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Fractional complex transform method for wave equations on Cantor sets within local fractional differential operator

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Abstract

This paper points out the fractional complex transform method for wave equations on Cantor sets within the local differential fractional operators. The proposed method is efficient to handle differential equations on Cantor sets.

Keywords: fractional complex transform method; wave equations; local fractional differential operators; Cantor sets

1 Introduction

In recent years, the local fractional calculus has attracted a lot of interest for scientists and engineers. Several sections of local fractional derivative had been introduced, *i.e.* the local fractional derivative structured by Kolwankar and Gangal [1–5], the modified Riemann-Liouville derivative given by Jumarie [3–7], the fractal derivative suggested by Parvate and Gangal [3–5, 8, 9], the fractal derivative considered by Chen *et al.* [3–5, 10–12], the generalized fractal derivative proposed Chen *et al.* [12], the local fractional derivative presented by Adda and Cresson [3–5, 13], the fractal derivative suggested by He [3–5, 14–16] and the local fractional derivative structured in [3–5, 17–25]. As a result, the local fractional calculus theory become important for modelling problems for fractal mathematics and engineering on Cantor sets and it plays a key role in many applications in several fields such as the theoretical physics [2, 3], the heat conduction theory [3, 14, 17], the fracture and elasticity mechanics [3, 19], the fluid mechanics [3, 26], signal analysis [4, 5], Fourier and wavelet analysis [4, 5], tensor analysis [3], complex analysis [4, 5], optimization method [3] and so on. For example, the local fractional Fokker-Planck equation was proposed in [2]. The fractal heat conduction problems were presented [3, 14]. The principles of virtual work, minimum potential and complementary energy in the mechanics of fractal media were investigated in [3, 19, 25]. The fluid flow in fractal porous medium was studied in [26]. The diffusion problems in fractal media was reported in [11, 24, 27]. Authors of [28] were concerned with one-dimension wave equation on Cantor sets, which read

$$\frac{\partial^{2\alpha} u(x, t)}{\partial t^{2\alpha}} + a^{2\alpha} \frac{\partial^{2\alpha} u(x, t)}{\partial x^{2\alpha}} = 0, \quad (1)$$

where the operator was described as local fractional operator given by [3–5, 17–23]

$$f^{(\alpha)}(x_0) = \left. \frac{d^\alpha f(x)}{dx^\alpha} \right|_{x=x_0} = \lim_{x \rightarrow x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha} \tag{2}$$

with $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1 + \alpha) \Delta (f(x) - f(x_0))$ and where local fractional operator of k th order was [3–5]

$$\frac{\partial^{k\alpha} f(x)}{\partial x^{k\alpha}} = \overbrace{\frac{\partial^\alpha}{\partial x^\alpha} \cdots \frac{\partial^\alpha}{\partial x^\alpha}}^{k \text{ times}} f(x). \tag{3}$$

However, it has not been extended to two-dimensional (2-D) and three-dimensional (3-D) wave equations on Cantor sets due to the structural behavior of complexity of Cantor materials.

Fractional complex transform method *via* the modified Riemann-Liouville derivative [3–7] was first proposed in 2010 [29–31] and was applied to model the heat conduction problem and differential equations [15, 32–34]. The fractional complex transform met some problems in applications when the modified Riemann-Liouville derivative [3–7] was adopted due to the complex chain rule [34]. Fractional complex transform [17, 35] *via* the local fractional derivative was first proposed by the following chain rule [3–5]

$$\frac{d^\alpha y(x)}{dx^\alpha} = f^{(1)}(g(x))g^{(\alpha)}(x), \tag{4}$$

where there exist $f^{(1)}(g(x))$ and $g^{(\alpha)}(x)$. The heat conduction problem in fractal media was processed by the fractional complex transform within local fractional derivative [17]. The similar results, but using different operators, can be seen in [36–38].

In this work, we consider 3-D wave equation on Cantor sets described by the local fractional derivative suggested in [3–5, 17–25], which is written in the form

$$\frac{\partial^{2\alpha} u(x, y, z, t)}{\partial t^{2\alpha}} + a^{2\alpha} \nabla^{2\alpha} u(x, y, z, t) = 0, \tag{5}$$

where local fractional Laplace operator is noted by [3–5, 39]

$$\nabla^{2\alpha} = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}. \tag{6}$$

The 2-D wave equation on Cantor sets can easily be obtained

$$\frac{\partial^{2\alpha} u(x, y, t)}{\partial t^{2\alpha}} + a^{2\alpha} \left(\frac{\partial^{2\alpha} u(x, y, t)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x, y, t)}{\partial y^{2\alpha}} \right) = 0. \tag{7}$$

Our attention is focused on convert the 3-D wave equation on Cantor sets by using the fractional complex transform method *via* local fractional derivatives.

The organization of this manuscript is given below: In Section 2, the fractional complex transform method *via* local fractional derivatives is presented. Section 3 shows the applications of fractional complex transform to convert 3-D and 2-D wave equations on Cantor sets. Conclusions end the manuscript in Section 4.

2 Fractional complex transform method via local fractional derivatives

In this section, we consider the fractional complex transform method for differential equations on Cantor sets [17, 35].

Proposition 1 *If*

$$\begin{cases} X = \frac{x^\alpha}{\Gamma(1+\alpha)}, \\ Y = \frac{y^\alpha}{\Gamma(1+\alpha)}, \end{cases} \tag{8}$$

where $0 < \alpha \leq 1$, then we have

$$\frac{\partial^\alpha U_1(x, y)}{\partial x^\alpha} + \frac{\partial^\alpha U_2(x, y)}{\partial y^\alpha} = 0,$$

such that

$$\frac{\partial U_1(X, Y)}{\partial X} + \frac{\partial U_2(X, Y)}{\partial Y} = 0, \tag{9}$$

where there exist the relations

$$\frac{\partial U_1(X, Y)}{\partial X}, \frac{\partial U_2(X, Y)}{\partial Y}, \frac{\partial^\alpha U_1(x, y)}{\partial x^\alpha}, \frac{\partial^\alpha U_2(x, y)}{\partial y^\alpha}.$$

Proof Let us consider the fractional complex transform (8), then we can write

$$\frac{\partial^\alpha U_1(x, y)}{\partial x^\alpha} = \frac{\partial U_1(X, Y)}{\partial X} \frac{\partial X}{\partial x^\alpha} + \frac{\partial U_1(X, Y)}{\partial Y} \frac{\partial Y}{\partial x^\alpha} = \frac{\partial U_1(X, Y)}{\partial X} \tag{10}$$

and finally we conclude

$$\frac{\partial^\alpha U_2(x, y)}{\partial y^\alpha} = \frac{\partial U_2(X, Y)}{\partial Y} \frac{\partial Y}{\partial y^\alpha} + \frac{\partial U_2(X, Y)}{\partial X} \frac{\partial X}{\partial y^\alpha} = \frac{\partial U_2(X, Y)}{\partial Y} \tag{11}$$

respectively. □

Proposition 2 *Assuming that (8) is valid, then we can transfer*

$$\frac{\partial^{2\alpha} U_2(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} U_1(x, y)}{\partial x^\alpha \partial y^\alpha} + \frac{\partial^{2\alpha} U_1(x, y)}{\partial y^\alpha \partial x^\alpha} + \frac{\partial^{2\alpha} U_2(x, y)}{\partial y^{2\alpha}} = 0 \tag{12}$$

into

$$\frac{\partial^2 U_2(X, Y)}{\partial X^2} + \frac{\partial^2 U_1(X, Y)}{\partial X \partial Y} + \frac{\partial^2 U_2(X, Y)}{\partial Y \partial X} + \frac{\partial^2 U_2(X, Y)}{\partial Y^2} = 0, \tag{13}$$

where there exist the following relations:

$$\begin{aligned} & \frac{\partial U_1(X, Y)}{\partial X}, \frac{\partial U_2(X, Y)}{\partial Y}, \frac{\partial^2 U_1(X, Y)}{\partial X^2}, \frac{\partial^2 U_1(X, Y)}{\partial Y \partial X}, \frac{\partial^2 U_2(X, Y)}{\partial X \partial Y}, \frac{\partial^2 U_2(X, Y)}{\partial Y^2}, \\ & \frac{\partial^\alpha U_1(x, y)}{\partial x^\alpha}, \frac{\partial^\alpha U_2(x, y)}{\partial y^\alpha}, \frac{\partial^{2\alpha} U_1(x, y)}{\partial x^{2\alpha}}, \frac{\partial^{2\alpha} U_1(x, y)}{\partial y^\alpha \partial x^\alpha}, \frac{\partial^{2\alpha} U_2(x, y)}{\partial x^\alpha \partial y^\alpha}, \frac{\partial^{2\alpha} U_2(x, y)}{\partial y^{2\alpha}}. \end{aligned}$$

Proof Taking the Cantorian coordination pairs (8) yields

$$\frac{\partial^{2\alpha} U_1(x, y)}{\partial x^{2\alpha}} = \frac{\partial^2 U_1(X, Y)}{\partial X^2} \frac{\partial^\alpha X}{\partial x^\alpha} + \frac{\partial^2 U_1(X, Y)}{\partial Y^2} \frac{\partial^\alpha Y}{\partial x^\alpha} = \frac{\partial^2 U_1(X, Y)}{\partial X^2}. \tag{14}$$

In a similar manner as in the previous proposition, we obtain

$$\frac{\partial^{2\alpha} U_2(x, y)}{\partial y^{2\alpha}} = \frac{\partial^2 U_2(X, Y)}{\partial Y^2} \frac{\partial^\alpha Y}{\partial y^\alpha} + \frac{\partial^2 U_2(X, Y)}{\partial X^2} \frac{\partial^\alpha X}{\partial y^\alpha} = \frac{\partial^2 U_2(X, Y)}{\partial X^2}, \tag{15}$$

$$\frac{\partial^{2\alpha} U_1(x, y)}{\partial y^\alpha \partial x^\alpha} = \frac{\partial^2 U_1(X, Y)}{\partial X \partial Y} \frac{\partial^\alpha Y}{\partial y^\alpha} + \frac{\partial^2 U_1(X, Y)}{\partial X \partial Y} \frac{\partial^\alpha Y}{\partial x^\alpha} = \frac{\partial^2 U_1(X, Y)}{\partial Y \partial X}, \tag{16}$$

$$\frac{\partial^{2\alpha} U_2(x, y)}{\partial x^\alpha \partial y^\alpha} = \frac{\partial^2 U_2(X, Y)}{\partial Y \partial X} \frac{\partial^\alpha X}{\partial x^\alpha} + \frac{\partial^2 U_2(X, Y)}{\partial X \partial Y} \frac{\partial^\alpha X}{\partial y^\alpha} = \frac{\partial^2 U_2(X, Y)}{\partial X \partial Y}. \tag{17}$$

Therefore, we conclude that

$$\frac{\partial^2 U_2(X, Y)}{\partial X^2} + \frac{\partial^2 U_1(X, Y)}{\partial X \partial Y} + \frac{\partial^2 U_2(X, Y)}{\partial Y \partial X} + \frac{\partial^2 U_2(X, Y)}{\partial Y^2} = 0. \tag{18}$$

□

Proposition 3 *Let us assume that there is the fractional complex transform*

$$\begin{cases} X = \frac{x^\alpha}{\Gamma(1+\alpha)}, \\ Y = \frac{y^\alpha}{\Gamma(1+\alpha)}, \\ Z = \frac{z^\alpha}{\Gamma(1+\alpha)} \end{cases} \tag{19}$$

and

$$\frac{\partial U_1(x, y, z)}{\partial x} + \frac{\partial U_2(x, y, z)}{\partial y} + \frac{\partial U_3(x, y, z)}{\partial z} = 0, \tag{20}$$

then

$$\frac{\partial^\alpha U_1(x, y, z)}{\partial x^\alpha} + \frac{\partial^\alpha U_2(x, y, z)}{\partial y^\alpha} + \frac{\partial^\alpha U_3(x, y, z)}{\partial z^\alpha} = 0. \tag{21}$$

Proof By using the fractional complex transform (19), we obtain

$$\begin{aligned} & \frac{\partial^\alpha U_1(x, y, z)}{\partial x^\alpha} \\ &= \frac{\partial U_1(X, Y, Z)}{\partial X} \frac{\partial^\alpha X}{\partial x^\alpha} + \frac{\partial U_1(X, Y, Z)}{\partial Y} \frac{\partial^\alpha Y}{\partial x^\alpha} + \frac{\partial U_1(X, Y, Z)}{\partial Z} \frac{\partial^\alpha Z}{\partial x^\alpha} \\ &= \frac{\partial U_1(X, Y, Z)}{\partial X}, \end{aligned} \tag{22}$$

$$\begin{aligned} & \frac{\partial^\alpha U_2(x, y, z)}{\partial y^\alpha} \\ &= \frac{\partial U_2(X, Y, Z)}{\partial X} \frac{\partial^\alpha X}{\partial y^\alpha} + \frac{\partial U_2(X, Y, Z)}{\partial Y} \frac{\partial^\alpha Y}{\partial y^\alpha} + \frac{\partial U_2(X, Y, Z)}{\partial Z} \frac{\partial^\alpha Z}{\partial y^\alpha} \\ &= \frac{\partial U_2(X, Y, Z)}{\partial Y}, \end{aligned} \tag{23}$$

$$\begin{aligned}
 & \frac{\partial^\alpha U_3(x, y, z)}{\partial z^\alpha} \\
 &= \frac{\partial U_3(X, Y, Z)}{\partial X} \frac{\partial^\alpha X}{\partial z^\alpha} + \frac{\partial U_3(X, Y, Z)}{\partial Y} \frac{\partial^\alpha Y}{\partial z^\alpha} + \frac{\partial U_3(X, Y, Z)}{\partial Z} \frac{\partial^\alpha Z}{\partial z^\alpha} \\
 &= \frac{\partial U_3(X, Y, Z)}{\partial X}.
 \end{aligned} \tag{24}$$

Clearly, from Eqs. (23)-(24), we directly obtain (21). □

Proposition 4 *Let us consider the following fractional complex transform (19) and*

$$\frac{\partial^2 U_1(x, y, z)}{\partial x^2} + \frac{\partial^2 U_2(x, y, z)}{\partial y^2} + \frac{\partial^2 U_3(x, y, z)}{\partial z^2} = 0, \tag{25}$$

thus, we obtain

$$\frac{\partial^{2\alpha} U_1(x, y, z)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} U_2(x, y, z)}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha} U_3(x, y, z)}{\partial z^{2\alpha}} = 0. \tag{26}$$

Proof Applying the fractional complex transform method (19), we obtain the following three equations:

$$\begin{aligned}
 & \frac{\partial^{2\alpha} U_1(x, y, z)}{\partial x^{2\alpha}} \\
 &= \frac{\partial^2 U_1(X, Y, Z)}{\partial X^2} \frac{\partial^\alpha X}{\partial x^\alpha} + \frac{\partial^2 U_1(X, Y, Z)}{\partial Y^2} \frac{\partial^\alpha Y}{\partial x^\alpha} + \frac{\partial^2 U_1(X, Y, Z)}{\partial Z^2} \frac{\partial^\alpha Z}{\partial x^\alpha} \\
 &= \frac{\partial^2 U_1(X, Y, Z)}{\partial X^2},
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 & \frac{\partial^{2\alpha} U_2(x, y, z)}{\partial x^{2\alpha}} \\
 &= \frac{\partial^2 U_2(X, Y, Z)}{\partial X^2} \frac{\partial^\alpha X}{\partial x^\alpha} + \frac{\partial^2 U_2(X, Y, Z)}{\partial Y^2} \frac{\partial^\alpha Y}{\partial x^\alpha} + \frac{\partial^2 U_2(X, Y, Z)}{\partial Z^2} \frac{\partial^\alpha Z}{\partial x^\alpha} \\
 &= \frac{\partial^2 U_2(X, Y, Z)}{\partial X^2},
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 & \frac{\partial^{2\alpha} U_3(x, y, z)}{\partial x^{2\alpha}} \\
 &= \frac{\partial^2 U_3(X, Y, Z)}{\partial X^2} \frac{\partial^\alpha X}{\partial x^\alpha} + \frac{\partial^2 U_3(X, Y, Z)}{\partial Y^2} \frac{\partial^\alpha Y}{\partial x^\alpha} + \frac{\partial^2 U_3(X, Y, Z)}{\partial Z^2} \frac{\partial^\alpha Z}{\partial x^\alpha} \\
 &= \frac{\partial^2 U_3(X, Y, Z)}{\partial X^2}.
 \end{aligned} \tag{29}$$

Finally, by taking into account Eqs. (8)-(10), we end up with (26). □

3 Wave equations on Cantor sets

In this section, the applications of fractional complex transform method to handle three-dimensional wave equations on Cantor sets are considered. Comparison between the fractional complex transform method, derived from modified R-L derivatives, and fractional complex transform *via* local fractional derivatives is investigated.

Let us consider 3-D wave equation on Cantor sets (3). We now determine the fractional complex transform *via* local fractional derivatives

$$\begin{cases} T = \frac{t^\alpha}{\Gamma(1+\alpha)}, \\ X = \frac{(ax)^\alpha}{\Gamma(1+\alpha)}, \\ Y = \frac{(ay)^\alpha}{\Gamma(1+\alpha)}, \\ Z = \frac{(az)^\alpha}{\Gamma(1+\alpha)} \end{cases} \quad (30)$$

such that

$$\frac{\partial^2 u(X, Y, Z, T)}{\partial T^2} + \nabla u(X, Y, Z, T) = 0, \quad (31)$$

where

$$\nabla = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}. \quad (32)$$

Let us consider the non-dimensional 2-D wave equation on Cantor sets.

We now determine the fractional complex transform *via* local fractional derivatives

$$\begin{cases} T = \frac{t^\alpha}{\Gamma(1+\alpha)}, \\ X = \frac{(ax)^\alpha}{\Gamma(1+\alpha)}, \\ Y = \frac{(ay)^\alpha}{\Gamma(1+\alpha)} \end{cases} \quad (33)$$

such that Eq. (6) is rewritten

$$\frac{\partial^2 u(X, Y, T)}{\partial T^2} + \nabla u(X, Y, T) = 0, \quad (34)$$

where

$$\nabla = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}.$$

If there is the mass function [3–5, 18, 28]

$$\gamma^\alpha [F, a, b] = \frac{1}{\Gamma(1+\alpha)} H^\alpha (F \cap (a, b)) = \frac{(b-a)^\alpha}{\Gamma(1+\alpha)}, \quad (35)$$

then we arrive at the following formula:

$$\gamma^\alpha [F, la, lb] = l^\alpha \frac{(b-a)^\alpha}{\Gamma(1+\alpha)}. \quad (36)$$

Following the above results, we revisit the fractional complex transform method *via* local fractional derivatives, Eq. (12) are suggested by

$$\begin{cases} T(t) = \frac{t^\alpha}{\Gamma(1+\alpha)} = \gamma^\alpha [F, 0, t], \\ X(x) = \frac{(ax)^\alpha}{\Gamma(1+\alpha)} = \gamma^\alpha [F, 0, ax], \\ Y(y) = \frac{(ay)^\alpha}{\Gamma(1+\alpha)} = \gamma^\alpha [F, 0, ay], \\ Z(z) = \frac{(az)^\alpha}{\Gamma(1+\alpha)} = \gamma^\alpha [F, 0, az]. \end{cases} \quad (37)$$

From [2–5, 18, 22, 23], we conclude that

$$\begin{aligned} |T(t_1) - T(t_2)| &\leq \varepsilon_1^\alpha, & |X(x_1) - X(x_2)| &\leq \varepsilon_2^\alpha, \\ |Y(y_1) - Y(y_2)| &\leq \varepsilon_3^\alpha, & |Z(z_1) - Z(z_2)| &\leq \varepsilon_4^\alpha \end{aligned}$$

for any $0 < \varepsilon_i$ and $\varepsilon_i \in \mathbb{R}$, which implies that fractal dimensions of transferring pairs are α .

4 Conclusions

In this manuscript, we consider that the fractional complex transform method is derived from the local fractional differential operator, which is set up on fractals. The obtained results on Cantor sets are related to physical phenomenon in Cantorian time-space. It is supposed that the transferring pairs are related to fractal measure that could be fundamental in understanding of the Cantorian monadic structure of quantum space time. The reported results have a potential application in observing the structure of differential equations from micro-physical to macro-physical behavior of substance in the real world.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Authors contributed equally and in writing this article. Authors read and approved the final manuscript.

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