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Research Article

Local Fractional Sumudu Transform with Application to IVPs on Cantor Sets

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Local fractional derivatives were investigated intensively during the last few years. The coupling method of Sumudu transform and local fractional calculus (called as the local fractional Sumudu transform) was suggested in this paper. The presented method is applied to find the nondifferentiable analytical solutions for initial value problems with local fractional derivative. The obtained results are given to show the advantages.

1. Introduction

Fractals are sets and their topological dimension exceeds the fractal dimensions. Mathematical techniques on fractal sets are presented (see, e.g., [1–4]). Nonlocal fractional derivative has many applications in fractional dynamical systems having memory properties. Fractional calculus has been applied to the phenomena with fractal structure [5–12]. Because of the limit of fractional calculus, the fractal calculus as a framework for the model of anomalous diffusion [13–16] had been constructed. The Newtonian mechanics, Maxwell's equations, and Hamiltonian mechanics on fractal sets [17–19] were generalized. The alternative definitions of calculus on fractal sets had been suggested in [20, 21] and the systems of Navier-Stokes equations on Cantor sets had been studied in [22]. Maxwell's equations on Cantor sets with local fractional vector calculus had been considered [23]. The local fractional Fourier analysis had been adapted to find Heisenberg uncertainty principle [24]. A family of local fractional Fredholm and Volterra integral equations was investigated in [25]. Local fractional variational iteration and decomposition methods for wave equation on Cantor sets were reported in [26]. The local fractional Laplace transforms were developed in [27–30].

The Sumudu transforms (ST) had been considered for application to solve differential equations and to deal with control engineering [31–37]. The aims of this paper are to couple the Sumudu transforms and the local fractional calculus (LFC) and to give some illustrative examples in order to show the advantages.

The structures of the paper are as follows. In Section 2, the local fractional derivatives and integrals are presented. In Section 3, the notions and properties of local fractional Sumudu transform are proposed. In Section 4, some examples for initial value problems are shown. Finally, the conclusions are given in Section 5.

2. Local Fractional Calculus and Polynomial Functions on Cantor Sets

In this section, we give the concepts of local fractional derivatives and integrals and polynomial functions on Cantor sets.

Definition 1 (see [20, 21, 24–26]). Let the function $f(x) \in C_\alpha(a,b)$, if there are

$$\left|f(x) - f(x_0)\right| < \varepsilon^\alpha, \quad 0 < \alpha \leq 1, \tag{1}$$

where $|x - x_0| < \delta$, for $\varepsilon > 0$ and $\varepsilon \in R$.

Definition 2 (see [20, 21, 24]). Let $f(x) \in C_\alpha(a,b)$. The local fractional derivative of $f(x)$ of order α in the interval $[a,b]$ is defined as

$$\frac{d^\alpha f(x_0)}{dx^\alpha} = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \tag{2}$$

where

$$\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1+\alpha)\left[f(x) - f(x_0)\right]. \tag{3}$$

The local fractional partial differential operator of order α ($0 < \alpha \leq 1$) was given by [20, 21]

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x_0, t) = \frac{\Delta^\alpha (u(x_0,t) - u(x_0,t_0))}{(t - t_0)^\alpha}, \tag{4}$$

where

$$\Delta^\alpha (u(x_0,t) - u(x_0,t_0)) \cong \Gamma(1+\alpha)\left[u(x_0,t) - u(x_0,t_0)\right]. \tag{5}$$

Definition 3 (see [20, 21, 24–26]). Let $f(x) \in C_\alpha[a,b]$. The local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is defined as

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha, \end{aligned} \tag{6}$$

where the partitions of the interval $[a,b]$ are denoted as (t_j, t_{j+1}), $j = 0, \ldots, N-1$, $t_0 = a$, and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_j, \ldots\}$.

Theorem 4 (local fractional Taylor' theorem (see [20, 21])). *Suppose that $f^{((k+1)\alpha)}(x) \in C_\alpha(a,b)$, for $k = 0, 1, \ldots, n$ and $0 < \alpha \leq 1$. Then, one has*

$$\begin{aligned} f(x) &= \sum_{k=0}^{n} \frac{f^{(k\alpha)}(x_0)}{\Gamma(1+k\alpha)}(x - x_0)^{k\alpha} \\ &\quad + \frac{f^{((n+1)\alpha)}(\xi)}{\Gamma(1+(n+1)\alpha)}(x - x_0)^{(n+1)\alpha} \end{aligned} \tag{7}$$

with $a < x_0 < \xi < x < b$, $\forall x \in (a,b)$, where

$$f^{((k+1)\alpha)}(x) = \overbrace{D_x^{(\alpha)} \ldots D_x^{(\alpha)}}^{k+1 \text{ times}} f(x). \tag{8}$$

Proof (see [20, 21]). Local fractional Mc-Laurin's series of the Mittag-Leffler functions on Cantor sets is given by [20, 21]

$$E_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{x^{\alpha k}}{\Gamma(1+k\alpha)}, \quad x \in R,\ 0 < \alpha \leq 1, \tag{9}$$

and local fractional Mc-Laurin's series of the Mittag-Leffler functions on Cantor sets with the parameter ζ reads as follows:

$$E_\alpha(\zeta^\alpha x^\alpha) = \sum_{k=0}^{\infty} \frac{\zeta^{k\alpha} x^{\alpha k}}{\Gamma(1+k\alpha)}, \quad x \in R,\ 0 < \alpha \leq 1. \tag{10}$$

As generalizations of (9) and (10), we have

$$f(x) = \sum_{k=0}^{\infty} a_k x^{\alpha k}, \tag{11}$$

where a_k ($k = 0, 1, 2, \ldots, n$) are coefficients of the generalized polynomial function on Cantor sets.

Making use of (10), we get

$$E_\alpha(i^\alpha x^\alpha) = \sum_{k=0}^{\infty} \frac{i^{k\alpha} x^{\alpha k}}{\Gamma(1+k\alpha)}, \tag{12}$$

where i^α is the imaginary unit with $E_\alpha(i^\alpha (2\pi)^\alpha) = 1$.

Let us consider the polynomial function on Cantor sets in the form

$$f(x) = \sum_{k=0}^{\infty} i^{\alpha k} x^{\alpha k}, \tag{13}$$

where $|x| < 1$.

Hence, we have the closed form of (13) as follows:

$$f(x) = \frac{1}{1 - i^\alpha x^\alpha}. \tag{14}$$

□

Definition 5. The local fractional Laplace transform of $f(x)$ of order α is defined as [27–30]

$$\begin{aligned} L_\alpha\{f(x)\} &= f_s^{L,\alpha}(s) \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x)(dx)^\alpha. \end{aligned} \tag{15}$$

If $F_\alpha\{f(x)\} \equiv f_\omega^{F,\alpha}(\omega)$, the inverse formula of (42) is defined as [27–30]

$$\begin{aligned} f(x) &= L_\alpha^{-1}\left\{f_s^{L,\alpha}(s)\right\} \\ &= \frac{1}{(2\pi)^\alpha} \int_{\beta - i\infty}^{\beta + i\infty} E_\alpha(s^\alpha x^\alpha) f_s^{L,\alpha}(s)(ds)^\alpha, \end{aligned} \tag{16}$$

where $f(x)$ is local fractional continuous, $s^\alpha = \beta^\alpha + i^\alpha \infty^\alpha$, and $\mathrm{Re}(s) = \beta > 0$.

Theorem 6 (see [21]). *If $L_\alpha\{f(x)\} = f_s^{L,\alpha}(s)$, then one has*

$$L_\alpha\left\{f^{(\alpha)}(x)\right\} = s^\alpha L_\alpha\{f(x)\} - f(0). \quad (17)$$

Proof. See [21]. □

Theorem 7 (see [21]). *If $L_\alpha\{f(x)\} = f_s^{L,\alpha}(s)$, then one has*

$$L_\alpha\left\{{}_0I_x^{(\alpha)}f(x)\right\} = \frac{1}{s^\alpha}L_\alpha\{f(x)\}. \quad (18)$$

Proof. See [21]. □

Theorem 8 (see [21]). *If $L_\alpha\{f_1(x)\} = f_{s,1}^{L,\alpha}(s)$ and $L_\alpha\{f_2(x)\} = f_{s,2}^{L,\alpha}(s)$, then one has*

$$L_\alpha\{f_1(x) * f_2(x)\} = f_{s,1}^{L,\alpha}(s)\, f_{s,2}^{L,\alpha}(s), \quad (19)$$

where

$$f_1(x) * f_2(x) = \frac{1}{\Gamma(1+\alpha)}\int_0^\infty f_1(t)\, f_2(x-t)\,(dt)^\alpha. \quad (20)$$

Proof. See [21]. □

3. Local Fractional Sumudu Transform

In this section, we derive the local fractional Sumudu transform (LFST) and some properties are discussed.

If there is a new transform operator $\mathrm{LFS}_\alpha : f(x) \to F(u)$, namely,

$$\mathrm{LFS}_\alpha\{f(x)\} = \mathrm{LFS}_\alpha\left\{\sum_{k=0}^{\infty} a_k x^{\alpha k}\right\} = \sum_{k=0}^{\infty}\Gamma(1+k\alpha)\, a_k z^{\alpha k}. \quad (21)$$

As typical examples, we have

$$\mathrm{LFS}_\alpha\{E_\alpha(i^\alpha x^\alpha)\} = \sum_{k=0}^{\infty} i^{\alpha k} z^{\alpha k},$$
$$\mathrm{LFS}_\alpha\left\{\frac{x^\alpha}{\Gamma(1+\alpha)}\right\} = z^\alpha. \quad (22)$$

As the generalized result, we give the following definition.

Definition 9. The local fractional Sumudu transform of $f(x)$ of order α is defined as

$$\mathrm{LFS}_\alpha\{f(x)\} = F_\alpha(z) =: \frac{1}{\Gamma(1+\alpha)} \times \int_0^\infty E_\alpha(-z^{-\alpha}x^\alpha)\frac{f(x)}{z^\alpha}(dx)^\alpha, \quad 0 < \alpha \le 1. \quad (23)$$

Following (23), its inverse formula is defined as

$$\mathrm{LFS}_\alpha^{-1}\{F_\alpha(z)\} = f(x), \quad 0 < \alpha \le 1. \quad (24)$$

Theorem 10 (linearity). *If $LFS_\alpha\{f(x)\} = F_\alpha(z)$ and $LFS_\alpha\{g(x)\} = G_\alpha(z)$, then one has*

$$LFS_\alpha\{f(x) + g(x)\} = F_\alpha(z) + G_\alpha(z). \quad (25)$$

Proof. As a direct result of the definition of local fractional Sumudu transform, we get the following result. □

Theorem 11 (local fractional Laplace-Sumudu duality). *If $L_\alpha\{f(x)\} = f_s^{L,\alpha}(s)$ and $LFS_\alpha\{f(x)\} = F_\alpha(z)$, then one has*

$$LFS_\alpha\{f(x)\} = \frac{1}{z^\alpha}L_\alpha\left\{f\left(\frac{1}{x}\right)\right\}, \quad (26)$$

$$L_\alpha\{f(x)\} = \frac{LFS_\alpha[f(1/s)]}{s^\alpha}. \quad (27)$$

Proof. Definitions of the local fractional Sumudu and Laplace transforms directly give the results. □

Theorem 12 (local fractional Sumudu transform of local fractional derivative). *If $LFS_\alpha\{f(x)\} = F_\alpha(z)$, then one has*

$$LFS_\alpha\left\{\frac{d^\alpha f(x)}{dx^\alpha}\right\} = \frac{F_\alpha(z) - f(0)}{z^\alpha}. \quad (28)$$

Proof. From (17) and (26), the local fractional Sumudu transform of the local fractional derivative of $f(x)$ read as

$$\mathrm{LFS}_\alpha\{H(x)\} = \frac{L_\alpha\{H(1/x)\}}{z^\alpha} = \frac{L_\alpha\{f(1/x)\}/z^\alpha - f(0)}{z^\alpha} = \frac{F_\alpha(z) - f(0)}{z^\alpha}, \quad (29)$$

where

$$H(x) = \frac{d^\alpha f(x)}{dx^\alpha}. \quad (30)$$

This completes the proof. □

As the direct result of (28), we have the following results. If $\mathrm{LFS}_\alpha\{f(x)\} = F_\alpha(z)$, then we have

$$\mathrm{LFS}_\alpha\left\{\frac{d^{n\alpha} f(x)}{dx^{n\alpha}}\right\} = \frac{1}{z^{n\alpha}}\left[F_\alpha(z) - \sum_{k=0}^{n-1} z^{k\alpha} f^{(k\alpha)}(0)\right]. \quad (31)$$

When $n = 2$, from (31), we get

$$\mathrm{LFS}_\alpha\left\{\frac{d^{2\alpha} f(x)}{dx^{2\alpha}}\right\} = \frac{1}{z^{2\alpha}}\left[F_\alpha(z) - f(0) - z^\alpha f^{(\alpha)}(0)\right]. \quad (32)$$

Theorem 13 (local fractional Sumudu transform of local fractional derivative). *If $LFS_\alpha\{f(x)\} = F_\alpha(z)$, then one has*

$$LFS_\alpha\left\{{}_0I_x^{(\alpha)} f(x)\right\} = z^\alpha F_\alpha(z). \quad (33)$$

Table 1: Local fractional Sumudu transform of special functions.

Mathematical operation in the t-domain	Corresponding operation in the z-domain	Remarks
a	a	a is a constant
$\frac{x^{\alpha}}{\Gamma(1+\alpha)}$	z^{α}	
$\sum_{k=0}^{\infty} a_k x^{\alpha k}$	$\sum_{k=0}^{\infty}\Gamma(1+k\alpha) a_k z^{\alpha k}$	
$E_{\alpha}(ax^{\alpha})$	$\frac{1}{1-az^{\alpha}}$	$E_{\alpha}(x^{\alpha}) = \sum_{k=0}^{\infty}\frac{x^{\alpha k}}{\Gamma(1+k\alpha)}$
$\sin_{\alpha}(ax^{\alpha})$	$\frac{az^{\alpha}}{1+a^2 z^{2\alpha}}$	$\sin_{\alpha} x^{\alpha} = \sum_{k=0}^{\infty}(-1)^k \frac{x^{\alpha(2k+1)}}{\Gamma[1+\alpha(2k+1)]}$
$\cos_{\alpha}(ax^{\alpha})$	$\frac{1}{1+a^2 z^{2\alpha}}$	$\cos_{\alpha} x^{\alpha} = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2\alpha k}}{\Gamma(1+2\alpha k)}$
$\sinh_{\alpha}(ax^{\alpha})$	$\frac{az^{\alpha}}{1-a^2 z^{2\alpha}}$	$\sinh_{\alpha} x^{\alpha} = \sum_{k=0}^{\infty}\frac{x^{\alpha(2k+1)}}{\Gamma[1+\alpha(2k+1)]}$
$\cosh_{\alpha}(ax^{\alpha})$	$\frac{1}{1-a^2 z^{2\alpha}}$	$\cosh_{\alpha} x^{\alpha} = \sum_{k=0}^{\infty}\frac{x^{2\alpha k}}{\Gamma(1+2\alpha k)}$

Proof. From (18) and (26), we have

$$L_{\alpha}\left\{ {}_0 I_x^{(\alpha)} f(x)\right\} = \frac{1}{s^{\alpha}} L_{\alpha}\{f(x)\} \tag{34}$$

so that

$$\mathrm{LFS}_{\alpha}\{B(x)\} = \frac{L_{\alpha}\{B(1/x)\}}{z^{\alpha}} = L_{\alpha}\left\{ f\left(\frac{1}{x}\right)\right\} = z^{\alpha} F_{\alpha}(z), \tag{35}$$

where

$$B(x) = {}_0 I_x^{(\alpha)} f(x). \tag{36}$$

This completes the proof. □

Theorem 14 (local fractional convolution). *If $LFS_{\alpha}\{f(x)\} = F_{\alpha}(z)$ and $LFS_{\alpha}\{g(x)\} = G_{\alpha}(z)$, then one has*

$$LFS_{\alpha}\{f(x) * g(x)\} = z^{\alpha} F_{\alpha}(z) G_{\alpha}(z), \tag{37}$$

where

$$f(x) * g(x) = \frac{1}{\Gamma(1+\alpha)} \int_0^{\infty} f(t) g(x-t)(dt)^{\alpha}. \tag{38}$$

Proof. From (19) and (26), we have

$$\begin{aligned} \mathrm{LFS}_{\alpha}\{f(x) * g(x)\} &= \frac{L_{\alpha}\{f(x) * g(x)\}}{z^{\alpha}} \\ &= \frac{L_{\alpha}\{f(1/x)\} L_{\alpha}\{g(1/x)\}}{z^{\alpha}} \\ &= z^{\alpha} F_{\alpha}(z) G_{\alpha}(z), \end{aligned} \tag{39}$$

where

$$\begin{aligned} F_{\alpha}(z) &= \frac{L_{\alpha}\{f(1/x)\}}{z^{\alpha}}, \\ G_{\alpha}(z) &= \frac{L_{\alpha}\{g(1/x)\}}{z^{\alpha}}. \end{aligned} \tag{40}$$

This completes the proof. □

In the following, we present some of the basic formulas which are in Table 1.

The above results are easily obtained by using local fractional Mc-Laurin's series of special functions.

4. Illustrative Examples

In this section, we give applications of the LFST to initial value problems.

Example 1. Let us consider the following initial value problems:

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}} = f(x), \tag{41}$$

subject to the initial value condition

$$f(0) = 5. \tag{42}$$

Taking the local fractional Sumudu transform gives

$$\frac{F_{\alpha}(z) - f(0)}{z^{\alpha}} = F_{\alpha}(z), \tag{43}$$

where

$$\mathrm{LFS}_{\alpha}\{f(x)\} = F_{\alpha}(z). \tag{44}$$

Making use of (43), we obtain

$$F_{\alpha}(z) = \frac{5}{1-z^{\alpha}}. \tag{45}$$

Hence, from (45), we get

$$f(x) = 5E_{\alpha}(x^{\alpha}) \tag{46}$$

and we draw its graphs as shown in Figure 1.

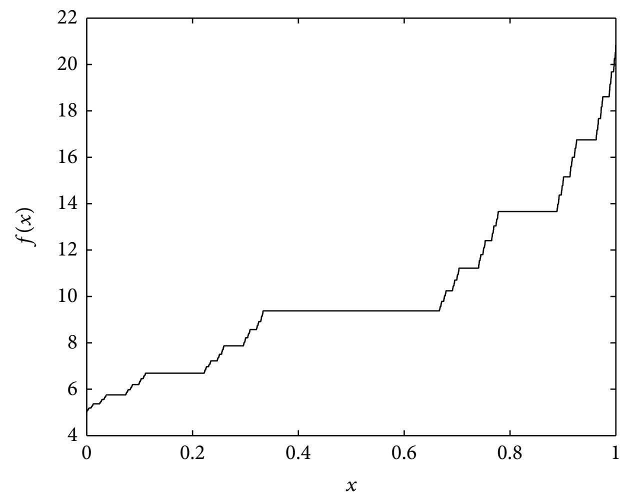

Figure 1: The plot of nondifferentiable solution of (41) with the parameter $\alpha = \ln 2/\ln 3$.

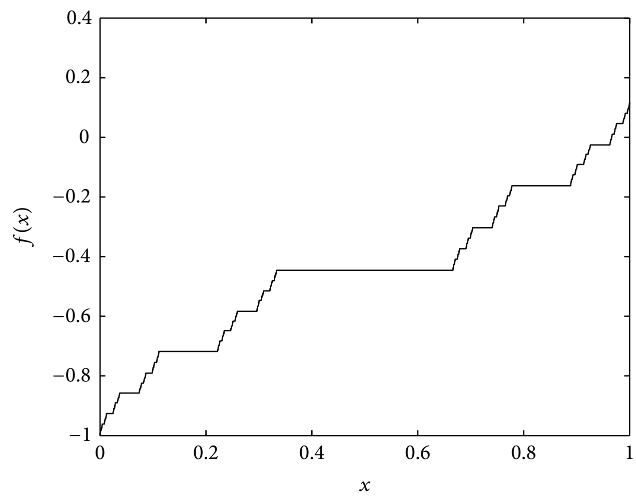

Figure 2: The plot of nondifferentiable solution of (47) with the parameter $\alpha = \ln 2/\ln 3$.

Example 2. We consider the following initial value problems:

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}} + f(x) = \frac{x^{\alpha}}{\Gamma(1+\alpha)} \tag{47}$$

and the initial boundary value reads as

$$f(0) = -1. \tag{48}$$

Taking the local fractional Sumudu transform, from (47) and (48), we have

$$\frac{F_{\alpha}(z) - f(0)}{z^{\alpha}} + F_{\alpha}(z) = z^{\alpha} \tag{49}$$

so that

$$F_{\alpha}(z) = z^{\alpha} - 1. \tag{50}$$

Therefore, the nondifferentiable solution of (47) is

$$f(x) = \frac{x^{\alpha}}{\Gamma(1+\alpha)} - 1 \tag{51}$$

and we draw its graphs as shown in Figure 2.

Example 3. We give the following initial value problems:

$$\frac{d^{2\alpha} f(x)}{dx^{2\alpha}} = f(x), \tag{52}$$

together with the initial value conditions

$$\begin{aligned} f^{(\alpha)}(0) &= 0, \\ f(0) &= 2. \end{aligned} \tag{53}$$

Taking the local fractional Sumudu transform, from (52), we obtain

$$\frac{1}{z^{2\alpha}}\left[F_{\alpha}(z) - f(0) - z^{\alpha} f^{(\alpha)}(0)\right] = F_{\alpha}(z), \tag{54}$$

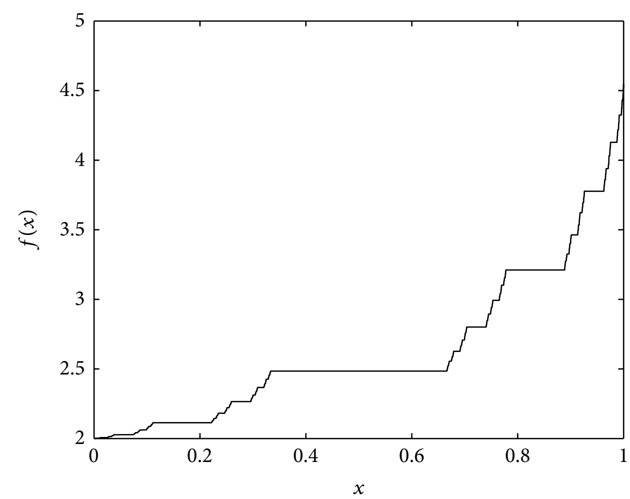

Figure 3: The plot of nondifferentiable solution of (52) with the parameter $\alpha = \ln 2/\ln 3$.

which leads to

$$F_{\alpha}(z) = \frac{f(0) + z^{\alpha} f^{(\alpha)}(0)}{1 - z^{2\alpha}} = \frac{2}{1 - z^{2\alpha}}. \tag{55}$$

Therefore, form (55), we give the nondifferentiable solution of (52)

$$f(x) = 2\cosh_{\alpha}(x^{\alpha}), \tag{56}$$

and we draw its graphs as shown in Figure 3.

5. Conclusions

In this work, we proposed the local fractional Sumudu transform based on the local fractional calculus and its results were discussed. Applications to initial value problems were presented and the nondifferentiable solutions are obtained. It is shown that it is an alternative method of local fractional Laplace transform to solve a class of local fractional differentiable equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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