# Diffraction by a black half plane: Modified theory of physical optics approach

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**Abstract:** The scattered fields from a black half plane which absorbs all the incoming electromagnetic energy are evaluated by defining a new modified theory of physical optics surface current. This current eliminates the reflected fields, coming from the first stationary point of the reflection integral and only creates a reflected diffracted field. The incident scattered fields are found from the same integral, written for the perfectly conducting half plane. The scattered fields are evaluated by using the stationary phase method and edge point technique. The evaluated fields are plotted numerically.



**OCIS codes:** (260.0260) Physical optics; (260.1960) Diffraction theory; (260.2110) Electromagnetic theory.

## References and links

1. F. Kottler, "Diffraction at a black screen. I. Kirchhof's theory," Prog. Opt. **4**, 281-314 (1965).
2. A. Sommerfeld, *Optics* (Academic Press, New York, 1954).
3. J. F. Nye, J. H. Hannay and W. Liang, "Diffraction by a black half-plane: theory and observation," Proc. Royal Soc. London A **449**, 515-535 (1995).
4. J. F. Nye and W. Liang, "Near-field diffraction by two slits in a black screen," Proc. Royal Soc. London A **454**, 1635-1658 (1998).
5. Y. Z. Umul, "Modified theory of physical optics," Opt. Express **12**, 4959-4972 (2004), http://www.opticsexpress.org/abstract.cfm?URI=OPEX-12-20-4959.
6. L. B. Felsen and N. Marcuwitz, *Radiation and scattering of waves* (IEEE Press, New York, 1994).
7. M. Berry, "Geometry of phase and polarization singularities, illustrated by edge diffraction and the tides," in *Singular Optics (optical Vertices): Fundamentals and Applications II*, Proc. SPIE **4403**, 1-12 (2001).

## 1. Introduction

A black screen that absorbs all the electromagnetic energy illuminating its surface and causes no reflection and transmission is a theoretical concept. An approximate approach to this problem is introduced by Kirchhoff and expanded by Kottler [1]. Kottler defined jump conditions across the black screen which relates the two sides of the screen to each other. Sommerfeld developed a different approach in the context of his mathematical theory of diffraction [2]. He thought the absorbed waves by the screen as traveling fields through the other sheets of the Riemann surfaces. So there is no reflection to the real space which is at the first Riemann surface, but the waves go through the conceptual spaces which forms the other Riemann surfaces. Nye and co-workers constructed some experiments in order to compare the related theories [3, 4].

This paper aims to deal with the problem of the black half plane by using the method of modified theory of physical optics (MTPO) [5]. The MTPO reflected field from a perfectly conducting (PEC) half plane will be considered and its general construction in terms of the critical points will be examined. Their relation with the geometrical optics (GO) and diffracted fields will be described. A new MTPO surface current will be defined by using the knowledge, obtained from the behavior of the MTPO integral. The total field (incident and

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diffracted) will be constructed by considering the MTPO surface current in the scattering integrals. These integrals will be evaluated asymptotically and the results will be plotted.

A time factor $e^{jwt}$ is assumed and suppressed throughout the paper.

## 2. General construction of the MTPO integral

The MTPO reflection integral for a PEC half plane can be written as

$$E_{rz} = -e^{j\frac{\pi}{4}} \frac{kE_i}{\sqrt{2\pi}} \int_0^{\infty} e^{jkx'\cos\phi_0} \frac{e^{-jkR}}{\sqrt{kR}} \sin\left(\frac{\beta+\phi_0}{2}\right) dx' \tag{1}$$

for an electric polarized incident plane wave [5]. $E_i$ is the amplitude of the incident plane wave, the expression of which can be written as $E_i \exp[jk(x\cos\phi_0 + y\sin\phi_0)]\vec{e}_z$. The geometry of the problem is given in Fig.1.

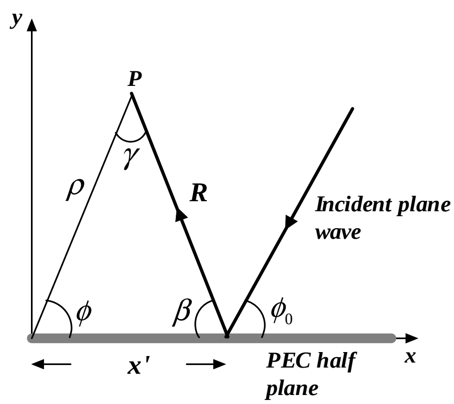


Fig. 1. Reflection geometry from a PEC half plane

It is possible to express Eq. (1) in the form of

$$I(k,\vec{r}) = \int_0^{\infty} f(\vec{r},x') e^{-jkg(\vec{r},x')} dx' \tag{2}$$

where $\vec{r}$ denotes the coordinates of the observation point. $k$ is the wave-number and has a large value for high frequencies $(k >> 1)$. The amplitude and phase functions of the integral in Eq. (2) can be defined as

$$f(\vec{r},x') = -e^{j\frac{\pi}{4}} \frac{kE_i}{\sqrt{2\pi}} \frac{\sin\left(\frac{\beta+\phi_0}{2}\right)}{\sqrt{kR}} \tag{3.a}$$

$$g(\vec{r},x') = R - x'\cos\phi_0 = \rho\cos\gamma + x'(\cos\beta - \cos\phi_0) \tag{3.b}$$

respectively. The critical points of Eq. (2) can be classified into two groups as the stationary phase points and the edge point. The stationary phase points are evaluated by equating the first derivative of Eq. (3.b) to zero. The equation of

$$\cos\phi_0 - \cos\beta = 0 \tag{4}$$

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is obtained as a result of the procedure pointed out above. The details of this operation are given in Ref. [5]. The stationary phase points can be found as

$$\beta_{s1} = \phi_0 \ , \ \beta_{s2} = 2\pi - \phi_0 \tag{5}$$

which are in the physical limits $(0 \le \phi \le 2\pi)$ of the problem. The contribution of these points can be evaluated by using the formula of

$$I \approx f(x_s) e^{-j\left[kg(x_s)+\frac{\pi}{4}\right]} \sqrt{\frac{2\pi}{kg''(x_s)}} \tag{6}$$

where $x_s$ and $g''(x_s)$ are the values of $x'$ and the second derivative of the phase function at the stationary point [6]. Equation (6) is equal to

$$I \approx -E_i e^{jk\rho\cos(\phi+\phi_0)} \tag{7}$$

for the first stationary phase point $(\beta_{s1})$. It is apparent that Eq. (7) shows the reflected GO waves from the PEC surface. Equation (6) is equal to zero at $\beta = \beta_{s2}$. This is the result of the term of $\sin[(\beta+\phi_0)/2]$ in Eq. (1). This term comes from the MTPO surface current [5]. The expression of $\sin\phi_0$ takes the place of the related term in the physical optics (PO) integral. For this reason, two scattered fields are observed at the plots of the PO integral.

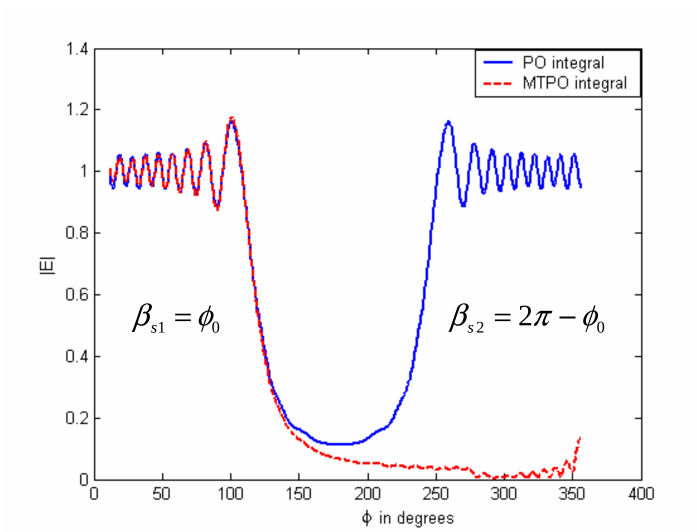


Fig. 2. Comparison of the PO and MTPO integrals for the places of the stationary points

Figure 2 shows the comparison of the PO and MTPO integrals. The angle of incidence $(\phi_0)$ is taken as $\pi/3$ and the observation point is at $6\lambda$. The first fields for $\phi \le \pi - \phi_0$ coincide, but PO gives a second GO field for $\phi \ge \pi + \phi_0$ where the MTPO integral approaches to zero. The field expression for $\phi \ge \pi + \phi_0$ represent the transmitted waves from the half plane. It is apparent that this field does not exist for the PEC half plane case.

It is clear from the above analysis that GO fields are related with the stationary phase points. The first stationary point gives the reflected fields. The stationary point of $\beta_{s2}$ represent the transmitted rays from the half plane. This result is important for the following analysis of the black half plane. The integral representation of the transmitted field can be expressed as

$$E_{rz} = e^{j\frac{\pi}{4}} \frac{kE_i}{\sqrt{2\pi}} \int_0^{\infty} e^{jkx'\cos\phi_0} \frac{e^{-jkR}}{\sqrt{kR}} \sin\left(\frac{\beta-\phi_0}{2}\right) dx' \tag{8}$$

where the term of $\sin[(\beta-\phi_0)/2]$ eliminates the reflected waves in Eq. (6).

The coordinates of the edge point can be defined as $x'=0$ and $\beta=\pi-\phi$. The influence of the edge point can be evaluated by using the formula of

$$I \approx -\frac{f(x_e)}{jkg'(x_e)} e^{-jkg(x_e)} \tag{9}$$

where $x_e$ is the value of $x'$ at the edge point of the integral. $x_e$ is equal to *0* for Eq. (2). This expression gives the field of

$$E_{rdz} = \frac{E_i}{\sqrt{2\pi}} \frac{\cos\frac{\phi-\phi_0}{2}}{\cos\phi+\cos\phi_0} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{10}$$

when applied to Eq. (2). Eq. (10) shows the exact reflected diffracted fields from a PEC half plane. The term of $\cos[(\phi-\phi_0/2)]$ comes from $\sin[(\beta+\phi_0)/2]$ and creates a transition region at the reflection boundary of $\phi=\pi-\phi_0$.

## 3. Boundary conditions for a black half plane

There is not any rigorous boundary condition, which will enable the exact solution, in literature for a black screen. The surface conditions will be examined in the point of view of PO in this section. According to the PO approach, the reflected waves are the result of the surface currents, which are created by the incident field on the scattering surface. There is an electric current density on the surface for the PEC half plane case and there will flow a magnetic current on a perfectly permeable surface. These current densities create the reflected electromagnetic fields. The MTPO surface current can be written as

$$\vec{J}_{MTPO} = \vec{e}_z \frac{2E_i}{Z_0} \sin\frac{\beta+\phi_0}{2} e^{jkx'\cos\phi_0} \tag{11}$$

for a PEC surface illuminated by an electric polarized incident plane wave. The related current density can be represented as

$$\vec{J}_{MTPO} = 2H_i\left(\vec{e}_x \cos\frac{\beta-\phi_0}{2} - \vec{e}_y \sin\frac{\beta-\phi_0}{2}\right) e^{jkx'\cos\phi_0} \tag{12}$$

when an incident magnetic polarized wave is considered. The surface currents will be constructed according to the approach of Kottler.

The method of MTPO considers two scattering integrals, the asymptotic evaluation of which give the GO and diffracted waves. The first one is the aperture integral, which gives the incident GO ($u_i$) and incident diffracted ($u_{di}$) fields as

$$u_A = u_i + u_{di} \tag{13}$$

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where *u* denotes the electric or magnetic field component. The second integral represents the reflected GO ($u_r$) and reflected diffracted ($u_{dr}$) fields by the formula of

$$u_S = u_r + u_{dr} \,. \tag{14}$$

According to Kottler, the boundary condition on a black screen can be written as

$$u\big|_{S+} - u\big|_{S-} = u_i \tag{15}$$

for *S+ and S-* represent the upper and lower surfaces of the black screen, respectively. The term of *u* is equal to the sum of $u_A$ and $u_S$. The condition in Eq. (15) can be expressed as

$$\left(u_S + u_i + u_{di}\right)\big|_{S+} - \left(u_S + u_{di}\right)\big|_{S-} = u_i \tag{16}$$

by using Eqs. (13) and (14) in Eq. (15). Since *S-* represents the shadow region, $u_i$ is not used in the second term at the left-hand side of Eq. (16). The conditions of

$$\left(u_S + u_{di}\right)\big|_{S+} = \left(u_S + u_{di}\right)\big|_{S-} \tag{17}$$

or

$$u_S\big|_{S\mp} = -u_{di}\big|_{S\mp} \tag{18}$$

can be obtained when Eq. (16) is taken into account. There will be only diffracted fields in the term of $u_S$, because the reflected waves vanish at the case of a black screen. The condition, in Eq. (18), can only be satisfied when the surface currents, flowing on the black screen, eliminate the reflected wave and give rise to the diffracted field.

A black half plane absorbs all the electromagnetic energy falling on its surface and causes no reflection and transmission. This proposal is valid when the electric and magnetic current densities are equal to zero. The related equations can be written as

$$\vec{J}_{es} = \vec{n} \times \vec{H}_T\big|_S = 0 \tag{19.a}$$

$$\vec{J}_{ms} = -\vec{n} \times \vec{E}_T\big|_S = 0 \tag{19.b}$$

where $\vec{E}_T$ and $\vec{H}_T$ are the total electric and magnetic fields. Equations (19.a) and (19.b) cause the MTPO integral to be equal to zero and the diffracted fields can not be evaluated. In order to obtain fields, that satisfies the condition in Eq. (18), the current components in Eqs. (19.a) and (19.b) must be different from zero. These equations will be represented as

$$\vec{J}_{es} = \vec{n}_1 \times \vec{H}_T\big|_S \neq 0 \tag{20.a}$$

$$\vec{J}_{ms} = -\vec{n}_1 \times \vec{E}_T\big|_S \neq 0 \tag{20.b}$$

for the MTPO reflection integrals. $\vec{n}_1$ is the modified unit normal vector of the surface. The main objective of the analysis is that the reflected fields from the black half plane are equal to zero, but the diffracted fields exist. This condition can be satisfied by multiplying the current density in Eq. (11) by the term of $\sin[(\beta - \phi_0)/2]$ since this term will eliminate the reflected GO fields by going to zero at the stationary point of the reflection integral. This case was discussed in Section 2. The analysis made in the second section show that $\sin[(\beta + \phi_0)/2]$

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affects the GO reflected fields directly at the stationary phase point. The reflected fields for a PEC half plane is different from zero since $\sin[(\beta+\phi_0)/2]$ is equal to $\sin\phi_0$ at the point of reflection, but when the term of $\sin[(\beta-\phi_0)/2]$ is multiplied with $\sin[(\beta+\phi_0)/2]$, the reflected fields at $\beta_{s1}$ will be canceled for the black half plane case. The current densities can be written as

$$\vec{J}_{es} = \vec{e}_z \frac{E_i}{Z_0} \sin\frac{\beta+\phi_0}{2} \sin\frac{\beta-\phi_0}{2} e^{jkx'\cos\phi_0} \tag{21.a}$$

$$\vec{J}_{ms} = -E_i\left(\vec{e}_x \cos\frac{\beta-\phi_0}{2} - \vec{e}_y \sin\frac{\beta-\phi_0}{2}\right)\sin\frac{\beta-\phi_0}{2} e^{jkx'\cos\phi_0} \tag{21.b}$$

for an electric polarized plane wave. Equation (21.a) is obtained by multiplying Eq. (11) by $\sin[(\beta-\phi_0)/2]$. Equation (21.b) can be written by considering Eq. (20.b) and the incident electric polarized plane wave.

The boundary conditions for a black half plane can be obtained by taking into account the PEC or perfectly magnetic conducting (PMC) surfaces. The boundary conditions for a black half plane can be written as

$$\vec{J}_{es} = \vec{n}\times\vec{H}_T\Big|_S = 0 \tag{22.a}$$

$$\vec{n}\times\vec{E}_T\Big|_S = 0 \tag{22.b}$$

by considering the conditions for a PEC surface. The tangential component of the electric field is equal to zero on a PEC body. The electric surface current must be also equal to zero when a black body is considered. The MTPO surface currents can be constructed as

$$\vec{J}_{es} = \vec{n}_1\times\vec{H}_T\Big|_S \neq 0 \tag{23.a}$$

$$\vec{n}_1\times\vec{E}_T\Big|_S = 0 \tag{23.b}$$

by considering Eq.(20.a). The electric surface current is equal to

$$\vec{J}_{es} = \vec{e}_z \frac{2E_i}{Z_0} \sin\frac{\beta+\phi_0}{2} \sin\frac{\beta-\phi_0}{2} e^{jkx'\cos\phi_0} \tag{24}$$

for this case. The integral can be written as

$$E_{sz} = -\frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_0^\infty e^{jkx'\cos\phi_0} \sin\frac{\beta-\phi_0}{2} \sin\frac{\beta+\phi_0}{2} \frac{e^{-jkR}}{\sqrt{kR}} dx' \tag{25}$$

for the scattered electric field by using Eq. (24) in Eq. (1).

The boundary conditions for a black half plane can also be represented as

$$\vec{n}\times\vec{H}_T\Big|_S = 0 \tag{26.a}$$

$$\vec{J}_{ms} = -\vec{n}\times\vec{E}_T\Big|_S = 0 \tag{26.b}$$

when a PMC surface is considered. These conditions can be expressed as

$$\vec{n}\times\vec{H}_T\Big|_S = 0 \tag{27.a}$$

$$\vec{J}_{ms} = -\vec{n}\times\vec{E}_T\Big|_S \neq 0 \tag{27.b}$$

for the MTPO approach. The magnetic surface current can be written as

$$\vec{J}_{ms} = -2E_i\left(\vec{e}_x\cos\frac{\beta-\phi_0}{2} - \vec{e}_y\sin\frac{\beta-\phi_0}{2}\right)\sin\frac{\beta-\phi_0}{2}e^{jkx'\cos\phi_0} \tag{28}$$

by considering Eq. (21.b). The electric field can be calculated by using the equation of $\vec{E} = -\left(\nabla\times\vec{F}\right)/\varepsilon_0$ where the vector potential can be represented as

$$\vec{F} = \frac{\varepsilon_0}{4\pi}\iint_s \vec{J}_{ms}\frac{e^{-jkR}}{R}dS'\,. \tag{29}$$

The scattered electric field can be found as

$$\vec{E}_s = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}}\int_0^\infty \nabla\times\left(\vec{J}_{ms}\frac{e^{-jkR}}{\sqrt{kR}}\right)dx' \tag{30}$$

by evaluating the $z'$ part of the integral, in Eq. (29), as in Ref. [5]. The Debye asymptotic expansion of the Hankel function, which comes from the evaluation of the $z'$ part, is used. The rotational operation can be performed as

$$jk\frac{e^{-jkR}}{\sqrt{kR}}\left(\frac{\partial R}{\partial x}\sin\frac{\beta-\phi_0}{2} + \frac{\partial R}{\partial y}\cos\frac{\beta-\phi_0}{2}\right) = jk\frac{e^{-jkR}}{\sqrt{kR}}\sin\frac{\beta+\phi_0}{2} \tag{31}$$

for $\left(\partial R/\partial x\right) = -\cos\beta$ and $\left(\partial R/\partial y\right) = \sin\beta$ for the geometry in Fig. 1. The curl is taken according to the coordinates of the observation point. As a result one obtains

$$E_{sz} = -\frac{kE_i}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\int_0^\infty e^{jkx'\cos\phi_0}\sin\frac{\beta-\phi_0}{2}\sin\frac{\beta+\phi_0}{2}\frac{e^{-jkR}}{\sqrt{kR}}dx' \tag{32}$$

for the scattered field from a black half plane. It is apparent that the three approaches are equivalent. The easiest method can be chosen according to the problem.

**4. Asymptotic evaluation of the integrals**

The MTPO scattering integrals will be evaluated asymptotically in order to obtain the GO and edge diffracted fields. The reflection integrals, obtained in the previous section will be used. The total MTPO integrals can be written as

$$E_{tz} = \frac{kE_i}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\left(\int_{-\infty}^{0} e^{jkx'\cos\phi_0}\sin\frac{\beta+\phi_0}{2}\frac{e^{-jkR}}{\sqrt{kR}}dx' - \int_0^\infty e^{jkx'\cos\phi_0}\sin\frac{\beta-\phi_0}{2}\sin\frac{\beta+\phi_0}{2}\frac{e^{-jkR}}{\sqrt{kR}}dx'\right) \tag{33}$$

for the black half plane problem. The first integral of Eq. (33) represents the incident scattered fields and was evaluated as


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$$E_{is} = e^{jk\rho\cos(\phi-\phi_0)} u(-\xi_i) - \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}\cos\frac{\phi-\phi_0}{2}} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{34}$$

in Ref. [5]. $\xi_i$ is the detour parameter of the incident diffracted field, which is equal to $-\sqrt{2k\rho}\cos[(\phi-\phi_0)/2]$. As mentioned in Section 2, the second integral of Eq. (33) has two stationary phase points and the contribution of these points are equal to zero according to the formula, given in Eq. (6). The effect of the edge point can be analyzed from Eq. (9). The resultant field can be written as

$$E_{dr} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{35}$$

and one obtains

$$E_{dt} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \left( 1 - \frac{1}{\cos\frac{\phi-\phi_0}{2}} \right) \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{36}$$

for the total diffracted field. The boundary condition, in Eq. (18), can be written as $E_{dr}\big|_{\phi=0,2\pi} = -E_{di}\big|_{\phi=0,2\pi}$ for an electric polarized plane wave illuminated black half plane. It is apparent that Eq. (36) does not satisfy this condition. A corrected form can be obtained as

$$E_{dtc} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \left( \frac{\mathrm{sgn}(-\xi_i)}{\cos\frac{\phi_0}{2}} - \frac{1}{\cos\frac{\phi-\phi_0}{2}} \right) \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{37}$$

where $\mathrm{sgn}(x)$ shows the signum function, which is equal to *-1* for *x<0* and *1* for *x>0*. This function is used since the incident diffracted field changes its sign at the shadow boundary ($\phi = \pi + \phi_0$). The total diffracted field in Eq. (30) satisfies the boundary condition, in Eq. (18). The diffracted fields, in Eqs.(36) and (37) approach to infinity at the shadow boundary. For this reason uniform field expressions will be obtained. The incident diffracted field component can be expressed as

$$E_{di} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{\pi}} \frac{e^{-j2k\rho\cos^2\frac{\phi-\phi_0}{2}}}{\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2}} e^{jk\rho\cos(\phi-\phi_0)} \tag{38}$$

by using the trigonometric identity of $1 = 2\cos^2(x/2) - \cos 2x$. Equation (38) can be written in terms of the first term in the asymptotic expansion of the Fresnel integral as

$$E_{di} = -\hat{F}\left( -\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2} \right) e^{jk\rho\cos(\phi-\phi_0)} \tag{39}$$


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where $\hat{F}(x)$ is equal to $e^{-j(x^2+\pi/4)}/(2\sqrt{\pi}x)$. The uniform diffracted field can be represented as

$$E_{diu} = -F(|\xi_i|)\mathrm{sgn}(\xi_i)e^{jk\rho\cos(\phi-\phi_0)} \tag{40}$$

by using the identity of $\hat{F}(x) \approx F(x) - u(-x) = F(|x|)\mathrm{sgn}(x)$. $F(x)$ is the Fresnel integral, which can be defined as

$$F(x) = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}}\int_x^{\infty} e^{-jt^2}dt\,. \tag{41}$$

The uniform total field can be given as

$$E_{Su} = E_i e^{jk\rho\cos(\phi-\phi_0)}F(\xi_i) + \frac{E_i e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}\cos\left(\phi_0/2\right)}\frac{e^{-jk\rho}}{\sqrt{k\rho}}\mathrm{sgn}(-\xi_i) \tag{42}$$

for a black half plane, illuminated by an electric polarized plane wave. The surface current of Eq. (24) can be rewritten as

$$\vec{J}_{es} = \vec{e}_z\frac{2E_i}{Z_0}\frac{\sin\frac{\beta+\phi_0}{2}\sin\frac{\beta-\phi_0}{2}}{\cos\frac{\phi_0}{2}}e^{jkx'\cos\phi_0}\,\mathrm{sgn}(-\xi_i) \tag{43}$$

by considering the modified reflected diffracted field in Eq. (37).

## 5. Numerical results

The total diffracted fields in Eqs. (36) and (37) and the total scattered fields will be plotted. These field expressions will be compared with the Sommerfeld solution, given in Ref. [7]. This solution can be written as

$$E_{Sz} = e^{jk\rho\cos(\phi-\phi_0)}F\left(-\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2}\right) \tag{44}$$

where the diffracted field of Eq. (44) can be evaluated as

$$E_{Sz} = e^{jk\rho\cos(\phi-\phi_0)}F\left(\left|\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2}\right|\right)\mathrm{sgn}\left(-\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2}\right). \tag{45}$$

by subtracting the incident field from Eq.(44) and using the identity of $F(x) - u(-x) = F(|x|)\mathrm{sgn}(x)$. The amplitude of the incident wave is equal to 1. It is apparent that Eq. (45) is the incident diffracted field in Eq. (28).


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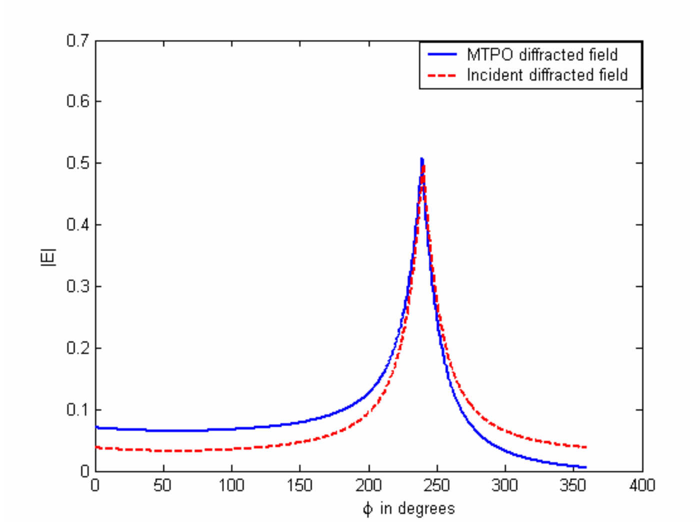


Fig. 3. Comparison of MTPO and incident diffracted fields

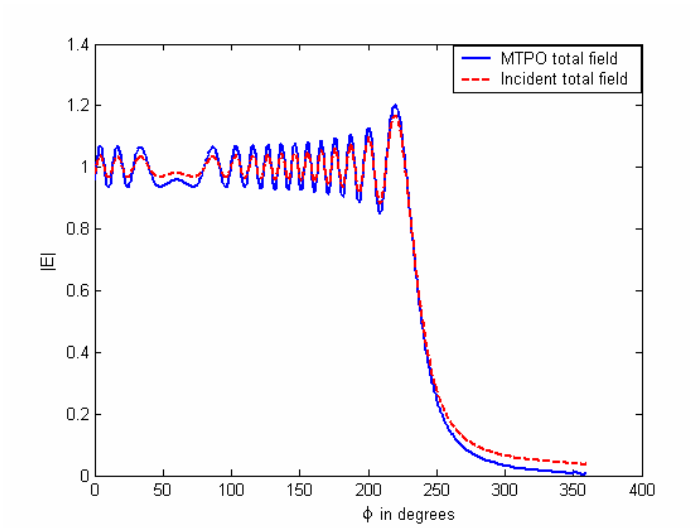


Fig. 4. Comparison of MTPO and incident total fields

Figure 3 shows the variation of the diffracted fields, in Eq. (36) versus the observation angle. The angle of incidence $(\phi_0)$ is taken as $60^0$ and $\rho$ is equal to $6\lambda$ where $\lambda$ is the wavelength. It can be observed that MTPO total diffracted field approaches to zero for $\phi = 2\pi$. This behavior satisfies the boundary condition given in Eq. (18). The same condition can be seen in Fig. 4, which represents the variation of the total scattered fields. There is a problem at $\phi = 0$ since the diffracted fields do not satisfy the related condition at the upper surface of the half plane. This problem is eliminated by obtaining the corrected reflected diffracted fields, in Eq. (37), which obey the condition of $E_{dr}\big|_{\phi=0,2\pi} = -E_{di}\big|_{\phi=0,2\pi}$. The total scattered field can be constructed by adding the incident wave to Eq. (37). The uniform expression of this field is given in Eq. (42).

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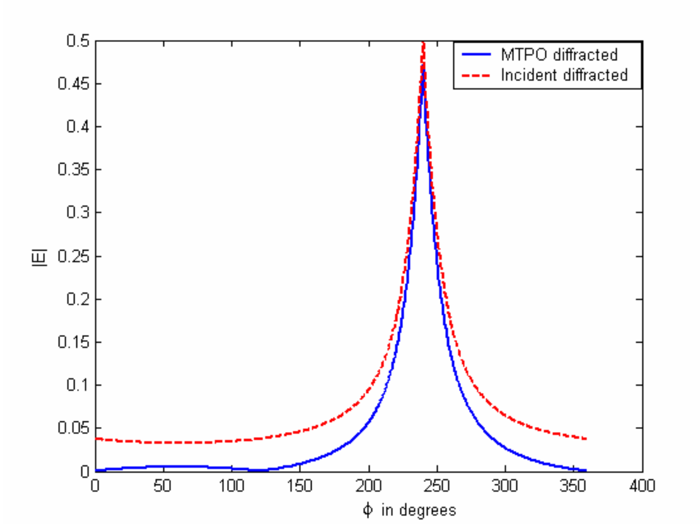

Fig. 5. MTPO and incident total diffracted fields

Figure 5 depicts the variation of total diffracted fields versus the observation angle. It can be seen that the corrected MTPO diffracted field, given in Eq. (37), satisfies the boundary conditions and also is equal to zero for $\phi = \pi - \phi_0$. This value represents the reflection boundary.

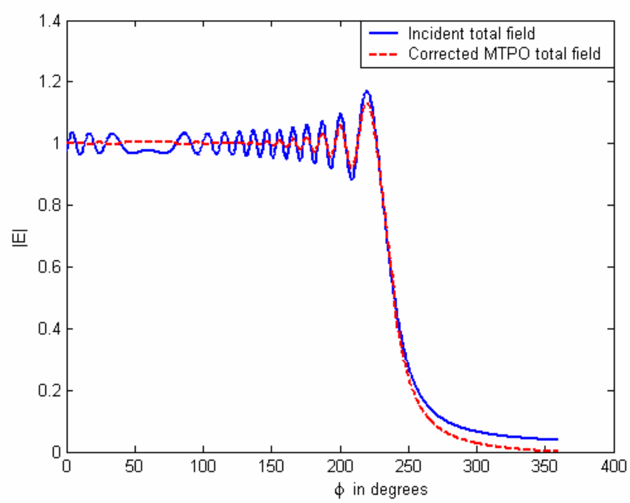

Fig. 6. MTPO and incident total fields

Figure 6 shows the variation of the total scattered fields with respect to the observation angle. It can be observed that the total MTPO field, given in Eq. (42), is equal to one, which is the value of the incident field at $\phi = 0$. The field approaches to zero for $\phi = 2\pi$.

MTPO diffracted field can also be expressed in terms of Fresnel function. The reflected diffracted field in Eq. (37), can be expressed as

$$E_{dr} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{\pi}} \frac{e^{-j2k\rho\cos^2\frac{\phi_0}{2}}}{\sqrt{2k\rho}\cos\frac{\phi_0}{2}} e^{jk\rho\cos\phi_0} \operatorname{sgn}(-\xi_i) \tag{46}$$

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by using the trigonometric identity of $1 = 2\cos^2(x/2) - \cos 2x$ . Equation (46) can be written in terms of the first term in the asymptotic expansion of the Fresnel integral as

$$E_{dr} = -\hat{F}\left(-\sqrt{2k\rho}\cos\frac{\phi_0}{2}\right)e^{jk\rho\cos\phi_0}\,\mathrm{sgn}(-\xi_i) \tag{47}$$

The diffracted field can be expressed as

$$E_{dr} = -F\left(\left|\sqrt{2k\rho}\cos\frac{\phi_0}{2}\right|\right)\mathrm{sgn}\left(-\sqrt{2k\rho}\cos\frac{\phi_0}{2}\right)\mathrm{sgn}(-\xi_i)e^{jk\rho\cos\phi_0} \tag{48}$$

by using the identity of $\hat{F}(x) \approx F(x) - u(-x) = F(|x|)\mathrm{sgn}(x)$.

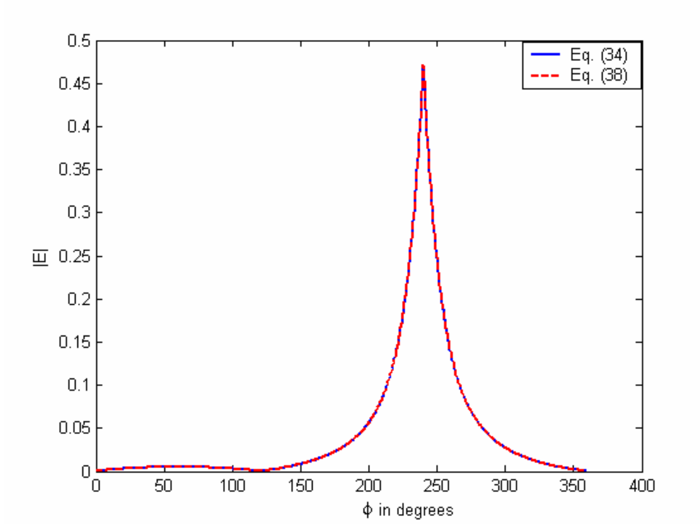


Fig.7 MTPO total diffracted fields

Figure 7 shows the comparison of the reflected diffracted fields, in Eqs. (37) and (48). It can be seen that they are harmonious.

## 6. Conclusion

In this work, electromagnetic scattering by a black half plane is examined with the method of MTPO. The surface currents are defined by using the Kottler's approach for the boundary conditions of a black screen. MTPO integral for the aperture part does not change but the integral of the black half plane is constructed by benefiting from the surface current, defined from the boundary conditions. The integrals are evaluated asymptotically by the methods of the stationary phase and the edge point. The numerical plot of the diffracted field shows that the evaluated edge diffraction coefficient of the black half plane approaches to zero for $\phi = 2\pi$, but nearly doubles its value at $\phi = 0$. The reason lies under the behavior of the edge diffraction coefficient. The coefficient has a negative value for the angle values smaller than the transition region value and takes positive values for greater angles. The diffraction coefficient in Eq. (36) is reconstructed in order to provide the mentioned property by considering the condition, given in Eq. (18). The resultant field expression satisfies the boundary conditions on the black half plane. The total diffracted field is equal to zero on the upper and lower surfaces of the black half plane and also goes to zero at the reflection boundary. A new surface current density is defined in Eq. (43) according to the corrected field expression.

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