NEW METHOD FOR INVESTIGATING THE DENSITY-DEPENDENT DIFFUSION NAGUMO EQUATION

by

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We apply reproducing kernel method to the density-dependent diffusion Nagumo equation. Powerful method has been applied by reproducing kernel functions. The approximations to the exact solution are obtained. In particular, series solutions are obtained. These solutions demonstrate the certainty of the method. The results acquired in this work conceive many attracted behaviors that assure further work on the Nagumo equation.

Key words: reproducing kernel functions, Nagumo equation

Introduction

Nagumo equation is presented as [1]:

$$\frac{\partial v}{\partial t}=\frac{\partial}{\partial x}\left(v^m\frac{\partial v}{\partial x}\right)+v(1-v)(v-\alpha),\quad \alpha\in\mathbb{R}, m\geq 1 \tag{1}$$

Utilizing the subalgebra $L_{1,1}$ we get the analogue variables and solutions $v(t,x)=V(\tau)$, $\tau=x-ct$. The reduced ODE is obtained:

$$\left[V(\tau)^m V'(\tau)\right]'+cV'(\tau)+V(\tau)[1-V(\tau)][V(\tau)-\alpha]=0 \tag{2}$$

Solutions of eq. (2) are traveling wave solutions of eq. (1). The natural conditions are given:

$$\lim_{\tau\to-\infty}V(\tau)=A,\quad \lim_{\tau\to\infty}V(\tau)=B \tag{3}$$

where $A,B\in\{0,1,\alpha\}$ are considered for investigating eq. (2). The initial conditions $V(0)=0.5$ and $V'(0)=\lambda,\quad (\lambda\in\mathbb{R})$ and particular $\lambda=0$, which are defined by the implementation at hand are used in our calculations.

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In the mathematics area, many problems have exact solutions and these models play the role of test problems to investigate the reliability and power of approximation methods. On the other hand, some applicable and significant models in physics and engineering have not exact solutions and we have to utilize just the powerful methods with strong mathematical background. The reproducing kernel method (RKM) is one of the most reliable method which can be analyzed it very deeply for many problems without exact solutions. To the best of our knowledge, the initial value problem of eq. (2) has not exact solution and is very important from the application side.

Reproducing kernel theory has valuable applications in integral equations, differential equations, probability, and statistics. This theory is implemented for many model problems in recent years. We use RKM to search the density-dependent diffusion Nagumo equation in this work.

The notion of reproducing kernel has been presented by Zaremba [2]. Mercer has introduced the following inequality [3]:

$$\sum_{p,q=1}^{n} m(x_p, t_q)\xi_i \xi_j \geq 0$$

He presented the reproducibility of the kernel:

$$v(t) = \langle v(x), m(x,t) \rangle$$

Aronszajn [4] reduced the studies of the formers and presented a systematic reproducing kernel theory containing the Bergman kernel function. For more details see [5-13].

Reproducing kernel functions

Definition 1. $W_2^1[0,1]$ is given:

$$W_2^1[0,1] = \{ f \in AC[0,1] : f' \in L^2[0,1] \}$$

where AC shows the space of absolutely continuous functions.

$$\langle f, g \rangle_{W_2^1} = \int_0^1 [f(\eta)g(\eta) + f'(\eta)g'(\eta)]\,\mathrm{d}\eta, \quad f, g \in W_2^1[0,1] \tag{4}$$

and

$$\|f\|_{W_2^1} = \sqrt{\langle f, f \rangle_{W_2^1}}, \quad f \in W_2^1[0,1] \tag{5}$$

are the inner product and the norm in $W_2^1[0,1]$, respectively. Reproducing kernel function $T_\eta(\varsigma)$ of $W_2^1[0,1]$ is given [3]:

$$T_\eta(\varsigma) = \frac{1}{2\sinh(1)}[\cosh(\eta + \varsigma - 1) + \cosh(|\eta - \varsigma| - 1)] \tag{6}$$

Definition 2. We describe the space ${}^oW_2^3[0,1]$ by:

$${}^oW_2^3[0,1] = \{ f \in AC[0,1] : f', f'' \in AC[0,1], f^{(3)} \in L^2[0,1], f(0) = 0 = f'(0) \}$$

$$\langle f, v \rangle_{{}^oW_2^3} = \sum_{i=0}^{2} f^{(i)}(0)v^{(i)}(0) + \int_0^1 f^{(3)}(\eta)v^{(3)}(\eta)\,\mathrm{d}\eta, \quad f, v \in {}^oW_2^3[0,1]$$

and

$$\|f\|_{{}^{o}W_2^3} = \sqrt{\langle f, f\rangle_{{}^{o}W_2^3}}, \quad f \in {}^{o}W_2^3[0,1]$$

are the inner product and the norm in ${}^{o}W_2^3[0,1]$, respectively.

Theorem 3. Reproducing kernel function r_ς of ${}^{o}W_2^3[0,1]$ is given:

$$r_\varsigma(\eta) = \begin{cases} \sum_{i=0}^{5} c_{i+1}(\varsigma)\eta^i, & 0 \le \eta < \varsigma \le 1 \\ \sum_{i=0}^{5} d_{i+1}(\varsigma)\eta^i, & 0 \le \varsigma < \eta \le 1 \end{cases} \tag{7}$$

where

$$c_1(\varsigma) = 0, \quad c_2(\varsigma) = 0, \quad c_3(\varsigma) = \frac{1}{4}\varsigma^2, \quad c_4(\varsigma) = \frac{1}{12}\varsigma^2$$

$$c_5(\varsigma) = -\frac{1}{24}\varsigma, \quad c_6(\varsigma) = \frac{1}{120}$$

$$d_1(\varsigma) = \frac{1}{120}\varsigma^5, \quad d_2(\varsigma) = -\frac{1}{24}\varsigma^4$$

$$d_3(\varsigma) = \frac{1}{12}\varsigma^3 + \frac{1}{4}\varsigma^2$$

$$d_4(\varsigma) = 0, \quad d_5(\varsigma) = 0, \quad d_6(\varsigma) = 0$$

Proof. Let $f \in {}^{o}W_2^3[0,1]$ and $0 \le \varsigma \le 1$. Define r_ς by eq. (7). We have:

$$r'_\varsigma(\eta) = \begin{cases} \sum_{i=0}^{4} (i+1)c_{i+1}(\varsigma)\eta^i, & 0 \le \eta < \varsigma \le 1 \\ \sum_{i=0}^{4} (i+1)d_{i+1}(\varsigma)\eta^i, & 0 \le \varsigma < \eta \le 1 \end{cases}$$

$$r''_\varsigma(\eta) = \begin{cases} \sum_{i=0}^{3} (i+1)(i+2)c_{i+2}(\varsigma)\eta^i, & 0 \le \eta < \varsigma \le 1 \\ \sum_{i=0}^{3} (i+1)(i+2)d_{i+2}(\varsigma)\eta^i, & 0 \le \varsigma < \eta \le 1 \end{cases}$$

$$r^{(3)}_\varsigma(\eta) = \begin{cases} \sum_{i=0}^{2} (i+1)(i+2)(i+3)c_{i+3}(\varsigma)\eta^i, & 0 \le \eta < \varsigma \le 1 \\ \sum_{i=0}^{2} (i+1)(i+2)(i+3)d_{i+3}(\varsigma)\eta^i, & 0 \le \varsigma < \eta \le 1 \end{cases}$$

$$r^{(4)}_\varsigma(\eta) = \begin{cases} \sum_{i=0}^{1} (i+1)(i+2)(i+3)(i+4)c_{i+4}(\varsigma)\eta^i, & 0 \le \eta < \varsigma \le 1 \\ \sum_{i=0}^{1} (i+1)(i+2)(i+3)(i+4)d_{i+4}(\varsigma)\eta^i, & 0 \le \varsigma < \eta \le 1 \end{cases}$$

and

$$r_{\varsigma}^{(5)}(\eta)=\begin{cases}120c_5(\varsigma), & 0\le\eta<\varsigma\le 1\\ 120d_5(\varsigma), & 0\le\varsigma<\eta\le 1\end{cases}$$

We get:

$$\begin{gathered}\left\langle f,r_{\varsigma}\right\rangle_{{}^{o}W_2^3}=\sum_{i=0}^{2}f^{(i)}(0)r_{\varsigma}^{(i)}(0)+\int_0^1 f^{(3)}(\eta)r_{\varsigma}^{(3)}(\eta)\mathrm{d}\eta=\\ =f'(0)r_{\varsigma}'(0)+f''(0)r_{\varsigma}''(0)+f''(1)r_{\varsigma}^{(3)}(1)-f''(0)r_{\varsigma}^{(3)}(0)-\\ -f'(1)r_{\varsigma}^{(4)}(1)+f'(0)r_{\varsigma}^{(4)}(0)+\int_0^1 f'(\eta)r_{\varsigma}^{(5)}(\eta)\mathrm{d}\eta=\\ =c_1(\varsigma)f'(0)+2c_2(\varsigma)f''(0)+\\ +6[d_3(\varsigma)+4d_4(\varsigma)+10d_5(\varsigma)]f''(1)-6c_3(\varsigma)f''(0)-\\ -24[d_4(\varsigma)+5d_5(\varsigma)]f'(1)+24c_4(\varsigma)f'(0)+\\ +\int_0^{\varsigma}120c_5(\varsigma)f'(\eta)\mathrm{d}\eta+\int_{\varsigma}^{1}120d_5(\varsigma)f'(\eta)\mathrm{d}\eta=\\ =[c_1(\varsigma)+24c_4(\varsigma)]f'(0)+2[c_2(\varsigma)-3c_3(\varsigma)]f''(0)+\\ +6[d_3(\varsigma)+4d_4(\varsigma)+10d_5(\varsigma)]f''(1)-24[d_4(\varsigma)+5d_5(\varsigma)]f'(1)+\\ +120[c_5(\varsigma)-d_5(\varsigma)]f(\varsigma)=\\ =f(\varsigma)\end{gathered}$$

Solutions in ${}^{o}W_2^3[0,1]$

The solution of eq. (1) is investigated in the ${}^{o}W_2^3[0,1]$ in this section. We define:

$$A:{}^{o}W_2^3[0,1]\to W_2^1[0,1]$$

as

$$Af(\eta)=0.5f''(\eta)+cf'(\eta)+0.25f(\eta) \tag{8}$$

model problem of eq. (2) changes to the following problem:

$$\begin{cases}Af=M(\eta,f), & \eta\in[0,1]\\ f(0)=0=f'(0)\end{cases} \tag{9}$$

Theorem 4. A is a bounded linear operator.

Proof. We will show $\|Lf\|_{W_2^1}^2\le M\|f\|_{{}^{o}W_2^3}^2$, where $P>0$. By eqs. (4) and (5), we obtain:

$$\|Af\|_{W_2^1}^2=\langle Af,Af\rangle_{W_2^1}=\int_0^1\left[Af(\eta)^2+Af'(\eta)^2\right]\mathrm{d}\eta$$

We get

$$f(\eta)=\left\langle f(\cdot),r_{\eta}(\cdot)\right\rangle_{{}^{o}W_2^3}$$

by reproducing property and

$$Af(\eta)=\left\langle f(\cdot),Ar_{\eta}(\cdot)\right\rangle_{{}^{o}W_2^3}$$

so

$$\left|Af(\eta)\right|\le\left\|f\right\|_{{}^{o}W_2^3}\left\|Ar_{\eta}\right\|_{{}^{o}W_2^3}=P_1\left\|u\right\|_{{}^{o}W_2^3}$$

where $P_1>0$. Therefore, we get:

$$\int_0^1\left[\left(Af\right)(\eta)\right]^2\mathrm{d}\eta\le P_1^2\left\|u\right\|_{{}^{o}W_2^3}^2$$

Since

$$(Af)'(\eta)=\left\langle f(\cdot),(Lr_{\eta})'(\cdot)\right\rangle_{{}^{o}W_2^3}$$

then

$$\left|(Af)'(\eta)\right|\le\left\|f\right\|_{{}^{o}W_2^3}\left\|(Ar_{\eta})'\right\|_{{}^{o}W_2^3}=P_2\left\|f\right\|_{{}^{o}W_2^3}$$

where $P_2>0$. Therefore, we have:

$$\left[(Af)'(\tau)\right]^2\le P_2^2\left\|f\right\|_{{}^{o}W_2^3}^2$$

and

$$\int_0^1\left[(Af)'(\eta)\right]^2\mathrm{d}\eta\le P_2^2\left\|f\right\|_{{}^{o}W_2^3}^2$$

that is

$$\left\|Af\right\|_{W_2^1}^2\le\int_0^1\left\{\left[(Af)(\eta)\right]^2+\left[(Af)'(\eta)\right]^2\right\}\mathrm{d}\eta\le\left(P_1^2+P_2^2\right)\left\|f\right\|_{{}^{o}W_2^3}^2=P\left\|f\right\|_{{}^{o}W_2^3}^2$$

where $P=P_1^2+P_2^2>0$ is a positive constant.

The main results

Let $\phi_i(\eta)=T_{\eta_i}(\eta)$ and $\psi_i(\eta)=A^*\phi_i(x)$, A^* is adjoint operator of L. The orthonormal system $\{\overline{\Psi}_i(\eta)\}_{i=1}^{\infty}$ of ${}^{o}W_2^3[0,1]$ can be achieved:

$$\overline{\psi}_i(\eta)=\sum_{k=1}^{i}\beta_{ik}\psi_k(\eta),\quad(\beta_{ii}>0,\quad i=1,2,\ldots)\tag{10}$$

Theorem 5. Let $\{\eta_i\}_{i=1}^{\infty}$ be dense in [0,1] and $\psi_i(\eta)=A_{\varsigma}r_{\eta}(\varsigma)\big|_{\varsigma=\eta_i}$. Then the sequence $\{\psi_i(\eta)\}_{i=1}^{\infty}$ is a complete system in ${}^{o}W_2^3[0,1]$.

Proof. By reproducing property and property of the operator we get:

$$\psi_i(\eta)=(A^*\phi_i)(\eta)=\left\langle(A^*\phi_i)(\varsigma),r_{\eta}(\varsigma)\right\rangle=\left\langle\phi_i)(\varsigma),A\varsigma r_{\eta}(\varsigma)\right\rangle=A_{\varsigma}r_{\eta}(\varsigma)\big|_{\varsigma=\eta_i}$$

It is clear that $\psi_i(\eta)\in{}^{o}W_2^3[0,1]$. For each fixed $f(\eta)\in{}^{o}W_2^3[0,1]$, let $\left\langle f(\eta),\psi_i(\eta)\right\rangle=0$, $(i=1,2,\ldots)$:

$$\left\langle f(\eta),(A^*\phi_i)(\eta)\right\rangle=\left\langle Af(\cdot),\phi_i(\cdot)\right\rangle=(Af)(\eta_i)=0$$

where $\{\eta_i\}_{i=1}^{\infty}$ is dense in $[0,1]$. Therefore, $(Af)(\eta)=0$. $u\equiv 0$ by the A^{-1}.

Theorem 6. If $f(\eta)$ is the exact solution of eq. (9), then:

$$f=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}M(\eta_k,f_k)\hat{\Psi}_i(\eta) \tag{11}$$

where $\{(\eta_i)\}_{i=1}^{\infty}$ is dense in [0,1].

Proof. We get:

$$f(\eta)=\sum_{i=1}^{\infty}\left\langle f(\eta),\hat{\Psi}_i(\eta)\right\rangle_{{}^{o}W_2^3}\hat{\Psi}_i(\eta)=$$

$$=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}\left\langle f(\eta),\Psi_k(\eta)\right\rangle_{{}^{o}W_2^3}\hat{\Psi}_i(\eta)=$$

$$=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}\left\langle f(\eta),A^{*}\phi_k(\eta)\right\rangle_{{}^{o}W_2^3}\hat{\Psi}_i(\eta)=$$

$$=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}\left\langle Af(\eta),\phi_k(\eta)\right\rangle_{W_2^1}\hat{\Psi}_i(\eta)=$$

$$=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}Af(\eta_k)\hat{\Psi}_i(\eta)=$$

$$=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}M(\eta_k,f_k)\hat{\Psi}_i(\eta)$$

from the eq. (10) and uniqueness of solution of eq. (9).

The approximate solution f_n can be achieved:

$$f_n=\sum_{i=1}^{n}\sum_{k=1}^{i}\beta_{ik}M(\eta_k,f_k)\hat{\Psi}_i(\eta) \tag{12}$$

Lemma 7. If $\|f_n-f\|_{{}^{o}W_2^3}\to 0$, $\eta_n\to\eta$, $(n\to\infty)$ and $M(\eta,f)$ is continuous for $\eta\in[0,1]$, then [3]:

$$M[\eta_n,f_{n-1}(\eta_n)]\to M[\eta,f(\eta)] \quad \text{as} \quad n\to\infty$$

Theorem 8. For any fixed $f_0(\eta)\in {}^{o}W_2^3[0,1]$ assume that the following conditions are hold:

– $$f_n(\eta)=\sum_{i=1}^{n}B_i\overline{\psi}_i(\eta) \tag{13}$$

$$B_i=\sum_{k=1}^{i}\beta_{ik}M[\eta_k,f_{k-1}(\eta_k)] \tag{14}$$

– $\|f_n\|_{{}^{o}W_2^3}$ is bounded
– $\{\eta_i\}_{i=1}^{\infty}$ is dense in $[0,1]$,
– $M(\eta,f)\in W_2^1[0,1]$ for any $f(\eta)\in{}^{o}W_2^3[0,1]$.

Then $f_n(\eta)$ in eq. (14) converges to the exact solution of eq. (11) in ${}^oW_2^3[0,1]$ and

$$f(\eta)=\sum_{i=1}^{\infty}B_i\overline{\psi}_i(\eta),$$

where B_i is given by eq. (14).

Proof. We demonstrate the convergence of $f_n(\eta)$ firstly. By eq. (13), we obtain:

$$f_{n+1}(\eta)=f_n(\eta)+B_{n+1}\hat{\Psi}_{n+1}(\eta) \tag{15}$$

from the orthonormality of $\{\hat{\Psi}_i\}_{i=1}^{\infty}$, we acquire:

$$\|f_{n+1}\|^2=\|f_n\|^2+B_{n+1}^2=\|f_{n-1}\|^2+B_n^2+A_{n+1}^2=\ldots=\sum_{i=1}^{n+1}B_i^2 \tag{16}$$

from boundedness of $\|f_n\|_{{}^oW_2^3}$, we get:

$$\sum_{i=1}^{\infty}B_i^2<\infty$$

i. e.,

$$\{B_i\}\in l^2,\quad (i=1,2,\ldots)$$

Let $m>n$, by $(f_m-f_{m-1})\perp(f_{m-1}-f_{m-2})\perp\ldots\perp(f_{n+1}-f_n)$, we acquire:

$$\|f_m-f_n\|_{{}^oW_2^3}^2=\|f_m-f_{m-1}+f_{m-1}-f_{m-2}+\ldots+f_{n+1}-f_n\|_{{}^oW_2^3}^2\le$$

$$\le\|f_m-f_{m-1}\|^2+\ldots+\|f_{n+1}-f_n\|_{{}^oW_2^3}^2=$$

$$=\sum_{i=n+1}^{m}B_i^2\to 0,\quad m,n\to\infty,$$

where $\perp$ denotes the orthogonality. Taking into consideration the completeness of ${}^oW_2^3[0,1]$, there exists $f(\eta)\in{}^oW_2^3[0,1]$, such that:

$$f_n(\eta)\to f(\eta)\quad\text{as}\quad n\to\infty$$

– Taking limits in eq. (10), gives:

$$f(\eta)=\sum_{i=1}^{\infty}B_i\overline{\psi}_i(\eta)$$

Since

$$(Af)(\eta_j)=\sum_{i=1}^{\infty}B_i\left\langle A\overline{\psi}_i(\eta),\phi_j(\eta)\right\rangle_{W_2^1}=\sum_{i=1}^{\infty}B_i\left\langle\overline{\psi}_i(\eta),A^*\phi_j(\eta)\right\rangle_{{}^oW_2^3}=\sum_{i=1}^{\infty}B_i\left\langle\overline{\psi}_i(\eta),\overline{\psi}_j(\eta)\right\rangle_{{}^oW_2^3}$$

we get:

$$\sum_{j=1}^{n}\beta_{nj}(Af)(\eta_j)=\sum_{i=1}^{\infty}B_i\left\langle\overline{\psi}_i(\eta),\sum_{j=1}^{n}\beta_{nj}\overline{\psi}_j(\eta)\right\rangle_{{}^oW_2^3}=\sum_{i=1}^{\infty}B_i\left\langle\overline{\psi}_i(\eta),\overline{\psi}_n(\eta)\right\rangle_{{}^oW_2^3}=B_n$$

If $n=1$, then:

$$Af(\eta_1)=M[\eta_1,f_0(\eta_1)] \tag{17}$$

If $n=2$, then:

$$\beta_{21}(Af)(\eta_1)+\beta_{22}(Af)(\eta_2)=\beta_{21}M[\eta_1,f_0(\eta_1)]+\beta_{22}M[\eta_2,f_1(\eta_2)] \tag{18}$$

From eqs. (17) and (18):

$$(Af)(\eta_2)=M[\eta_2,f_1(\eta_2)]$$

Additionally, it is simple to show by induction that:

$$(Af)(\eta_j)=M[\eta_j,f_{j-1}(\eta_j)] \tag{19}$$

Therefore, we get:

$$(Af)(\varsigma)=M[\varsigma,f(\varsigma)]$$

that is, $f(\eta)$ is the solution of eq. (9) and:

$$f(\eta)=\sum_{i=1}^{\infty}B_i\overline{\psi}_i$$

where B_i are given by eq. (14). This completes the proof.

Theorem 9. If $f\in {}^{o}W_2^3[0,1]$ then:

$$\|f_n-f\|_{{}^{o}W_2^3}\to 0,\quad n\to\infty$$

Additionally a sequence $\|f_n-f\|_{{}^{o}W_2^3}$ is monotonically decreasing in n.

Proof. We acquire:

$$\|f_n-f\|_{{}^{o}W_2^3}=\left\|\sum_{i=n+1}^{\infty}\sum_{k=1}^{i}\beta_{ik}f(\eta_k,f_k,)\hat{\Psi}_i\right\|_{{}^{o}W_2^3}$$

by eqs. (11) and (12). Thus, we get:

$$\|f_n-f\|_{{}^{o}W_2^3}\to 0,\quad n\to\infty$$

$$\|f_n-f\|_{{}^{o}W_2^3}^2=\left\|\sum_{i=n+1}^{\infty}\sum_{k=1}^{i}\beta_{ik}f(\eta_k,f_k)\hat{\Psi}_i\right\|_{{}^{o}W_2^3}^2=\sum_{i=n+1}^{\infty}\left[\sum_{k=1}^{i}\beta_{ik}M(\eta_k,f_k)\hat{\Psi}_i\right]^2$$

Obviously, $\|f_n-f\|_{{}^{o}W_2^3}$ is monotonically decreasing in n. Now, we are ready to show the effectiveness and accuracy of the presented technique in this section. We found approximate solutions of the Nagumo equation. Approximate solutions are given in tabs. 1-4. We calculated all our results by MAPLE 16. We used:

$$x_i=\frac{i}{m},\quad i=1,2,3,\cdots,m,$$

for our numerical results.

Table 1. Approximate solutions of $V(\tau)$ when $m = 1$, $\alpha = 0.6$, $V(0) = 0.5$, $V'(0) = 0$ for varying c

x/*c*	4	2	1	0.8	0.6
0	0.500000	0.500000	0.500000	0.500000	0.500000
5	0.524629	0.542982	0.568164	0.577177	0.588971
10	0.544890	0.569795	0.592220	0.596909	0.600339
15	0.559630	0.583898	0.598078	0.599581	0.600019
20	0.570374	0.591380	0.599523	0.599943	0.599998
25	0.578224	0.595373	0.599881	0.599992	0.599999
30	0.583973	0.597513	0.599970	0.599998	0.600000
35	0.588109	0.598569	0.599982	0.599999	0.600000
40	0.590711	0.598650	0.599936	0.599994	0.599999
45	0.590711	0.597755	0.599831	0.599986	0.599999

Table 2. Approximate solutions of $V(\tau)$ when $m = 1$, $\alpha = 0.2$, $V(0) = 0.5$, $V'(0) = 0$ for varying c

x/*c*	0.4	0.6	1.0	2.0
0	0.500000	0.500000	0.500000	0.500000
5	0.105788	0.207712	0.290385	0.370545
10	0.156800	0.197117	0.229971	0.295967
15	0.200621	0.199541	0.211500	0.257317
20	0.200577	0.199935	0.204636	0.235579
25	0.199916	0.199991	0.201905	0.222629
30	0.200000	0.199999	0.200788	0.214616
35	0.199972	0.199998	0.200404	0.209759
40	0.199836	0.199991	0.200641	0.207804

Table 3. Approximate solutions of $V(\tau)$ when $m = 1$, $c = 0.3$, $V(0) = 0.5$, $V'(0) = 0$ for varying α

x/*α*	0.3	0.4	0.5	0.6	0.7	0.8
0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5	0.211963	0.369630	0.500000	0.615833	0.720430	0.812370
10	0.303934	0.405871	0.500000	0.596365	0.697911	0.809419
15	0.300229	0.398670	0.500000	0.600775	0.699829	0.796569
20	0.299599	0.400288	0.500000	0.599832	0.700185	0.800677
25	0.300160	0.399936	0.500000	0.600035	0.699926	0.799949

Table 4. Approximate solutions of $V(\tau)$ when $m = 1, c = 1.0, V(0) = 0.5, V'(0) = 0$ for varying α

x/*α*	0.3	0.4	0.5	0.6	0.7	0.8
0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5	0.359637	0.430018	0.500000	0.568164	0.633221	0.694019
10	0.316199	0.407192	0.500000	0.592220	0.681381	0.764472
15	0.304763	0.401768	0.500000	0.598078	0.694499	0.786410
20	0.301433	0.400437	0.500000	0.599523	0.698343	0.794531
25	0.300434	0.400108	0.500000	0.599881	0.699497	

Conclusion

We discussed the RKM for investigating the the density-dependent diffusion Nagumo equation in this paper. An example was chosen to present the computational accuracy. As shown in tabs. 1-4 this method is very accurate. We obtained some significant reproducing kernel functions in this work. We proved many useful theorems in the paper.

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