Nonlinear Analysis: Modelling and Control, 2019, Vol. 24, No. 1, 20–33 ISSN 1392-5113
https://doi.org/10.15388/NA.2019.1.2

Optical solitons and modulation instability analysis to the quadratic-cubic nonlinear Schrödinger equation

Mustafa Inc[a], **Aliyu Isa Aliyu**[a,b], **Abdullahi Yusuf**[a,b], **Dumitru Baleanu**[c,d]

[a]Department of Mathematics, Firat University,
23119 Elaziğ, Turkey
minc@firat.edu.tr

[b]Department of Mathematics, Federal University Dutse,
7156 Jigawa, Nigeria
aliyu.isa@fud.edu.ng; yusufabdullahi@fud.edu.ng

[c]Department of Mathematics, Cankaya University,
Öğretmenler Cad. 1406530, Ankara, Turkey

[d]Institute of Space Sciences,
Magurele, Bucharest, Romania
dumitru@cankaya.edu.tr

Received: November 11, 2017 / **Revised:** May 16, 2018 / **Published online:** December 14, 2018

Abstract. This paper obtains the dark, bright, dark-bright, dark-singular optical and singular soliton solutions to the nonlinear Schrödinger equation with quadratic-cubic nonlinearity (QC-NLSE), which describes the propagation of solitons through optical fibers. The adopted integration scheme is the sine-Gordon expansion method (SGEM). Further more, the modulation instability analysis (MI) of the equation is studied based on the standard linear-stability analysis, and the MI gain spectrum is got. Physical interpretations of the acquired results are demonstrated. It is hoped that the results reported in this paper can enrich the nonlinear dynamical behaviors of the PNSE.

Keywords: sine-Gordon expansion method, optical solitons, modulation instability.

1 Introduction

Nonlinear Schrödinger equations (NLSEs) appear in various areas of engineering sciences, physical and biological sciences. In particular, the NLSEs appears in fluid dynamics, nonlinear optics, plasma and nuclear physics [1, 30, 56]. The previous studies provide a test bed to investigate soliton solutions in presence of many different nonlinearities [16]. These days QC-NLSE has gathered significant attention. This paper will study a newly proposed form of nonlinearity called the quadratic-cubic law. The law first appeared in 2011 [22]. The model supports soliton solutions (both temporal and spatial) with applications in soliton lasers, optical communications, ultra fast soliton switches and logic gate devices [47]. Recently, we observe many new progresses in the field of nonlinear optics [2–15, 17–26, 28, 29, 31–46, 48–55, 57–60]. The NLSE with quadratic-cubic nonlinearity is given by [6, 19, 25, 27, 47, 54]:

$$\mathrm{i}\psi_t + a\psi_{xx} - b_1\psi|\psi| + b_2\psi|\psi|^2 = 0, \quad \mathrm{i} = \sqrt{-1}. \tag{1}$$

In Eq. (1), t and x are the independent variables representing the temporal and spatial variables, respectively. $\psi(x,t)$ is the dependent variable. The real-valued constant a represents group velocity dispersion (GVD), while b_1 and b_2 are real-valued constants. The chaotic phenomena of the equation was studied in [27]. In [47], the analytical self-similar wave solutions of the equation were constructed. In [54], the method of undetermined coefficients was adopted to extract the soliton solutions and the conservation laws of the equation were reported. In [19], the He's semi-inverse variational principle was adopted to study the equation. In [6], the Jaccobi elliptic function ansatz method was used to study the equation. In [25], the extended trial equation method was utilized to retrieve some soliton solutions of the equation.

This paper determines the dark, bright, dark-singular and a new dark-bright soliton solutions to the model by SGEM [23, 57]. The SGEM is a very strong approach to to retrieve the soliton solutions of nonlinear models. It has been used to study several NLSEs by several authors in [23, 34, 36–39, 57]. One advantage of this approach is that it has the capability to retrieve dark-bright or combined optical solitons and combined singular solitons, which are special types of soliton solutions combining the futures of bright and dark optical solitons in one expression, and dark-singular and singular solitons in another expression. The subsequent section gives a full description of the integration scheme that will be applied retrieve the soliton solutions of the governing equation. Finally, the MI of the equation will be studied using the standard linear-stability analysis [1, 2, 36, 48].

2 Description of the sine-Gordon expansion method

Consider the following sine-Gordon equation:

$$\psi_{xt} = \alpha \sin\psi, \tag{2}$$

where α is a non-zero constant. We apply the transformation

$$\psi(x,t) = u(\xi), \quad \xi = \eta(x + vt), \tag{3}$$

where v is the traveling wave velocity. Substituting Eq. (3) into Eq. (2), we obtain

$$u'' = \frac{\alpha}{v\eta^2}\sin\bigl(u(\xi)\bigr). \tag{4}$$

Equation (4) can be simplified to give

$$\left[\left(\frac{u(\xi)}{2}\right)'\right]^2 = \frac{\alpha}{v\eta^2}\sin^2\frac{u(\xi)}{2} + K, \tag{5}$$

where K is a constant of integration. By letting $K = 0$, $w(\xi) = u(\xi)/2$ and $f^2 = \alpha/(v\eta^2)$, Eq. (5) reduces to

$$w'(\xi)^2 = f^2 \sin^2\bigl(w(\xi)\bigr), \tag{6}$$

and in a more simplified form gives

$$w'(\xi) = f \sin\bigl(w(\xi)\bigr). \tag{7}$$

Setting $f = 1$ in Eq. (7), we get

$$w'(\xi) = \sin\bigl(w(\xi)\bigr). \tag{8}$$

Equation (8) has the following solutions:

$$\sin\bigl(w(\xi)\bigr) = \operatorname{sech}\xi \quad \text{or} \quad \cos\bigl(w(\xi)\bigr) = \tanh\xi, \tag{9}$$

and

$$\sin\bigl(w(\xi)\bigr) = \mathrm{i}\operatorname{csch}\xi \quad \text{or} \quad \cos\bigl(w(\xi)r\bigr) = \coth\xi. \tag{10}$$

To obtain the solution of the nonlinear partial differential equation of the form

$$P(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \psi_{xt}, \dots) = 0, \tag{11}$$

we use the following series solution:

$$u(w) = \sum_{j=1}^{n} \cos^{j-1} w \cdot [B_j \sin w + A_j \cos w] + A_0. \tag{12}$$

From Eqs. (9) and (10), the solution of Eq. (12) can be written as

$$u_1(\xi) = \sum_{j=1}^{n} \tanh^{j-1} \xi \cdot [B_j \operatorname{sech}\xi + A_j \tanh\xi] + A_0, \tag{13}$$

and

$$u_2(\xi) = \sum_{j=1}^{n} \coth^{j-1} \xi \cdot [B_j \operatorname{csch}\xi + A_j \coth\xi] + A_0. \tag{14}$$

To obtain the value of n, we use the balancing principle. Substituting n into Eq. (12) and putting of the result into the reduced ordinary differential equation using Eq. (8) give a system of algebraic equations. Equating the coefficients of $\sin^i w$ and $\cos^i w$ to zero and solving the result gives the values of A_i, B_i and v. Subsequently, one can retrieve the soliton solutions of Eq. (11).

2.1 Application to Eq. (1)

To solve Eq. (1), we apply the transformation

$$\psi(x,t) = u(\xi)\mathrm{e}^{\mathrm{i}\phi(x,t)}, \quad \xi = K(x - vt), \tag{15}$$

where

$$\phi = -kx + \omega t + \theta. \tag{16}$$

In Eq. (16), $\phi(x,t)$ represents the phase component, k is the frequency, ω represents the wave number, θ represents the phase constant. In Eq. (15), v is the velocity, while K represents the width of the traveling wave [54]. Putting Eq. (15) in Eq. (1) and separating into real and imaginary components, a pair of equation is acquired. The imaginary part yields

$$v = -2ak, \tag{17}$$

the speed of the soliton stays the same irrespective of the type of soliton in question real part, therefore,

$$\left(ak^2 + \omega\right)u + b_1u^2 - b_2u^3 - aK^2u'' = 0. \tag{18}$$

Balancing the terms of u^3 and u'' in Eqs. (18) and (19) gives $n = 1$. Substituting $n = 1$ into Eq. (12), we obtain

$$u(w) = B_1 \sin w + A_1 \cos w + A_0. \tag{19}$$

Substituting Eq. (19) and the necessary derivatives into Eq. (18) using Eq. (8) substituting trigonometric identities where necessary, we obtain the following algebraic expression:

$$\begin{aligned}
&ak^2A_0 + \omega A_0 + ak^2\cos wA_1 + \omega\cos wA_1 + 2aK^2\cos w\sin^2 wA_1 \\
&\quad + A_0^2b_1 + 2\cos wA_0A_1b_1 + A_1^2b_1 - \sin^2 wA_1^2b_1 - A_0^3b_2 - 3\cos wA_0^2A_1 \\
&\quad + b_2 - 3A_0A_1^2b_2 + 3\sin^2 wA_0A_1^2b_2 - \cos wA_1^3b_2 + \cos w\sin^2 wA_1^3b_2 \\
&\quad + ak^2\sin wB_1 + aK^2\sin wB_1 + \omega\sin wB_1 - 2aK^2\cos^2 w\sin w \\
&\quad + B_1 + 2\sin wA_0b_1B_1 + 2\cos w\sin wA_1b_1B_1 - 3\sin wA_0^2b_2B_1 \\
&\quad - 6\cos w\sin wA_0A_1b_2B_1 - 3\cos^2 w\sin wA_1^2b_2B_1 + \sin^2 w \\
&\quad + b_1B_1^2 - 3\sin^2 wA_0b_2B_1^2 - 3\cos w\sin^2 wA_1b_2B_1^2 - \sin wb_2B_1^3 \\
&\quad + \cos^2 w\sin wb_2B_1^3 = 0.
\end{aligned} \tag{20}$$

Equating each summation of the coefficients of trigonometric functions having the same power to zero, we obtain the following independent parametric equations.

$\cos w$:

$$A_1\left(ak^2 + \omega + 2A_0b_1 - 3A_0^2b_2 - A_1^2b_2\right) = 0, \tag{21}$$

constants:

$$A_0^2b_1 + A_1^2b_1 - A_0^3b_2 + A_0\left(ak^2 + \omega - 3A_1^2b_2\right) = 0, \tag{22}$$

$\sin w \cos w$:

$$2A_1(b_1 - 3A_0b_2)B_1 = 0, \tag{23}$$

$\sin^2 w$:

$$-(b_1 - 3A_0b_2)\left(A_1^2 - B_1^2\right) = 0, \tag{24}$$

$\sin w$:

$$B_1\left(ak^2 + aK^2 + \omega + 2A_0b_1 - 3A_0^2b_2 - b_2B_1^2\right) = 0, \tag{25}$$

$\sin w \cos^2 w$:

$$B_1\left(-2aK^2 - 3A_1^2b_2 + b_2B_1^2\right) = 0, \tag{26}$$

$\sin^2 w \cos w$:

$$A_1\left(2aK^2 + A_1^2b_2 - 3b_2B_1^2\right) = 0. \tag{27}$$

Solving Eqs. (21)–(27), we obtain the following families.

With the aid of Mathematica, from Eqs. (21)–(27) we can get

Case 1. $A_0 = b_1/3b_2$, $A_1 = \pm b_1/(3b_2)$, $a = b_1^2/(18K^2b_2)$, $B_1 = 0$, $\omega = -2b_1^2/(9b_2) + k^2b_1^2/(18K^2b_2)$.

Case 2. $A_0 = b_1/(3b_2)$, $A_1 = 0$, $a = b_1^2/(9K^2b_2)$, $B_1 = \pm\sqrt{2}K\sqrt{b_1^2/(K^2b_2)}/(3\sqrt{b_2})$, $\omega = -2b_1^2/(9b_2) - k^2b_1^2/(9K^2b_2)$.

Case 3. $A_0 = b_1/(3b_2)$, $A_1 = \pm b_1/(3b_2)$, $a = -2b_1^2/(9K^2b_2)$, $B_1 = \mathrm{i}b_1/(3b_2)$, $\omega = -2b_1^2/(9b_2) + 2k^2b_1^2/(9K^2b_2)$.

2.1.1 *Dark optical soliton*

From the coefficients in case 1 above we obtain the dark optical solitary solution

$$\psi(x,t) = \left\{\frac{b_1}{3b_2} \pm \frac{b_1}{3b_2}\tanh\left[Kx - \frac{2ktb_1^2}{18Kb_2}\right]\right\} \times \exp\left\{\mathrm{i}\left(-kx + \left(-\frac{2b_1^2}{9b_2} + \frac{k^2b_1^2}{18K^2b_2}\right)t + \theta\right)\right\}. \tag{28}$$

2.1.2 *Bright optical soliton*

From the coefficients in case 2 above we obtain the bright optical solitary solution

$$\psi(x,t) = \left\{\frac{b_1}{3b_2} \pm \frac{\sqrt{2}K\sqrt{\frac{b_1^2}{K^2b_2}}}{3\sqrt{b_2}}\operatorname{sech}\left[Kx - \frac{2ktb_1^2}{9Kb_2}\right]\right\} \times \exp\left\{\mathrm{i}\left(-kx + \left(-\frac{2b_1^2}{9b_2} - \frac{k^2b_1^2}{9K^2b_2}\right)t + \theta\right)\right\}. \tag{29}$$

2.1.3 Dark-bright optical soliton

From the coefficients in case 3 above we obtain the dark optical solitary solution

$$\psi(x,t) = \left\{\frac{b_1}{3b_2} \pm \mathrm{i}\frac{b_1}{3b_2}\operatorname{sech}\left[Kx - \frac{2ktb_1^2}{9Kb_2}\right] \pm \frac{b_1}{3b_2}\tanh\left[Kx - \frac{2ktb_1^2}{9Kb_2}\right]\right\} \times \exp\left\{\mathrm{i}\left(-kx + \left(-\frac{2b_1^2}{9b_2} + \frac{2k^2b_1^2}{9K^2b_2}\right)t + \theta\right)\right\}. \quad (30)$$

2.1.4 Singular solitons

From the coefficients in case 1 we obtain the dark-singular solution

$$\psi(x,t) = \left\{\frac{b_1}{3b_2} \pm \frac{b_1}{3b_2}\coth\left[Kx - \frac{2ktb_1^2}{18Kb_2}\right]\right\} \times \exp\left\{\mathrm{i}\left(-kx + \left(-\frac{2b_1^2}{9b_2} + \frac{k^2b_1^2}{18K^2b_2}\right)t + \theta\right)\right\}, \quad (31)$$

while the coefficients in case 2 give the dark-singular solitary wave

$$\psi(x,t) = \left\{\frac{b_1}{3b_2} - \mathrm{i}\frac{\sqrt{2}K\sqrt{\frac{b_1^2}{K^2b_2}}}{3\sqrt{b_2}}\operatorname{csch}\left[Kx - \frac{2ktb_1^2}{9Kb_2}\right]\right\} \times \exp\left\{\mathrm{i}\left(-kx + \left(-\frac{2b_1^2}{9b_2} - \frac{k^2b_1^2}{9K^2b_2}\right)t + \theta\right)\right\}. \quad (32)$$

2.1.5 Combined singular soliton

From the coefficients in case 3 above we acquire the dark optical solitary wave

$$\psi(x,t) = \left\{\frac{b_1}{3b_2} - \frac{b_1}{3b_2}\operatorname{csch}\left[Kx - \frac{2ktb_1^2}{9Kb_2}\right] \pm \frac{b_1}{3b_2}\coth\left[Kx - \frac{2ktb_1^2}{9Kb_2}\right]\right\} \times \exp\left\{\mathrm{i}\left(-kx + \left(-\frac{2b_1^2}{9b_2} + \frac{2k^2b_1^2}{9K^2b_2}\right)t + \theta\right)\right\}. \quad (33)$$

3 Physical expressions, discussion and comparative study

In this section, we compare the obtained solutions in this manuscript with the existing results in [6,19,25,54]. We observe that some of the solutions in this manuscript are newly constructed solutions. It is observed that our solutions are related to the physical features of optical and singular soliton solutions, which play a vital role in understanding various physical phenomena in nonlinear systems. The dark optical soliton Eq. (28) and the bright optical soliton Eq. (28) are similar to the ones obtained in [6, 19, 25, 54]. Singular soliton solutions similar to Eq. (28) and Eq. (28) have also been reported in [47].

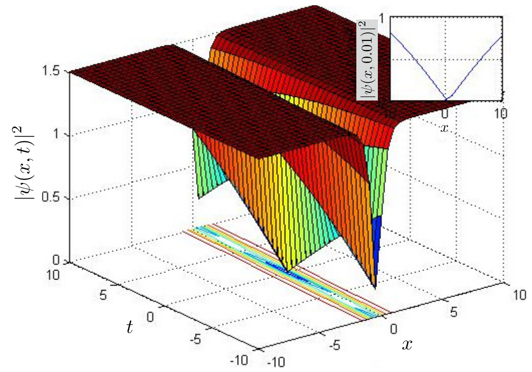

Figure 1. The 3D and 2D evolution of dark soliton Eq. (28) for some chosen parameters mentioned in the text.

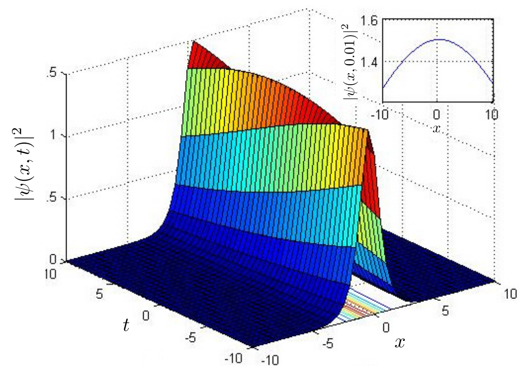

Figure 2. The 3D and 2D evolution of intensity of the bright soliton Eq. (29) for some chosen parameters mentioned in the text.

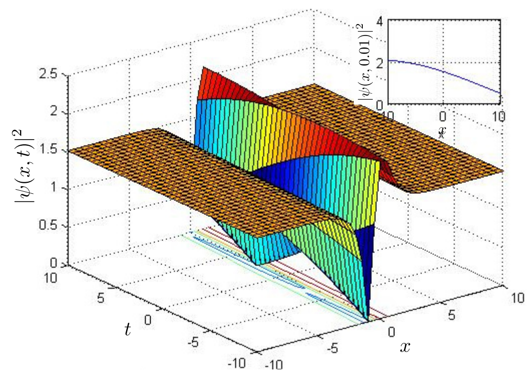

Figure 3. The 3D and 2D evolution of intensity of the dark-bright soliton Eq. (30) for some chosen parameters mentioned in the text.

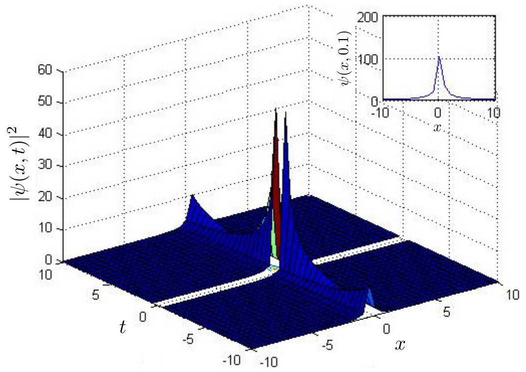

Figure 4. The 3D and 2D evolution of intensity of the dark-bright soliton Eq. (31) for some chosen parameters mentioned in the text.

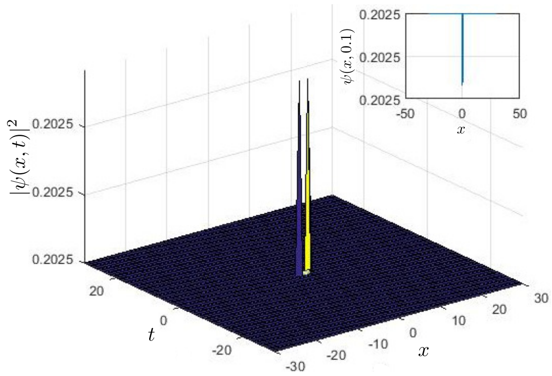

Figure 5. The 3D and 2D evolution of intensity of the dark-singular soliton Eq. (32) for some chosen parameters mentioned in the text.

Finally, the dark-bright optical soliton Eq. (30) and the combined singular soliton Eq. (33) are, to the best of our knowledge, new forms of solutions introduced into the literature for the first time in this work. To understand the physical nature and evolution of the optical soliton solutions Eqs. (28)–(33), we consider Figs. 1–5 for the dispersion parameters k, K, b_1 and b_2. For the dark optical soliton Eq. (28) and the dark-singular soliton Eq. (28), we choose $k = 0.2$, $K = 1$, $b_1 = 0.1$ and $b_2 = 0.4$. Plotting the 3D profiles of the bright optical soliton Eq. (28) and the singular soliton Eq. (28), we choose $k = 0.2$, $K = 1$, $b_1 = 0.2$ and $b_2 = 0.1$. On close observation, the intensity profiles of the optical soliton solutions Eqs. (28)–(33) are demonstrated in Figs. 1–5. We have found that the wave number k, associated with the phase component, plays a significant role in defining the trajectory of the pulses as they propagate through the waveguide. This may find applications in some optical devices such as counter, amplifier, etc. Moreover, we have verified that the solutions obtained in this paper are indeed solutions of Eq. (1), this is carried out with the assistance of Wolfram Mathematica 9.

4 Modulation instability analysis

In the previous section, optical solitons to Eq. (1) are got by employing the complex envelope function ansatz. Now we will study the MI. We apply the linear stability analysis technique. We suppose that Eq. (1) has the perturbed steady-state solution of the form

$$\psi(x,t) = \left[\sqrt{P_0} + \rho(x,t)\right] \cdot \mathrm{e}^{\mathrm{i}\phi_{NL}}, \quad \phi_{NL} = P_0 x, \tag{34}$$

where P_0 represent the incident power. We investigate the evolution of the perturbation $\rho(x,t)$ using the concept of linear stability analysis. Substituting Eq. (34) into Eq. (1) and linearizing the result in $\rho(x,t)$, we acquire

$$\mathrm{i}\rho_t + a\rho_{xx} - b_1\sqrt{P_0}\,(\rho + \rho^*) + b_2 P_0(\rho + \rho^*) = 0. \tag{35}$$

The linear equation Eq. (35) can be solved in the frequency domain easily. But because of the ρ^* component, the Fourier terms at frequencies Ω and $-\Omega$ are coupled. So, we seek for

$$\rho(t,x) = a_1 \cdot \mathrm{e}^{\mathrm{i}(Kx - \Omega t)} + a_2 \cdot \mathrm{e}^{-\mathrm{i}(Kx - \Omega t)}, \tag{36}$$

where K is the wave number, Ω is the frequency of the perturbation, respectively. Eqs. (35) and (36) give a set of two homogeneous equations in a_1 and a_2. Substituting Eq. (36) into Eq. (35), we get the following system of equations for a_1 and a_2 upon separating the coefficients of $\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}$ and $\mathrm{e}^{-\mathrm{i}(Kx-\Omega t)}$:

$$\begin{aligned} &\left(aK^2 - \Omega\right)a_1 + (a_1 + a_2)b_1\sqrt{P_0} - (a_1 + a_2)b_2 P_0 = 0, \\ &\left(aK^2 + \Omega\right)a_2 + (a_1 + a_2)b_1\sqrt{P_0} - (a_1 + a_2)b_2 P_0 = 0. \end{aligned} \tag{37}$$

From Eq. (37) one can easily obtain the following coefficient matrix of a_1 and a_2:

$$\begin{pmatrix} aK^2 - \Omega + b_1\sqrt{P_0} - b_2 P_0 & b_1\sqrt{P_0} - b_2 P_0 \\ b_1\sqrt{P_0} - b_2 P_0 & aK^2 + \Omega + b_1\sqrt{P_0} - b_2 P_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{38}$$

The coefficient matrix Eq. (38) has a nontrivial solution if the determinant vanishes. By expanding the determinant, we obtain the following dispersion relation:

$$a^2K^4 - \Omega^2 + 2aK^2 b_1\sqrt{P_0} - 2aK^2 b_2 P_0 = 0. \tag{39}$$

The dispersion relation Eq. (39) has the following solution:

$$K = \frac{1}{a}\sqrt{-ab_1\sqrt{P_0} + ab_2P_0 - \sqrt{a^2\left(\Omega^2 + b_1^2P_0 - 2b_1b_2P_0^{3/2} + b_2^2P_0^2\right)}}. \tag{40}$$

The stability of the steady state is determined by Eq. (40). If the wave number K has an imaginary part, the steady-state solution is unstable since the perturbation grows exponentially. But if the wave number K is real, the steady state is stable against small

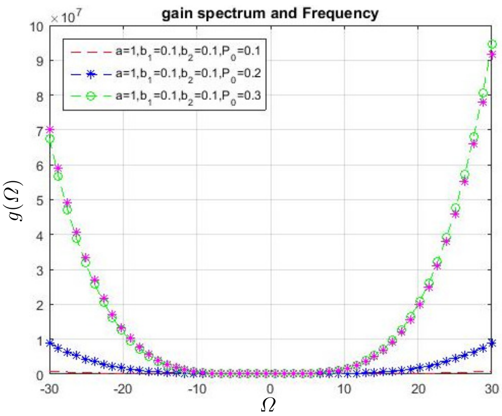

Figure 6. Regions of modulation instability gain spectrum Eq. (1) for different values.

perturbations. Thus, the necessary condition necessary for the existence of modulation instability to occur from Eq. (36) is when either

$$P_0 < 0, \qquad a^2\left(\Omega^2 + b_1^2 P_0 - 2b_1 b_2 P_0^{3/2} + b_2^2 P_0^2\right) < 0,$$

or

$$-ab_1\sqrt{P_0} + ab_2 P_0 - \sqrt{a^2\left(\Omega^2 + b_1^2 P_0 - 2b_1 b_2 P_0^{3/2} + b_2^2 P_0^2\right)} < 0.$$

Finally, we obtain the MI gain spectrum as

$$\begin{aligned} g(\Omega) &= 2\operatorname{Im} k \\ &= 2\left\{\frac{1}{a}\sqrt{-ab_1\sqrt{P_0} + ab_2 P_0 - \sqrt{a^2\left(\Omega^2 + b_1^2 P_0 - 2b_1 b_2 P_0^{3/2} + b_2^2 P_0^2\right)}}\right\}. \end{aligned}$$

The MI gain is significantly affected by the incident power P_0. From Fig. 3 it can be seen that the MI growth rates increases with increase in incidence power values. The main reason is due to increase in the gain along the fiber length. It is observed that two different side bands appear in the MI gain, but the intensity of the MI gain remains closely the same with increase of incident power.

5 Concluding remarks

This paper secured the dark, bright, dark-singular and dark-bright or combined and singular solitons to the QC-NLSE in optical fibers. The adopted integration algorithm is the sine-Gordon equation expansion method. Acquired dark-bright optical soliton and the combined singular soliton solutions are added to existing solutions in the literature. By applying the concept of linear stability analysis, the modulation instability analysis is studied and the MI gain spectrum is reported. All the acquired solutions satisfy the original equation. The results of the paper are truly encouraging to conduct further research in this avenue. Some interesting figures are also presented in Figs. 1–3.

References

1. G.P. Agrawal, *Nonlinear Fiber Optics*, Academic Press, New York, 1995.
2. G.P. Agrawal, P.L. Baldeck, R.R. Alfano, Modulation instability induced by cross-phase modulation in optical fibers, *Phys. Rev. A*, **39**:3406–3413, 1989.
3. M.M. Al-Qurashi, A. Yusuf, A.I. Aliyu, M. Inc, Optical and other solitons for the fourth-order dispersive nonlinear schrodinger equation with dual-power law nonlinearity, *Superlattices Microstruct.*, **105**:183–197, 2017.
4. A.H. Arnous, M. Mirzazadeh, M. Eslami, Exact solutions of the Drinfel–Sokolov–Wilson equation using the Backlund transformation of Riccati equation and trial function approach, *Pramana J. Phys.*, **86**:1153–1160, 2016.
5. A.H. Arnous, M.Z. Ullah, M. Asma, S.P. Moshokoa, Q. Zhou, M. Mirzazadeh, A. Biswas, M. Belic, Dark and singular dispersive optical solitons of Schrödinger–Hirota equation by modified simple equation method, *Optik*, **136**:445–450, 2017.
6. E.C. Aslan, M. Inc, Soliton solutions of NLSE with quadratic-cubic nonlinearity and stability analysis, *Waves Random Complex Media*, **27**:594–601, 2017.
7. M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Optical soliton pertubation with quadratic-cubic nonlinearity by semi-inverse variational principle, *Proc. Rom. Acad., Ser. A, Math. Phys. Tech. Sci. Inf. Sci.*, **18**:331–336, 2017.
8. M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Optical soliton pertubation with quadratic-cubic nonlinearity by the method of undetermined coefficient, *J. Optoelectron. Adv. Mater.*, **19**:699–703, 2017.
9. M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Optical soliton pertubation with quadratic-cubic nonlinearity by traveling wave hypothesis, *Optoelectron. Adv. Mater. Rapid Commun.*, **11**:517–519, 2017.
10. H.O. Bakodah, A.A. Al Qarni, M.A. Banaja, Q. Zhou, S.P. Moshokoa, A. Biswas, Bright and dark thirring optical solitons with improved adomian decomposition scheme, *Optik*, **130**:1115–1123, 2017.
11. D. Baleanu, M. Inc, A. Yusuf, A.I. Aliyu, Optical solitons, nonlinear self-adjointness and conservation laws for Kundu–Eckhaus equation, *Chin. J. Phys.*, **55**:2341–2355, 2017.
12. I. Bernstein, E. Zerrad, Q. Zhou, A. Biswas, N. Melikechi, Dispersive optical solitons with Schrödinger–Hirota equation by traveling wave hypothesis, *Optoelectron. Adv. Mater. Rapid Commun.*, **9**:792–797, 2015.
13. A.H. Bhrawy, A.A. Alshaery, E.M. Hila, W.N. Manrakhan, M. Savescu, A. Biswas, Dispersive optical solitons with Schrödinger–Hirota equation, *Optoelectron. Adv. Mater. Rapid Commun.*, **23**:1450014, 2014.
14. A.H. Bhrawy, A.A. Alshaery, E.M. Hila, W.N. Manrakhan, M. Savescu, A. Biswas, Dispersive optical solitons with Schrödinger–Hirota equation, *J. Nonlinear Opt. Phys. Mater.*, **23**:1450014, 2014.

15. A. Biswas, Soliton solutions of the perturbed resonant nonlinear Schrödinger's equation with full nonlinearity by semi-inverse variational principle, *Quant. Phys. Lett.*, **1**(2):79–86, 2012.
16. A. Biswas, S. Konar, *Introduction to Non-Kerr Law Optical Solitons*, CRC Press, Boca Raton, FL, 2006.
17. A. Biswas, E. Topkara, S. Johnson, E. Zerrad, S. Kunar, Optical soliton pertubation in non-Kerr law media: Travelling wave solution, *J. Nonlinear Opt. Phys. Mater.*, **24**:309–331, 2011.
18. A. Biswas, M.Z. Ullah, M. Asma, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons with quadratic-cubic nonlinearity by semi-inverse varitional principle, *Optik*, **139**:16–19, 2017.
19. A. Biswas, M.Z. Ullah, M. Asmac, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons with quadratic-cubic nonlinearity by semi-inverse variational principle, *Optik*, **139**:16–19, 2017.
20. A. Biswas, M.Z. Ullah, Q. Zhou, S.P. Moshokoa, H. Triki, M. Belic, Resonance optical solitons with quadratic-cubic nonlinearity by semi-inverse variational principle, *Optik*, **145**:18–21, 2017.
21. A. Biswas, Q. Zhou, M.Z. Ullah, M. Asma, S.P. Moshokoa, M. Belic, Perturbation theory and optical soliton cooling with anti-cubic nonlinearity, *Optik*, **142**(3):73–76, 2017.
22. N. Cheemaa, M. Younis, New and more general traveling wave solutions for nonlinear Schrödinger's equation, *Waves Random Complex Media*, **26**:84–91, 2016.
23. Y. Chen, Z. Yan, A simple transformation for nonlinear waves, *Chaos Solitons Fractals*, **26**:399–406, 2005.
24. M. Ekici, M. Mirzazadeh, M. Eslami, Q. Zhou, S.P. Moshokoa, A. Biswas, M. Belic, Optical soliton perturbation with fractional-temporal evolution by first integral method with conformable fractional derivatives, *Optik*, **127**(3):10659–10669, 2016.
25. M. Ekici, Q. Zhou, A. Sonmezoglu, S.P. Moshok, M.Z. Ullah, H. Triki, A. Biswas, M. Belic, Optical solitons in nonlinear negative-index materials with quadratic-cubic nonlinearity, *Superlattices Microstruct.*, **107**:176–182, 2017.
26. M. Ekici, Q. Zhou, A. Sonmezoglu, S.P. Moshokoa, M.Z. Ullah, H. Triki, A. Biswas, M. Belic, Optical solitons in nonlinear negative-index materials with quadratic-cubic nonlinearity, *Superlattices Microstruct.*, **109**:176–182, 2017.
27. J. Fujioka, E. Cortes, R. Perez-Pascual, R.F. Rodriguez, A. Espinosa, B.A. Malomed, Chaotic solitons in the quadratic-cubic nonlinear Schrödinger equation under nonlinearity management, *Chaos*, **21**:63–70, 2011.
28. X. Geng, Y. Lv, Darboux transformation for an integrable generalization of the nonlinear Schrödinger equation, *Nonlinear Dyn.*, **69**:1621–1630, 2012.
29. I. Haddouche, L. Cherbi, A. Biswas, Highly sensitive optical immunosensor for bacteria detection in water, *Optoelectron. Adv. Mater. Rapid Commun.*, **11**:46–50, 2017.
30. A. Hasegawa, Y. Kodama, *Solitons in Optical Communication*, Clarendon Press, Oxford, 1995.
31. M. Inc, A.I. Aliyu, A. Yusuf, Optical solitons to the nonlinear Schrödinger's equation with spatio-temporal dispersion using complex amplitude ansatz, *Optik*, **64**:2273–2280, 2017.
32. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Dark and combined optical solitons, and modulation instability analysis in dispersive metamaterial, *Optik*, **157**:484–491, 2017.

33. M. Inc, A.I Aliyu, A. Yusuf, D. Baleanu, New solitary wave solutions and conservation laws to the Kudryashov–Sinelshchikov equation, *Optik*, **142**:665–673, 2017.

34. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Optical solitary waves, conservation laws and modulation instability analysis to the nonlinear Schrödinger's equation in compressional dispersive Alven waves, *Optik*, **155**:257–266, 2017.

35. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Combined optical solitary waves and conservation laws for nonlinear Chen–Lee–Liu equation in optical fibers, *Optik*, **158**:297–304, 2018.

36. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Dispersive optical solitons and modulation instability analysis of Schrödinger–Hirota equation with spatio-temporal dispersion and Kerr law nonlinearity, *Superlattices Microstruct.*, **113**:319–327, 2018.

37. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Optical solitons for Biswas–Milovic model in nonlinear optics by sine-Gordon equation method, *Optik*, **157**:267–274, 2018.

38. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Optical solitons for complex Ginzburg–Landau model in nonlinear optics, *Optik*, **158**:368–375, 2018.

39. M. Inc, A.I. Aliyu, A. Yusuf, D. Baleanu, Optical solitons to the resonance nonlinear Schrödinger equation by sine-Gordon equation method, *Superlattices Microstruct.*, **113**:541–549, 2018.

40. M. Inc, A. Yusuf, A.I. Aliyu, Dark optical and other soliton solutions for the three different nonlinear Schrödinger's equations, *Opt. Quantum Electron.*, **49**:354, 2017.

41. M.A. Jawad, M.D. Petkovic, A. Biswas, Modified simple equation method for nonlinear evolution equations, *Appl. Math. Comput.*, **217**:869–877, 2010.

42. A.H. Kara, P. Razborova, A. Biswas, Solitons and conservation laws of coupled Ostrovsky equation for internal waves, *Appl. Math. Comput.*, **25**:95–99, 2015.

43. M. Mirzazadeh, R.T. Alqahtani, A. Biswas, Optical soliton pertubation with quadratic-cubic nonlinearity by Riccati–Bernoulli sub-ODE method and Kudryashov's scheme, *Optik*, **145**(3):74–78, 2017.

44. M. Mirzazadeh, M. Eslami, A. Biswas, Dispersive optical solitons by Kudryashov's method, *Optik*, **125**(3):6874–6880, 2014.

45. M. Mirzazadeh, M.F. Mahmood, F.B. Majid, A. Biswas, M. Belic, Optical solitons in birefringent fibers with Riccati equation method, *Optoelectron. Adv. Mater. Rapid Commun.*, **9**(3):1032–1036, 2015.

46. M. Mirzazadeh, Q. Zhou, E. Zerrad, M.F. Mahmood, A. Biswas, M. Belic, Bifurcation analysis and bright soliton of generalized resonant dispersive nonlinear schrodinger's equation, *Optoelectron. Adv. Mater. Rapid Commun.*, **9**:1342–1346, 2015.

47. R. Pal, S. Loomba, C.N. Kumar, Chirped self-similar waves for quadratic-cubic nonlinear Schrödinger equation, *Ann. Phys.*, **387**:213–221, 2017.

48. M. Saha, A.K. Sarma, Solitary wave solutions and modulation instability analysis of the nonlinear Schrödinger equation with higher order dispersion and nonlinear terms, *Commun. Nonlinear Sci. Numer. Simul.*, **18**:2420–2425, 2003.

49. E. Yaşar, Y. Yıldı rım, Q. Zhou, S.P. Moshokoa, M.Z. Ullah, H. Triki, A. Biswas, M. Belic, Perturbed dark and singular optical solitons in polarization preserving fibers by modified simple equation method, *Superlattices Microstruct.*, **2**(3):79–84, 2012.

50. M. Savescu, A.A. Alshaery, E.M. Hilal, A.H. Bhrawy, Q. Zhou, A. Biswas, Optical soliton pertubation in non-Kerr law media: Travelling wave solution, *Optoelectron. Adv. Mater. Rapid Commun.*, **9**:14–19, 2015.

51. A.R. Seadawy, Approximation solutions of derivative nonlinear Schrödinger equation with computational applications by variational method, *Eur. Phys. J. Plus*, **130**:1–10, 2015.

52. A.R. Seadawy, Ionic acoustic solitary wave solutions of two-dimensional nonlinear Kadomtsev–Petviashvili–Burgers equations in quntum plasma, *Math. Methods Appl. Sci.*, **40**(3):1598–1607, 2017.

53. A.R. Seadawy, Modulation instability analysis for the generalized derivative higher order nonlinear Schrödinger equation and its the bright and dark soliton solutions, *J. Electromagn. Waves Appl.*, **31**:1353–1362, 2017.

54. H. Triki, A. Biswas, S.P. Moshokoa, M. Belic, Optical solitons and conservation laws with quadratic-cubic nonlinearity, *Optik*, **128**:63–70, 2017.

55. H. Triki, A. Yıldı rım, T. Hayat, O.M. Aldossary, A. Biswas, 1-soliton solution of the generalized resonant nonlinear dispersive Schrödinger's equation with time-dependent coefficients, *Adv. Sci. Lett.*, **16**(1):309–312, 2012.

56. G.B. Whitham, *Linear and Nonlinear Waves*, John Whiley, New York, 174.

57. C. Yan, Z. Yan, New exact solutions of $(2+1)$-dimensional Gardner equation via the new sine-Gordon equation expansion method, *Phys. Lett. A*, **224**:77–84, 1996.

58. X. Yanan, M. Savescu, K.R. Khan, M.F. Mahmood, A. Biswas, M. Belic, Soliton propagation through nanoscale waveguides in optical metamaterials, *Opt. Laser Technol.*, **77**:177–186, 2016.

59. M. Younis, H. Rehman, S.T. Rizvi, Optical soliton like pulses in ring cavity fibers of carbon nanotubes, *J. Nanoelectron. Optoelectron.*, **11**(3):276–279, 2017.

60. M. Younis, H. Rehman, S.T. Rizvi, S.A. Mahmood, Dark and singular optical solitons perturbation with fractional temporal evolution, *Superlattices Microstruct.*, **106**:156–162, 2017.