-\beta)}{(2-\beta)M(\beta)}\|K(t,x) - K(t,y)\| + \frac{2\beta}{(2-\beta)M(\beta)}\int_0^t \|(K(s,x) - K(s,y))\|ds \tag{53}$$

By employing the Lipschitz conditions of kernel, we obtain

$$\|x(t) - y(t)\| \leq \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho\|x(t) - y(t)\| + \frac{2\beta}{(2-\beta)M(\beta)}\rho t\|x(t) - y(t)\| \tag{54}$$

This gives

$$\|x(t) - y(t)\|\left(1 - \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho - \frac{2\beta}{(2-\beta)M(\beta)}\rho t\right) \leq 0 \tag{55}$$

Theorem 5. If the following condition holds, then fractional logistic equation (3) has a unique solution

$$\left(1 - \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho - \frac{2\beta}{(2-\beta)M(\beta)}\rho t\right) > 0 \tag{56}$$

Proof. If the aforesaid condition holds, then

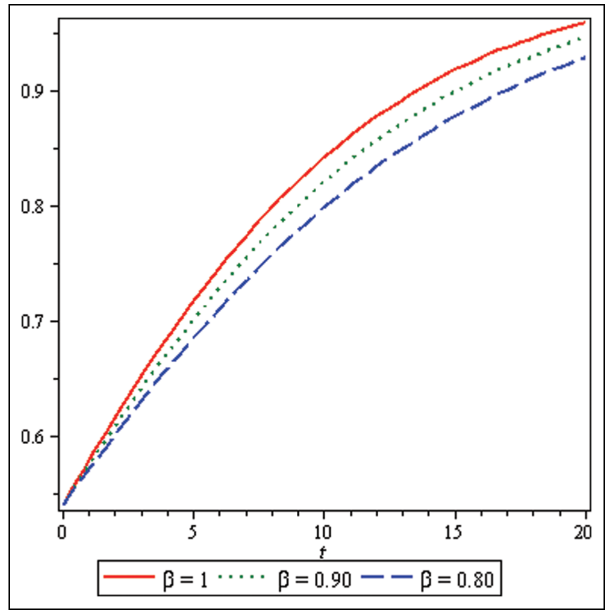

Figure 1. The response of solution $x(t)$ versus t at $\lambda = 1/3$ for distinct values of β.

$$\|x(t) - y(t)\|\left(1 - \frac{2(1-\beta)}{(2-\beta)M(\beta)}\rho - \frac{2\beta}{(2-\beta)M(\beta)}\rho t\right) \leq 0 \tag{57}$$

which implies that

$$\|x(t) - y(t)\| = 0$$

Then, we get

$$x(t) = y(t) \tag{58}$$

Hence, we proved the uniqueness of the solution of equation (3).

Numerical results and discussions

Here, we compute the numerical solution of fractional model of logistic equation (3) using perturbation-iterative technique and Padé approximation.[24] For the numerical calculation, the initial condition is taken as $x(0) = 0.5$. In Figures 1 and 2, growth of population $x(t)$ is investigated with respect to various values of β and $\lambda = 1/3$ and $\lambda = 1/2$, respectively. The graphical representations show that the model depends notably to the fractional order. From Figures 1 and 2, we can observe that the growth of population increases with increasing value of order of time-fractional derivative β. Thus, the fractional model narrates a new characteristic at $\beta = 0.80$ and $\beta = 0.90$ that was invisible when modeling at $\beta = 1$.

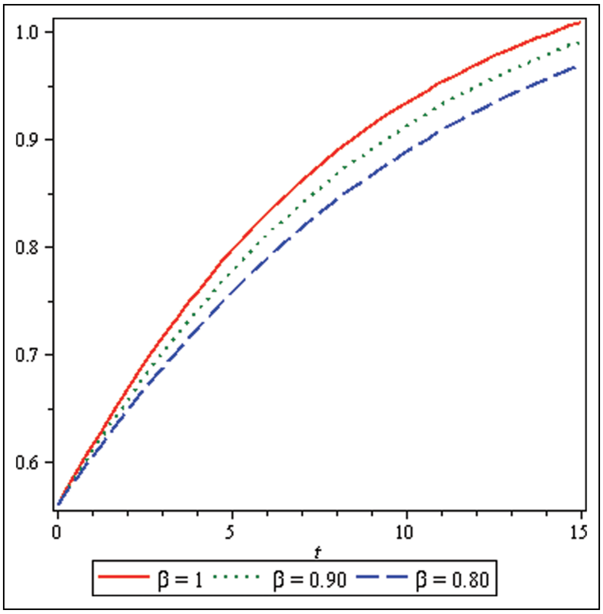

Figure 2. The behavior of the solution $x(t)$ versus t at $\lambda = 1/2$ for distinct values of β.

Conclusion

In this article, we have studied the logistic equation involving a novel Caputo–Fabrizio fractional derivative. The stability analysis of model is conducted. The existence and uniqueness of the solution of logistic equation of fractional order are shown. The numerical solution is obtained using an iterative scheme for the arbitrary order model. The most important part of this study is to analyze the fractional logistic equation and related issues. It is also observed that the order of time-fractional derivative significantly affects the population growth. Hence, we conclude that the proposed fractional model is very useful and efficient to describe the real-world problems in a better and systematic manner.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors extend their appreciation to the International Scientific Partnership Program ISPP at King Saud University for funding this research work through ISPP# 63.

References

1. Strogatz SH. *Nonlinear dynamics and chaos*. Kolkata, India: Levant Books, 2007.
2. Podlubny I. *Fractional differential equations*. New York: Academic Press, 1999.
3. Caputo M. *Elasticita e Dissipazione*. Bologna: Zanichelli, 1969.
4. Yang XJ. *Advanced local fractional calculus and its applications*. New York: World Science, 2012.
5. Caputo M and Fabrizio M. A new definition of fractional derivative without singular kernel. *Prog Fract Diff Appl* 2015; 1: 73–85.
6. Losada J and Nieto JJ. Properties of the new fractional derivative without singular kernel. *Prog Fract Diff Appl* 2015; 1: 87–92.
7. Golmankhaneh AK and Baleanu D. New derivatives on the fractal subset of real-line. *Entropy* 2016; 18: 1.
8. Baleanu D, Guvenc ZB and Machado JAT (eds). *New trends in nanotechnology and fractional calculus applications*. Dordrecht; Heidelberg; London; New York: Springer, 2010.
9. Kilbas AA, Srivastava HM and Trujillo JJ. *Theory and applications of fractional differential equations*. Amsterdam: Elsevier, 2006.
10. Bulut H, Baskonus HM and Belgacem FBM. The analytical solutions of some fractional ordinary differential equations by Sumudu transform method. *Abstr Appl Anal* 2013; 2013: 203875 (6 pp.).
11. Atangana A and Alkahtani BST. Analysis of the Keller–Segel model with a fractional derivative without singular kernel. *Entropy* 2015; 17: 4439–4453.
12. Alkahtani BST and Atangana A. Analysis of non-homogeneous heat model with new trend of derivative with fractional order. *Chaos Solit Fractal* 2016; 89: 566–571.
13. Atangana A. On the new fractional derivative and application to nonlinear Fisher's reaction-diffusion equation. *Appl Math Comput* 2016; 273: 948–956.
14. Singh J, Kumar D and Nieto JJ. A reliable algorithm for local fractional Tricomi equation arising in fractal transonic flow. *Entropy* 2016; 18: 206.
15. Kumar D, Singh J and Baleanu D. Numerical computation of a fractional model of differential-difference equation. *J Comput Nonlin Dyn* 2016; 11: 061004 (6 pp.).
16. Choudhary A, Kumar D and Singh J. Numerical simulation of a fractional model of temperature distribution and heat flux in the semi infinite solid. *Alexandria Eng J* 2016; 55: 87–91.
17. Yang XJ, Machado JAT, Baleanu D, et al. On exact traveling-wave solutions for local fractional Korteweg-de Vries equation. *Chaos* 2016; 26: 084312.
18. Yang XJ, Zhang ZZ and Srivastava HM. Some new applications for heat and fluid flows via fractional derivatives without singular kernel. *Therm Sci* 2016; 20: 833–839.
19. Yang XJ, Srivastava HM and Machado JAT. A new fractional derivative without singular kernel: application to the modelling of the steady heat flow. *Therm Sci* 2016; 20: 753–756.
20. Hristov J. Transient heat diffusion with a non-singular fading memory: from the Cattaneo constitutive equation with Jeffrey's kernel to the Caputo-Fabrizio time-fractional derivative. *Therm Sci* 2016; 20: 757–762.

21. Golmankhaneh AK, Arefi R and Baleanu D. Synchronization in a nonidentical fractional order of a proposed modified system. *J Vib Control* 2015; 21: 1154–1161.
22. El-Sayed AMA, El-Mesiry AEM and El-Saka HAA. On the fractional-order logistic equation. *Appl Math Lett* 2007; 20: 817–823.
23. Momani S and Qaralleh R. Numerical approximations and Padé approximants for a fractional population growth model. *Appl Math Model* 2007; 31: 1907–1914.
24. Boyd JP. Padé approximants algorithm for solving nonlinear ordinary differential equation boundary value problems on an unbounded domain. *Comput Phys* 1997; 11: 299–303.