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New analytical wave structures for the (3 + 1)-dimensional Kadomtsev-Petviashvili and the generalized Boussinesq models and their applications

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ABSTRACT

Different types of soliton wave solutions for the (3 + 1)-dimensional Kadomtsev-Petviashvili and the generalized Boussinesq equations are investigated via the solitary wave ansatz method. These solutions are classified into three categories, namely solitary wave, shock wave, and singular wave solutions. The corresponding integrability criteria, termed as constraint conditions, obviously arise from the study. Moreover, the influences of the free parameters and interaction properties in these solutions are discussed graphically for physical interests and possible applications.

Introduction

Traveling wave solutions of the nonlinear evolution equations (NLEEs) are of utmost important through the wave phenomena since they act as a bridge between mathematics and its applications in different branches of science [1–13].

In the last decay, soliton wave solutions and its characteristics have been investigated and applied in many fields, such as ocean engineering [14,15], optical fibers [16,17], materials [18,19], fluid dynamics [20], and so on. This kind of wave solutions has various forms such as solitary waves, shock waves, singular waves, cnoidal waves, snoidal waves, cuspons, and peakons.

The most appropriate way to comprehend the dynamics of the NLEEs is to find their exact solutions [21–32]. Different approaches are used in literature for calculating the exact solutions for the NLEEs. Among these method; the improved fractional sub-equation method [33,34], Kudryashov method and its extended form [35–38], the unified method [39–41] and its generalized scheme [42–47], the homotopy perturbation method [48,49], and the new extended trial equation method [50,51].

The main purpose of this paper is to find the solitary wave (which is sufficiently short in duration and locally irregular given in space disturbances), shock wave (it is a type of propagating disturbance that moves faster than the other waves in the medium), and singular wave (this is a type of traveling wave solutions has blow up phenomenon) solutions for the (3 + 1)-dimensional Kadomtsev-Petviashvili [52–54] and the generalized Boussinesq [54,55] equations via the solitary wave ansatz method [56,57].

The governing equations are:

The (3 + 1)-dimensional Kadomtsev-Petviashvili equation (3D-KPE) [52–54]

The 3D-KPE was first introduced in 1970 by Boris B. Kadomtsev and Vladimir I. Petviashvili [58]. The 3D-KPE describes the water waves of long wavelength with weakly nonlinear restoring forces, waves in ferromagnetic media, and two-dimensional matter-wave pulses in Bose-Einstein condensates. Due to its significance, it have been studied extensively in the literature [52–54].

$$u_{tx} + \nu_1(uu_x)_x + \nu_2 u_{xxxx} + \nu_3 u_{yy} + \nu_4 u_{zz} = 0, \tag{1}$$

where $u = u(x, y, z, t)$ is a real valued function in its arguments and the coefficients $\nu_1 = 6, \nu_2 = 1,$ and $\nu_3 = \nu_4 = \pm 3$. The coefficients $\nu_3 = \nu_4 = \pm 3$ are used for weak surface tension and strong surface tension, respectively [59–61].

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The generalized Boussinesq equation (GBE) [54,55]

The GBE was used in coastal engineering as a computer model for the simulation of water waves in shallow seas and seaports [62]. Furthermore, the GBE arises in the study of water waves [63], anharmonic lattice waves [64], and dense lattices [65].

$$v_{tt} - \delta_1 (v^2)_{xx} - \delta_2 v_{xx} - \delta_3 v_{xxxx} - \delta_4 v_{yy} - \delta_5 v_{zz} = 0, \tag{2}$$

where $v = v(x, y, z, t)$ is an elevation on the free surface of fluid, the coefficients δ_1, δ_2, δ_3, δ_4, and δ_5 are real constants depend on the depth of fluid and characteristic speed of long waves.

This article has been arranged as follows: in Section "Solution to the 3D-KPE" and "Solution to the GBE", various soliton solutions for the above two equations have been investigated and the properties for these solutions are described with some figures. In Section "Conclusions", the conclusions have been drawn.

Solution to the 3D-KPE

In this section, the solitary wave, shock wave and singular wave solutions for (1) are calculated.

Solitary wave solutions

In order to calculate the solitary wave solution, suppose

$$u(x, y, z, t) = \frac{A}{\cosh^\lambda \psi} \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - vt, \tag{3}$$

where α, β, γ are the inverse widths, A is the amplitude and v is the velocity of the solitary wave, λ is a constant to be determined later. By using (3)

$$u_{tx} = \frac{A\lambda(\lambda+1)\alpha v}{\cosh^{\lambda+2}\psi} - \frac{A\lambda^2 v}{\cosh^\lambda \psi},$$
$$(uu_x)_x = \frac{2A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi} - \frac{A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi},$$
$$u_{xxxx} = \frac{A\lambda^4\alpha^4}{\cosh^\lambda\psi} - \frac{2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4}{\cosh^{\lambda+2}\psi} + \frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4}{\cosh^{\lambda+4}\psi},$$
$$u_{yy} = \frac{A\lambda^2\beta^2}{\cosh^\lambda\psi} - \frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi},$$
$$u_{zz} = \frac{A\lambda^2\gamma^2}{\cosh^\lambda\psi} - \frac{A\lambda(\lambda+1)\gamma^2}{\cosh^{\lambda+2}\psi},$$

substituting above values into (1)

$$\frac{A\lambda(\lambda+1)\alpha v}{\cosh^{\lambda+2}\psi} - \frac{A\lambda^2 v}{\cosh^\lambda\psi} + v_1\frac{2A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi} - v_1\frac{A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi} - v_2\frac{2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4}{\cosh^{\lambda+2}\psi} + v_2\frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4}{\cosh^{\lambda+4}\psi} + v_2\frac{A\lambda^4\alpha^4}{\cosh^\lambda\psi} + v_3\frac{A\lambda^2\beta^2}{\cosh^\lambda\psi} - v_3\frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi} + v_4\frac{A\lambda^2\gamma^2}{\cosh^\lambda\psi} - v_4\frac{A\lambda(\lambda+1)\gamma^2}{\cosh^{\lambda+2}\psi} = 0.$$

By comparing the powers 2λ, $\lambda + 2$ and $\lambda + 4$, $2\lambda + 2$

$$A\lambda(\lambda+1)\alpha v + 2v_1A^2\lambda^2\alpha^2 - 2v_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4 - v_3A\lambda(\lambda+1)\beta^2 - v_4A\lambda(\lambda+1)\gamma^2 = 0,$$
$$-v_1A^2\lambda(2\lambda+1)\alpha^2 + v_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0,$$

set $\lambda = 2$

$$A = \frac{12\alpha^2 v_2}{v_1}, \quad v = \frac{4\alpha^4 v_2 + \beta^2 v_3 + \gamma^2 v_4}{\alpha}.$$

Thus

$$u_1(x, y, z, t) = \frac{A}{\cosh^2(\alpha x + \beta y + \gamma z - vt)}. \tag{4}$$

Shock wave solutions

To calculate the shock wave soliton, suppose

$$u(x, y, z, t) = A\tanh^\lambda\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - vt \quad \text{and} \quad \lambda > 0, \tag{5}$$

from (5)

$$u_{tx} = -A\lambda(\lambda-1)\alpha v\tanh^{\lambda-2}\psi + 2A\lambda^2\alpha v\tanh^\lambda\psi - A\lambda(\lambda+1)\alpha v\tanh^{\lambda+2}\psi,$$
$$(uu_x)_x = A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi - 4A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi + A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi,$$
$$u_{xxxx} = A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\tanh^{\lambda-4}\psi - 4A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^4\tanh^{\lambda-2}\psi - 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\tanh^{\lambda+2}\psi + 2A\lambda^2(3\lambda^2+5)\alpha^4\tanh^\lambda\psi + A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\tanh^{\lambda+4}\psi,$$
$$u_{yy} = A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi - 2A\lambda^2\beta^2\tanh^\lambda\psi + A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi,$$
$$u_{zz} = A\lambda(\lambda-1)\gamma^2\tanh^{\lambda-2}\psi - 2A\lambda^2\gamma^2\tanh^\lambda\psi + A\lambda(\lambda+1)\gamma^2\tanh^{\lambda+2}\psi,$$

substituting above values into (1)

$$-A\lambda(\lambda-1)\alpha v\tanh^{\lambda-2}\psi + 2A\lambda^2\alpha v\tanh^\lambda\psi - A\lambda(\lambda+1)\alpha v\tanh^{\lambda+2}\psi + v_1A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi - 4v_1A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi + v_1A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi + v_2A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\tanh^{\lambda-4}\psi - 4v_2A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^4\tanh^{\lambda-2}\psi - 4v_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\tanh^{\lambda+2}\psi + 2v_2A\lambda^2(3\lambda^2+5)\alpha^4\tanh^\lambda\psi + v_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\tanh^{\lambda+4}\psi + v_3A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi - 2v_3A\lambda^2\beta^2\tanh^\lambda\psi + v_3A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi + v_4A\lambda(\lambda-1)\gamma^2\tanh^{\lambda-2}\psi - 2v_4A\lambda^2\gamma^2\tanh^\lambda\psi + v_4A\lambda(\lambda+1)\gamma^2\tanh^{\lambda+2}\psi = 0.$$

By comparing the powers 2λ, $\lambda + 2$ and $\lambda + 4$, $2\lambda + 2$

$$-A\lambda(\lambda+1)\alpha v - 4v_1A^2\lambda^2\alpha^2 - 4v_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4 + v_3A\lambda(\lambda+1)\beta^2 + v_4A\lambda(\lambda+1)\gamma^2 = 0,$$
$$v_1A^2\lambda(2\lambda+1)\alpha^2 + v_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0,$$

set $\lambda = 2$

$$A = -\frac{12\alpha^2 v_2}{v_1}, \quad v = \frac{-8\alpha^4 v_2 + \beta^2 v_3 + \gamma^2 v_4}{\alpha}.$$

Thus

$$u_2(x, y, z, t) = A\tanh^2(\alpha x + \beta y + \gamma z - vt). \tag{6}$$

Singular wave Form-I

For the singular wave Form I solution, suppose

$$u(x, y, z, t) = A\coth^\lambda\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - vt \quad \text{and} \quad \lambda > 0, \tag{7}$$

from (7)

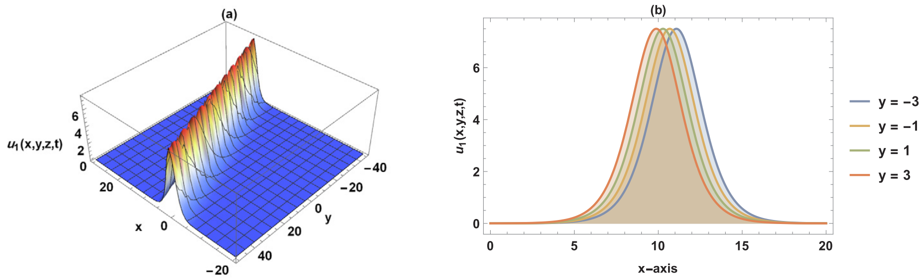

Fig. 1. $u_1(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

$$u_{tx} = -A\lambda(\lambda-1)\alpha\nu\coth^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\coth^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\coth^{\lambda+2}\psi,$$

$$(uu_x)_x = A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi - 4A^2\lambda^2\alpha^2\coth^{2\lambda}\psi + A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi,$$

$$u_{xxxx} = A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\coth^{\lambda-4}\psi - 4A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^4\coth^{\lambda-2}\psi - 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\coth^{\lambda+2}\psi + 2A\lambda^2(3\lambda^2+5)\alpha^4\coth^{\lambda}\psi + A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\coth^{\lambda+4}\psi,$$

$$u_{yy} = A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi - 2A\lambda^2\beta^2\coth^{\lambda}\psi + A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi,$$

$$u_{zz} = A\lambda(\lambda-1)\gamma^2\coth^{\lambda-2}\psi - 2A\lambda^2\gamma^2\coth^{\lambda}\psi + A\lambda(\lambda+1)\gamma^2\coth^{\lambda+2}\psi,$$

substituting above values into (1)

$$-A\lambda(\lambda-1)\alpha\nu\coth^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\coth^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\coth^{\lambda+2}\psi + \nu_1A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi - 4\nu_1A^2\lambda^2\alpha^2\coth^{2\lambda}\psi + \nu_1A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi + \nu_2A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\coth^{\lambda-4}\psi - 4\nu_2A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^4\coth^{\lambda-2}\psi - 4\nu_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\coth^{\lambda+2}\psi + 2\nu_2A\lambda^2(3\lambda^2+5)\alpha^4\coth^{\lambda}\psi + \nu_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\coth^{\lambda+4}\psi + \nu_3A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi - 2\nu_3A\lambda^2\beta^2\coth^{\lambda}\psi + \nu_3A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi + \nu_4A\lambda(\lambda-1)\gamma^2\coth^{\lambda-2}\psi - 2\nu_4A\lambda^2\gamma^2\coth^{\lambda}\psi + \nu_4A\lambda(\lambda+1)\gamma^2\coth^{\lambda+2}\psi = 0.$$

By comparing the powers 2λ, $\lambda+2$ and $\lambda+4$, $2\lambda+2$

$$-A\lambda(\lambda+1)\alpha\nu - 4\nu_1A^2\lambda^2\alpha^2 - 4\nu_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4 + \nu_3A\lambda(\lambda+1)\beta^2 + \nu_4A\lambda(\lambda+1)\gamma^2 = 0,$$
$$\nu_1A^2\lambda(2\lambda+1)\alpha^2 + \nu_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0,$$

set $\lambda = 2$

$$A = -\frac{12\alpha^2\nu_2}{\nu_1}, \quad \nu = \frac{-8\alpha^4\nu_2 + \beta^2\nu_3 + \gamma^2\nu_4}{\alpha}.$$

Thus

$$u_3(x, y, z, t) = A\coth^2(\alpha x + \beta y + \gamma z - \nu t). \tag{8}$$

Singular wave Form-II

For the singular wave Form II solution, suppose

$$u(x, y, z, t) = A\mathrm{csch}^{\lambda}\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t \quad \text{and} \quad \lambda > 0, \tag{9}$$

from (9)

$$u_{tx} = -A\lambda(\lambda+1)\alpha\nu\mathrm{csch}^{\lambda+2}\psi - A\lambda^2\alpha\nu\mathrm{csch}^{\lambda}\psi,$$
$$(uu_x)_x = 2A^2\lambda^2\alpha^2\mathrm{csch}^{2\lambda}\psi + A^2\lambda(2\lambda+1)\alpha^2\mathrm{csch}^{2\lambda+2}\psi,$$
$$u_{xxxx} = A\lambda^4\alpha^4\mathrm{csch}^{\lambda}\psi + 2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\mathrm{csch}^{\lambda+2}\psi + A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\mathrm{csch}^{\lambda+4}\psi,$$
$$u_{yy} = A\lambda(\lambda+1)\beta^2\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\beta^2\mathrm{csch}^{\lambda}\psi,$$
$$u_{zz} = A\lambda(\lambda+1)\gamma^2\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\gamma^2\mathrm{csch}^{\lambda}\psi,$$

substituting above values into (1)

$$-A\lambda(\lambda+1)\alpha\nu\mathrm{csch}^{\lambda+2}\psi - A\lambda^2\alpha\nu\mathrm{csch}^{\lambda}\psi + 2\nu_1A^2\lambda^2\alpha^2\mathrm{csch}^{2\lambda}\psi + \nu_1A^2\lambda(2\lambda+1)\alpha^2\mathrm{csch}^{2\lambda+2}\psi + \nu_2A\lambda^4\alpha^4\mathrm{csch}^{\lambda}\psi + 2\nu_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\mathrm{csch}^{\lambda+2}\psi + \nu_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\mathrm{csch}^{\lambda+4}\psi + \nu_3A\lambda(\lambda+1)\beta^2\mathrm{csch}^{\lambda+2}\psi + \nu_3A\lambda^2\beta^2\mathrm{csch}^{\lambda}\psi + \nu_4A\lambda(\lambda+1)\gamma^2\mathrm{csch}^{\lambda+2}\psi + \nu_4A\lambda^2\gamma^2\mathrm{csch}^{\lambda}\psi = 0.$$

By comparing the powers 2λ, $\lambda+2$ and $\lambda+4$, $2\lambda+2$

$$-A\lambda(\lambda+1)\alpha\nu + 2\nu_1A^2\lambda^2\alpha^2 + 2\nu_2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4 + \nu_3A\lambda(\lambda+1)\beta^2 + \nu_4A\lambda(\lambda+1)\gamma^2 = 0. \quad \nu_1A^2\lambda(2\lambda+1)\alpha^2 + \nu_2A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0,$$

set $\lambda = 2$

$$A = -\frac{12\alpha^2\nu_2}{\nu_1}, \quad \nu = \frac{4\alpha^4\nu_2 + \beta^2\nu_3 + \gamma^2\nu_4}{\alpha}.$$

Thus

$$u_4(x, y, z, t) = A\mathrm{csch}^2(\alpha x + \beta y + \gamma z - \nu t). \tag{10}$$

Figs. 1–4 depicts the 3D and 2D charts of the solution given by $u_i(x, y, z, t)$, $i = 1, 2, 3, 4$ respectively with the parameters $\alpha = 0.5$, $\beta = 0.1$, $\gamma = 0.1$, $\nu_1 = 2$, $\nu_2 = 5$, and $\nu_3 = \nu_4 = 3$.

Fig. (1)(a) and (b) represent a bright soliton wave which is a stable

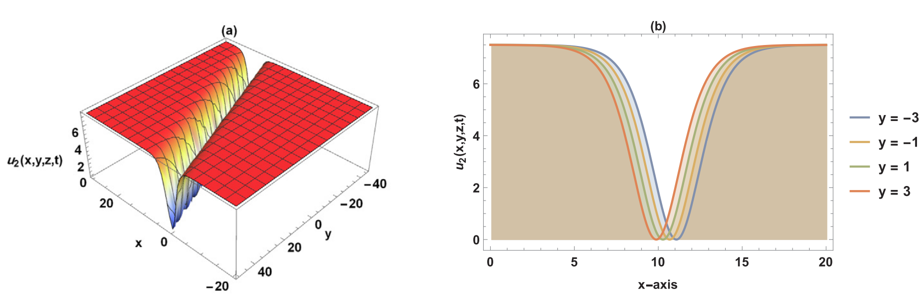

Fig. 2. $u_2(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

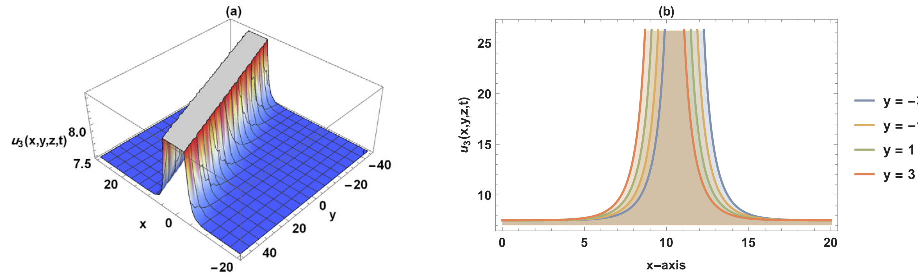

Fig. 3. $u_3(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

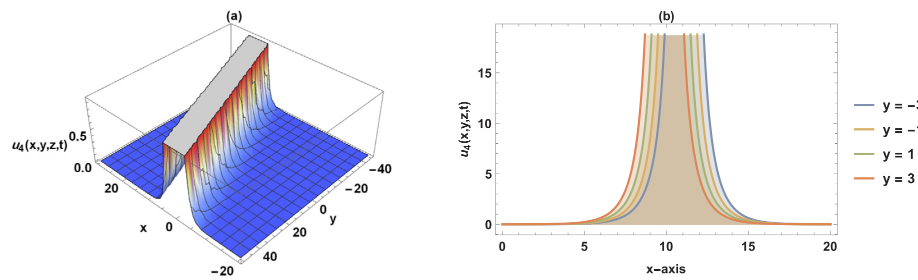

Fig. 4. $u_4(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

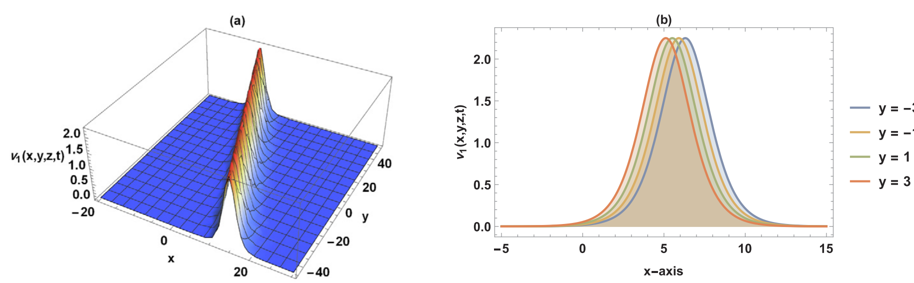

Fig. 5. $v_1(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

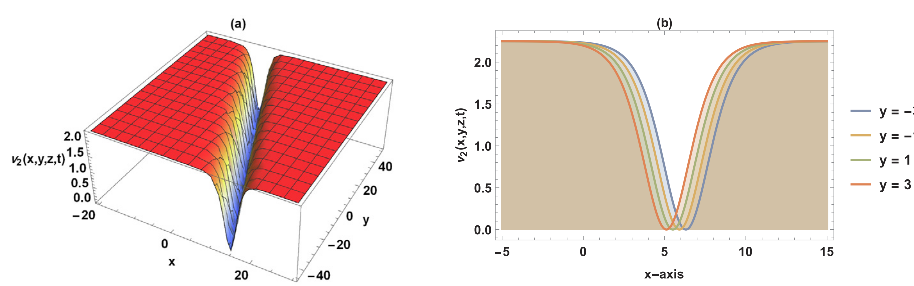

Fig. 6. $v_2(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

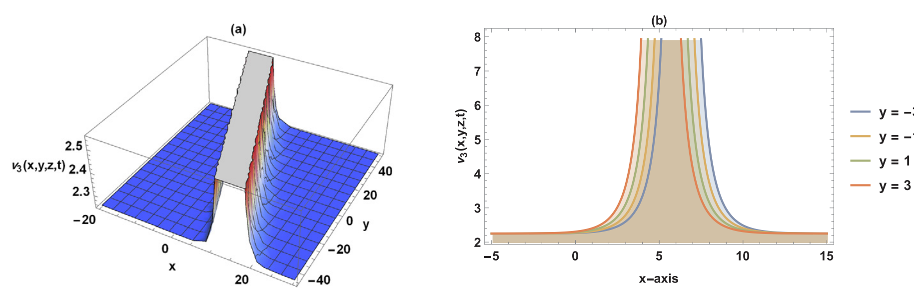

Fig. 7. $v_3(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

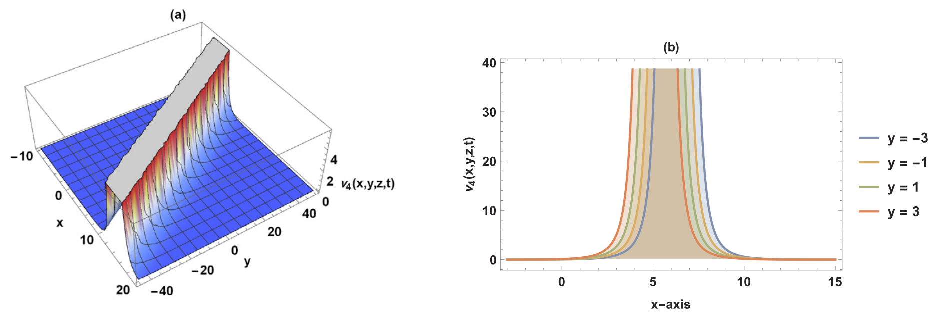

Fig. 8. $v_4(x, y, z, t)$ when $z = 0$, $t = 2$ in 3D- and 2D-plots.

solution while Fig. (2)(a) and (b) represent a dark soliton wave which is also a stable solution (shock wave solution).

Figs. (3)(a), (b), (4)(a), and (b) represent singular wave solutions which are not stable.

Solution to the GBE

In this section, the solitary wave, shock wave and singular wave solutions for (2) are obtained.

Solitary wave solutions

In order to calculate the solitary wave solution, suppose

$$v(x, y, z, t) = \frac{A}{\cosh^{\lambda}\psi} \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t, \tag{11}$$

from (11)

$$v_{tt} = \frac{A\lambda^2\nu^2}{\cosh^{\lambda}\psi} - \frac{A\lambda(\lambda+1)\nu^2}{\cosh^{\lambda+2}\psi},$$
$$(v^2)_{xx} = \frac{4A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi} - \frac{2A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi},$$
$$v_{xx} = \frac{A\lambda^2\alpha^2}{\cosh^{\lambda}\psi} - \frac{A\lambda(\lambda+1)\alpha^2}{\cosh^{\lambda+2}\psi},$$
$$v_{xxxx} = \frac{A\lambda^4\alpha^4}{\cosh^{\lambda}\psi} - \frac{2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4}{\cosh^{\lambda+2}\psi} + \frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4}{\cosh^{\lambda+4}\psi},$$
$$v_{yy} = \frac{A\lambda^2\beta^2}{\cosh^{\lambda}\psi} - \frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi},$$
$$v_{zz} = \frac{A\lambda^2\gamma^2}{\cosh^{\lambda}\psi} - \frac{A\lambda(\lambda+1)\gamma^2}{\cosh^{\lambda+2}\psi},$$

substituting above values into (2)

$$\frac{A\lambda^2\nu^2}{\cosh^{\lambda}\psi} - \frac{A\lambda(\lambda+1)\nu^2}{\cosh^{\lambda+2}\psi} - \delta_1\frac{4A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi} + \delta_1\frac{2A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi} - \delta_2\frac{A\lambda^2\alpha^2}{\cosh^{\lambda}\psi} + \delta_2\frac{A\lambda(\lambda+1)\alpha^2}{\cosh^{\lambda+2}\psi} - \delta_3\frac{A\lambda^4\alpha^4}{\cosh^{\lambda}\psi} + \delta_3\frac{2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4}{\cosh^{\lambda+2}\psi} - \delta_3\frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4}{\cosh^{\lambda+4}\psi} - \delta_4\frac{A\lambda^2\beta^2}{\cosh^{\lambda}\psi} + \delta_4\frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi} - \delta_5\frac{A\lambda^2\gamma^2}{\cosh^{\lambda}\psi} + \delta_5\frac{A\lambda(\lambda+1)\gamma^2}{\cosh^{\lambda+2}\psi} = 0.$$

By comparing the powers 2λ, $\lambda + 2$ and $\lambda + 4$, $2\lambda + 2$

$$-A\lambda(\lambda+1)\nu^2 - 4\delta_1A^2\lambda^2\alpha^2 + \delta_2A\lambda(\lambda+1)\alpha^2 + 2\delta_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4 + \delta_4A\lambda(\lambda+1)\beta^2 + \delta_5A\lambda(\lambda+1)\gamma^2 = 0,$$
$$2\delta_1A^2\lambda(2\lambda+1)\alpha^2 - \delta_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0,$$

set $\lambda = 2$

$$A = \frac{6\alpha^2\delta_3}{\delta_1}, \quad \nu = \pm\sqrt{\alpha^2\delta_2 + 4\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5}.$$

Thus

$$v_1(x, y, z, t) = \frac{A}{\cosh^2(\alpha x + \beta x + \gamma y - \nu t)}. \tag{12}$$

and the condition for the existence of the solution is $(\alpha^2\delta_2 + 4\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5) > 0$.

Shock wave solitons

In order to calculate the shock wave soliton, suppose

$$v(x, y, z, t) = A\tanh^{\lambda}\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t \quad \text{and} \quad \lambda > 0, \tag{13}$$

from (13)

$$v_{tt} = A\lambda(\lambda-1)\nu^2\tanh^{\lambda-2}\psi - 2A\lambda^2\nu^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\nu^2\tanh^{\lambda+2}\psi,$$
$$(v^2)_{xx} = 2A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi - 8A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi,$$
$$v_{xx} = A\lambda(\lambda-1)\alpha^2\tanh^{\lambda-2}\psi - 2A\lambda^2\alpha^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\alpha^2\tanh^{\lambda+2}\psi.$$

$$v_{xxxx} = A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\tanh^{\lambda-4}\psi - 4\lambda(\lambda-1)(\lambda2 - 2\lambda + 2)\alpha^4\tanh^{\lambda-2}\psi - 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\tanh^{\lambda+2}\psi + 2A\lambda^2(3\lambda^2+5)\alpha^4\tanh^{\lambda}\psi + A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\tanh^{\lambda+4}\psi,$$
$$v_{yy} = A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi - 2A\lambda^2\beta^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi,$$
$$v_{zz} = A\lambda(\lambda-1)\gamma^2\tanh^{\lambda-2}\psi - 2A\lambda^2\gamma^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\gamma^2\tanh^{\lambda+2}\psi,$$

substituting above values into (2)

$$A\lambda(\lambda-1)\nu^2\tanh^{\lambda-2}\psi - 2A\lambda^2\nu^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\nu^2\tanh^{\lambda+2}\psi - 2\delta_1A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi + 8\delta_1A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi - 2\delta_1A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi - \delta_2A\lambda(\lambda-1)\alpha^2\tanh^{\lambda-2}\psi + 2\delta_2A\lambda^2\alpha^2\tanh^{\lambda}\psi - \delta_2A\lambda(\lambda+1)\alpha^2\tanh^{\lambda+2}\psi - \delta_3A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\tanh^{\lambda-4}\psi + 4\delta_3\lambda(\lambda-1)(\lambda2 - 2\lambda + 2)\alpha^4\tanh^{\lambda-2}\psi + 4\delta_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\tanh^{\lambda+2}\psi - 2\delta_3A\lambda^2(3\lambda^2+5)\alpha^4\tanh^{\lambda}\psi - \delta_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\tanh^{\lambda+4}\psi - \delta_4A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi + 2\delta_4A\lambda^2\beta^2\tanh^{\lambda}\psi - \delta_4A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi - \delta_5A\lambda(\lambda-1)\gamma^2\tanh^{\lambda-2}\psi + 2\delta_5A\lambda^2\gamma^2\tanh^{\lambda}\psi - \delta_5A\lambda(\lambda+1)\gamma^2\tanh^{\lambda+2}\psi = 0.$$

By comparing the powers 2λ, $\lambda + 2$ and $\lambda + 4$, $2\lambda + 2$

$$A\lambda(\lambda+1)\nu^2 + 8\delta_1A^2\lambda^2\alpha^2 - \delta_2A\lambda(\lambda+1)\alpha^2 + 4\delta_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4 - \delta_4A\lambda(\lambda+1)\beta^2 - \delta_5A\lambda(\lambda+1)\gamma^2 = 0,$$
$$+2\delta_1A^2\lambda(2\lambda+1)\alpha^2 + \delta_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0,$$

set $\lambda = 2$

$$A = -\frac{6\alpha^2\delta_3}{\delta_1}, \quad \nu = \pm\sqrt{\alpha^2\delta_2 - 8\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5}.$$

Thus

$$v_2(x, y, z, t) = A\tanh^2(\alpha x + \beta y + \gamma z - \nu t), \tag{14}$$

and the condition for the existence of the solution is $(\alpha^2\delta_2 - 8\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5) > 0$.

Singular wave Form-I

To calculate the singular wave Form I solution, suppose

$$v(x, y, z, t) = A\coth^\lambda\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t \quad \text{and} \quad \lambda > 0, \tag{15}$$

from (15)

$$\begin{aligned} v_{tt} &= A\lambda(\lambda-1)\nu^2\coth^{\lambda-2}\psi - 2A\lambda^2\nu^2\coth^\lambda\psi + A\lambda(\lambda+1)\nu^2\coth^{\lambda+2}\psi,\\ (v^2)_{xx} &= 2A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi - 8A^2\lambda^2\alpha^2\coth^{2\lambda}\psi\\ &\quad + 2A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi,\\ v_{xx} &= A\lambda(\lambda-1)\alpha^2\coth^{\lambda-2}\psi - 2A\lambda^2\alpha^2\coth^\lambda\psi + A\lambda(\lambda+1)\alpha^2\coth^{\lambda+2}\psi,\\ v_{xxxx} &= A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\coth^{\lambda-4}\psi - 4\lambda(\lambda-1)(\lambda 2 - 2\lambda\\ &\quad + 2)\alpha^4\coth^{\lambda-2}\psi - 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\coth^{\lambda+2}\psi\\ &\quad + 2A\lambda^2(3\lambda^2+5)\alpha^4\coth^\lambda\psi + A\lambda(\lambda+1)(\lambda+2)(\lambda\\ &\quad + 3)\alpha^4\coth^{\lambda+4}\psi,\\ v_{yy} &= A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi - 2A\lambda^2\beta^2\coth^\lambda\psi + A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi,\\ v_{zz} &= A\lambda(\lambda-1)\gamma^2\coth^{\lambda-2}\psi - 2A\lambda^2\gamma^2\coth^\lambda\psi + A\lambda(\lambda+1)\gamma^2\coth^{\lambda+2}\psi, \end{aligned}$$

substituting above values into (2)

$$\begin{aligned} &A\lambda(\lambda-1)\nu^2\coth^{\lambda-2}\psi - 2A\lambda^2\nu^2\coth^\lambda\psi + A\lambda(\lambda+1)\nu^2\coth^{\lambda+2}\psi - 2\delta_1A^2\\ &\quad \lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi + 8\delta_1A^2\lambda^2\alpha^2\coth^{2\lambda}\psi - 2\delta_1A^2\lambda(2\lambda+1)\\ &\quad \alpha^2\coth^{2\lambda+2}\psi - \delta_2A\lambda(\lambda-1)\alpha^2\coth^{\lambda-2}\psi + 2\delta_2A\lambda^2\alpha^2\coth^\lambda\psi - \delta_2A\lambda\\ &\quad (\lambda+1)\alpha^2\coth^{\lambda+2}\psi - \delta_3A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^4\coth^{\lambda-4}\psi + 4\delta_3\lambda\\ &\quad (\lambda-1)(\lambda 2 - 2\lambda + 2)\alpha^4\coth^{\lambda-2}\psi + 4\delta_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\\ &\quad \alpha^4\coth^{\lambda+2}\psi - 2\delta_3A\lambda^2(3\lambda^2+5)\alpha^4\coth^\lambda\psi - \delta_3A\lambda(\lambda+1)(\lambda+2)\\ &\quad (\lambda+3)\alpha^4\coth^{\lambda+4}\psi - \delta_4A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi + 2\delta_4A\lambda^2\beta^2\\ &\quad \coth^\lambda\psi - \delta_4A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi - \delta_5A\lambda(\lambda-1)\gamma^2\coth^{\lambda-2}\psi + 2\delta_5A\\ &\quad \lambda^2\gamma^2\coth^\lambda\psi - \delta_5A\lambda(\lambda+1)\gamma^2\coth^{\lambda+2}\psi = 0. \end{aligned}$$

By comparing the powers 2λ, $\lambda+2$ and $\lambda+4$, $2\lambda+2$

$$\begin{aligned} &A\lambda(\lambda+1)\nu^2 + 8\delta_1A^2\lambda^2\alpha^2 - \delta_2A\lambda(\lambda+1)\alpha^2 + 4\delta_3A\lambda(\lambda+1)\\ &\quad (\lambda^2+2\lambda+2)\alpha^4 - \delta_4A\lambda(\lambda+1)\beta^2 - \delta_5A\lambda(\lambda+1)\gamma^2 = 0,\\ &+2\delta_1A^2\lambda(2\lambda+1)\alpha^2 + \delta_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0, \end{aligned}$$

set $\lambda = 2$

$$A = -\frac{6\alpha^2\delta_3}{\delta_1}, \quad \nu = \pm\sqrt{\alpha^2\delta_2 - 8\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5}$$

Thus

$$v_3(x, y, z, t) = A\coth^2(\alpha x + \beta y + \gamma z - \nu t). \tag{16}$$

and the condition for the existence of the solution is $(\alpha^2\delta_2 - 8\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5) > 0$.

Singular wave Form-II

To calculate the singular wave Form II solution, suppose

$$v(x, y, z, t) = A\text{csch}^\lambda\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t \quad \text{and} \quad \lambda > 0, \tag{17}$$

from (17)

$$\begin{aligned} v_{tt} &= A\lambda^2\nu^2\text{csch}^\lambda\psi + A\lambda(\lambda+1)\nu^2\text{csch}^{\lambda+2}\psi,\\ (v^2)_{xx} &= 4A^2\lambda^2\alpha^2\text{csch}^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\text{csch}^{2\lambda+2}\psi,\\ v_{xx} &= A\lambda(\lambda+1)\alpha^2\text{csch}^{\lambda+2}\psi + A\lambda^2\alpha^2\text{csch}^\lambda\psi,\\ v_{xxxx} &= A\lambda^4\alpha^4\text{csch}^\lambda\psi + 2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^4\text{csch}^{\lambda+2}\psi\\ &\quad + A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\text{csch}^{\lambda+4}\psi,\\ v_{yy} &= A\lambda(\lambda+1)\beta^2\text{csch}^{\lambda+2}\psi + A\lambda^2\beta^2\text{csch}^\lambda\psi,\\ v_{zz} &= A\lambda(\lambda+1)\gamma^2\text{csch}^{\lambda+2}\psi + A\lambda^2\gamma^2\text{csch}^\lambda\psi, \end{aligned}$$

substituting above values into (2)

$$\begin{aligned} &A\lambda^2\nu^2\text{csch}^\lambda\psi + A\lambda(\lambda+1)\nu^2\text{csch}^{\lambda+2}\psi - 4\delta_1A^2\lambda^2\alpha^2\text{csch}^{2\lambda}\psi - 2\delta_1A^2\lambda\\ &\quad (2\lambda+1)\alpha^2\text{csch}^{2\lambda+2}\psi - \delta_2A\lambda(\lambda+1)\alpha^2\text{csch}^{\lambda+2}\psi - \delta_2A\lambda^2\alpha^2\\ &\quad \text{csch}^\lambda\psi - \delta_3A\lambda^4\alpha^4\text{csch}^\lambda\psi - 2\delta_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\\ &\quad \alpha^4\text{csch}^{\lambda+2}\psi - \delta_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4\text{csch}^{\lambda+4}\psi - \delta_4A\lambda(\lambda+1)\\ &\quad \beta^2\text{csch}^{\lambda+2}\psi - \delta_4A\lambda^2\beta^2\text{csch}^\lambda\psi - \delta_5A\lambda(\lambda+1)\gamma^2\text{csch}^{\lambda+2}\psi - \delta_5A\lambda^2\gamma^2\\ &\quad \text{csch}^\lambda\psi = 0. \end{aligned}$$

By comparing the powers 2λ, $\lambda+2$ and $\lambda+4$, $2\lambda+2$

$$\begin{aligned} &A\lambda(\lambda+1)\nu^2 - 4\delta_1A^2\lambda^2\alpha^2 - \delta_2A\lambda(\lambda+1)\alpha^2 - 2\delta_3A\lambda(\lambda+1)\\ &\quad (\lambda^2+2\lambda+2)\alpha^4 - \delta_4A\lambda(\lambda+1)\beta^2 - \delta_5A\lambda(\lambda+1)\gamma^2 = 0,\\ &2\delta_1A^2\lambda(2\lambda+1)\alpha^2 + \delta_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^4 = 0, \end{aligned}$$

set $\lambda = 2$

$$A = -\frac{6\alpha^2\delta_3}{\delta_1}, \quad \nu = \pm\sqrt{\alpha^2\delta_2 + 4\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5}.$$

Thus

$$v_4(x, y, z, t) = A\text{csch}^2(\alpha x + \beta y + \gamma z - \nu t), \tag{18}$$

and the condition for the existence of the solution is $(\alpha^2\delta_2 + 4\alpha^4\delta_3 + \beta^2\delta_4 + \gamma^2\delta_5) > 0$.

Figs. 5–8 depicts the 3D and 2D charts of the solution given by $v_i(x, y, z, t)$, $i = 1, 2, 3, 4$ respectively with the parameters $\alpha = 0.5$, $\beta = 0.1$, $\gamma = 0.1$, $\delta_1 = 2$, $\delta_2 = 5$, $\delta_3 = \delta_4 = 3$, and $\delta_5 = 1$.

The same discussion as in Figs. 1–4 can be investigated here.

Conclusions

This paper had investigated the analytical solutions to the (3 + 1)-Dimensional Kadomtsev-Petviashvili and the generalized Boussinesq equations with the help of the solitary wave ansatz method. These solutions included solitary wave, shock wave, and singular wave solutions. The dynamical behavior and the propagation of these solutions are discussed in a graded index waveguide by choosing suitable parameters. To our best of knowledge, the discussion and results in this work, comparing with the other results in literature, are new and had different wave structures. The obtained solutions can be critical to under stand attributes of the 3D-KPE and the GBE which are important in different branches of science where these equations are used to describe some physical phenomenon.

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