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Research Article

Local Fractional Variational Iteration and Decomposition Methods for Wave Equation on Cantor Sets within Local Fractional Operators

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We perform a comparison between the fractional iteration and decomposition methods applied to the wave equation on Cantor set. The operators are taken in the local sense. The results illustrate the significant features of the two methods which are both very effective and straightforward for solving the differential equations with local fractional derivative.

1. Introduction

Many problems of physics and engineering are expressed by ordinary and partial differential equations, which are termed boundary value problems. We can mention, for example, the wave, the Laplace, the Klein-Gordon, the Schrodinger's, the telegraph, the Advection, the Burgers, the KdV, the Boussinesq, and the Fisher equations and others [1].

Recently, the fractional calculus theory was recognized to be a good tool for modeling complex problems demonstrating its applicability in numerical scientific disciplines. Boundary value problems for the fractional differential equations have been the focus of several studies due to their frequent appearance in various areas, such as fractional diffusion and wave [2], fractional telegraph [3], fractional KdV [4], fractional Schrödinger [5], fractional evolution [6], fractional Navier-Stokes [7], fractional Heisenberg [8], fractional Klein-Gordon [9], and fractional Fisher equations [10].

Several analytical and numerical techniques were successfully applied to deal with differential equations, fractional differential equations, and local fractional differential equations (see, e.g., [1–36] and the references therein). The techniques include the heat-balance integral [11], the fractional Fourier [12], the fractional Laplace transform [12], the harmonic wavelet [13, 14], the local fractional Fourier and Laplace transform [15], local fractional variational iteration [16, 17], the local fractional decomposition [18], and the generalized local fractional Fourier transform [19] methods.

Recently, the wave equation on Cantor sets (local fractional wave equation) was given by [35]

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - a^{2\alpha} \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \tag{1}$$

where the operators are local fractional ones [16–19, 35, 36].

Following (1), a wave equation on Cantor sets was proposed as follows [36]:

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \tag{2}$$

where $u(x,t)$ is a fractal wave function.

In this paper, our purpose is to compare the local fractional variational iteration and decomposition methods for solving the local fractional differential equations. For illustrating the concepts we adopt one example for solving the wave equation on Cantor sets with local fractional operator.

Bearing these ideas in mind, the paper is organized as follows. In Section 2, we present basic definitions and provide some properties of local fractional derivative and integration. In Section 3, we introduce the local fractional variational iteration and the decomposition methods. In Section 4, we discuss one application. Finally, in Section 5 we outline the main conclusions.

2. Mathematical Tools

We recall in this section the notations and some properties of the local fractional operators [15–19, 35, 36].

Definition 1 (see [15–19, 35, 36]). The function $f(x)$ is local fractional continuous, if it is valid for

$$\left| f(x) - f(x_0) \right| < \varepsilon^{\alpha}, \tag{3}$$

where $|x - x_0| < \delta$, for $\varepsilon > 0$ and $\varepsilon \in R$.

We notice that there are existence conditions of local fractional continuities that operating functions are right-hand and left-hand local fractional continuity. Meanwhile, the right-hand local fractional continuity is equal to its left-hand local fractional continuity. For more details, see [35].

Following (4), we have [15–19, 35, 36]

$$\rho^{\alpha} |x - x_0|^{\alpha} \le \left| f(x) - f(x_0) \right| \le \kappa^{\alpha} |x - x_0|^{\alpha} \tag{4}$$

with $|x - x_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta, \kappa, \rho \in R$.

For a fractal set F, there is a fractal measure [35]

$$H^{\alpha}\left(F \cap (x, x_0)\right) = (x - x_0)^{\alpha}, \tag{5}$$

where $f(x)$ presents a bi-Lipschitz mapping with fractal dimension α and H^{α} denotes a Hausdorff dimension.

We verify that there is a measure

$$H^{1}\left(F \cap (x, x_0)\right) = x - x_0 \tag{6}$$

in the case of $\alpha = 1$ and $f(x)$ is a Lipschitz mapping. If F is a Cantor set, we have $H^{\ln 2/\ln 3}(F \cap (x, x_0)) = (x - x_0)^{\ln 2/\ln 3}$ with $\alpha = \ln 2/\ln 3$.

Definition 2 (see [15–19, 35, 36]). The local fractional derivative of $f(x)$ at $x = x_0$ is defined as [16–20]

$$f^{(\alpha)}(x_0) = \left.\frac{d^{\alpha} f(x)}{dx^{\alpha}}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^{\alpha}\left(f(x) - f(x_0)\right)}{(x - x_0)^{\alpha}}, \tag{7}$$

where

$$\Delta^{\alpha}\left(f(x) - f(x_0)\right) \cong \Gamma(1+\alpha)\, \Delta\left(f(x) - f(x_0)\right). \tag{8}$$

We find that the existence condition for local fractional derivative of $f(x)$ is that the right-hand local fractional derivative is equal to the left-hand local fractional derivative (see, e.g., [16, 35] and the references therein).

Definition 3 (see [15–19, 35, 36]). A partition of the interval $[a, b]$ is denoted as (t_j, t_{j+1}), $j = 0, \ldots, N-1$, $t_0 = a$, and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_j, \ldots\}$. Local fractional integral of $f(x)$ in the interval $[a, b]$ is given by

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)\,(dt)^{\alpha} \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f\left(t_j\right) \left(\Delta t_j\right)^{\alpha}. \end{aligned} \tag{9}$$

If the functions are local fractional continuous then the local fractional derivatives and integrals exist. That is to say, operating functions have nondifferentiable and fractal properties (see [35] and the references therein).

Some properties of local fractional derivative and integrals are given in [35].

3. Analytical Methods

In order to illustrate two analytical methods, we investigate the nonlinear local fractional equation as follows:

$$L_{\alpha}^{(n)} u + R_{\alpha} u = 0, \tag{10}$$

where $L_{\alpha}^{(n)}$ is linear local fractional operators, respectively, with $n = 1, 2$ and R_{α} is linear local fractional operators of order less than $L_{\alpha}^{(n)}$.

3.1. Local Fractional Variational Iteration Method. The local fractional variational iteration algorithm is given by [16, 17] on the line of the formalism suggested in [35]

$$\begin{aligned} u_{n+1}(t) &= u_n(t) + \frac{1}{\Gamma(1+\alpha)} \\ &\quad \times \int_0^t \frac{\lambda^{\alpha}}{\Gamma(1+\alpha)} \left\{ L_{\alpha}^{(n)} u_n(s) + R_{\alpha} u_n(s) \right\} (ds)^{\alpha}. \end{aligned} \tag{11}$$

Here, we can construct a correction functional as follows [16, 17]:

$$\begin{aligned} u_{n+1}(t) &= u_n(t) + \frac{1}{\Gamma(1+\alpha)} \\ &\quad \times \int_0^t \frac{\lambda^{\alpha}}{\Gamma(1+\alpha)} \left\{ L_{\alpha}^{(n)} u_n(s) + R_{\alpha} \tilde{u}_n(s) \right\} (ds)^{\alpha}, \end{aligned} \tag{12}$$

where $\tilde{u}_n$ is considered as a restricted local fractional variation; that is, $\delta^{\alpha}\tilde{u}_n = 0$ (for more details, see [35]).

For $n = 2$, we have

$$\lambda^{\alpha} = \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)}, \tag{13}$$

so that iteration is expressed as

$$\begin{aligned} u_{n+1}(t) &= u_n(t) + \frac{1}{\Gamma(1+\alpha)} \\ &\quad \times \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \left\{ L_{\alpha}^{(n)} u_n(s) + R_{\alpha} u_n(s) \right\} (ds)^{\alpha}. \end{aligned} \tag{14}$$

Finally, the solution is

$$u(x) = \lim_{n\to\infty} u_n(x). \tag{15}$$

3.2. Local Fractional Decomposition Method. When $L_{\alpha}^{(n)}$ in (10) is a local fractional differential operator of order 2α, we denote it as

$$\begin{aligned} L_{\alpha}^{(n)} &= L_{xx}^{(2\alpha)} = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}, \\ R_{\alpha} u(t) &= \frac{\partial^{\alpha}}{\partial x^{\alpha}} u(t) + f(t). \end{aligned} \tag{16}$$

By defining the n-fold local fractional integral operator

$$L_{\alpha}^{(-2)} m(s) = {}_0I_x^{(\alpha)}\, {}_0I_x^{(\alpha)} m(s) \tag{17}$$

we get

$$L_{\alpha}^{(-2)} L_{\alpha}^{(2)} u(s) = L_{\alpha}^{(-2)} R_{\alpha} u(s). \tag{18}$$

Thus,

$$u(s) = r(x) + L_{\alpha}^{(-2)} R_{\alpha} u(s), \tag{19}$$

where the term $r(x)$ is to be determined from the fractal initial conditions.

Therefore, we get the iterative formula as follows:

$$u(x) = u_0(x) + L_{\alpha}^{(-2)} R_{\alpha} u(s), \tag{20}$$

with $u_0(x) = r(x)$.

Hence, for $n \geq 0$, we have the following recurrence relationship:

$$\begin{aligned} u_{n+1}(x) &= L_{\alpha}^{(-2)} R_{\alpha} u_n(s), \\ u_0(x) &= r(x). \end{aligned} \tag{21}$$

Finally, the solution can be constructed as

$$u(x) = \lim_{n\to\infty} \phi_n(x) = \lim_{n\to\infty} \sum_{n=0}^{\infty} u_n(x). \tag{22}$$

For more details, see [18].

4. An Illustrative Example

In this section one example for wave equation is presented in order to demonstrate the simplicity and the efficiency of the above methods.

In (2), we consider the following initial and boundary conditions:

$$\frac{\partial^{\alpha} u(x,0)}{\partial t^{\alpha}} = 0, \quad u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{23}$$

Using (14) we have the iterative formula

$$\begin{aligned} &u_{n+1}(x,t) \\ &\quad = u_n(x,t) + \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_n(x,s)}{\partial s^{2\alpha}} (ds)^{\alpha} \\ &\qquad - \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u_n(x,s)}{\partial x^{2\alpha}} (ds)^{\alpha}, \end{aligned} \tag{24}$$

where the initial value is given by

$$u_0(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{25}$$

Thus, after computing (23) we obtain

$$\begin{aligned} &u_1(x,t) \\ &\quad = u_0(x,t) + \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_0(x,s)}{\partial s^{2\alpha}} (ds)^{\alpha} \\ &\qquad - \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u_0(x,s)}{\partial x^{2\alpha}} (ds)^{\alpha} \\ &\quad = u_0(x,t) + \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \left\{ -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \right\} (ds)^{\alpha} \\ &\quad = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(1 + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \right), \\ &u_2(x,t) \\ &\quad = u_1(x,t) + \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_1(x,s)}{\partial s^{2\alpha}} (ds)^{\alpha} \\ &\qquad - \frac{1}{\Gamma(1+\alpha)} \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} u_1(x,s)}{\partial x^{2\alpha}} (ds)^{\alpha} \\ &\quad = u_1(x,t) + \frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_0^t \frac{(s-t)^{\alpha}}{\Gamma(1+\alpha)} \left\{ -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{s^{2\alpha}}{\Gamma(1+2\alpha)} \right\} (ds)^{\alpha} \\ &\quad = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \left(1 + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{4\alpha}}{\Gamma(1+4\alpha)} \right), \end{aligned}$$

$$\begin{aligned}
&u_3(x,t)\\
&= u_2(x,t) + \frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u_2(x,s)}{\partial s^{2\alpha}}(ds)^\alpha\\
&\quad - \frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u_2(x,s)}{\partial x^{2\alpha}}(ds)^\alpha\\
&= u_2(x,t) + \frac{1}{\Gamma(1+\alpha)}\\
&\quad \times \int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\left\{-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{4\alpha}}{\Gamma(1+4\alpha)}\right\}(ds)^\alpha\\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\sum_{i=0}^{3}\frac{t^{2i\alpha}}{\Gamma(1+2i\alpha)},\\
&u_4(x,t)\\
&= u_3(x,t) + \frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u_3(x,s)}{\partial s^{2\alpha}}(ds)^\alpha\\
&\quad - \frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u_3(x,s)}{\partial x^{2\alpha}}(ds)^\alpha\\
&= u_3(x,t) + \frac{1}{\Gamma(1+\alpha)}\\
&\quad \times \int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\left\{-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{6\alpha}}{\Gamma(1+6\alpha)}\right\}(ds)^\alpha\\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\sum_{i=0}^{4}\frac{t^{2i\alpha}}{\Gamma(1+2i\alpha)},\\
&\qquad\vdots
\end{aligned} \tag{26}$$

$$\begin{aligned}
&u_n(x,t)\\
&= u_{n-1}(x,t) + \frac{1}{\Gamma(1+\alpha)}\int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u_{n-1}(x,s)}{\partial s^{2\alpha}}(ds)^\alpha\\
&\quad - \frac{1}{\Gamma(1+\alpha)}\\
&\quad \times \int_0^t \frac{(s-t)^\alpha}{\Gamma(1+\alpha)}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u_{n-1}(x,s)}{\partial x^{2\alpha}}(ds)^\alpha\\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\sum_{i=0}^{n}\frac{t^{2i\alpha}}{\Gamma(1+2i\alpha)}.
\end{aligned} \tag{27}$$

Hence, from (27) we obtain the solution of (3) as

$$u(x) = \lim_{n\to\infty} u_n(x) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\cosh_\alpha(t^\alpha). \tag{28}$$

Here, from (21) we get

$$\begin{aligned}
u_{n+1}(x,t) &= {}_0I_t^{(\alpha)}\,{}_0I_t^{(\alpha)}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u_n(x,s)}{\partial x^{2\alpha}},\\
u_0(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}.
\end{aligned} \tag{29}$$

Therefore, from (29) we give the components as follows:

$$\begin{aligned}
u_0(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)},\\
u_1(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{2\alpha}}{\Gamma(1+2\alpha)},\\
u_2(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{4\alpha}}{\Gamma(1+4\alpha)},\\
u_3(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{3\alpha}}{\Gamma(1+6\alpha)},\\
u_4(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{8\alpha}}{\Gamma(1+8\alpha)},\\
&\ \ \vdots\\
u_n(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{2n\alpha}}{\Gamma(1+2n\alpha)}.
\end{aligned} \tag{30}$$

Consequently, the exact solution is given by

$$\begin{aligned}
u(x,t) &= \lim_{n\to\infty}\sum_{n=0}^{\infty}u_n(x,t)\\
&= \lim_{n\to\infty}\sum_{n=0}^{\infty}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{2n\alpha}}{\Gamma(1+2n\alpha)}\\
&= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\cosh_\alpha(x^\alpha),
\end{aligned} \tag{31}$$

where

$$\cosh_\alpha(x^\alpha) = \sum_{n=0}^{\infty}\frac{t^{2n\alpha}}{\Gamma(1+2n\alpha)}. \tag{32}$$

The solution of (2) for $\alpha = \ln 2/\ln 3$ is depicted in Figure 1.

5. Conclusions

In this work, we developed a comparison between the variational iteration method and the decomposition method within local fractional operators. The two approaches constitute efficient tools to handle the approximation solutions for differential equations on Cantor sets with local fractional derivative. We notice that the fractional variational iteration method gives the several successive approximate formulas using the iteration of the correction local fractional functional. However, the local fractional decomposition method

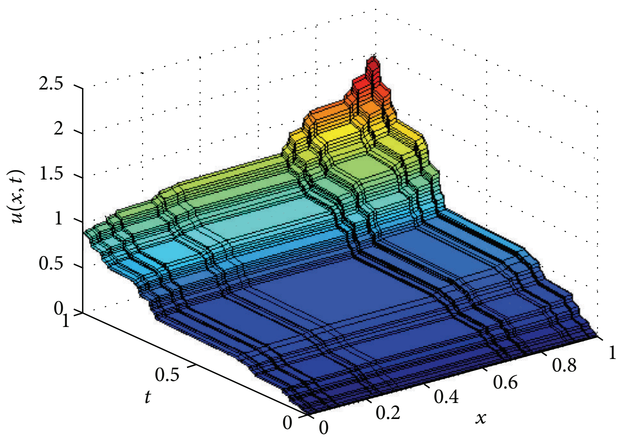

FIGURE 1: Graph of $u(x, t)$ for $\alpha = \ln 2/\ln 3$.

provides the components of the exact solution, which is local fractional continuous function, where these components are also local fractional continuous functions. Both the variational iteration method and the decomposition method within local fractional operators provide the solution in successive components. The methods are structured to get the local fractional series solution, which is a nondifferentiable function.

Conflict of Interests

The authors declare that there is no conflict of interests regarding publication of this paper.

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