



Research Article

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New analytical solutions for conformable fractional PDEs arising in mathematical physics by exp-function method

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Abstract: Modelling of physical systems mathematically, produces nonlinear evolution equations. Most of the physical systems in nature are intrinsically nonlinear, therefore modelling such systems mathematically leads us to nonlinear evolution equations. The analysis of the wave solutions corresponding to the nonlinear partial differential equations (NPDEs), has a vital role for studying the nonlinear physical events. This article is written with the intention of finding the wave solutions of Nizhnik-Novikov-Veselov and Klein-Gordon equations. For this purpose, the exp-function method, which is based on a series of exponential functions, is employed as a tool. This method is an useful and suitable tool to obtain the analytical solutions of a considerable number of nonlinear FDEs within a conformable derivative.

Keywords: exp-function method, Nizhnik-Novikov-Veselov equation, Klein-Gordon equation, conformable derivative

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1 Introduction

The nonlinear equations have a wide application area in various fields such as biomathematics, hydrodynamics, plasma physics, nonlinear optics, fluid dynamics, high-

energy physics, elastic media, optical fibers, chemical physics, chemical kinematics, and geochemistry. In order to extract the hidden physical information of the nonlinear phenomenon, it often proves very attractive to solve the nonlinear equation describing that complex event. So, as a consequence of this attractiveness, many powerful methods to seek for exact solutions to the nonlinear partial differential equations, which can lead us to further applications, were suggested, such as the tanh-function technique [1], F-expansion approach [2], Jacobian elliptic function approach [3], G'/G expansion technique [4], first integral method [5], the hyperbolic function approach [6], extended G'/G expansion method [7], modified Kudryashov technique [8], etc.

This paper is committed to the study of NPDEs which are called the Nizhnik-Novikov-Veselov and Klein-Gordon equations. The nonlinear Klein-Gordon equation which is the relativistic wave equation version of the Schrödinger's equation, plays an important role in mathematical physics. The scientists who are very interested in this equation [10], are so because of its applications: in studying solitons [9] in condensed matter physics; in searching the interaction of solitons in a collisionless plasma, and the recurrence of initial states. The Nizhnik-Novikov-Veselov equation system, which is an isotropic extension of the (1+1)-dimensional KdV equation, is studied by many authors [11–13].

Fractional calculus is developing fast as a field of mathematical analysis, and it possess various definitions of fractional operators, e.g. Riemann-Liouville, Caputo, and Grünwald-Letnikov [14–16]. These operators were utilized in various fields such as diffusion problems, waves in liquids and gases, mechanics, bioengineering, etc. [17] Nowadays, a well-behaved simple derivative called the conformable derivative was suggested in [18].

Definition We consider $h : [0, \infty) \rightarrow \mathbb{R}$ be a function. The β^{th} order "conformable derivative" of h is written as,

$$T_{\beta}(h)(s) = \lim_{\varepsilon \rightarrow 0} \frac{h(s + \varepsilon s^{1-\beta}) - h(s)}{\varepsilon},$$

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such that $s > 0$, $\beta \in (0, 1)$. If h is β -differentiable in some $(0, c)$, $c > 0$ and $\lim_{t \rightarrow 0^+} h^{(\beta)}(s)$ exists then we define $h^{(\beta)}(0) = \lim_{t \rightarrow 0^+} h^{(\beta)}(s)$. The "conformable integral" is written as:

$$I_{\beta}^c(h)(s) = \int_c^t \frac{h(x)}{x^{1-\beta}} dx.$$

Here the integral denotes the well known Riemann improper integral, and $\beta \in (0, 1]$. We recall that the properties of a conformable derivative can be seen in [18]. Although this conformable derivative has a flaw, which is the conformable derivative of any differentiable function at origin is zero, a great number of studies [19, 20] have been made over this subject based on the lucidity and easy applicability of this definition. As an example: higher order integration, relations between integration and differentiation; sequential differentiation and integration, and chain rule, have been presented in [21]. The basic tools of differentiation and integration for conformable time-scale calculus have been developed in [22]. In [23] the authors tried to give the exact solution for the heat equation where the derivatives are conformable derivatives. In [24] there were discussed some mechanical problems such as the fractional harmonic oscillator and the fractional damped oscillator problems, together with the forced one dimensional oscillator dynamics. On top of these studies, in [26] was employed the first integral method, which is based on the ring theory of commutative algebra, while obtaining the analytical solutions of the Wu-Zhang system by means of a conformable derivative. In [28] the sub-equation approach was used to get the new exact solutions for the fractional Schrodinger equation. This type of investigation shows the effects of quantic nonlinearity on the ultrashort optical solitons pulse propagation in the presence of non-Kerr media. Studies on this subject grow day by day, some examples of these studies can be see in references [25, 27, 29].

Following the enhanced version of the conformable derivative, called the beta-derivative, the integral is expressed in [30, 31]. Many studies have been carried out and shown in [32–34].

2 Presentation of exp-function method

Let us discuss the following nonlinear conformable time equation [35]

$$F\left(u, \frac{\partial^\alpha v}{\partial t^\alpha}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial^{2\alpha} v}{\partial t^{2\alpha}}, \frac{\partial^2 v}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \dots\right) = 0. \quad (1)$$

Introducing the new wave variable, namely

$$v(x, y, t) = v(\eta), \quad \eta = \gamma x + \beta y + m \frac{t^\alpha}{\alpha} \quad (2)$$

where γ, β, m denote arbitrary constants to be found later. By using chain rule [21], we have

$$\frac{\partial^\alpha(\cdot)}{\partial t^\alpha} = m \frac{d(\cdot)}{d\eta}, \quad \frac{\partial(\cdot)}{\partial x} = \gamma \frac{d(\cdot)}{d\eta}, \quad \frac{\partial(\cdot)}{\partial y} = \beta \frac{d(\cdot)}{d\eta}, \dots \quad (3)$$

Then Eq.(1) becomes an ordinary differential equation as

$$Q(v, v_\eta, v_{\eta\eta}, v_{\eta\eta\eta}, v_{\eta\eta\eta\eta}, \dots) = 0. \quad (4)$$

Utilizing the exp-function method, it is suppose that the wave solution can be written as

$$v(\eta) = \frac{a_{c_1} e^{c_1 \eta} + \dots + a_{-d} e^{-d \eta}}{b_p e^{p \eta} + \dots + a_{-q} e^{-q \eta}}. \quad (5)$$

Here p, q, c_1 and d are positive integers to be determined later, a_n and b_m are unknown constants. We mention that the Eq. (5) has a very significant role in determining the analytical solution of the nonlinear wave equations. To arrange the highest order values of c_1 and p , the linear term of Eq. (4) is balanced with the highest order nonlinear term. In a similar manner, to find q and d , we balance the lowest order linear term of Eq. (4) with lowest order nonlinear term. After this procedure we obtain the generalized travelling wave solutions to nonlinear conformable evolution equations with fractional order. To show the applicability and convenience of the procedure on conformable PDEs, we handle the $(2 + 1)$ -dimensional Nizhnik-Novikov-Veselov(NNV) equation system and the Klein-Gordon equation.

3 New travelling wave solutions of the $(2 + 1)$ -dimensional Nizhnik-Novikov-Veselov equation

Regarding $(2 + 1)$ dimensional time fractional NNV equation, namely

$$\begin{aligned} \frac{\partial^\alpha u}{\partial t^\alpha} &= \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3} + 3 \frac{\partial(uv)}{\partial x} + 3 \frac{\partial(uw)}{\partial y}, \quad (6) \\ \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y}, \\ \frac{\partial u}{\partial y} &= \frac{\partial w}{\partial x} \end{aligned}$$

in which $\frac{\partial^\alpha}{\partial t^\alpha}$ denotes the conformable derivative with fractional order. By utilizing the new wave variable (2) for all

unknown functions, applying the chain rule [21], and integrating once with respect to η , Eq. set (6) can be rewritten as follows

$$\begin{aligned} mu &= (\gamma^3 + \beta^3)u'' + 3\gamma uv + 3\beta uw, \\ \frac{\gamma}{\beta}u &= v, \\ \frac{\beta}{\gamma}u &= w. \end{aligned} \tag{7}$$

Taking into account the second and third equation in the first equation of (7), we obtain

$$mu = (\gamma^3 + \beta^3)u'' + \left(\frac{3\gamma^2}{\beta} + \frac{3\beta^2}{\gamma}\right)u^2 \tag{8}$$

where double prime means the derivative with respect to η . The next step is to suppose that the expression of the solution of (8) becomes

$$u(\eta) = \frac{a_c e^{c\eta} + \dots + a_{-d} e^{-d\eta}}{b_p e^{p\eta} + \dots + a_{-q} e^{-q\eta}}. \tag{9}$$

Using (8) and the ansatz (9), we handle

$$u'' = \frac{l_1 e^{(c+3p)\eta} + \dots}{l_2 e^{4p\eta} + \dots} \tag{10}$$

and

$$u^2 = \frac{l_3 e^{2c\eta} + \dots}{l_4 e^{2p\eta} + \dots} = \frac{l_3 e^{(2c+2p)\eta} + \dots}{l_4 e^{4p\eta} + \dots} \tag{11}$$

where l_i are coefficients used for convenience. By balancing the highest order of Exp-function in Eqs. (10) and (11), we conclude that $c+3p = 2c+2p$, which implies that $c = p$. The next step is to evaluate d and q . Hence we get,

$$u'' = \dots + \frac{s_1 e^{-(d+3q)\eta} + \dots}{s_2 e^{-4q\eta}} \tag{12}$$

and

$$u^2 = \dots + \frac{s_3 e^{-2d\eta} + \dots}{s_4 e^{-2q\eta}} = \frac{s_3 e^{-(2d+2q)\eta} + \dots}{s_4 e^{-4q\eta} + \dots} \tag{13}$$

where s_i are coefficients used for easiness. As before from (14) and (15) we report that $-d - 3q = -2d - 2q$ which give us $d = q$.

In the consideration of the obtained data set, we can freely choose the values of c and d . For easiness, lets set $c = p = 1$ and $d = q = 1$. Then Eq. (9) becomes

$$u(\eta) = \frac{a_1 e^\eta + a_0 + a_{-1} e^{-\eta}}{e^\eta + b_0 + b_{-1} e^{-\eta}}. \tag{14}$$

Introducing Eq. (14) into Eq. (8) and equating the coefficients of $e^{n\eta}$ we get a system of algebraic equations possessing the following solution

$$m = -\gamma^3 - \beta^3, a_1 = -\frac{\beta\gamma}{3}, a_0 = \frac{2}{3}\beta\gamma b_0, \tag{15}$$

$$a_{-1} = -\frac{1}{12}\beta\gamma b_0^2, b_{-1} = \frac{1}{4}b_0^2$$

Inserting values of m, a_1, a_0, a_{-1} and b_{-1} into Eq. (14), we obtain

$$u(\eta) = \frac{-\frac{\beta\gamma}{3}e^\eta + \frac{2}{3}\beta\gamma b_0 - \frac{1}{12}\beta\gamma b_0^2 e^{-\eta}}{e^\eta + b_0 + \frac{1}{4}b_0^2 e^{-\eta}}. \tag{16}$$

where b_0 is nonzero parameter and $\eta = \gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}$.

By simplifying Eq. (16), the new travelling wave solutions of conformable time fractional NNV system read as

$$\begin{aligned} u(x, y, t) &= \frac{\beta\gamma (8b_0 - (4 + b_0^2))}{A} \\ &\cdot \frac{\cosh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)}{A} \\ &+ \frac{(-4 + b_0^2) \sinh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)}{A}, \end{aligned}$$

where $A = 3\left(4b_0 + (4 + b_0^2) \cosh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right) - (-4 + b_0^2) \sinh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)\right)$,

$$\begin{aligned} v(x, y, t) &= \frac{\gamma^2 (8b_0 + (4b_0^2))}{B} \\ &\cdot \frac{\cosh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)}{B} \\ &+ \frac{(-4 + b_0^2) \sinh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)}{B}, \end{aligned}$$

where $B = 3\left(4b_0 + (4 + b_0^2) \cosh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right) - (-4 + b_0^2) \sinh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)\right)$,

$$\begin{aligned} w(x, y, t) &= \frac{\beta^2 (8b_0 + (4b_0^2))}{C} \\ &\cdot \frac{\cosh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)}{C} \\ &+ \frac{(-4 + b_0^2) \sinh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)}{C}, \end{aligned}$$

where $C = 3\left(4b_0 + (4 + b_0^2) \cosh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right) - (-4 + b_0^2) \sinh\left(\gamma x + \beta y - (\gamma^3 + \beta^3)\frac{t^\alpha}{\alpha}\right)\right)$.

4 New exact solutions of Klein-Gordon equation

Now, we discuss the following equation with quadratic nonlinearity, namely

$$\frac{\partial^\alpha}{\partial t^\alpha} \frac{\partial^\alpha u}{\partial t^\alpha} - \frac{\partial^2 u}{\partial x^2} + \beta u - \sigma u^2 = 0 \tag{17}$$

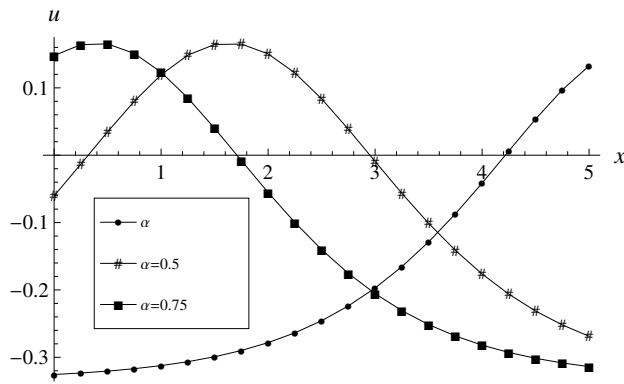


Figure 1: Graphic of $u(x, y, t)$ for $t = 0.5, \beta = 1, \gamma = 1, b_0 = 1, y = 0.5$ and various values of α

where $\frac{\partial^\alpha}{\partial t^\alpha}$ denotes the conformable derivative. Introducing $\eta = \gamma x + m \frac{t^\alpha}{\alpha}$, we can convert Eq. (17) into an ODE as

$$(m^2 - \gamma^2)u'' + \beta u - \sigma u^2 = 0 \tag{18}$$

by using the chain rule [21].

Assume that (18) admits a solution which can be written as

$$u(\eta) = \frac{a_c e^{c\eta} + \dots + a_{-d} e^{-d\eta}}{b_p e^{p\eta} + \dots + a_{-q} e^{-q\eta}}. \tag{19}$$

To obtain c and p we use the Eq. (18) and the ansatz (19). Thus, we get the following

$$u'' = \frac{l_1 e^{(c+3p)\eta} + \dots}{l_2 e^{4p\eta} + \dots} \tag{20}$$

and

$$u^2 = \frac{l_3 e^{2c\eta} + \dots}{l_4 e^{2p\eta} + \dots} = \frac{l_3 e^{(2c+2p)\eta} + \dots}{l_4 e^{4p\eta} + \dots} \tag{21}$$

where l_i denotes the coefficients used for convenience. As before utilizing (20) and (21), we obtain $c + 3p = 2c + 2p$, which implies $c = p$. In the same way, to examine the values of d and q , let us balancing the lowest order terms u'' and u^2 in Eq. (18). Again by utilizing ansatz (19), we conclude

$$u'' = \dots + \frac{s_1 e^{-(d+3q)\eta} + \dots}{s_2 e^{-4q\eta}} \tag{22}$$

and

$$u^2 = \dots + \frac{s_3 e^{-2d\eta} + \dots}{s_4 e^{-2q\eta}} = \frac{s_3 e^{-(2d+2q)\eta} + \dots}{s_4 e^{-4q\eta}} \tag{23}$$

where s_i are coefficients used for easiness. Balancing the lowest order of Exp-funciton in Eqs. (22) and (23) we conclude $-d - 3q = -2d - 2q$, therefore $d = q$.

In the view of the obtained results, we can freely set the values of c and d . Lets determine the values as $c = p = 1$

and $d = q = 1$. For these determined values Eq. (19) can be expressed as

$$u(\eta) = \frac{a_1 e^\eta + a_0 + a_{-1} e^{-\eta}}{b_1 e^\eta + b_0 + b_{-1} e^{-\eta}}. \tag{24}$$

Subrogating (18) and equating the coefficients of $e^{n\eta}$ there occurs a system of algebraic equations. By inspection we conclude that the solution of this system reads as

$$m = \sqrt{\gamma^2 + \beta}, \quad a_1 = \frac{b_0^2 \beta}{4b_{-1} \sigma}, \quad a_0 = -\frac{2b_0 \beta}{\sigma}, \\ a_{-1} = \frac{b_2 \beta}{\sigma}, \quad b_1 = \frac{b_0^2}{b_{-1}}, \quad b_{-1} = b_{-1}$$

Substituting the values which are obtained above into Eq. (24) and simplifying (17) we report that

$$u(x, t) = \frac{\frac{b_0^2 \beta}{4b_{-1} \sigma} e^{\gamma x + m \frac{t^\alpha}{\alpha}} + \frac{2b_0 \beta}{\sigma} + \frac{b_2 \beta}{\sigma} e^{-(\gamma x + m \frac{t^\alpha}{\alpha})}}{\frac{b_0^2}{b_{-1}} e^\eta + b_0 + b_{-1} e^{-(\gamma x + m \frac{t^\alpha}{\alpha})}}. \tag{25}$$

By setting $b_0 = 2$ and $b_{-1} = 1$, the solution can be obtained as

$$u(x, t) = \frac{\beta}{\sigma} - \frac{3\beta}{\sigma \left(1 + \text{Cosh} \left(x\gamma + \frac{t^\alpha \sqrt{\beta + \gamma^2}}{\alpha} \right) \right)}$$

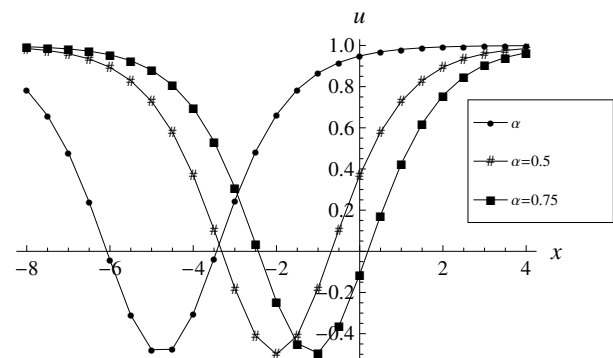


Figure 2: Graphic of $u(x, t)$ for $t = 0.5, \beta = 1, \gamma = 1, \sigma = 1$ and various values of α

5 Conclusion

We have employed the exp-function method to get the travelling wave solutions of the Nizhnik-Novikov-Veselov and Klein-Gordon equations. As a consequence it is understood that the exp-function method is a very useful and powerful mathematical tool for finding the travelling wave solutions of the conformable NPDEs.

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