Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 202650, 6 pages
http://dx.doi.org/10.1155/2013/202650

Research Article

A Local Fractional Variational Iteration Method for Laplace Equation within Local Fractional Operators

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Received 15 November 2012; Accepted 27 February 2013

Academic Editor: Syed Tauseef Mohyud-Din

The local fractional variational iteration method for local fractional Laplace equation is investigated in this paper. The operators are described in the sense of local fractional operators. The obtained results reveal that the method is very effective.

1. Introduction

As it is known, the partial differential equations [1, 2] and fractional differential equations [3–5] appear in many areas of science and engineering. As a result, various kinds of analytical methods and numerical methods were developed [6–8]. For example, the variational iteration method [9–15] was applied to solve differential equations [16–18], integral equations [19], and numerous applications to different nonlinear equations in physics and engineering. Also, the fractional variational iteration method [20–23] and the fractional complex transform [24–27] were discussed recently. The efficient techniques have successfully addressed a wide class of nonlinear problems for differential equations; see [28–36] and the references therein. We notice that the developed methods are very convenient, efficient, and accurate.

Recently, the local fractional variational iteration method [37] is derived from local fractional operators [38–48]. The method, which accurately computes the solutions in a local fractional series form or in an exact form, presents interest to applied sciences for problems where the other methods cannot be applied properly.

In this paper, we investigate the application of local fractional variational iteration method for solving the local fractional Laplace equations [49] with the different fractal conditions.

This paper is organized as follows.

In Section 2, the basic mathematical tools are reviewed. Section 3 presents briefly the local fractional variational iteration method based on local fractional variational for fractal Lagrange multipliers. Section 4 presents solutions to the local fractional Laplace equations with differential fractal conditions.

2. Mathematical Fundamentals

In this section, we present few mathematical fundamentals of local fractional calculus and introduce the basic notions of local fractional continuity, local fractional derivative, and local fractional integral of nondifferential functions.

2.1. Local Fractional Continuity

Lemma 1 (see [42]). *Let F be a subset of the real line and a fractal. If $f:(F,d)\to(\Omega',d')$ is a bi-Lipschitz mapping, then there is, for constants $\rho,\tau>0$ and $F\subset R$,*

$$\rho^s H^s(F)\le H^s(f(F))\le\tau^s H^s(F) \tag{1}$$

such that for all $x_1,x_2\in F$,

$$\rho^\alpha|x_1-x_2|^\alpha\le|f(x_1)-f(x_2)|\le\tau^\alpha|x_1-x_2|^\alpha. \tag{2}$$

As a direct result of Lemma 1, one has [42]

$$|f(x_1)-f(x_2)|\le\tau^\alpha|x_1-x_2|^\alpha \tag{3}$$

such that

$$|f(x_1)-f(x_2)|<\varepsilon^\alpha, \tag{4}$$

where α is fractal dimension of F.

Suppose that there is [38–43]

$$|f(x)-f(x_0)|<\varepsilon^\alpha \tag{5}$$

with $|x-x_0|<\delta$, for ε, $\delta>0$ and $\varepsilon,\delta\in R$, then $f(x)$ is called local fractional continuous at $x=x_0$ and it is denoted by

$$\lim_{x\to x_0}f(x)=f(x_0). \tag{6}$$

Suppose that the function $f(x)$ is satisfied the condition (5) *for $x\in(a,b)$, and hence it is called a local fractional continuous on the interval (a,b), denoted by*

$$f(x)\in C_\alpha(a,b). \tag{7}$$

2.2. Local Fractional Derivatives and Integrals. Suppose that $f(x)\in C_\alpha(a,b)$, then the local fractional derivative of $f(x)$ of order α at $x=x_0$ is given by [37–43]

$$\begin{aligned}D_x^{(\alpha)}f(x_0)=f^{(\alpha)}(x_0)&=\left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0}\\&=\lim_{x\to x_0}\frac{\Delta^\alpha(f(x)-f(x_0))}{(x-x_0)^\alpha},\end{aligned} \tag{8}$$

where $\Delta^\alpha(f(x)-f(x_0))\cong\Gamma(1+\alpha)\Delta(f(x)-f(x_0))$.

There is [38–40]

$$f(x)\in D_x^{(\alpha)}(a,b) \tag{9}$$

if

$$f^{(\alpha)}(x)=D_x^{(\alpha)}f(x) \tag{10}$$

for any $x\in(a,b)$.

Local fractional derivative of high order is written in the form [38–40]

$$f^{(k\alpha)}(x)=\overbrace{D_x^{(\alpha)}\cdots D_x^{(\alpha)}}^{k\text{ times}}f(x), \tag{11}$$

and local fractional partial derivative of high order is [38–40]

$$\frac{\partial^{k\alpha}}{\partial x^{k\alpha}}f(x)=\overbrace{\frac{\partial^\alpha}{\partial x^\alpha}\cdots\frac{\partial^\alpha}{\partial x^\alpha}}^{k\text{ times}}f(x). \tag{12}$$

Let a function $f(x)$ satisfy the condition (7). Local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is given by [37–43]

$$\begin{aligned}{}_aI_b^{(\alpha)}f(x)&=\frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^\alpha\\&=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta t\to0}\sum_{j=0}^{j=N-1}f(t_j)(\Delta t_j)^\alpha,\end{aligned} \tag{13}$$

where $\Delta t_j=t_{j+1}-t_j$, $\Delta t=\max\{\Delta t_1,\Delta t_2,\Delta t_j,\ldots\}$, and $[t_j,t_{j+1}]$, $j=0,\ldots,N-1$, $t_0=a$, $t_N=b$, is a partition of the interval $[a,b]$. For other definition of local fractional derivative, see [44–48].

There exists [38–40]

$$f(x)\in I_x^{(\alpha)}(a,b) \tag{14}$$

if

$$f^{(\alpha)}(x)={}_aI_x^{(\alpha)}f(x) \tag{15}$$

for any $x\in(a,b)$.

Local fractional multiple integrals of $f(x)$ is written in the form [40]

$${}_{x_0}I_x^{(k\alpha)}f(x)=\overbrace{{}_{x_0}I_x^{(\alpha)}\cdots{}_{x_0}I_x^{(\alpha)}}^{k\text{ times}}f(x) \tag{16}$$

if (7) is valid for $x\in(a,b)$.

3. Local Fractional Variational Iteration Method

In this section, we introduce the local fractional variational iteration method derived from the local fractional variational approach for fractal Lagrange multipliers [40].

Let us consider the local fractional variational approach in the one-dimensional case through the following local fractional functional, which reads [40]

$$I(y)={}_aI_b^{(\alpha)}f\left(x,y(x),y^{(\alpha)}(x)\right), \tag{17}$$

where $y^{(\alpha)}(x)$ is taken in local fractional differential operator and $a\le x\le b$.

The local fractional variational derivative is given by [40]

$$\delta^\alpha I={}_aI_b^{(\alpha)}\left\{\left(\frac{\partial f}{\partial y}-\frac{d^\alpha}{dx^\alpha}\left(\frac{\partial f}{\partial y^{(\alpha)}}\right)\right)\eta(x)\right\}, \tag{18}$$

where δ^α is local fractional variation signal and $\eta(a)=\eta(b)=0$.

The nonlinear local fractional equation reads as

$$L_\alpha u + N_\alpha u = 0, \tag{19}$$

where L_α and N_α are linear and nonlinear local fractional operators, respectively.

Local fractional variational iteration algorithm can be written as [37]

$$u_{n+1}(t) = u_n(t) + {}_{t_0}I_t^{(\alpha)}\left\{\xi^\alpha\left[L_\alpha u_n(s) + N_\alpha u_n(s)\right]\right\}. \tag{20}$$

Here, we can construct a correction functional as follows [37]:

$$u_{n+1}(t) = u_n(t) + {}_{t_0}I_t^{(\alpha)}\left\{\xi^\alpha\left[L_\alpha u_n(s) + N_\alpha \tilde{u}_n(s)\right]\right\}, \tag{21}$$

where $\tilde{u}_n$ is considered as a restricted local fractional variation and ξ^α is a fractal Lagrange multiplier; that is, $\delta^\alpha \tilde{u}_n = 0$ [37, 40].

Having determined the fractal Lagrangian multipliers, the successive approximations u_{n+1}, $n \geq 0$, of the solution u will be readily obtained upon using any selective fractal function u_0. Consequently, we have the solution

$$u = \lim_{n\to\infty} u_n. \tag{22}$$

Here, this technology is called the local fractional variational method [37]. We notice that the classical variation is recovered in case of local fractional variation when the fractal dimension is equal to 1. Besides, the convergence of local fractional variational process and its algorithms were taken into account [37].

4. Solutions to Local Fractional Laplace Equation in Fractal Timespace

The local fractional Laplace equation (see [38–40] and the references therein) is one of the important differential equations with local fractional derivatives. In the following, we consider solutions to local fractional Laplace equations in fractal timespace.

Case 1. Let us start with local fractional Laplace equation given by

$$\frac{\partial^{2\alpha}T(x,t)}{\partial t^{2\alpha}} + \frac{\partial^{2\alpha}T(x,t)}{\partial x^{2\alpha}} = 0 \tag{23}$$

and subject to the fractal value conditions

$$\frac{\partial^\alpha}{\partial t^\alpha}T(x,0) = 0, \quad T(x,0) = -E_\alpha(x^\alpha). \tag{24}$$

A corrected local fractional functional for (24) reads as

$$\begin{aligned} &u_{n+1}(x,t) \\ &\quad = u_n(x,t) \\ &\qquad + {}_0I_t^{(\alpha)}\left\{\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_n(x,\tau)}{\partial \tau^{2\alpha}} + \frac{\partial^{2\alpha}T_n(x,\tau)}{\partial x^{2\alpha}}\right)\right\}. \end{aligned} \tag{25}$$

Taking into account the properties of the local fractional derivative, we obtain

$$\begin{aligned} &\delta^\alpha u_{n+1}(x,t) \\ &\quad = \delta^\alpha u_n(x,t) \\ &\qquad + \delta^\alpha\, {}_0I_t^{(\alpha)}\left\{\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_n(x,\tau)}{\partial \tau^{2\alpha}} + \frac{\partial^{2\alpha}T_n(x,\tau)}{\partial x^{2\alpha}}\right)\right\}. \end{aligned} \tag{26}$$

Hence, from (25)-(26) we get

$$\begin{aligned} &\delta^\alpha u_{n+1}(x,t) \\ &\quad = \delta^\alpha u_n(x,t) + \frac{\lambda^\alpha}{\Gamma(1+\alpha)}\delta^\alpha u_n^{(\alpha)}(x,t)\Big|_{\tau=t} \\ &\qquad - \left[\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\right]^{(\alpha)}\delta^\alpha u_n(x,t)\Big|_{\tau=t} \\ &\qquad - \left(\delta^\alpha u_n(x,\tau)\right)\, {}_0I_t^{(\alpha)}\left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\right)^{(2\alpha)} \\ &\quad = \delta^\alpha u_n(x,t) + \frac{\lambda^\alpha}{\Gamma(1+\alpha)}\delta^\alpha u_n^{(\alpha)}\Big|_{\tau=t} \\ &\qquad - \left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\right)^{(\alpha)}\delta^\alpha u_n(x,t)\Big|_{\tau=t} \\ &\qquad + \left(\delta^\alpha u_n(x,\tau)\right)\, {}_0I_t^{(\alpha)}\left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\right)^{(2\alpha)} = 0. \end{aligned} \tag{27}$$

As a result, from (27) we can derive

$$\begin{gathered} \left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\right)^{(2\alpha)} = 0, \qquad \frac{\lambda^\alpha}{\Gamma(1+\alpha)}\Big|_{\tau=t} = 0, \\ \left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)}\right)^{(\alpha)} = 1. \end{gathered} \tag{28}$$

We have $\lambda = \tau - t$ such that the fractal Lagrange multiplier reads as

$$\frac{\lambda^\alpha}{\Gamma(1+\alpha)} = \frac{(\tau - t)^\alpha}{\Gamma(1+\alpha)}. \tag{29}$$

From (24) we take the initial value, which reads as

$$u_0(x,t) = -E_\alpha(x^\alpha). \tag{30}$$

By using (25) we structure a local fractional iteration procedure as

$$\begin{aligned} &u_{n+1}(x,t) \\ &\quad = u_n(x,t) \\ &\qquad + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_n(x,\tau)}{\partial \tau^{2\alpha}} + \frac{\partial^{2\alpha}T_n(x,\tau)}{\partial x^{2\alpha}}\right)\right\}. \end{aligned} \tag{31}$$

Hence, we can derive the first approximation term as

$$\begin{aligned}
u_1(x,t) &= u_0(x,t) + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_0(x,\tau)}{\partial\tau^{2\alpha}}+\frac{\partial^{2\alpha}T_0(x,\tau)}{\partial x^{2\alpha}}\right)\right\} \\
&= -E_\alpha(x^\alpha) + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}(-E_\alpha(x^\alpha))\right\} \\
&= E_\alpha(x^\alpha)\left(-1+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right).
\end{aligned} \tag{32}$$

The second approximation can be calculated in the similar way, which is

$$\begin{aligned}
u_2(x,t) &= u_1(x,t) + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_1(x,\tau)}{\partial\tau^{2\alpha}}+\frac{\partial^{2\alpha}T_1(x,\tau)}{\partial x^{2\alpha}}\right)\right\} \\
&= E_\alpha(x^\alpha)\left(-1+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right) + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{t^{2\alpha}E_\alpha(x^\alpha)}{\Gamma(1+2\alpha)}\right)\right\} \\
&= E_\alpha(x^\alpha)\left(-1+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{t^{4\alpha}}{\Gamma(1+4\alpha)}\right).
\end{aligned} \tag{33}$$

Proceeding in this manner, we get

$$u_n(x,t) = E_\alpha(x^\alpha)\left(\sum_{k=0}^{n}(-1)^k\frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)}\right). \tag{34}$$

Thus, the final solution reads as

$$\begin{aligned}
u(x,t) &= \lim_{n\to\infty}u_n(x,t) \\
&= E_\alpha(x^\alpha)\left(\sum_{k=0}^{\infty}(-1)^k\frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)}\right) \\
&= -E_\alpha(x^\alpha)\cos_\alpha(t^\alpha).
\end{aligned} \tag{35}$$

Case 2. Consider the local fractional Laplace equation as

$$\frac{\partial^{2\alpha}T(x,t)}{\partial t^{2\alpha}}+\frac{\partial^{2\alpha}T(x,t)}{\partial x^{2\alpha}}=0 \tag{36}$$

subject to fractal value conditions given by

$$\frac{\partial^\alpha}{\partial t^\alpha}T(x,0) = -E_\alpha(x^\alpha), \quad T(x,0)=0. \tag{37}$$

Now we can structure the same local fractional iteration procedure (31).

By using (36)-(37) we take an initial value as

$$u_0(x,t) = -\frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)}. \tag{38}$$

The first approximation term reads as

$$\begin{aligned}
u_1(x,t) &= u_0(x,t) + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_0(x,\tau)}{\partial\tau^{2\alpha}}+\frac{\partial^{2\alpha}T_0(x,\tau)}{\partial x^{2\alpha}}\right)\right\} \\
&= -\frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)} + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(-\frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)}\right)\right\} \\
&= -\frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)}+\frac{t^{3\alpha}E_\alpha(x^\alpha)}{\Gamma(1+3\alpha)}.
\end{aligned} \tag{39}$$

In the same manner, the second approximation is given by

$$\begin{aligned}
u_2(x,t) &= u_1(x,t) + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}T_1(x,\tau)}{\partial\tau^{2\alpha}}+\frac{\partial^{2\alpha}T_1(x,\tau)}{\partial x^{2\alpha}}\right)\right\} \\
&= -\frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)}+\frac{t^{3\alpha}E_\alpha(x^\alpha)}{\Gamma(1+3\alpha)} + {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{t^{3\alpha}E_\alpha(x^\alpha)}{\Gamma(1+3\alpha)}\right)\right\} \\
&= -\frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)}+\frac{t^{3\alpha}E_\alpha(x^\alpha)}{\Gamma(1+3\alpha)}-\frac{t^{5\alpha}E_\alpha(x^\alpha)}{\Gamma(1+5\alpha)}.
\end{aligned} \tag{40}$$

Finally, we can obtain the local fractional series solution as follows:

$$u_n(x,t) = E_\alpha(x^\alpha)\left(\sum_{k=0}^{n}(-1)^k\frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)}\right). \tag{41}$$

Thus, the expression of the final solution is given by

$$\begin{aligned}
u(x,t) &= \lim_{n\to\infty}u_n(x,t) \\
&= E_\alpha(x^\alpha)\left(\sum_{i=0}^{\infty}(-1)^k\frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)}\right) \\
&= -E_\alpha(x^\alpha)\sin_\alpha(t^\alpha).
\end{aligned} \tag{42}$$

As is known, the Mittag-Leffler function in fractal space can be written in the form

$$\begin{aligned}
&\left|E_\alpha(x^\alpha)-E_\alpha(x_0^\alpha)\right| \le E_\alpha(x_0^\alpha)\left|x-x_0\right|^\alpha < \varepsilon^\alpha, \\
&\left|\sin_\alpha(t^\alpha)-\sin_\alpha(t_0^\alpha)\right| < \left|\cos_\alpha(x_0^\alpha)\right|\left|t-t_0\right|^\alpha < \varepsilon^\alpha.
\end{aligned} \tag{43}$$

Hence, the fractal dimensions of both $E_\alpha(x^\alpha)$ and $\cos_\alpha(t^\alpha)$ are equal to α.

5. Conclusions

Local fractional calculus is set up on fractals and the local fractional variational iteration method is derived from local fractional calculus. This new technique is efficient for the applied scientists to process these differential and integral equations with the local fractional operators. The variational iteration method [9–19, 27] is derived from fractional calculus and classical calculus; the fractional variational iteration method [20–22, 27] is derived from the modified fractional derivative, while the local fractional variational iteration method [37] is derived from the local fractional calculus [37–43]. Other methods for local fractional ordinary and partial differential equations were considered in [27].

In this paper, two outstanding examples of applications of the local fractional variational iteration method to the local fractional Laplace equations are investigated in detail. The reliable obtained results are complementary with the ones presented in the literature.

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