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Research Article

The Existence of Solution for a k-Dimensional System of Multiterm Fractional Integrodifferential Equations with Antiperiodic Boundary Value Problems

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There are many published papers about fractional integrodifferential equations and system of fractional differential equations. The goal of this paper is to show that we can investigate more complicated ones by using an appropriate basic theory. In this way, we prove the existence and uniqueness of solution for a k-dimensional system of multiterm fractional integrodifferential equations with antiperiodic boundary conditions by applying some standard fixed point results. An illustrative example is also presented.

1. Introduction

Fractional differential equations have recently been studied by many researchers for a variety of problems (see, e.g., [1–33] and the references therein). Antiperiodic boundary value problems occur in the mathematical modeling of a variety of physical processes (see, e.g., [3–6, 29, 30] and the references therein). On the other hand, the study of a coupled system of fractional order is also very significant because this kind of system can often occur in applications (see, e.g., [7, 15, 24, 28, 29] and the references therein). We are going to investigate a complicated case in this work. Let $T > 0$ and $I = [0, T]$. In this paper, we study the existence and uniqueness of solution for the k-dimensional system of multiterm fractional integrodifferential equations

$$\begin{aligned}
{}^{c}D^{\alpha_1} x_1(t) = f_1\Big(& t, x_1(t), x_2(t), \dots, x_k(t), \\
& \phi_{11}x_1(t), \phi_{12}x_2(t), \dots, \phi_{1k}x_k(t), \\
& {}^{c}D^{\mu_{11}} x_1(t), {}^{c}D^{\mu_{12}} x_2(t), \dots, \\
& {}^{c}D^{\mu_{1k}} x_k(t), {}^{c}D^{\beta_{11}} x_1(t), \\
& {}^{c}D^{\beta_{12}} x_2(t), \dots, {}^{c}D^{\beta_{1k}} x_k(t)\Big), \\
{}^{c}D^{\alpha_2} x_2(t) = f_2\Big(& t, x_1(t), x_2(t), \dots, x_k(t), \\
& \phi_{21}x_1(t), \phi_{22}x_2(t), \dots, \phi_{2k}x_k(t), \\
& {}^{c}D^{\mu_{21}} x_1(t), {}^{c}D^{\mu_{22}} x_2(t), \dots, \\
& {}^{c}D^{\mu_{2k}} x_k(t), {}^{c}D^{\beta_{21}} x_1(t), \\
& {}^{c}D^{\beta_{22}} x_2(t), \dots, {}^{c}D^{\beta_{2k}} x_k(t)\Big), \\
& \vdots \\
{}^{c}D^{\alpha_k} x_k(t) = f_k\Big(& t, x_1(t), x_2(t), \dots, x_k(t), \\
& \phi_{k1}x_1(t), \phi_{k2}x_2(t), \dots, \\
& \phi_{kk}x_k(t), {}^{c}D^{\mu_{k1}} x_1(t),
\end{aligned}$$

$$ {}^cD^{\mu_{k2}}x_2(t),\ldots,{}^cD^{\mu_{kk}}x_k(t), $$
$$ {}^cD^{\beta_{k1}}x_1(t), $$
$$ {}^cD^{\beta_{k2}}x_2(t),\ldots,{}^cD^{\beta_{kk}}x_k(t)\Big), $$
$$ (t\in I), \tag{1} $$

with antiperiodic boundary conditions $x_i(0) = -x_i(T)$, ${}^cD^{p_i}x_i(0) = -{}^cD^{p_i}x_i(T)$, and ${}^cD^{q_i}x_i(0) = -{}^cD^{q_i}x_i(T)$ for $i = 1,2,\ldots,k$, where ${}^cD$ denotes the Caputo fractional derivative, $\alpha_i \in (2,3]$, $p_i,\mu_{ij} \in (0,1)$, $q_i,\beta_{ij} \in (1,2)$ for $i,j = 1,2,\ldots,k$, $(\phi_{ij}x_j)(t) = \int_0^t \lambda_{ij}(t,s)x_j(s)ds$, and $f_j \in C(I\times\mathbb{R}^{4k},\mathbb{R})$, $\lambda_{ij} : I\times I \to [0,\infty)$ are continuous functions for all $i,j = 1,2,\ldots,k$. Hereafter, we will use vector notations. Define the space $X = \{u(t) : u(t) \in C^2(I)\}$ endowed with the norm $\|x\|_X = \sup_{t\in I}|x(t)| + \sup_{t\in I}|x'(t)| + \sup_{t\in I}|x''(t)|$. In fact, $(X,\|\cdot\|_X)$ and the product space $(X^k = \underbrace{X\times X\times\cdots\times X}_{k},\|\cdot\|_*)$ endowed with the norm $\|(x_1,x_2,\ldots,x_k)\|_* = \|x_1\|_X + \|x_2\|_X + \cdots + \|x_k\|_X$ are Banach spaces. The Riemann-Liouville fractional integral of order q is defined by $I^qf(t) = (1/\Gamma(q))\int_0^t (f(s)/(t-s)^{1-q})ds$ ($t > 0$ and $q > 0$), provided the integral exists. The Caputo derivative of order q for a function $f \in C^n([0,\infty),\mathbb{R})$ is defined by ${}^cD^qf(t) = (1/\Gamma(n-q))\int_0^t (f^{(n)}(s)/(t-s)^{q+1-n})\,ds = I^{n-q}f^{(n)}(t)$ for $t > 0$ and $n-1 < q < n$ [25]. Recently, Wang et al. proved the following result [30].

Lemma 1. *For each $y \in C([0,T])$, the unique solution of the boundary value problem*

$$ \begin{gathered} {}^cD^{\alpha}x(t) = y(t), \quad (t\in[0,T],\ T>0,\ 2<\alpha\le 3) \\ x(0) = -x(T), \qquad {}^cD^{p}x(0) = -{}^cD^{p}x(T), \\ {}^cD^{q}x(0) = -{}^cD^{q}x(T), \quad (0<p<1,\ 1<q<2) \end{gathered} \tag{2} $$

is given by $x(t) = \int_0^T G_\alpha(t,s)y(s)ds$, where $G_\alpha(t,s)$ is Green's function defined as

$$ G_\alpha(t,s) = \begin{cases} \dfrac{(t-s)^{\alpha-1}-(1/2)(T-s)^{\alpha-1}}{\Gamma(\alpha)} + \dfrac{\Gamma(2-p)(T-2t)(T-s)^{\alpha-p-1}}{2\Gamma(\alpha-p)T^{1-p}} - \dfrac{[pT^2-4Tt+2(2-p)t^2]\Gamma(3-q)(T-s)^{\alpha-q-1}}{4(2-p)\Gamma(\alpha-q)T^{2-q}}, & s\le t, \\ -\dfrac{(T-s)^{\alpha-1}}{2\Gamma(\alpha)} + \dfrac{\Gamma(2-p)(T-2t)(T-s)^{\alpha-p-1}}{2\Gamma(\alpha-p)T^{1-p}} - \dfrac{[pT^2-4Tt+2(2-p)t^2]\Gamma(3-q)(T-s)^{\alpha-q-1}}{4(2-p)\Gamma(\alpha-q)T^{2-q}}, & t\le s. \end{cases} \tag{3} $$

One can find the next result in [34].

Theorem 2. *Let E be a Banach space and $T : E \to E$ a completely continuous operator. Suppose that the set $V = \{u \in E : u = \mu Tu,\ 0\le\mu\le 1\}$ is bounded. Then T has a fixed point in E.*

We will use the last two results for solving the problem (1).

2. Main Results

Now, we are ready to state and prove our main results. For each $i = 1,2,\ldots,k$, put

$$ \begin{aligned} M_i = {} & \left(\frac{3}{2\Gamma(\alpha_i+1)} + \frac{\Gamma(2-p_i)}{2\Gamma(\alpha_i-p_i+1)} + \frac{(4-p_i)\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i} \\ & + \left(\frac{1}{\Gamma(\alpha_i)} + \frac{\Gamma(2-p_i)}{\Gamma(\alpha_i-p_i+1)} + \frac{\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-1} \\ & + \left(\frac{1}{\Gamma(\alpha_i-1)} + \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-2} \end{aligned} \tag{4} $$

and $M = \min_{1\le j\le k}\{1-\sum_{i=1}^k M_i(b_{ij}+c_{ij}\lambda_{ij}^0+d_{ij}(T^{1-\mu_{ij}}/\Gamma(2-\mu_{ij}))+e_{ij}(T^{2-\beta_{ij}}/\Gamma(3-\beta_{ij})))\}$, where $\lambda_{ij}^0 = \sup_{t\in I}|\int_0^t \lambda_{ij}(t,s)ds|$ for all $i,j = 1,2,\ldots,k$. Define the operator $T : X^k \to X^k$ by

$$ T(x)(t) = \begin{pmatrix} T_1(x)(t) \\ T_2(x)(t) \\ \vdots \\ T_k(x)(t) \end{pmatrix}, \tag{5} $$

where $x = (x_1,x_2,\ldots,x_k)$ and

$$ T_i(x)(t) = \int_0^T G_{\alpha_i}(t,s)\tilde{f}_i(s,x(s))\,ds \tag{6} $$

for $i = 1,2,\ldots,k$, where

$$ \begin{aligned} \tilde{f}_i(s,x(s)) = {} & f_i\Big(s,x_1(s),x_2(s),\ldots,x_k(s), \\ & \phi_{i1}x_1(s),\phi_{i2}x_2(s),\ldots,\phi_{ik}x_k(s), \\ & {}^cD^{\mu_{i1}}x_1(s),{}^cD^{\mu_{i2}}x_2(s),\ldots,{}^cD^{\mu_{ik}}x_k(s), \\ & {}^cD^{\beta_{i1}}x_1(s),{}^cD^{\beta_{i2}}x_2(s),\ldots,{}^cD^{\beta_{ik}}x_k(s)\Big). \end{aligned} \tag{7} $$

Thus, for each $i = 1, 2, \ldots, k$, we have

$$(T_i(x))'(t) = \frac{1}{\Gamma(\alpha_i - 1)} \int_0^t (t-s)^{\alpha_i - 2} \widetilde{f}_i(s, x(s))\, ds - \frac{\Gamma(2-p_i)}{T^{1-p_i}\Gamma(\alpha_i - p_i)} \times \int_0^T (T-s)^{\alpha_i - p_i - 1} \widetilde{f}_i(s, x(s))\, ds + \frac{[T - (2-p_i)t]\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i - q_i)T^{2-q_i}} \times \int_0^T (T-s)^{\alpha_i - q_i - 1} \widetilde{f}_i(s, x(s))\, ds, \quad (8)$$

$$(T_i(x))''(t) = \frac{1}{\Gamma(\alpha_i - 2)} \int_0^t (t-s)^{\alpha_i - 3} \widetilde{f}_i(s, x(s))\, ds - \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i - q_i)T^{2-q_i}} \times \int_0^T (T-s)^{\alpha_i - q_i - 1} \widetilde{f}_i(s, x(s))\, ds. \quad (9)$$

Theorem 3. *The operator $T : X^k \to X^k$ is completely continuous.*

Proof. First, we show that the operator $T : X^k \to X^k$ is continuous. Let $0 < \mu_{ij} < 1$ and $1 < \beta_{ij} < 2$ for $i, j = 1, 2, \ldots, k$ and let $\{(x_1^n, x_2^n, \ldots, x_k^n)\}$ be a sequence in X^k such that $(x_1^n, x_2^n, \ldots, x_k^n) \to (x_1^0, x_2^0, \ldots, x_k^0)$. Then, we have

$$\begin{aligned}
&\sup_{t\in I} \left| {}^cD^{\mu_{ij}} x_j^n(t) - {}^cD^{\mu_{ij}} x_j^0(t) \right| \\
&= \sup_{t\in I} \left| \frac{1}{\Gamma(1-\mu_{ij})} \int_0^t (t-s)^{-\mu_{ij}} (x_j^n)'(s)\, ds - \frac{1}{\Gamma(1-\mu_{ij})} \int_0^t (t-s)^{-\mu_{ij}} (x_j^0)'(s)\, ds \right| \\
&= \sup_{t\in I} \left| \frac{1}{\Gamma(1-\mu_{ij})} \times \int_0^t (t-s)^{-\mu_{ij}} \left[(x_j^n)'(s) - (x_j^0)'(s) \right] ds \right| \\
&\le \frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} \sup_{t\in I} \left| (x_j^n)'(t) - (x_j^0)'(t) \right| \\
&\le \frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} \left\| x_j^n - x_j^0 \right\|, \\
&\sup_{t\in I} \left| {}^cD^{\beta_{ij}} x_j^n(t) - {}^cD^{\beta_{ij}} x_j^0(t) \right| \\
&= \sup_{t\in I} \left| \frac{1}{\Gamma(2-\beta_{ij})} \int_0^t (t-s)^{1-\beta_{ij}} (x_j^n)''(s)\, ds - \frac{1}{\Gamma(2-\beta_{ij})} \int_0^t (t-s)^{1-\beta_{ij}} (x_j^0)''(s)\, ds \right| \\
&= \sup_{t\in I} \left| \frac{1}{\Gamma(2-\beta_{ij})} \times \int_0^t (t-s)^{1-\beta_{ij}} \left[(x_j^n)''(s) - (x_j^0)''(s) \right] ds \right| \\
&\le \frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})} \sup_{t\in I} \left| (x_j^n)''(t) - (x_j^0)''(t) \right| \\
&\le \frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})} \left\| x_j^n - x_j^0 \right\|, \\
&\sup_{t\in I} \left| \phi_{ij} x_j^n(t) - \phi_{ij} x_j^0(t) \right| \\
&= \sup_{t\in I} \left| \int_0^t \lambda_{ij}(t,s) x_j^n(s)\, ds - \int_0^t \lambda_{ij}(t,s) x_j^0(s)\, ds \right| \\
&\le \sup_{t\in I} \left| \int_0^t \lambda_{ij}(t,s)\, ds \right| \sup_{t\in I} \left| x_j^n(t) - x_j^0(t) \right| \\
&\le \lambda_{ij}^0 \left\| x_j^n - x_j^0 \right\|
\end{aligned} \quad (10)$$

for $i, j = 1, 2, \ldots, k$. Since $\|x_j^n - x_j^0\|_X \to 0$ for $j = 1, 2, \ldots, k$, the sequences $\{{}^cD^{\mu_{ij}} x_j^n(t)\}$, $\{{}^cD^{\beta_{ij}} x_j^n(t)\}$, and $\{\phi_{ij} x_j^n(t)\}$ converge uniformly on $[0, T]$ and also $\lim_{n\to\infty} {}^cD^{\mu_{ij}} x_j^n(t) = {}^cD^{\mu_{ij}} x_j^0(t)$, $\lim_{n\to\infty} {}^cD^{\beta_{ij}} x_j^n(t) = {}^cD^{\beta_{ij}} x_j^0(t)$, and $\lim_{n\to\infty} \phi_{ij} x_j^n(t) = \phi_{ij} x_j^0(t)$ converge uniformly on $[0, T]$ for $i, j = 1, 2, \ldots, k$. Since

$$\begin{aligned}
&\left\| T(x_1^n, x_2^n, \ldots, x_k^n) - T(x_1^0, x_2^0, \ldots, x_k^0) \right\|_* \\
&= \sup_{t\in I} \left| T_1(x_1^n, x_2^n, \ldots, x_k^n)(t) - T_1(x_1^0, x_2^0, \ldots, x_k^0)(t) \right| \\
&+ \sup_{t\in I} \left| T_2(x_1^n, x_2^n, \ldots, x_k^n)(t) - T_2(x_1^0, x_2^0, \ldots, x_k^0)(t) \right|
\end{aligned}$$

$$
\begin{aligned}
&+\cdots+\sup_{t\in I}\left|T_k\left(x_1^n,x_2^n,\ldots,x_k^n\right)(t)\right.\\
&\qquad\left.-T_k\left(x_1^0,x_2^0,\ldots,x_k^0\right)(t)\right|\\
&+\sup_{t\in I}\left|\left(T_1\left(x_1^n,x_2^n,\ldots,x_k^n\right)\right)'(t)\right.\\
&\qquad\left.-\left(T_1\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right)'(t)\right|\\
&+\sup_{t\in I}\left|\left(T_2\left(x_1^n,x_2^n,\ldots,x_k^n\right)\right)'(t)\right.\\
&\qquad\left.-\left(T_2\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right)'(t)\right|\\
&+\cdots+\sup_{t\in I}\left|\left(T_k\left(x_1^n,x_2^n,\ldots,x_k^n\right)\right)'(t)\right.\\
&\qquad\left.-\left(T_k\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right)'(t)\right|\\
&+\sup_{t\in I}\left|\left(T_1\left(x_1^n,x_2^n,\ldots,x_k^n\right)\right)''(t)\right.\\
&\qquad\left.-\left(T_1\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right)''(t)\right|\\
&+\sup_{t\in I}\left|\left(T_2\left(x_1^n,x_2^n,\ldots,x_k^n\right)\right)''(t)\right.\\
&\qquad\left.-\left(T_2\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right)''(t)\right|\\
&+\cdots+\sup_{t\in I}\left|\left(T_k\left(x_1^n,x_2^n,\ldots,x_k^n\right)\right)''(t)\right.\\
&\qquad\left.-\left(T_k\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right)''(t)\right|,
\end{aligned}
\tag{11}
$$

by using the above inequalities and the continuity of f_i ($i=1,2,\ldots,k$), we get

$$
\left\|T\left(x_1^n,x_2^n,\ldots,x_k^n\right)-T\left(x_1^0,x_2^0,\ldots,x_k^0\right)\right\|_*\longrightarrow 0. \tag{12}
$$

Thus, T is continuous in X^k. Let Ω be a bounded subset of X^k. Choose positive constants $l_1>0,\ldots,l_k>0$ such that $|\widetilde{f}_i(s,x(s))|\le l_i$ for all $x=(x_1,x_2,\ldots,x_k)\in\Omega$ and $i=1,2,\ldots,k$. Thus, for each $x=(x_1,x_2,\ldots,x_k)\in\Omega$ we have

$$
\begin{aligned}
\left|T_i(x)(t)\right| &\le \frac{1}{\Gamma(\alpha_i)}\int_0^t(t-s)^{\alpha_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&+\frac{1}{2\Gamma(\alpha_i)}\int_0^T(T-s)^{\alpha_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&+\frac{\Gamma(2-p_i)\left|T-2t\right|}{2T^{1-p_i}\Gamma(\alpha_i-p_i)}\\
&\times\int_0^T(T-s)^{\alpha_i-p_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&+\frac{\left|p_iT^2-4Tt+2(2-p_i)t^2\right|\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}}\\
&\times\int_0^T(T-s)^{\alpha_i-q_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&\le\frac{l_i}{\Gamma(\alpha_i)}\int_0^t(t-s)^{\alpha_i-1}ds\\
&+\frac{l_i}{2\Gamma(\alpha_i)}\int_0^T(T-s)^{\alpha_i-1}ds\\
&+l_i\frac{\Gamma(2-p_i)\left|T-2t\right|}{2T^{1-p_i}\Gamma(\alpha_i-p_i)}\int_0^T(T-s)^{\alpha_i-p_i-1}ds\\
&+l_i\frac{\left|p_iT^2-4Tt+2(2-p_i)t^2\right|\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}}\\
&\times\int_0^T(T-s)^{\alpha_i-q_i-1}ds\\
&\le\left(\frac{3}{2\Gamma(\alpha_i+1)}+\frac{\Gamma(2-p_i)}{2\Gamma(\alpha_i-p_i+1)}\right.\\
&\qquad\left.+\frac{(4-p_i)\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i}l_i,\\
\left|\left(T_i(x)\right)'(t)\right| &\le\frac{1}{\Gamma(\alpha_i-1)}\int_0^t(t-s)^{\alpha_i-2}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&+\frac{\Gamma(2-p_i)}{T^{1-p_i}\Gamma(\alpha_i-p_i)}\\
&\times\int_0^T(T-s)^{\alpha_i-p_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&+\frac{\left|T-(2-p_i)t\right|\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}}\\
&\times\int_0^T(T-s)^{\alpha_i-q_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&\le\left(\frac{1}{\Gamma(\alpha_i)}+\frac{\Gamma(2-p_i)}{\Gamma(\alpha_i-p_i+1)}\right.\\
&\qquad\left.+\frac{\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-1}l_i,\\
\left|\left(T_i(x)\right)''(t)\right| &\le\frac{1}{\Gamma(\alpha_i-2)}\int_0^t(t-s)^{\alpha_i-3}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&+\frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i)T^{2-q_i}}\\
&\times\int_0^T(T-s)^{\alpha_i-q_i-1}\left|\widetilde{f}_i(s,x(s))\right|ds\\
&\le\left(\frac{1}{\Gamma(\alpha_i-1)}+\frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-2}l_i
\end{aligned}
\tag{13}
$$

for all $i = 1, 2, \ldots, k$. Hence,

$$\begin{aligned}\|T_i(x)\| &\le \left(\frac{3}{2\Gamma(\alpha_i+1)} + \frac{\Gamma(2-p_i)}{2\Gamma(\alpha_i-p_i+1)} + \frac{(4-p_i)\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i}l_i \\ &\quad + \left(\frac{1}{\Gamma(\alpha_i)} + \frac{\Gamma(2-p_i)}{\Gamma(\alpha_i-p_i+1)} + \frac{\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-1}l_i \\ &\quad + \left(\frac{1}{\Gamma(\alpha_i-1)} + \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-2}l_i \\ &= M_i l_i \end{aligned} \tag{14}$$

for all $i = 1, 2, \ldots, k$ and so $\|T(x)\|_* \le \sum_{i=1}^{k} M_i l_i = M'$. This implies that the operator T is uniformly bounded. Now, we show that $T\Omega$ is an equicontinuous set. Let $0 \le t_1 < t_2 \le T$. Then, we have

$$\begin{aligned}
&\left|T_i(x)(t_2) - T_i(x)(t_1)\right| \\
&= \left|\frac{1}{\Gamma(\alpha_i)}\int_0^{t_2}(t_2-s)^{\alpha_i-1}\tilde{f}_i(s,x(s))\,ds - \frac{1}{\Gamma(\alpha_i)}\int_0^{t_1}(t_1-s)^{\alpha_i-1}\tilde{f}_i(s,x(s))\,ds \right.\\
&\quad - \frac{\Gamma(2-p_i)(t_2-t_1)}{T^{1-p_i}\Gamma(\alpha_i-p_i)} \times \int_0^T (T-s)^{\alpha_i-p_i-1}\tilde{f}_i(s,x(s))\,ds \\
&\quad \left. + \frac{\left[2T(t_2-t_1) - (2-p_i)\left(t_2^2-t_1^2\right)\right]\Gamma(3-q_i)}{2(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}} \times \int_0^T (T-s)^{\alpha_i-q_i-1}\tilde{f}_i(s,x(s))\,ds\right| \\
&\le \frac{l_i}{\Gamma(\alpha_i)}\int_0^{t_1}\left[(t_2-s)^{\alpha_i-1} - (t_1-s)^{\alpha_i-1}\right]ds + \frac{l_i}{\Gamma(\alpha_i)}\int_{t_1}^{t_2}(t_2-s)^{\alpha_i-1}ds \\
&\quad + \left[\frac{\Gamma(2-p_i)T^{\alpha_i-1}l_i}{\Gamma(\alpha_i-p_i+1)} + \frac{\Gamma(3-q_i)(3-p_i)T^{\alpha_i-1}l_i}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right] \times (t_2-t_1) \\
&= \frac{l_i}{\Gamma(\alpha_i+1)}\left(t_2^{\alpha_i} - t_1^{\alpha_i}\right) \\
&\quad + \left[\frac{\Gamma(2-p_i)T^{\alpha_i-1}l_i}{\Gamma(\alpha_i-p_i+1)} + \frac{\Gamma(3-q_i)(3-p_i)T^{\alpha_i-1}l_i}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right] \times (t_2-t_1), \\
&\left|(T_i(x))'(t_2) - (T_i(x))'(t_1)\right| \\
&\le \left|\frac{1}{\Gamma(\alpha_i-1)}\int_0^{t_2}(t_2-s)^{\alpha_i-2}\tilde{f}_i(s,x(s))\,ds - \frac{1}{\Gamma(\alpha_i-1)}\int_0^{t_1}(t_1-s)^{\alpha_i-2}\tilde{f}_i(s,x(s))\,ds \right.\\
&\quad \left. - \frac{\Gamma(3-q_i)(t_2-t_1)}{\Gamma(\alpha_i-q_i)T^{2-q_i}} \times \int_0^T (T-s)^{\alpha_i-q_i-1}\tilde{f}_i(s,x(s))\,ds\right| \\
&\le \frac{l_i}{\Gamma(\alpha_i-1)}\int_0^{t_1}\left[(t_2-s)^{\alpha_i-2} - (t_1-s)^{\alpha_i-2}\right]ds + \frac{l_i}{\Gamma(\alpha_i-1)}\int_{t_1}^{t_2}(t_2-s)^{\alpha_i-2}ds \\
&\quad + \frac{\Gamma(3-q_i)T^{\alpha_i-2}l_i}{\Gamma(\alpha_i-q_i+1)}(t_2-t_1) \\
&= \frac{l_i}{\Gamma(\alpha_i)}\left(t_2^{\alpha_i-1} - t_1^{\alpha_i-1}\right) + \frac{\Gamma(3-q_i)T^{\alpha_i-2}l_i}{\Gamma(\alpha_i-q_i+1)}(t_2-t_1), \\
&\left|(T_i(x))''(t_2) - (T_i(x))''(t_1)\right| \\
&\le \frac{1}{\Gamma(\alpha_i-2)}\left|\int_0^{t_2}(t_2-s)^{\alpha_i-3}\tilde{f}_i(s,x(s))\,ds - \int_0^{t_1}(t_1-s)^{\alpha_i-3}\tilde{f}_i(s,x(s))\,ds\right| \\
&\le \frac{l_i}{\Gamma(\alpha_i-2)}\int_0^{t_1}\left[(t_1-s)^{\alpha_i-3} - (t_2-s)^{\alpha_i-3}\right]ds + \frac{l_i}{\Gamma(\alpha_i-2)}\int_{t_1}^{t_2}(t_2-s)^{\alpha_i-3}ds \\
&= \frac{l_i}{\Gamma(\alpha_i-1)}\left[2(t_2-t_1)^{\alpha_i-2} + t_1^{\alpha_i-2} - t_2^{\alpha_i-2}\right]
\end{aligned} \tag{15}$$

for all $i = 1, \ldots, k$. As $t_2 \to t_1$, the right-hand side of the above inequalities tends to zero. Thus, by using the Arzela-Ascoli theorem one can conclude that the operator $T : X^k \to X^k$ is completely continuous. This completes the proof. □

Theorem 4. *Assume that there exist positive constants $a_i > 0$, $b_{ij} \geq 0$, $c_{ij} \geq 0$, $d_{ij} \geq 0$, and $e_{ij} \geq 0$ $(i, j = 1, 2, \ldots, k)$ such that*

$$\begin{aligned} &\left|f_i(t, x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_k, z_1, z_2, \ldots, z_k, w_1, w_2, \ldots, w_k)\right| \\ &\leq a_i + \sum_{j=1}^{k} b_{ij}\left|x_j\right| + \sum_{j=1}^{k} c_{ij}\left|y_j\right| + \sum_{j=1}^{k} d_{ij}\left|z_j\right| + \sum_{j=1}^{k} e_{ij}\left|w_j\right| \end{aligned} \tag{16}$$

and $\sum_{i=1}^{k} M_i(b_{ij} + c_{ij}\lambda_{ij}^0 + d_{ij}(T^{1-\mu_{ij}}/\Gamma(2-\mu_{ij})) + e_{ij}(T^{2-\beta_{ij}}/\Gamma(3-\beta_{ij}))) < 1$ *for all* $x_i, y_i, z_i, w_i \in \mathbb{R}$, $t \in I$, *and* $i, j = 1, 2, \ldots, k$. *Then problem* (1) *has at least one solution.*

Proof. First, we show that $V = \{x = (x_1, x_2, \ldots, x_k) \in X^k : x = \mu T(x) \text{ for some } \mu \in [0,1]\}$ is bounded. Let $x = (x_1, x_2, \ldots, x_k) \in V$. Then, for each $t \in I$ we have

$$\begin{aligned} x_i(t) &= \frac{1}{\Gamma(\alpha_i)} \int_0^t \mu(t-s)^{\alpha_i-1} \widetilde{f}_i(s, x(s))\, ds - \frac{1}{2\Gamma(\alpha_i)} \int_0^T \mu(T-s)^{\alpha_i-1} \widetilde{f}_i(s, x(s))\, ds \\ &+ \frac{\Gamma(2-p_i)(T-2t)}{2T^{1-p_i}\Gamma(\alpha_i - p_i)} \times \int_0^T \mu(T-s)^{\alpha_i-p_i-1} \widetilde{f}_i(s, x(s))\, ds \\ &- \frac{\left[p_i T^2 - 4Tt + 2(2-p_i)t^2\right]\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i - q_i)T^{2-q_i}} \times \int_0^T \mu(T-s)^{\alpha_i-q_i-1} \widetilde{f}_i(s, x(s))\, ds. \end{aligned} \tag{17}$$

Hence,

$$\begin{aligned} x_i'(t) &= \frac{1}{\Gamma(\alpha_i - 1)} \int_0^t \mu(t-s)^{\alpha_i-2} \widetilde{f}_i(s, x(s))\, ds - \frac{\Gamma(2-p_i)}{T^{1-p_i}\Gamma(\alpha_i - p_i)} \int_0^T \mu(T-s)^{\alpha_i-p_i-1} \widetilde{f}_i(s, x(s))\, ds \\ &+ \frac{\left[T - (2-p_i)t\right]\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i - q_i)T^{2-q_i}} \times \int_0^T \mu(T-s)^{\alpha_i-q_i-1} \widetilde{f}_i(s, x(s))\, ds, \end{aligned} \tag{18}$$

$$\begin{aligned} x_i''(t) &= \frac{1}{\Gamma(\alpha_i - 2)} \int_0^t \mu(t-s)^{\alpha_i-3} \widetilde{f}_i(s, x(s))\, ds \\ &- \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i - q_i)T^{2-q_i}} \times \int_0^T \mu(T-s)^{\alpha_i-q_i-1} \widetilde{f}_i(s, x(s))\, ds \end{aligned} \tag{19}$$

for all $i = 1, 2, \ldots, k$. Thus, we get

$$\begin{aligned} \left|x_i(t)\right| &= \mu\left|T_i(x)(t)\right| \\ &\leq \left[a_i + \sum_{j=1}^{k} b_{ij}\left\|x_j\right\| + \sum_{j=1}^{k} c_{ij}\lambda_{ij}^0\left\|x_j\right\| + \sum_{j=1}^{k} d_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}\left\|x_j\right\| + \sum_{j=1}^{k} e_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\left\|x_j\right\|\right] \\ &\times \left(\frac{3}{2\Gamma(\alpha_i+1)} + \frac{\Gamma(2-p_i)}{2\Gamma(\alpha_i - p_i + 1)} + \frac{(4-p_i)\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i - q_i + 1)}\right) T^{\alpha_i}, \\ \left|x_i'(t)\right| &= \mu\left|(T_i(x))'(t)\right| \\ &\leq \left[a_i + \sum_{j=1}^{k} b_{ij}\left\|x_j\right\| + \sum_{j=1}^{k} c_{ij}\lambda_{ij}^0\left\|x_j\right\| + \sum_{j=1}^{k} d_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}\left\|x_j\right\| + \sum_{j=1}^{k} e_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\left\|x_j\right\|\right] \\ &\times \left(\frac{1}{\Gamma(\alpha_i)} + \frac{\Gamma(2-p_i)}{\Gamma(\alpha_i - p_i + 1)} + \frac{\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i - q_i + 1)}\right) T^{\alpha_i-1}, \\ \left|x_i''(t)\right| &= \mu\left|(T_i(x))''(t)\right| \\ &\leq \left[a_i + \sum_{j=1}^{k} b_{ij}\left\|x_j\right\| + \sum_{j=1}^{k} c_{ij}\lambda_{ij}^0\left\|x_j\right\| + \sum_{j=1}^{k} d_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}\left\|x_j\right\| + \sum_{j=1}^{k} e_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\left\|x_j\right\|\right] \\ &\times \left(\frac{1}{\Gamma(\alpha_i - 1)} + \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i - q_i + 1)}\right) T^{\alpha_i-2}. \end{aligned} \tag{20}$$

Hence, $\|x_i\| \leq M_i[a_i + \sum_{j=1}^{k} b_{ij}\|x_j\| + \sum_{j=1}^{k} c_{ij}\lambda_{ij}^0\|x_j\| + \sum_{j=1}^{k} d_{ij}(T^{1-\mu_{ij}}/\Gamma(2-\mu_{ij}))\|x_j\| + \sum_{j=1}^{k} e_{ij}(T^{2-\beta_{ij}}/\Gamma(3-\beta_{ij}))\|x_j\|]$ for $i = 1, 2, \ldots, k$. This implies that

$$\sum_{i=1}^{k}\|x_i\| \leq \sum_{j=1}^{k}\left[\sum_{i=1}^{k} M_i\left(b_{ij} + c_{ij}\lambda_{ij}^0 + d_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} + e_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\right)\right]\|x_j\| + \sum_{i=1}^{k} M_i a_i \tag{21}$$

and so $\|x = (x_1, x_2, \ldots, x_k)\|_* \leq (\sum_{i=1}^{k} M_i a_i)/M$. Therefore, the set V is bounded. Now by using Theorem 2, the operator T has at least one fixed point. This implies that the problem (1) has at least one solution. □

Theorem 5. *Suppose that there exist nonnegative constants $\eta_{ij} \geq 0$, $\theta_{ij} \geq 0$, $\nu_{ij} \geq 0$, and $\xi_{ij} \geq 0$ for $i, j = 1, 2, \ldots, k$ such that*

$$\begin{aligned}&\left|f_i(t, x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_k, z_1, z_2, \ldots, z_k, w_1, w_2, \ldots, w_k)\right.\\&\quad\left. - f_i(t, x_1', x_2', \ldots, x_k', y_1', y_2', \ldots, y_k', z_1', z_2', \ldots, z_k', w_1', w_2', \ldots, w_k')\right|\\&\leq \sum_{j=1}^{k}\eta_{ij}\left|x_j - x_j'\right| + \sum_{j=1}^{k}\theta_{ij}\left|y_j - y_j'\right| + \sum_{j=1}^{k}\nu_{ij}\left|z_j - z_j'\right| + \sum_{j=1}^{k}\xi_{ij}\left|w_j - w_j'\right|,\end{aligned} \tag{22}$$

$$\sum_{j=1}^{k}\eta_{ij} + \sum_{j=1}^{k}\theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^{k}\nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} + \sum_{j=1}^{k}\xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})} \leq \frac{1}{2kM_i} \tag{23}$$

for all $t \in I$, $x_i, y_i, z_i, w_i, x_i', y_i', z_i', w_i' \in \mathbb{R}$, and $i = 1, 2, \ldots, k$. Then the problem (1) *has a unique solution.*

Proof. Let $N_i = \sup_{t\in I}|f_i(t, 0, 0, \ldots, 0)| < \infty$ for $i = 1, 2, \ldots, k$, $r \geq 2k\max_{1\leq i\leq k}\{M_i N_i\}$, and

$$B_r = \left\{x = (x_1, x_2, \ldots, x_k) \in X^k \mid \|x\|_* \leq r\right\}. \tag{24}$$

We show that $TB_r \subseteq B_r$. Let $x = (x_1, x_2, \ldots, x_k) \in B_r$. Then

$$\begin{aligned}\left|T_i(x)(t)\right| &\leq \frac{1}{\Gamma(\alpha_i)}\int_0^t (t-s)^{\alpha_i-1}\left|\tilde{f}_i(s, x(s))\right| ds + \frac{1}{2\Gamma(\alpha_i)}\int_0^T (T-s)^{\alpha_i-1}\left|\tilde{f}_i(s, x(s))\right| ds\\&\quad + \frac{\Gamma(2-p_i)|T-2t|}{2T^{1-p_i}\Gamma(\alpha_i-p_i)} \times \int_0^T (T-s)^{\alpha_i-p_i-1}\left|\tilde{f}_i(s, x(s))\right| ds\\&\quad + \frac{\left|p_iT^2 - 4Tt + 2(2-p_i)t^2\right|\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}} \times \int_0^T (T-s)^{\alpha_i-q_i-1}\left|\tilde{f}_i(s, x(s))\right| ds\\&\leq \frac{1}{\Gamma(\alpha_i)}\int_0^t (t-s)^{\alpha_i-1} \times \left[\left|\tilde{f}_i(s, x(s)) - f_i(s, 0, 0, \ldots, 0)\right| + \left|f_i(s, 0, 0, \ldots, 0)\right|\right] ds\\&\quad + \frac{1}{2\Gamma(\alpha_i)}\int_0^T (T-s)^{\alpha_i-1} \times \left[\left|\tilde{f}_i(s, x(s)) - f_i(s, 0, 0, \ldots, 0)\right| + \left|f_i(s, 0, 0, \ldots, 0)\right|\right] ds\\&\quad + \frac{\Gamma(2-p_i)|T-2t|}{2T^{1-p_i}\Gamma(\alpha_i-p_i)} \times \int_0^T (T-s)^{\alpha_i-p_i-1} \times \left[\left|\tilde{f}_i(s, x(s)) - f_i(s, 0, 0, \ldots, 0)\right| + \left|f_i(s, 0, 0, \ldots, 0)\right|\right] ds\\&\quad + \frac{\left|p_iT^2 - 4Tt + 2(2-p_i)t^2\right|\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}} \times \int_0^T (T-s)^{\alpha_i-q_i-1} \times \left[\left|\tilde{f}_i(s, x(s)) - f_i(s, 0, 0, \ldots, 0)\right| + \left|f_i(s, 0, 0, \ldots, 0)\right|\right] ds\\&\leq \left(\left[\sum_{j=1}^{k}\eta_{ij} + \sum_{j=1}^{k}\theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^{k}\nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}\right.\right.\end{aligned}$$

$$+\sum_{j=1}^{k}\xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\Bigg]r+N_i\Bigg)$$
$$\times\left(\frac{3}{2\Gamma(\alpha_i+1)}+\frac{\Gamma(2-p_i)}{2\Gamma(\alpha_i-p_i+1)}+\frac{(4-p_i)\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i},$$

$$\begin{aligned}\left|(T_i(x))'(t)\right| &\le \frac{1}{\Gamma(\alpha_i-1)}\times\int_0^t(t-s)^{\alpha_i-2}\left|\tilde{f}_i(s,x(s))\right|ds\\
&+\frac{\Gamma(2-p_i)}{T^{1-p_i}\Gamma(\alpha_i-p_i)}\times\int_0^T(T-s)^{\alpha_i-p_i-1}\left|\tilde{f}_i(s,x(s))\right|ds\\
&+\frac{|T-(2-p_i)t|\,\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}}\times\int_0^T(T-s)^{\alpha_i-q_i-1}\left|\tilde{f}_i(s,x(s))\right|ds\\
&\le\frac{1}{\Gamma(\alpha_i-1)}\times\int_0^t(t-s)^{\alpha_i-2}\times\left[\left|\tilde{f}_i(s,x(s))-f_i(s,0,0,\ldots,0)\right|+\left|f_i(s,0,0,\ldots,0)\right|\right]ds\\
&+\frac{\Gamma(2-p_i)}{T^{1-p_i}\Gamma(\alpha_i-p_i)}\times\int_0^T(T-s)^{\alpha_i-p_i-1}\times\left[\left|\tilde{f}_i(s,x(s))-f_i(s,0,0,\ldots,0)\right|+\left|f_i(s,0,0,\ldots,0)\right|\right]ds\\
&+\frac{|T-(2-p_i)t|\,\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i)T^{2-q_i}}\times\int_0^T(T-s)^{\alpha_i-q_i-1}\times\left[\left|\tilde{f}_i(s,x(s))-f_i(s,0,0,\ldots,0)\right|+\left|f_i(s,0,0,\ldots,0)\right|\right]ds\\
&\le\Bigg(\Bigg[\sum_{j=1}^{k}\eta_{ij}+\sum_{j=1}^{k}\theta_{ij}\lambda_{ij}^{0}+\sum_{j=1}^{k}\nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}+\sum_{j=1}^{k}\xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\Bigg]r+N_i\Bigg)\\
&\times\left(\frac{1}{\Gamma(\alpha_i)}+\frac{\Gamma(2-p_i)}{\Gamma(\alpha_i-p_i+1)}+\frac{\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-1},\end{aligned}$$

$$\begin{aligned}\left|(T_i(x))''(t)\right| &\le \frac{1}{\Gamma(\alpha_i-2)}\times\int_0^t(t-s)^{\alpha_i-3}\left|\tilde{f}_i(s,x(s))\right|ds\\
&+\frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i)T^{2-q_i}}\times\int_0^T(T-s)^{\alpha_i-q_i-1}\left|\tilde{f}_i(s,x(s))\right|ds\\
&\le\frac{1}{\Gamma(\alpha_i-2)}\times\int_0^t(t-s)^{\alpha_i-3}\times\left[\left|\tilde{f}_i(s,x(s))-f_i(s,0,0,\ldots,0)\right|+\left|f_i(s,0,0,\ldots,0)\right|\right]ds\\
&+\frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i)T^{2-q_i}}\times\int_0^T(T-s)^{\alpha_i-q_i-1}\times\left[\left|\tilde{f}_i(s,x(s))-f_i(s,0,0,\ldots,0)\right|+\left|f_i(s,0,0,\ldots,0)\right|\right]ds\\
&\le\Bigg(\Bigg[\sum_{j=1}^{k}\eta_{ij}+\sum_{j=1}^{k}\theta_{ij}\lambda_{ij}^{0}+\sum_{j=1}^{k}\nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}+\sum_{j=1}^{k}\xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\Bigg]r+N_i\Bigg)\\
&\times\left(\frac{1}{\Gamma(\alpha_i-1)}+\frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-2}\end{aligned}\qquad(25)$$

for $i=1,2,\ldots,k$. Hence,

$$\|T_i(x)\|\le\Bigg(\Bigg[\sum_{j=1}^{k}\eta_{ij}+\sum_{j=1}^{k}\theta_{ij}\lambda_{ij}^{0}+\sum_{j=1}^{k}\nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})}+\sum_{j=1}^{k}\xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\Bigg]r+N_i\Bigg)M_i\le\frac{r}{k}\qquad(26)$$

for $i = 1, 2, \ldots, k$. Hence, $\|T(x)\|_* \le r$. Now, for each $x = (x_1, x_2, \ldots, x_k)$, $y = (y_1, y_2, \ldots, y_k) \in X^k$, and $t \in I$ we get

$$\begin{aligned}
&\left|T_i(x)(t) - T_i(y)(t)\right| \\
&\quad \le \frac{1}{\Gamma(\alpha_i)} \int_0^t (t-s)^{\alpha_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad + \frac{1}{2\Gamma(\alpha_i)} \\
&\quad \times \int_0^T (T-s)^{\alpha_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad + \frac{\Gamma(2-p_i)|T-2t|}{2T^{1-p_i}\Gamma(\alpha_i - p_i)} \\
&\quad \times \int_0^T (T-s)^{\alpha_i-p_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad + \frac{\left|p_iT^2 - 4Tt + 2(2-p_i)t^2\right|\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i - q_i)T^{2-q_i}} \\
&\quad \times \int_0^T (T-s)^{\alpha_i-q_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad \le \left(\sum_{j=1}^k \eta_{ij} + \sum_{j=1}^k \theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^k \nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} + \sum_{j=1}^k \xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\right) \\
&\quad \times \left(\frac{3}{2\Gamma(\alpha_i+1)} + \frac{\Gamma(2-p_i)}{2\Gamma(\alpha_i-p_i+1)} + \frac{(4-p_i)\Gamma(3-q_i)}{4(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i} \\
&\quad \times \left(\sum_{j=1}^k \left\|x_j - y_j\right\|\right), \\
&\left|(T_i(x))'(t) - (T_i(y))'(t)\right| \\
&\quad \le \frac{1}{\Gamma(\alpha_i - 1)} \\
&\quad \times \int_0^t (t-s)^{\alpha_i-2} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad + \frac{\Gamma(2-p_i)}{T^{1-p_i}\Gamma(\alpha_i - p_i)} \\
&\quad \times \int_0^T (T-s)^{\alpha_i-p_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad + \frac{\left|T - (2-p_i)t\right|\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i - q_i)T^{2-q_i}} \\
&\quad \times \int_0^T (T-s)^{\alpha_i-q_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad \le \left(\sum_{j=1}^k \eta_{ij} + \sum_{j=1}^k \theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^k \nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} + \sum_{j=1}^k \xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\right) \\
&\quad \times \left(\frac{1}{\Gamma(\alpha_i)} + \frac{\Gamma(2-p_i)}{\Gamma(\alpha_i-p_i+1)} + \frac{\Gamma(3-q_i)}{(2-p_i)\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-1} \\
&\quad \times \left(\sum_{j=1}^k \left\|x_j - y_j\right\|\right), \\
&\left|(T_i(x))''(t) - (T_i(y))''(t)\right| \\
&\quad \le \frac{1}{\Gamma(\alpha_i - 2)} \\
&\quad \times \int_0^t (t-s)^{\alpha_i-3} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad + \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i - q_i)T^{2-q_i}} \\
&\quad \times \int_0^T (T-s)^{\alpha_i-q_i-1} \left|\tilde{f}_i(s,x(s)) - \tilde{f}_i(s,y(s))\right| ds \\
&\quad \le \left(\sum_{j=1}^k \eta_{ij} + \sum_{j=1}^k \theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^k \nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} + \sum_{j=1}^k \xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\right) \\
&\quad \times \left(\frac{1}{\Gamma(\alpha_i-1)} + \frac{\Gamma(3-q_i)}{\Gamma(\alpha_i-q_i+1)}\right)T^{\alpha_i-2} \\
&\quad \times \left(\sum_{j=1}^k \left\|x_j - y_j\right\|\right)
\end{aligned} \tag{27}$$

for $i = 1, 2, \ldots, k$. This implies that

$$\begin{aligned}
&\left\|T_i(x) - T_i(y)\right\| \\
&\quad \le M_i\left(\sum_{j=1}^k \eta_{ij} + \sum_{j=1}^k \theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^k \nu_{ij}\frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} + \sum_{j=1}^k \xi_{ij}\frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})}\right) \times \left(\sum_{j=1}^k \left\|x_j - y_j\right\|\right)
\end{aligned} \tag{28}$$

for $i = 1, 2, \ldots, k$ and so

$$\begin{aligned} &\|T(x) - T(y)\|_* \\ &\quad \leq \left[\sum_{i=1}^{k} M_i \left(\sum_{j=1}^{k} \eta_{ij} + \sum_{j=1}^{k} \theta_{ij} \lambda_{ij}^0 + \sum_{j=1}^{k} \nu_{ij} \frac{T^{1-\mu_{ij}}}{\Gamma(2-\mu_{ij})} \right.\right. \\ &\qquad \left.\left. + \sum_{j=1}^{k} \xi_{ij} \frac{T^{2-\beta_{ij}}}{\Gamma(3-\beta_{ij})} \right) \right] \times \left(\sum_{j=1}^{k} \|x_j - y_j\| \right). \end{aligned} \tag{29}$$

Since $\sum_{i=1}^{k} M_i(\sum_{j=1}^{k} \eta_{ij} + \sum_{j=1}^{k} \theta_{ij}\lambda_{ij}^0 + \sum_{j=1}^{k} \nu_{ij}(T^{1-\mu_{ij}}/\Gamma(2-\mu_{ij})) + \sum_{j=1}^{k} \xi_{ij}(T^{2-\beta_{ij}}/\Gamma(3-\beta_{ij}))) < 1$, T is a contraction and so, by using the Banach contraction principle, T has a unique fixed point. Now, one can easily get that the problem (1) has a unique solution. □

3. Example

Here, we provide an example to illustrate one of our results. It is considerable that there are some examples which provide nonuniqueness of solutions for some fractional differential equations (see, e.g., [32]).

Example 1. Consider the following 3-dimensional system:

$$\begin{aligned} &{}^cD^{\alpha_1} x_1(t) \\ &\quad = 12e^{-\cos^2 t} \\ &\qquad + e^{-\pi t}\left(x_1(t) + x_2(t) + x_3(t)\right) \\ &\qquad \times \Big(9121\sqrt{\pi} \\ &\qquad\quad \times \left(1 + \left| {}^cD^{\mu_{11}} x_1(t) + {}^cD^{\mu_{12}} x_2(t) + {}^cD^{\mu_{13}} x_3(t) \right|\right)\Big)^{-1} \\ &\qquad + \frac{5 + \cos \pi t}{8124(t+4)^2} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{e^{-|x_k(t)|\sin^2(t^3+4)}}{2^k k!} \ln\left(1 + |\phi_{1k} x_k(t)|\right) \right) \\ &\qquad + \frac{8e^{-12\sin^2(x_2(t))}\cos^3 t}{2450\left(2t + 387e^3\right)} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{e^{-kt}\cos k\pi}{4^k k!} {}^cD^{\mu_{1k}} x_k(t) \right) \\ &\qquad + \frac{1}{2130(t+6)^3} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{e^{-(2+\cos t)} \sin\left({}^cD^{\mu_{1k}} x_k(t)\right) {}^cD^{\beta_{1k}} x_k(t)}{\sqrt[k+1]{e}\left(2 + \left|x_k(t) + {}^cD^{\mu_{1k}} x_k(t)\right|\right)} \right), \\ &{}^cD^{\alpha_2} x_2(t) \\ &\quad = 5t^2 + \frac{\cos^3 t}{6421\pi\left(2 + \left|x_1(t) + {}^cD^{\beta_{21}} x_1(t)\right|\right)} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{(t-1/2)^4 x_k(t)}{3 + \sin x_k(t)} \right) + \frac{(3-t)^{1/2}}{5180} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{\left({}^cD^{\beta_{2k}} x_k(t)\right)^3}{1 + \left|{}^cD^{\beta_{2k}} x_k(t)\right|^3} \phi_{2k} x_k(t) \right) \\ &\qquad + \frac{e^{-5t}\sin\left({}^cD^{\beta_{22}} x_2(t)\right)}{1145\left(1 + e^t\right)} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{e^{-\pi t(1+|x_k(t)+\phi_{2k}x_k(t)|)}\, {}^cD^{\mu_{2k}} x_k(t)}{\sqrt{17 + \sin\left({}^cD^{\mu_{2k}} x_k(t)\right)}\left(1 + \left|{}^cD^{\mu_{2k}} x_k(t)\right|\right)} \right) \\ &\qquad + \frac{e^{(1-\cos^2 x_1(t))^2}}{2918\left(1 + |x_2(t)|\right)} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{\ln\left(1 + \left|{}^cD^{\beta_{2k}} x_k(t)\right|\right)}{\ln\left(e + \cos^2 x_k(t) + \sin^2\left({}^cD^{\beta_{2k}} x_k(t)\right)\right)} \right), \\ &{}^cD^{\alpha_3} x_3(t) \\ &\quad = 15\cos^2 t + \frac{6 + \cos t}{360(t+7)^5} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{x_k(t) + {}^cD^{\beta_{3k}} x_k(t)}{2 + |x_k(t)| + \cos\left({}^cD^{\mu_{3k}} x_k(t)\right)} \right) \\ &\qquad + \frac{e^{-\cos^4 t}}{1200\pi\left(17 + \left|x_3(t) + \phi_{33} x_3(t)\right|\right)} \\ &\qquad \times \left(\sum_{k=1}^{3} \frac{\ln\left(1 + \left|\phi_{3k} x_k(t) + {}^cD^{\mu_{3k}} x_k(t)\right|\right)}{12 + \sin\left({}^cD^{\mu_{3k}} x_k(t)\right) + \left|{}^cD^{\beta_{3k}} x_k(t)\right|} \right), \\ &\qquad\qquad x_i(0) = -x_i(1), \\ &\qquad\qquad {}^cD^{p_i} x_i(0) = -{}^cD^{p_i} x_i(1), \\ &\qquad\qquad {}^cD^{q_i} x_i(0) = -{}^cD^{q_i} x_i(1), \\ &\qquad\qquad\qquad i = 1, 2, 3, \end{aligned} \tag{30}$$

where $p_1 = 1/9$, $p_2 = 3/5$, $p_3 = 1/7$, $q_1 = 3/2$, $q_2 = 11/10$, $q_3 = 7/6$, $\alpha_1 = 17/8$, $\alpha_2 = 7/3$, $\alpha_3 = 12/5$, $\mu_{11} = 1/3$, $\mu_{12} = 1/15$, $\mu_{13} = 1/20$, $\mu_{21} = 1/19$, $\mu_{22} = 1/17$, $\mu_{23} = 1/12$, $\mu_{31} = 1/14$, $\mu_{32} = 2/7$, $\mu_{33} = 4/13$, $\beta_{11} = 8/7$, $\beta_{12} = 13/12$, $\beta_{13} = 15/13$, $\beta_{21} = 6/5$, $\beta_{22} = 15/8$, $\beta_{23} = 5/4$, $\beta_{31} = 10/9$, $\beta_{32} = 7/5$, $\beta_{33} = 12/11$, $\phi_{11}x_1(t) = \int_0^t ((t-s)e^{(t-s)^2}/919)x_1(s)ds$, $\phi_{21}x_1(t) = \int_0^t ((t-s)^3 e^{(t-s)^4}/727)x_1(s)ds$, $\phi_{31}x_1(t) = \int_0^t ((t-s)^2 e^{(t-s)^3}/500)x_1(s)ds$,

$\phi_{12}x_2(t) = \int_0^t (4s^3\sqrt{s^4+7}/831)x_2(s)ds$, $\phi_{22}x_2(t) = \int_0^t((3s+6)/700(s^2+5s+4))x_2(s)ds$, $\phi_{32}x_2(t) = \int_0^t(6s/901(4+s^2))x_2(s)ds$, $\phi_{13}x_3(t) = \int_0^t(e^{(t-s)/2}/925)x_3(s)ds$, $\phi_{23}x_3(t) = \int_0^t(e^{(t-s)/3}/1875)x_3(s)ds$, and $\phi_{33}x_3(t) = \int_0^t(e^{(t-s)/4}/3200)x_3(s)ds$, with $\lambda_{11}^0 = (e-1)/1838$, $\lambda_{21}^0 = (e-1)/2908$, $\lambda_{31}^0 = (e-1)/1500$, $\lambda_{12}^0 = 2(8\sqrt{8}-7\sqrt{7})/2493$, $\lambda_{22}^0 = (\ln 2 + 2\ln(5/4))/700$, $\lambda_{32}^0 = 3\ln(5/4)/901$, $\lambda_{13}^0 = 2(\sqrt{e}-1)/925$, $\lambda_{23}^0 = (\sqrt[3]{e}-1)/625$, and $\lambda_{33}^0 = (\sqrt[4]{e}-1)/800$. Then, we have

$$\begin{aligned}
&\Big|f_1\Big(t, x_1(t), x_2(t), x_3(t), \phi_{11}x_1(t), \\
&\quad \phi_{12}x_2(t), \phi_{13}x_3(t), {}^cD^{\mu_{11}}x_1(t), \\
&\quad {}^cD^{\mu_{12}}x_2(t), {}^cD^{\mu_{13}}x_3(t), \\
&\quad {}^cD^{\beta_{11}}x_1(t), {}^cD^{\beta_{12}}x_2(t), {}^cD^{\beta_{13}}x_3(t)\Big)\Big| \\
&\le 12 + \sum_{j=1}^{3}\frac{1}{9121\sqrt{\pi}}\left|x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{3}{8124\times 8\times 2^j\times j!}\left|\phi_{1j}x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{8}{2450\times 387e^3\times 4^j\times j!}\left|{}^cD^{\mu_{1j}}x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{1}{4260\times 6^3\times \sqrt[j+1]{e}}\left|{}^cD^{\beta_{1j}}x_j(t)\right|, \\
&\Big|f_2\Big(t, x_1(t), x_2(t), x_3(t), \phi_{21}x_1(t), \\
&\quad \phi_{22}x_2(t), \phi_{23}x_3(t), {}^cD^{\mu_{21}}x_1(t), \\
&\quad {}^cD^{\mu_{22}}x_2(t), {}^cD^{\mu_{23}}x_3(t), {}^cD^{\beta_{21}}x_1(t), \\
&\quad {}^cD^{\beta_{22}}x_2(t), {}^cD^{\beta_{23}}x_3(t)\Big)\Big| \\
&\le 5 + \sum_{j=1}^{3}\frac{1}{6421\times 64\times\pi}\left|x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{\sqrt{3}}{5180}\left|\phi_{2j}x_j(t)\right| + \sum_{j=1}^{3}\frac{1}{1145\times 4}\left|{}^cD^{\mu_{2j}}x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{e}{2918}\left|{}^cD^{\beta_{2j}}x_j(t)\right|, \\
&\Big|f_3\Big(t, x_1(t), x_2(t), x_3(t), \phi_{31}x_1(t), \\
&\quad \phi_{32}x_2(t), \phi_{33}x_3(t), {}^cD^{\mu_{31}}x_1(t), \\
&\quad {}^cD^{\mu_{32}}x_2(t), {}^cD^{\mu_{33}}x_3(t), \\
&\quad {}^cD^{\beta_{31}}x_1(t), {}^cD^{\beta_{32}}x_2(t), {}^cD^{\beta_{33}}x_3(t)\Big)\Big| \\
&\le 15 + \sum_{j=1}^{3}\frac{1}{7^4\times 360}\left|x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{1}{1200\pi\times 17\times 11}\left|\phi_{3j}x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{1}{1200\pi\times 17\times 11}\left|{}^cD^{\mu_{3j}}x_j(t)\right| \\
&\quad + \sum_{j=1}^{3}\frac{1}{7^4\times 360}\left|{}^cD^{\beta_{3j}}x_j(t)\right|.
\end{aligned} \tag{31}$$

Here, $M_1 \cong 5.401859055$, $M_2 \cong 5.327781471$, $M_3 \cong 4.713515099$,

$$\begin{aligned}
&\sum_{i=1}^{3}M_i\left(b_{i1} + c_{i1}\lambda_{i1}^0 + \frac{d_{i1}}{\Gamma(2-\mu_{i1})} + \frac{e_{i1}}{\Gamma(3-\beta_{i1})}\right) \\
&\quad \cong 0.015425405 < 1, \\
&\sum_{i=1}^{3}M_i\left(b_{i2} + c_{i2}\lambda_{i2}^0 + \frac{d_{i2}}{\Gamma(2-\mu_{i2})} + \frac{e_{i2}}{\Gamma(3-\beta_{i2})}\right) \\
&\quad \cong 0.036796259 < 1, \\
&\sum_{i=1}^{3}M_i\left(b_{i3} + c_{i3}\lambda_{i3}^0 + \frac{d_{i3}}{\Gamma(2-\mu_{i3})} + \frac{e_{i3}}{\Gamma(3-\beta_{i3})}\right) \\
&\quad \cong 0.01958673 < 1.
\end{aligned} \tag{32}$$

Thus, by using Theorem 4, 3-dimensional system (30) has at least one solution.

4. Conclusions

Fractional integrodifferential equations, system of fractional differential equations, and their applications represent a topic of high interest in the area of fractional calculus and its applications in various fields of science and engineering. Antiperiodic boundary value problems occur in the mathematical modeling of a variety of physical processes. The goal of this paper is to investigate a complicated case by using an appropriate basic theory. In this way, we prove the existence and uniqueness of solution for a new k-dimensional system of multiterm fractional integrodifferential equations with antiperiodic boundary conditions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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