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# Laminar Forced Convection Over An Inclined Flat Plate With Unheated Starting Length

*Araştırma Makalesi / Research Article*

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## ABSTRACT

Two-dimensional laminar forced convection over an inclined flat plate with an unheated starting length was investigated numerically for both constant surface temperature and constant heat flux boundary conditions. The numerical study was implemented using the commercial software ANSYS Fluent 15.0. Air is used as working fluid. The influence of Reynolds number, inclination angle and the length of unheated plate on velocity and temperature distributions, surface temperature, surface heat flux and local Nusselt number was investigated. The results show that Reynolds number, inclination angle and the length of unheated region of plate play important role on heat transfer from the plate. It is seen that Nusselt number increases with increasing Reynolds number and inclination angle of inclined flat plate but decreases with increasing the length of unheated region of plate.

**Keywords: Laminar flow, forced convection, inclined flat plate, unheated starting, numerical analysis.**

# Isıtılmamış Başlangıç Uzunluğuna Sahip Bir Eğik Düz Plaka Üzerindeki Laminar Zorlanmış Konveksiyon

## ÖZ

Isıtılmamış başlangıç uzunluğuna sahip bir eğik düz plaka üzerindeki iki boyutlu laminar zorlanmış konveksiyon sabit yüzey sıcaklığı ve sabit yüzey ısı akısı sınır şartları için sayısal olarak incelenmiştir. Sayısal çalışma ANSYS Fluent 15.0 ticari paket programı kullanılarak gerçekleştirilmiştir. Çalışma akışkanı olarak hava kullanılmıştır. Reynolds sayısının, ısıtılmamış plaka uzunluğunun ve plaka eğim açısının hız ve sıcaklık dağılımları, yüzey sıcaklığı, yüzey ısı akısı ve yerel Nusselt sayısı üzerindeki etkisi incelenmiştir. Sonuçlar Reynolds sayısının, plaka eğim açısının ve düz plakanın ısıtılmamış bölge uzunluğunun plakadan olan ısı transferi için önemli olduğunu göstermiştir. Nusselt sayısının artan Reynolds sayısı ve düz plaka eğim açısı ile arttığı, fakat düz plakanın ısıtılmamış bölge uzunluğunun artması ile azaldığı görülmüştür.

**Anahtar Kelimeler: Laminar akış, zorlanmış konveksiyon, eğik düz plaka, ısıtılmamış uzunluk, numerik analiz**

## 1. INTRODUCTION

The flow and heat transfer over a flat plate are encountered in various engineering applications. In some of these applications, a heat source is placed at a distance from the leading edge of the plate. In mean time, the plate can be inclined at relative to the flow direction. Such an application is commonly seen in cooling of electronic components. Operating temperatures of electronic components are significant for their reliability. Thus, low temperature is necessary for improving the life of electronic components.

Convection heat transfer over a flat plate has been studied by a number of investigators. Nagendra [1] investigated the laminar transient forced convection heat transfer over a horizontal isothermal flat plate. Two-dimensional vortex shedding for transient and asymptotically steady separated flow over an inclined plate was investigated by Sarpkaya [2]. Dey and Nath [3] examined the forced convection heat transfer over a horizontal semi-infinite flat plate with an unheated starting length. Ma *et al*. [4] conducted a study to investigate the two-dimensional forced convection over a horizontal flat plate with an unheated starting length using matched asymptotic expansions. Ameel [5] analytically investigated the parallel flow heat transfer over a horizontal flat plate with an unheated starting length for both constant temperature and constant heat flux boundary conditions and for both laminar and turbulent flow regimes. Free convection flow over an inclined plate was investigated by Chamkha [6]. Vynnycky *et al*. [7] analytically and experimentally investigated the two-dimensional forced convection flow over a horizontal flat plate. Yovanovich and Teertstra [8] numerically investigated laminar forced convection over horizontal isothermal rectangular plates. Umur and Karagöz [9] carried out a numerical study to investigate the flow and heat transfer characteristics over a horizontal flat plate with an unheated starting length for

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two-dimensional laminar and turbulent flows under constant surface temperature boundary condition. The effect of free stream turbulence on heat transfer coefficient, velocity profile and temperature profile on plates has been reviewed by Kondjoyan *et al*. [10]. Nan *et al*. [11] conducted an experimental study to investigate the turbulent flow over a horizontal flat plate. Kondjoyan *et al*. [12] performed an experimental study to see the effect of free stream turbulence intensity on velocity and thermal boundary layers for flow over a flat plate with an unheated section. Rebay *et al*. [13] numerically investigated the forced convection over a horizontal microstructure having an unheated section for constant heat flux boundary condition. Juncu [14] conducted a numerical study using finite difference method to investigate transient, conjugate laminar forced convection heat transfer from a finite flat plate. Li and Nalim [15] numerically investigated the transient thermal boundary layer response owing to a convected non-uniform temperature fluid for laminar flow on a semi-infinite plate. Palani [16] conducted a numerical study to investigate the free convection over a semi-infinite inclined plate with variable surface temperature. Kumar and Mullick [17] experimentally investigated the laminar convective heat transfer from a square flat plate. Lam and Wei [18] numerically investigated the vortex shedding from an inclined flat plate for turbulent flow. Palani and Kim [19] conducted a numerical study to investigate the natural convection over a semi-infinite inclined plate with variable surface temperature. Li *et al*. [20] numerically investigated turbulent compressible flow over a two-dimensional semi-infinite flat plate using two-layer eddy viscosity model. Malvandi *et al*. [21] analytically investigated the entropy generation for the steady two-dimensional flow over an isothermal flat plate. Kumar [22] conducted a numerical study to investigate the heat and mass transfer over an isothermal inclined plate at constant concentration gradient and with heat source. Kumar [23] analytically investigated the steady laminar flow over an inclined plate at a prescribed heat flux with chemical reaction. Samanta and Guha [24] carried out a study to investigate the fluid flow and heat transfer characteristics for the laminar forced convection over a horizontal plate using similarity analysis. A numerical study was performed by Islam *et al*. [25] to study the mass transfer through porous medium with an inclined plate. Jana and Das [26] studied the magnetohydrodynamic (MHD) slip flow and heat transfer over a flat plate with convective surface heat flux at the boundary and temperature dependent fluid properties. Uddin *et al*. [27] conducted a theoretical and numerical study of two-dimensional steady laminar flow over a moving horizontal plate. Magnetohydrodynamic mixed convective flow and heat transfer over an inclined plate was investigated by Shanmugapriya [28]. Film flow over an inclined plate was numerically investigated by Singh et al. [29] using volume of fluid method.

Literature survey indicates that there is a lack of information about laminar forced convection heat transfer over an inclined flat plate with an unheated starting length. Literature survey shows that current studies which are related to the heat transfer over an inclined plate are about the free convection and film flow. That is, it is seen that literature survey does not involve forced convection heat transfer over an inclined flat plate with an unheated starting length. Thus, the novelty of this study is the investigation of laminar forced convection over an inclined flat plate with an unheated starting length. Therefore, in this study two-dimensional laminar forced convection over an inclined flat plate with an unheated starting length has been numerically investigated for both constant surface temperature and constant surface heat flux boundary conditions. The effect of Reynolds number, the length of unheated starting length and the inclination angle of flat plate on heat transfer, surface temperature, surface heat flux, and velocity and temperature profiles over inclined flat plate has been investigated numerically.

## 2. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

The schematic geometry and coordinate system used in the computational domain are shown in Figure 1.

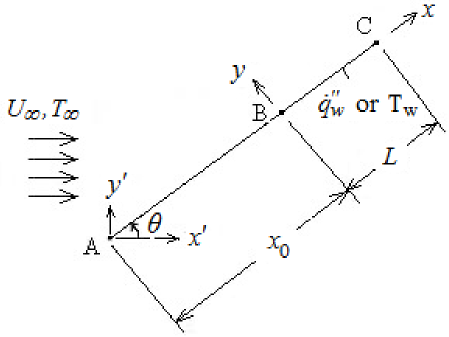


**Figure 1.** The geometry and coordinate system of problem

As seen in this figure, problem consists of an inclined semi-infinite flat plate and a flow over it. The flow approaches to the inclined plate horizontally with a uniform velocity $U_\infty$ and temperature $T_\infty$ in the direction of $x'$. The part of the plate starting from the leading edge, region AB, is unheated. The length of this unheated region is denoted by $x_0$. The part of the plate after distance $x_0$ from the leading edge, region BC, is exposed to a constant temperature or constant heat flux boundary condition. In other words, laminar velocity boundary layer development starts at the leading edge (at $x$=-$x_0$) while thermal boundary layer development begins at $x$=0 which is the beginning of the heating section. The length of the heated surface $L$ is chosen as 50mm, and the ratio of the unheated starting length to the heated length, $x_0/L$, is taken as 1 and 10. $x/L$ changes between 0 and 1.0. $\theta$ shown in Figure 1 is the inclination angle of the flat plate and measured from horizontal. The value of $\theta$ is changed between 0° and 75° to investigate the effect of inclination angle on the heat transfer over an inclined flat plate with an unheated starting length. Fluid flows parallel over the plate when $\theta$ is zero.

For steady laminar flow of a constant-property viscous flow neglecting the buoyancy effects and viscous dissipation the governing continuity, momentum, and energy equations can be written as follows:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\rho\left(D\mathbf{V}/Dt\right) = -\nabla p + \mu\nabla^2\mathbf{V} \tag{2}$$

$$\rho c_p\left(DT/Dt\right) = k\nabla^2 T \tag{3}$$

where $\mathbf{V}$, $\rho$, $p$, $\mu$, $c_p$, $k$ and $T$ designate the velocity vector, density, pressure, dynamic viscosity, specific heat, thermal conductivity, and temperature, respectively.

### 2.1. Boundary Conditions

Computational domain used for numerical solutions is shown in Figure 2. The computational domain is indicated by AEDCBA plane.

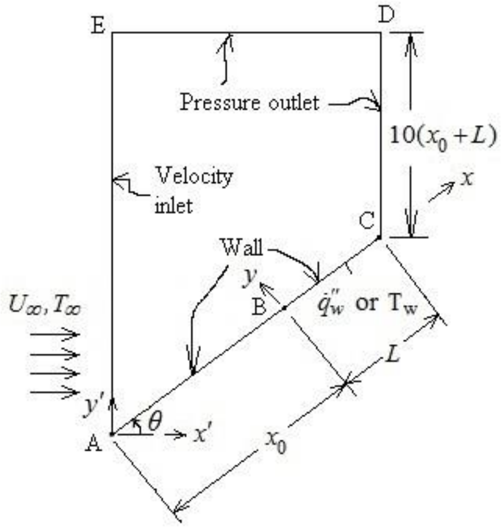


**Figure 2.** Schematic view of the computational domain and boundary conditions

On the left side of the computational domain, i.e. at $x=-x_0$ (AE surface), fluid enters the domain with uniform velocity and temperature, i.e.

$$u = U_\infty,\quad v = 0,\quad T = T_\infty = 293\text{K} \tag{4}$$

On the right side of the computational domain, i.e. at $x=L$ (DC surface), pressure outlet boundary condition is applied. On this boundary, the gage pressure is fixed to zero. Therefore, boundary conditions at the outlet are

$$\partial u/\partial x = 0,\quad \partial v/\partial x = 0,\quad \partial T/\partial x = 0,\quad p_{\text{gage}} = 0 \tag{5}$$

On the top surface, i.e. at $y=10(x_0+L)$ from the trailing edge of the plate (ED surface), pressure outlet boundary condition is considered and gage pressure is taken as zero, i.e.

$$\partial u/\partial y = 0,\quad \partial v/\partial y = 0,\quad \partial T/\partial y = 0,\quad p_{\text{gage}} = 0 \tag{6}$$

On the unheated surface (AB surface, i.e. $-x_0 \le x \le 0$), no slip velocity condition is applied, i.e.

$$u = 0,\quad v = 0,\quad T = T_\infty \tag{7}$$

On the heated surface (BC surface, $0 \le x \le L$), no slip velocity condition is used. For temperature, two different cases are considered: *i*) constant temperature ($T_w$=328K), *ii*) constant heat flux ( $\dot{q}''_{w,x}$ = 400Wm$^{-2}$ ). These can be expressed as

$$u = 0,\quad v = 0,\quad T = T_w \text{ or } \left(\partial T/\partial n\right)_w = -\dot{q}''_w\big/k \tag{8}$$

### 2.2. Calculation of Reynolds and Nusselt numbers

The Reynolds number based on $x_0+x$ on the heated surface is defined as

$$Re_{x_0+x} = U_\infty(x_0+x)/\nu \tag{9}$$

Thus, the maximum Reynolds number at the trailing edge of the plate $Re_{x0+L}$ is about $15.5\times10^3$ and $85\times10^3$ for $x_0/L$=1 and 10, respectively. Thus, flow is laminar at the trailing edge of the plate. Local Nusselt number based on $x_0+x$ is calculated as

$$Nu_{x_0+x} = \dot{q}''_{w,x}\left(x_0+x\right)/\left(k\left(T_{w,x}-T_\infty\right)\right) \tag{10}$$

All fluid properties are taken at the free stream temperature $T_\infty$ [30].

### 2.3. Numerical Solution

Numerical solutions were carried out using commercial software ANSYS Fluent 15.0. The computational scheme used by ANSYS Fluent Inc. is based on the finite volume discretization method. A typical mesh distribution is shown in Figure 3 for inclination angle of $\theta$=45°.

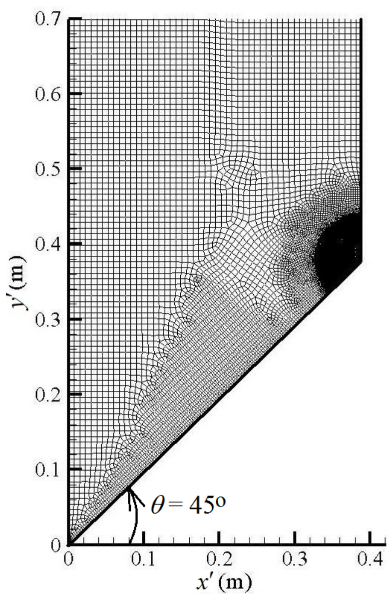


**Figure 3.** Mesh distribution of the computational domain

Computations were carried out for laminar flow. Steady pressure-based solver is employed with second order upwind scheme for convective terms in the mass, momentum, and energy equations. SIMPLE-algorithm is utilized for pressure-velocity coupling. No convergence problems are observed. To attain converged solution, each equation is iterated until the residuals fall below $1\times10^{-6}$.

The grid independence study is performed by changing the cell size. Surface temperature and local Nusselt number along the heated plate are plotted for five different mesh systems in Figure 4(a) and 4(b), respectively, at $Re_{x0}$=77120 for $\theta$=45° and $x_0/L$=10 at constant surface heat flux boundary condition. Mesh size is changed between 35197 and 82465 (mesh1-mesh5). No significant change occurs when mesh number changes from 68651 (mesh4) to 82465 (mesh5). Thus, mesh4 is considered as the optimum mesh. This mesh is used for other Reynolds numbers $Re_{x0}$ at $\theta$=45° and $x_0/L$=10. Mesh independency analysis is also performed when inclination angle and type of boundary condition of plate were changed.

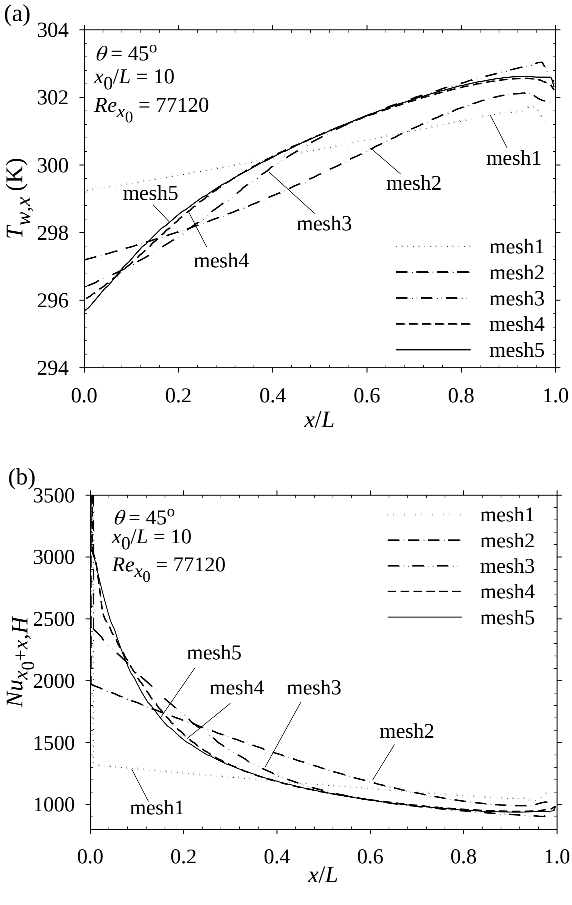


**Figure 4.** Surface temperature (a) and (b) local Nusselt number along the heated plate

## 3. CODE VALIDATION

Local Nusselt number $Nu_{x0+x}$ for laminar parallel flow over a horizontal flat plate, i.e. $\theta$=0°, with an unheated starting length is given for constant surface temperature and constant heat flux boundary conditions, respectively, as [30]

$$Nu_{x_0+x,T} = \frac{0.332\, Re_{x_0+x}^{\ 0.5}\, Pr^{1/3}}{\left[1\text{-}\left(x_0/\left(x_0+x\right)\right)^{3/4}\right]^{1/3}} \tag{11}$$

$$Nu_{x_0+x,\mathrm{H}} = \frac{0.453\, Re_{x_0+x}^{\ 0.5}\, Pr^{1/3}}{\left[1\text{-}\left(x_0/\left(x_0+x\right)\right)^{3/4}\right]^{1/3}} \tag{12}$$

To assess the accuracy of the present results, the computed $Nu_{x0+x}$ for constant surface temperature and constant surface heat flux boundary conditions are plotted along the heated flat plate in Figure 5(a) and 5(b) for $\theta$=0°, $x_0/L$=10 and $Re_{x0}$=77120. It is seen that present results are in good agreement with the literature results.

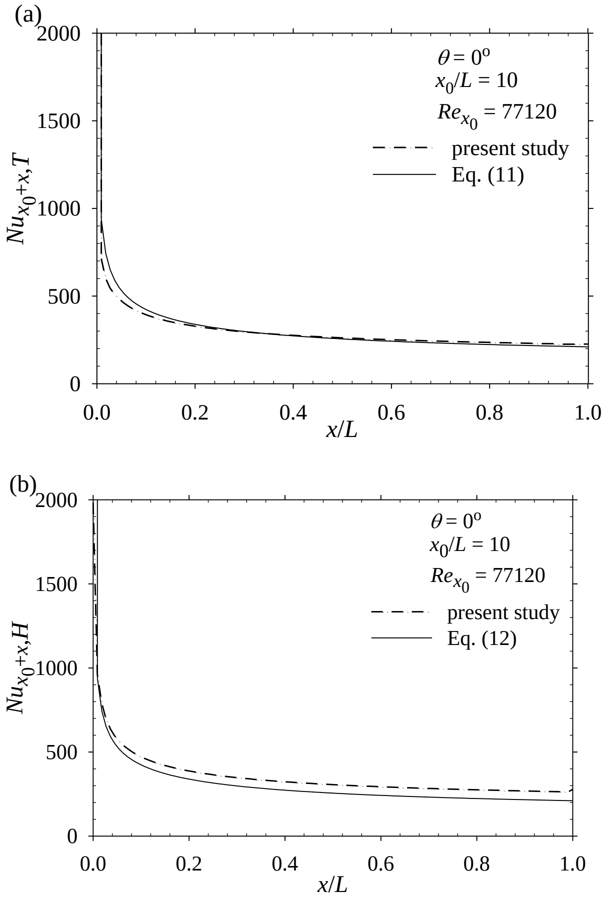


**Figure 5.** $Nu_{x0+x}$ along the heated section for (a) constant temperature and (b) constant heat flux

## 4. RESULTS AND DISCUSSION

After showing the validation of numerical results, the effects of Reynolds number, length of the unheated starting length and inclination angle of flat plate on flow and temperature fields, surface temperature, surface heat flux and local Nusselt number were investigated. To carry out these analysis, simulations were performed for the Reynolds numbers $Re_{x0}$ changing from 7710 to 77120, for the inclination angle of 0°≤$\theta$≤75°, and for the ratio of the unheated starting length to the heated length of $x_0/L$= 1 and 10.

Typical dimensionless $x$-velocity $u/U_\infty$ on the heated surface at a position of $x/L$=0.20 is plotted along $y$-axis in Figure 6(a) for three different Reynolds numbers $Re_{x0}$ for $x_0/L$=10 at $\theta$=0°. Dimensionless velocity over flat plate changes between zero and unity. As seen in Figure 6(a), velocity boundary layer thickness increases with decreasing Reynolds number. Velocity boundary layer thickness is defined as the distance at which 99% of the free stream velocity magnitude is achieved. Based on this definition, velocity boundary layer thicknesses for the Reynolds number $Re_{x0}$ of 7710, 46120 and 77120 are obtained as 18.20mm, 8.14mm and 6.02mm, respectively. In Figure 6(b), typical dimensionless temperature $(T-T_w)/(T_\infty-T_w)$ on the heated surface at a given position $x/L$=0.20 is plotted for three different $Re_{x0}$. It is seen that dimensionless temperature on the heated surface changes from zero to unity in the thermal boundary layer, and it remains constant outside of the boundary layer. Thermal boundary layer thickness is defined as the distance from surface at which $(T-T_w)/(T_\infty-T_w)$=0.99. Thus, thermal boundary layer thicknesses for $Re_{x0}$=7710, 46120 and 77120 are 6.31mm, 3.04mm and 1.94mm, respectively. That is, temperature boundary layer thickness decreases when flow velocity increases. Thus, the decreasing thermal boundary layer thickness results in increasing heat transfer.

Dimensionless axial velocity $u/U_\infty$ is plotted in Figure 7(a) on the heated surface at three different positions at $\theta$=0° for $x_0/L$=10 and $Re_{x0}$=46120. It is seen that dimensionless velocity changes from zero to unity in the hydrodynamic boundary layer, and it remains constant at unity out of the boundary layer. Velocity boundary layer thickness increases along the heated surface. In other words, velocity boundary layer thicknesses for $Re_{x0}$=46120 at $x/L$=0.20, 0.50 and 1 (near the leading edge, at the middle, and at the trailing edge of the heated surface) are 8.42mm, 8.79mm and 9.32mm, respectively. Likewise, typical dimensionless temperature $(T-T_w)/(T_\infty-T_w)$ is shown in Figure 7(b) on the heated surface at three different positions at $\theta$=0° for $x_0/L$=10 and $Re_{x0}$=46120. Dimensionless temperature varies between zero and unity inside thermal boundary layer, and it takes value of unity outside the thermal boundary layer. Also, it is seen that thermal boundary layer thickness increases from leading edge to the trailing edge over the heated surface. Thermal boundary layer thicknesses for $Re_{x0}$=46120 at $x/L$=0.20, 0.50 and 1.0 are 3.04mm, 3.91mm and 4.82mm, respectively.

The effect of the length of unheated starting length on surface temperature and thermal boundary layer are designated in Figure 8. In Figure 8(a), typical surface temperature is plotted along heated length at constant heat flux boundary condition for three different free stream velocities at $\theta$=0° for $x_0/L$=1 and 10. It is seen that unheated starting length affects the surface temperature of the heated section. The surface temperature of the heated section for short unheated section is smaller than that of long unheated starting section. That is, more heat is transferred from the surface for short unheated section. Dimensionless temperature $(T-T_w)/(T_\infty-T_w)$ at $x/L$=0.20 for $\theta$=0° is plotted in Figure 8(b) to see the effect of the length of unheated section on thermal boundary layer thickness for two different free stream velocities. Thermal boundary layer thicknesses at $x/L$=0.20 for $x_0/L$=1 are 4.23mm and 1.40mm for $U_\infty$=0.276m·s$^{-1}$ and 2.76m·s$^{-1}$, respectively. However, thermal boundary layer thicknesses at $x/L$=0.20 for $x_0/L$=10 are 6.31mm and 1.94mm for $U_\infty$=0.276m·s$^{-1}$ and 2.76m·s$^{-1}$, respectively. In other words, thermal boundary layer thickness for $x_0/L$=10 is greater than that of $x_0/L$=1. Therefore, increasing thermal boundary layer thickness results in decreasing heat transfer. It is seen that unheated section length affects the thermal boundary layer thickness. In other words, velocity boundary layer affects thermal boundary layer.

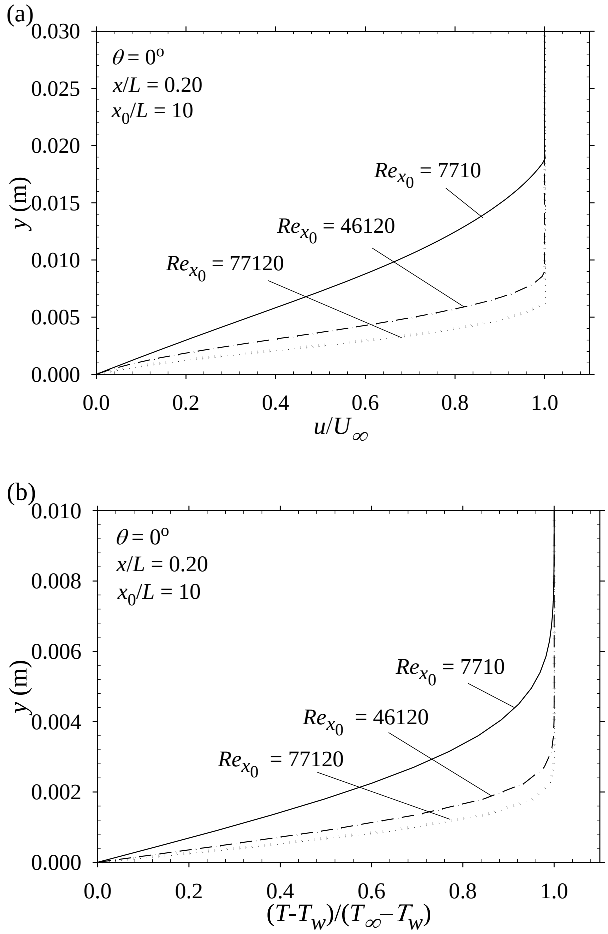


**Figure 6.** Variation of (a) $u/U_\infty$ and (b) $(T-T_w)/(T_\infty-T_w)$ on the heated surface at $x/L$=0.20 for $\theta$=0° and $x_0/L$=10

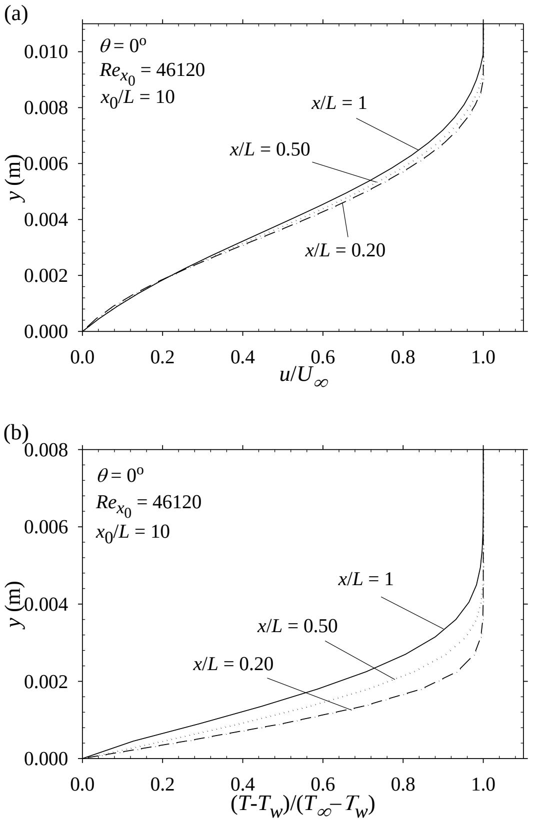


**Figure 7.** (a) $u/U_\infty$ and (b) $(T\text{-}T_w)/(T_\infty\text{-}T_w)$ along $y$-axis at three different $x/L$ for $\theta$=0°, $x_0/L$=10 and $Re_{x0}$=46120

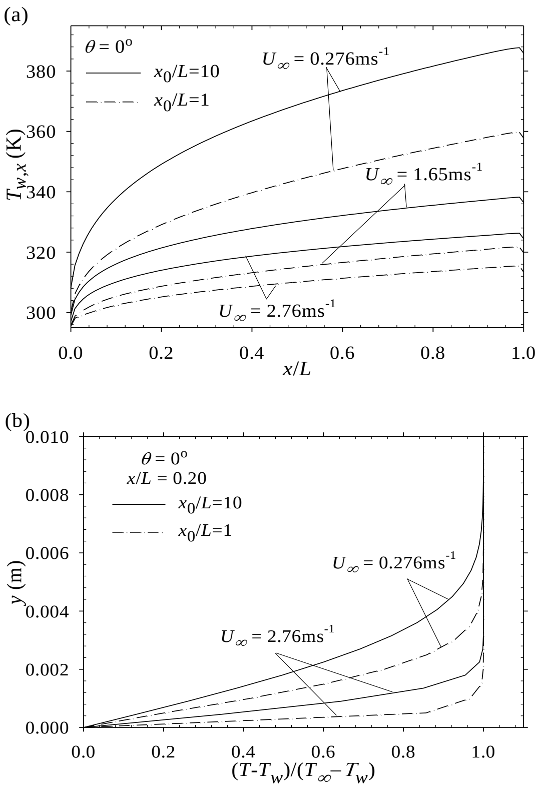


**Figure 8.** The effect of unheated starting length on (a) surface temperature and (b) thermal boundary layer thickness

Typical temperature contours over heated surface are plotted in Figure 9 for three different Reynolds numbers at constant heat flux boundary condition for $\theta$=0° and $x_0/L$=10. It is seen that thermal boundary layer thickness decreases while Reynolds number increases, as expected. Similar result can be seen in Figure 6(a). Thus, heat transfer increases as Reynolds number increases.

Typical temperature contours are shown in Figure 10 over the heated surface at different inclination angles of flat plate for $Re_{x0}$=7710 at $x_0/L$=10. As seen in Figure 10, inclination angle affects the thermal boundary layer. That is, thermal boundary layer thickness decreases while inclination angle increases, and this causes an increase in heat transfer.

The effects of Reynolds number $Re_{x0}$ on surface temperature and heat flux are shown in Figure 11(a) and 11(b) for constant heat flux and constant surface temperature boundary conditions, respectively. Figure 11(a) shows the variation of the surface temperature with Reynolds number along the heated plate for $\theta$=0° and $x_0/L$=10 at constant heat flux boundary condition. It is observed in Figure 11(a) that, at a given location $x/L$ (e.g. at $x/L$=0.80), surface temperature increases when Reynolds number decreases for constant heat flux boundary condition. As for Figure 11(b), variation of surface heat flux along the heated plate is given at different Reynolds numbers for constant surface temperature boundary condition at $\theta$=0° and $x_0/L$=10. It is seen that heat flux on the surface decreases along the duct. It is also seen that surface heat flux at a given position on the plate increases with increasing Reynolds number.

The variation of local Nusselt number along the heated plate for different Reynolds numbers $Re_{x0}$ at $x_0/L$=10 and $\theta$=0° is shown in Figure 12(a) and 12(b) for constant surface temperature and constant heat flux boundary conditions, respectively. It is seen that the local Nusselt number decreases along the heated plate. At a given position, it is also seen that local Nusselt number increases with increasing Reynolds number. This result can also be seen in Figure 6(b) due to the decreasing of thermal boundary layer thickness.

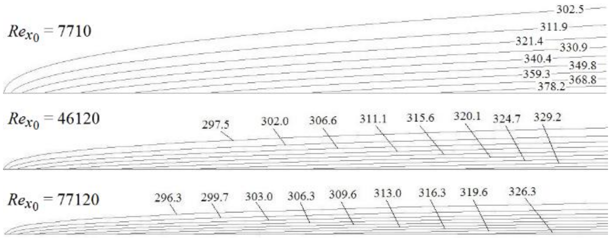


**Figure 9.** Temperature contours over heated surface for different Reynolds numbers at constant heat flux for $\theta$=0º

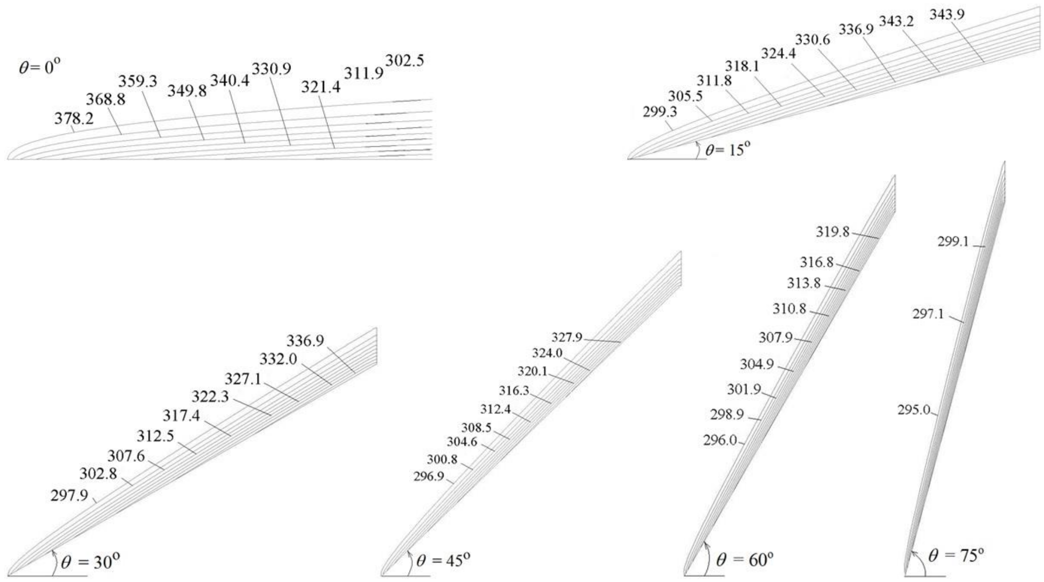


**Figure 10.** Typical temperature contours over the heated surface for different $\theta$ values at $Re_{x0}$=7710

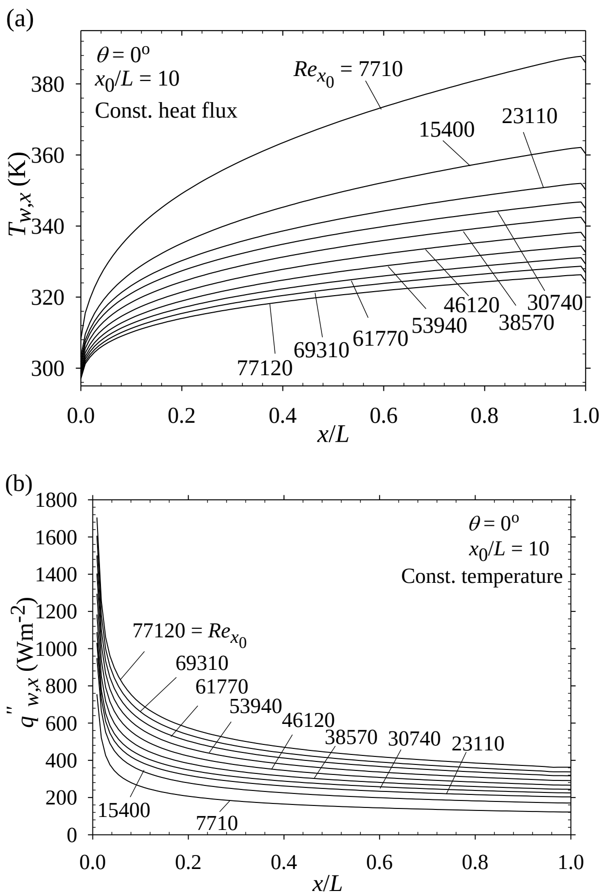


**Figure 11.** Variation of surface temperature (a) and heat flux (b) along the heated surface at $\theta$=0° and $x_0/L$=10 for different $Re_{x0}$ at constant surface heat flux and constant temperature

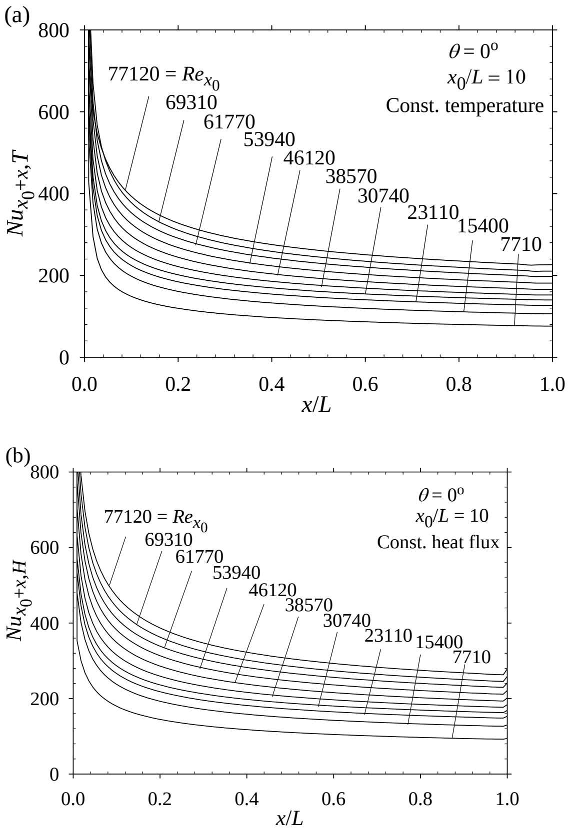


**Figure 12.** Variation of local Nusselt number along heated plate for different Reynolds numbers at constant temperature (a) and heat flux (b)

Comparison of local Nusselt number for constant surface heat flux and constant surface temperature boundary conditions is designated along heated surface in Figure 13 for $x_0/L$=10 and $\theta$=0° at $Re_{x0}$=77120. It is seen that local Nusselt number for constant heat flux boundary condition is higher than that of constant surface temperature boundary condition at a given position. In other words, heat transfer coefficient for constant heat flux boundary condition is about 20 percent higher than that of constant surface temperature boundary condition at the trailing edge of the heated surface.

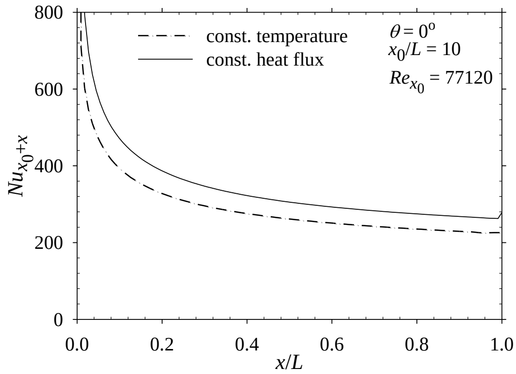


**Figure 13.** Local Nusselt number along heated surface

The effect of $\theta$ on surface temperature and heat flux is given in Figure 14(a) and 14(b) for constant heat flux and constant surface temperature at $Re_{x0}$=7710 for $x_0/L$=10. It is seen that surface temperature increases along heated surface. It is also seen that surface temperature depends on $\theta$. Surface temperature decreases with increasing $\theta$ at a given position for constant surface heat flux. With regard to Figure 14(b), surface heat flux is plotted along heated surface at different $\theta$. Heat flux begins with a high value at the beginning of the heated surface and decreases along heated surface. At a given position, it is seen that, e.g. at $x/L$=0.8, heat flux increases with increasing $\theta$ at same constant surface temperature.

The effect of $\theta$ on local Nusselt number is shown in Figure 15(a) and 15(b) along heated surface for $x_0/L$=10 and $Re_{x0}$=7710. In Figure 15(a), typical local Nusselt number for constant surface temperature is plotted along heated surface for different $\theta$. It is seen that local Nusselt number decreases along the heated surface. Also, it is seen that local Nusselt number increases with increasing $\theta$ at a constant position. Similar result can be seen in Figure 10 as well. In other words, maximum Nusselt number is obtained for $\theta$=75°. In Figure 15(b), local Nusselt number for constant surface heat flux is shown along heated surface at different $\theta$ values for $x_0/L$=10 and $Re_{x0}$=7710. It is seen that local Nusselt number has similar behavior as the case of constant surface temperature. As seen in Figure 15, inclination angle of flat plate affects the local Nusselt number.

## 5. CONCLUSION

Two-dimensional laminar flow and forced convection on an inclined flat plate with an unheated starting length are numerically investigated under both constant surface temperature and constant heat flux boundary conditions.

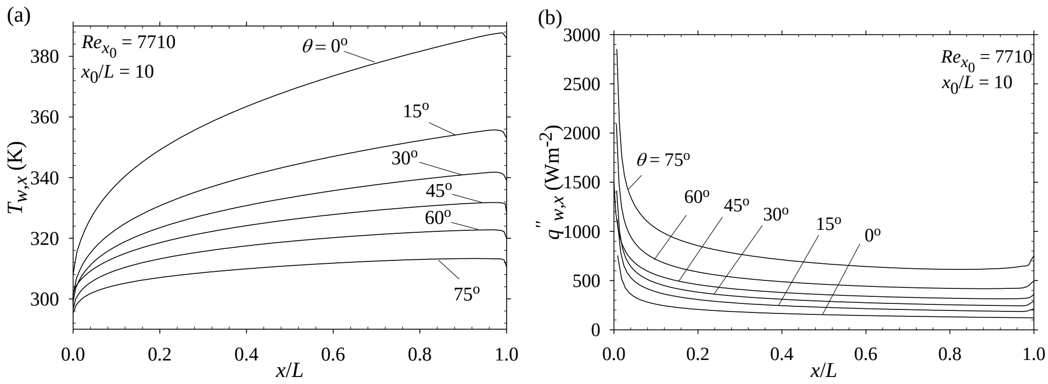


**Figure 14.** Variation of (a) surface temperature and (b) heat flux along heated surface

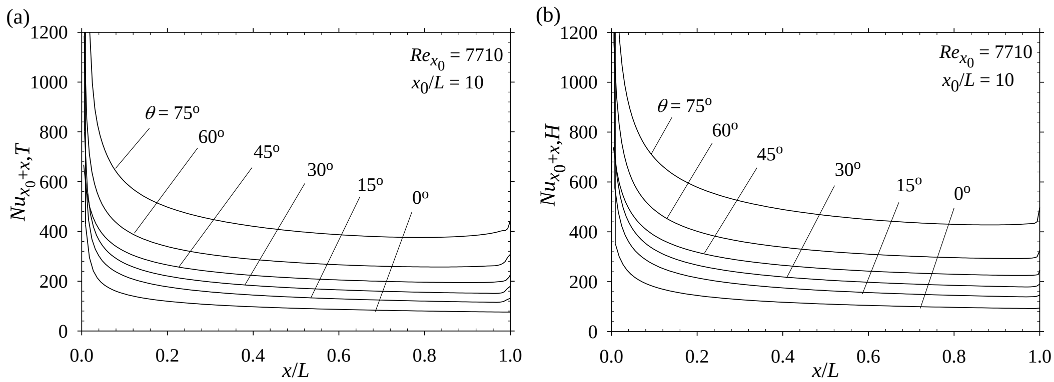


**Figure 15.** Local Nusselt number versus $x/L$ for $Re_{x0}$=7710 at constant temperature (a) and heat flux (b)

Numerical analysis has been carried out for different Reynolds numbers ($7710 \leq Re_{x0} \leq 77120$), different inclination angles of flat plate ($0° \leq \theta \leq 75°$), and two different unheated starting length ($x_0/L$=1 and 10). Air is used as the working fluid ($Pr$=0.71). Some conclusions are as follows:

- It is seen that the inclination angle, unheated starting length of flat plate and Reynolds number affect the heat transfer.
- Heat transfer decreases with increasing unheated starting length but increases with increasing Reynolds number and inclination angle of flat plate.
- It is seen that velocity boundary layer affects the thermal boundary layer.
- Unheated starting length affects the surface temperature of the heated section.
- Thermal boundary layer thickness for $x_0/L$=10 is greater than that of $x_0/L$=1.
- Heat transfer increases with increasing inclination angle of inclined flat plate due to decreasing thermal boundary layer thickness.
- Heat transfer coefficient for constant heat flux boundary condition is greater than that of constant temperature boundary condition.
- Hydrodynamic and thermal boundary layers decrease with increasing Reynolds number as expected.

## REFERENCES


[1] Nagendra H. R., "Transient forced convection heat transfer from an isothermal flat plate", ***AIAA Journal***, 11: 876-878, (1973).

[2] Sarpkaya T., "An inviscid model of two-dimensional vortex shedding for transient and asymptotically steady separated flow over an inclined plate", ***Journal of Fluid Mechanics***, 68: 109-128, (1975).

[3] Dey J. and Nath G., "Forced convection heat transfer over a semi-infinite plate", ***International Journal of Heat and Mass Transfer***, 25: 1774-1776, (1982).

[4] Ma S. W., Gerner F. M. and Tsuei Y. G., "Composite expansions on forced convection over a flat plate with an unheated starting length", ***International Journal of Heat and Mass Transfer***, 35: 3275-3289, (1992).

[5] Ameel T. A., "Average effects of forced convection over a flat plate with an unheated starting length", ***International Communications in Heat and Mass Transfer***, 24: 1113-1120, (1997).

[6] Chamkha A. J., "Hydromagnetic Free convection flow over an inclined plate caused by solar radiation", ***Journal of Thermophysics and Heat Transfer***, 19: 312-314, (1997).

[7] Vynnycky M., Kimura S., Kanev K. and Pop I., "Forced convection heat transfer from a flat plate: The conjugate problem", ***International Journal of Heat and Mass Transfer***, 41: 45-59, (1998).

[8] Yovanovich M. M. and Teertstra P., "Laminar forced convection from isothermal rectangular plates from small to large Reynolds numbers", ***American Institute of Aeronautics and Astronautics***, AIAA-98-2675: 1-14, (1998).

[9] Umur H. and Karagöz I., "An investigation of external flows with various pressures and surfaces", ***International Communications in Heat and Mass Transfer***, 26: 411-419, (1999).

[10] Kondjoyan A., Péneau F. and Boisson H. C., "Effect of high free stream turbulence on heat transfer between plates and air flows: A review of existing experimental results", ***International Journal of Thermal Sciences***, 41: 1–16, (2002).

[11] Nan J., Yu-chun W., Whei S. and Zhen-dong W., "Experimental study of measurement for dissipation rate scaling exponent in heated wall turbulence", ***Applied Mathematics and Mechanics***, 23: 1035-1044, (2002).

[12] Kondjoyan A., Péneau F. and Boisson H. C., "Development of flat-plate thermal and velocity boundary layers under highly turbulent and instable air flows: Reynolds numbers ranging from 8400 to 127000", ***International Journal of Thermal Sciences***, 43: 1091–1100, (2004).

[13] Rebay M., Padet J. and Kakaç S., "Forced convection from a microstructure on a flat plate", ***Heat and Mass Transfer***, 43: 309-317, (2007).

[14] Juncu Gh., "Unsteady conjugate forced convection heat/mass transfer from a finite flat plate", ***International Journal of Thermal Sciences***, 47: 972–984, (2008).

[15] Li H. and Nalim M. R., "Thermal-boundary-layer response to convected far-field fluid temperature changes", ***Journal of Heat Transfer-Transactions of the ASME***, 130: 101001-101001-6, (2008).

[16] Palani G., "Convection on flow past an inclined plate with variable surface temperatures in water at 4°C", ***Journal of Engineering Annals of Faculty of Engineering Hunedoara***, 6: 75-82, (2008).

[17] Kumar S. and Mullick S. C., "Wind heat transfer coefficient in solar collectors in outdoor conditions", ***Solar Energy***, 84: 956–963, (2010).

[18] Lam K. M. and Wei C. T., "Numerical simulation of vortex shedding from an inclined flat plate", ***Engineering Applications of Computational Fluid Mechanics***, 4: 569-579, (2010).

[19] Palani G. and Kim K.Y., "Viscous dissipation effects on heat transfer in flow over an inclined plate", ***Journal of Applied Mechanics and Technical Physics***, 51: 241–248, (2010).

[20] Li H., Nalim M. R. and Merkle C. L., "Transient thermal response of turbulent compressible boundary layers", ***Journal of Heat Transfer-Transactions of the ASME***, 133: 081701-081701-8, (2011).

[21] Malvandi A., Ganji D. D., Hedayati F., Kaffash H. M. and Jamshidi M., "Series solution of entropy generation toward an isothermal flat plate", ***Thermal Science***, 16: 1289-1295, (2012).

[22] Kumar H., "Heat and mass transfer over an isothermal inclined plate at constant concentration gradient and with heat source", ***World Applied Sciences Journal***, 24: 364-369, (2013).

[23] Kumar H., "An analytical solution to the problem of radiative heat and mass transfer over an inclined plate at a prescribed heat flux with chemical reaction", ***Journal of the Serbian Chemical Society***, 78: 873–881, (2013).

[24] Samanta S. and Guha A., "Similarity theory for forced convection over horizontal plates", ***Journal of Thermophysics and Heat Transfer***, 27: 506-514, (2013).

[25] Islam M., Akter F. and Islam A., "Mass Transfer Flow Through an Inclined Plate with Porous Medium", ***American Journal of Applied Mathematics***, 3: 215-220, (2015).

[26] Jana S. and Das K., "Influence of variable fluid properties, thermal radiation and chemical reaction on MHD slip flow over a flat plate", ***Italian Journal of Pure and Applied Mathematics***, 34: 29-44, (2015).

[27] Uddin M. J., Khan W. A., Ismail A. I. and Hamad M. A. A., "New similarity solution of boundary layer flow along a continuously moving convectively heated horizontal plate by deductive group method", ***Thermal Science***, 19: 1017-1024, (2015).

[28] Shanmugapriya M., "Magnetohydrodynamic (MHD) mixed convective flow and heat transfer over an inclined plate with radiation effect", ***ARPN Journal of Engineering and Applied Sciences***, 11: 2130-2136, (2016).

[29] Singh R. K., Galvin J. E. and Sun X., "Three-dimensional simulation of rivulet and film flows over an inclined plate: Effects of solvent properties and contact angle", ***Chemical Engineering Science***, 142: 244-257, (2016).

[30] Incropera F. P., DeWitt D. P., Theodore L. B. and Adrienne S. L., "***Principles of heat and mass transfer***", John Wiley and Sons Inc., Singapore, (2013).