ARTICLE IN PRESS

Alexandria Engineering Journal (2020) xxx, xxx–xxx

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena

Syed Tauseef Saeed [a,*], **Muhammad Bilal Riaz** [b,e], **Dumitru Baleanu** [c,f], **Kashif Ali Abro** [d,e]

[a] *Department of Sciences & Humanities, National University of Computer & Emerging Sciences, 54000 Lahore, Pakistan*
[b] *Department of Mathematics, University of Management and Technology Lahore, Pakistan*
[c] *Department of Mathematics, Cankaya University, Turkey*
[d] *Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Pakistan*
[e] *Institute for Groundwater Studies (IGS), University of the Free State, South Africa*
[f] *Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan*

Received 22 December 2019; revised 7 February 2020; accepted 8 February 2020

KEYWORDS

Modern fractional operator;
Inversion algorithm;
Dufour effect;
Mass transfer;
Convolution;
Convection flow

Abstract In this manuscript, we have obtained closed form solution using Laplace transform, inversion algorithm and convolution theorem. The study of mass transfer flow of an incompressible fluid is carried out near vertical channel. Recently, new classes of differential operators have been introduced and recognized to be efficient in capturing processes following the decay law and the crossover behaviors. For the study of heat and mass transfer, we applied the newly differential operators say Atangana-Baleanu (*ABC*) and Caputo-Fabrizio (*CF*) to model such flow. This model for temperature, concentration and velocity gradient is presented in dimensionless form. The obtained solutions have been plotted for various values physical parameters like $\alpha, D_f, G_m, G_r, S_c$ and P_r on temperature and velocity profile. Our results suggest that for the variation of time the velocity behavior for *CF* and *ABC* are reversible. Finally, an incremental value of prandtl number is observed for decrease in the velocity field which reflects the control of thickness of momentum and enlargement of thermal conductivity. Further, dynamical analysis of fluid with memory effect are efficient for *ABC* as compared to *CF*.

1. Introduction

Heat and mass transfer left a significant impact on non-Newtonian fluid. The flow of non-Newtonian fluids under assumption of heat generation analyzed in different applications of engineering. The engineering processes find numerous

* Corresponding author.
E-mail address: tauseefsaeed301@gmail.com (S.T. Saeed).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

https://doi.org/10.1016/j.aej.2020.02.012
1110-0168

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

application on heat and mass transfer in a flow fluid across channel such as fire engine, nuclear energy, petroleum reservoir and transport phenomena. In many systems, the combined effect of temperature and concentration forces often used in modeling of transport phenomena. Furthermore electric circuits, heat exchanger, solar panels and thermal protection system are major application of heat transfer which directed impact on fluid system and its geometry [1–8]. Recently, changes in temperature has significant role in free convection flow. Magnetic field effect on infinite plate with heat and mass transfer by convection flow discussed by Chamkha et al. [9]. Further, Chamkha et al. [10] analyzed the laminar and convection flow with heat and concentration field inside the rectangular enclosure. In the presence of heat radiation in MHD convective flow through permeable surface with ramped wall temperature investigated by Ismail et al. [11]. Influence of heat effect on electrically and chemically fluid under vertical channel investigated by Umavathi et al. [12]. Chamkha et al. [13] presented the micropolar fluid on heated vertical preamble surface under effect of chemical and radiation parameter. The solution of *PDE* for heat generation can be obtained by using implicit finite difference schemes.

Soret and Dufour reaction are two significant circumstances in the process of heat and mass transfer. Concentration and temperature profile generate Dufour and Soret effect respectively. Both effects are used to discuss the Fick's law. The analytical solution of free convection flow to mass transfer in upended channel with consideration of Dufour effect discussed by Jha et al. [14]. Impact of heat radiation and chemical reaction on viscous incompressible nano-fluid through a preamble surface with Dufour and Soret effect investigated by Reddy et al. [15]. Narahari et al. [16] presented the solution of natural convection flow with ramped wall temperature in the presence of thermal and mass diffusion between the parallel plates. Furthermore, Author [17] discussed the influence of ramped wall temperature, heat and chemical reaction on fluid convection flow over infinite plate. He notice that the velocity profile decrease in the case of chemical reaction, inclination of angle and heat source increase.

In fluid dynamics, fractional derivatives models have been analyzed for viscoelastic materials, such as glassy state and polymers. Recently, different real life problems have been discussed through fractional time derivative operator, namely Caputo-Fabrizio (*CF*) time fractional derivative and Atangana-Baleanu (*ABC*) time fractional derivative [18–20]. Fractional calculus is an emerging field which is based on different types of kernels. The main significance of kernels is to provide a better description of the dynamics among complex systems, for instance, collecting the memory at whole and partial domain of certain processes. The non-locality of the new kernel analyzed the memory structure with alternate scales. Furthermore, the study of thermal sciences with non-local exchange plays an important role in non-singular kernel fractional derivative. *CF* and *ABC* time fractional operators discussed with non-singular kernel [21,32]. Fahd et al. [34] discussed the classification of ordinary differential equations in the frame of Atangana-Baleanu fractional derivative. Further authors [33] investigate the existence theory and numerical solutions to smoking model using Caputo-Fabrizio fractional derivative.

Recently, Riaz et al. [22] investigate the effect of ramped wall velocity on MHD fractional Oldroyd-B fluid using fractional time derivative operators. furthermore, Authors [23] investigate the influence of chemical reaction with ramped temperature condition on MHD free convection flow over a vertical plate using non-singular kernel. The solution of MHD Oldroyd-B fluid with time dependent boundary conditions using classical model and fractional operators analyzed by Riaz et al. [30]. The mass transfer has the impact on rate of heat transfer and mass transfer depends upon concentration differences. Imran et al. [24] applied Caputo derivative on differential type fluid and investigated solutions for temperature, concentration and velocity. Heat and mass transfer analysis for second grade fluid with thermophoresis and thermal radiation analyzed by Das et al. [25]. Ali [35] discussed the novel method for a fractional derivative with non-local and non-singular kernel. Furthermore, the solutions of the linear and nonlinear differential equations within the generalized fractional derivatives is investigated by Ali et al. [36]. Some remarkable work regarding heat and mass transfer phenomena for viscous fluid has been done by researchers [26–29].

The aim of present article to analyze the closed form solution for unsteady free convection under the presence of mass transfer with Dufour effect. *CF* and *ABC* modern fractional operator are used to governs the flow of fluid using partial differential equations (*PDE*). *LT* is used to determine the solution of given *PDE* for concentration, temperature and velocity profile by satisfying *IC′s* and *BC′s* of non-integer order derivatives. To see the behavior of fluid flow, fractional order model is good to explain the dynamics and memory effect with respect to classical model. Closed form solutions investigate in this article are interpreted graphically and computationally by software Mathcad-15 to study the impact of different pertinent parameters.

2. Fractional formation of natural convection flow through a channel

Free convection and mass transfer fluid flow in a vertical channel are considered with Dufour effect. Let x-axis be the vertical direction in which initial temperature T^*_∞ and C^*_∞ are consider as a constant on both plates when $t^* \leq 0$. Here, a condition is invoked for fluid to be at rest between two bounding walls, this is because a Dufour effect with systems of equations shows the influence of temperature and concentration. Meanwhile, at $t^* = 0^+$, concentration and temperature are given as $C^*_\infty + \frac{(C^*_0 + C^*_\infty)t}{t_0}$ and $T^*_\infty + \frac{(T^*_0 + T^*_\infty)t}{t_0}$. We set the physical quantities on spatial y^* and time t^* variables between two walls having infinite length. Within such assumptions, governing equations under Boussinesq's approximation with Dufour effect are derived as:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta T - g\beta T_\infty + g\beta C - g\beta C_\infty, \tag{2.1}$$

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) + D^*\left(\frac{\partial^2 C}{\partial y^2}\right), \tag{2.2}$$

$$\frac{\partial C}{\partial t} = D_m\left(\frac{\partial^2 C}{\partial y^2}\right), \tag{2.3}$$

subject to following employed conditions as defined below

$$u(y,0) = 0, T(y,0) = T_\infty, C(y,0) = C_\infty, \tag{2.4}$$

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

$$u(y,t) = U_o \cos(wt), T(y,t) = T_w, C(y,t) = C_w, \tag{2.5}$$

$$u(y,t) \to 0, T(y,t) \to T_\infty, C(y,t) \to C_\infty. \tag{2.6}$$

In order to avoid lengthiness and exaggerations among governing equations, we introduce the dimensionless functions and parameters in Eqs. (2.1)–(2.6) as illustrated below:

$$y^* = \frac{y}{l}, u^* = \frac{u}{u_o}, t^* = \frac{t}{t_0}, t_0 = \frac{l^2}{\upsilon}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{2.7}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, S_c = \frac{\upsilon}{D_m}, P_r = \frac{\upsilon}{\alpha}. \tag{2.8}$$

The optimal format of set of dimensionless governing equations have been obtained in terms of non-fractional approaches, we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi, \tag{2.9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial y^2}\right) + \frac{D_f}{P_r}\left(\frac{\partial^2 \phi}{\partial y^2}\right), \tag{2.10}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c}\left(\frac{\partial^2 C}{\partial y^2}\right), \tag{2.11}$$

with appropriate conditions

$$u(y,0) = \theta(y,0) = \phi(y,0) = 0, y \geqslant 0, \tag{2.12}$$

$$u(y,t) = \cos(wt), \theta(y,t) = \phi(y,t) = 1, y = 0, \tag{2.13}$$

$$u(\infty,t) = \theta(\infty,t) = \phi(\infty,t) = 0, y \to \infty. \tag{2.14}$$

2.1. Development of governing equations in terms of CF derivative

Before developing the non-fractional governing equations in terms of fractional differential operator, we define the time fractional differential operators with its Laplace transform. The *CF* and its Laplace in Caputo sense is defined as [31]

$$^{CF}D_t^{\varsigma}h(y,t) = \frac{1}{1-\varsigma}\int_0^t exp\left(-\frac{\varsigma(t-\varsigma)}{1-\varsigma}\right)\frac{\partial h(y,\tau)}{\partial \tau}d\tau, 0 < \varsigma < 1. \tag{2.15}$$

The *LT* of *CF* derivative is

$$\mathcal{L}\left(^{CF}D_t^{\varsigma}h(y,t)\right) = \frac{s\mathcal{L}(h(y,t)) - h(y,0)}{(1-\varsigma)s + \varsigma}. \tag{2.16}$$

2.2. Development of governing equations in terms of ABC derivative

In this context, The *ABC* derivative and its Laplace in Caputo sense is defined as [31]

$$^{ABC}D_t^{\varsigma}h(y,t) = \frac{1}{1-\varsigma}\int_0^t \mathbf{E}_{\varsigma}\left(-\frac{\varsigma(t-\varsigma)^{\varsigma}}{1-\varsigma}\right)\frac{\partial h(y,\tau)}{\partial \tau}d\tau. \tag{2.17}$$

The *LT* of *ABC* derivative is

$$\mathcal{L}\left(^{ABC}D_t^{\alpha}h(y,t)\right) = \frac{s^{\varsigma}\mathcal{L}(h(y,t)) - s^{\varsigma-1}h(y,0)}{(1-\varsigma)s^{\varsigma} + \varsigma}. \tag{2.18}$$

3. Optimal solutions based on modern fractional approaches

3.1. Fractional optimality of concentration via CF derivative

Generating Eq. (2.11) for the fractionalized form, we imposed Eq. (2.15) on Eq. (2.11), we arrive at

$$\frac{\partial^\alpha \phi}{\partial t^\alpha} - \frac{1}{S_c}\left(\frac{\partial^2 C}{\partial y^2}\right) = 0, \tag{3.19}$$

Eq. (3.19) can be manipulated by means of several methods but we prefer to employee a powerful technique namely Laplace transform on Eq. (3.19) with imposed conditions as discussed in Eqs. (2.12)–(2.14). This mathematical process is resulted the suitable expression as

$$S_c\left(\frac{q}{q - \alpha q + \alpha}\right)\bar{\phi}(y,q) = \frac{\partial^2 \bar{\phi}(y,q)}{\partial y^2}. \tag{3.20}$$

The second order partial differential equation say (3.20) can have the incomplete solution as investigated in Eq. (3.20)

$$\bar{\phi}(y,q) = c_1 e^{-y\sqrt{\frac{Scq}{q-\alpha q+\alpha}}} + c_2 e^{y\sqrt{\frac{Scq}{q-\alpha q+\alpha}}}. \tag{3.21}$$

With the help of boundary conditions for concentration, c_1 and c_2 can be traced out in the following equation:

$$\bar{\phi}(y,q) = \frac{e^{-y\sqrt{\frac{Scq}{q-\alpha q+\alpha}}}}{q}. \tag{3.22}$$

Inverting an exponential equation through Laplace transform subject to the an Appendix (A1), we get final solution of concentration equation as:

$$\phi(y,t) = 1 - \frac{2\gamma S_c}{\pi}\int_0^\infty \frac{\sin(ys)}{s(\gamma S_c + s^2)} e^{\left(\frac{-\alpha\gamma t s^2}{\gamma S_c + s^2}\right)} ds. \tag{3.23}$$

Here, γ is letting parameter can be taken as $\gamma = \frac{1}{1-\alpha}$.

3.2. Fractional Optimality of Temperature via CF derivative

Generating Eq. (2.10) for the fractionalized form, we imposed Eqs. (2.15) and (2.16) on Eq. (2.10), we arrive at

$$\left(\frac{q}{q - \alpha q + \alpha}\right)\bar{\theta}(y,q) = \frac{1}{P_r}\left(\frac{\partial^2 \bar{\theta}(y,q)}{\partial y^2}\right) + \frac{D_f.S_c}{P_r}\left(\frac{q}{q - \alpha q + \alpha}\right)\bar{\phi}(y,q). \tag{3.24}$$

The solution of second order partial differential equation say (3.20) is analyzed as:

$$\bar{\theta}(y,q) = c_1 e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}} + c_2 e^{-y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}} - \frac{A_1}{q}e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}. \tag{3.25}$$

With the help of boundary conditions for temperature, constants c_1 and c_2 can be traced out in the following equation:

$$\bar{\theta}(y,q) = \frac{1}{q}e^{-y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}} + \frac{A_1}{q} \times \left(e^{-y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}} - e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}\right). \tag{3.26}$$

Inverting an exponential equation through Laplace transform subject to the an Appendix (A1), we get final solution of temperature equation as

$$\theta(y,t) = \delta(y,t,\gamma P_r,\gamma\alpha) + A_1 \times (\delta(y,t,\gamma P_r,\gamma\alpha) - \delta(y,t,\gamma S_c,\gamma\alpha)). \tag{3.27}$$

3.3. Fractional optimality of velocity field via CF derivative

Generating Eq. (2.9) for the fractionalized form, we imposed Eqs. (2.15) and (2.16) on Eq. (2.9), we arrive at

$$\frac{q}{q-\alpha q+\alpha}\bar{u}(y,q)=\frac{\partial^2\bar{u}(y,q)}{\partial y^2}+G_r\times\bar{\theta}(y,q)+G_m\times\bar{\phi}(y,q), \tag{3.28}$$

by substitution of $\bar{\theta}$ and $\bar{\phi}$ into (3.28). With minor simplification, we get:

$$\begin{aligned}\bar{u}(y,q)&=c_1e^{y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}+c_2e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-\frac{B_1(q+\alpha\gamma)}{q^2}e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}}\\&-\frac{B_2(q+\alpha\gamma)}{q^2}e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}}+\frac{B_3(q+\alpha\gamma)}{q^2}e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}\\&-\frac{B_4(q+\alpha\gamma)}{q^2}e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}.\end{aligned} \tag{3.29}$$

With the help of boundary conditions for velocity field, c_1 and c_2 can be traced out in the following equation:

$$\begin{aligned}\bar{u}(y,q)&=\left(\frac{q}{q^2+w^2}\right)e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}\\&+\frac{B_1}{q}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}}\right)\\&+\frac{B_1\alpha\gamma}{q^2}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}}\right)\\&+\frac{B_2}{q}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}}\right)\\&+\frac{B_2\alpha\gamma}{q^2}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Prq}{q+\alpha\gamma}}}\right)\\&-\frac{B_3}{q}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}\right)\\&-\frac{B_3\alpha\gamma}{q^2}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}\right)\\&+\frac{B_4}{q}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}\right)\\&+\frac{B_4\alpha\gamma}{q^2}\left(e^{-y\sqrt{\frac{\gamma q}{q+\alpha\gamma}}}-e^{y\sqrt{\frac{\gamma Scq}{q+\alpha\gamma}}}\right).\end{aligned} \tag{3.30}$$

Inverting an exponential equation through Laplace transform subject to the an Appendix (A2)–(A5), we get final solution for velocity equation as:

$$\begin{aligned}u(y,t)&=\cos(wt)\times\frac{1}{t}*\Xi(y,t,\gamma,\gamma\alpha,0)\\&+B_1(\Xi(y,t,\gamma,\gamma\alpha,0)-\Xi(y,t,P_r\gamma,\gamma\alpha,0))\\&+B_1\alpha\gamma(\vartheta(y,t,\gamma,\alpha\gamma)-\vartheta(y,t,\gamma P_r,\gamma\alpha))\\&+B_2(\Xi(y,t,\gamma,\gamma\alpha,0)-\Xi(y,t,P_r\gamma,\gamma\alpha,0))\\&+B_2\alpha\gamma(\vartheta(y,t,\gamma,\alpha\gamma)-\vartheta(y,t,\gamma P_r,\gamma\alpha))\\&-B_3(\Xi(y,t,\gamma,\gamma\alpha,0)-\Xi(y,t,S_c\gamma,\gamma\alpha,0))\\&-B_3\alpha\gamma(\vartheta(y,t,\gamma,\alpha\gamma)-\vartheta(y,t,\gamma S_c,\gamma\alpha))\\&+B_4(\Xi(y,t,\gamma,\gamma\alpha,0)-\Xi(y,t,S_c\gamma,\gamma\alpha,0))\\&B_4\alpha\gamma(\vartheta(y,t,\gamma,\alpha\gamma)-\vartheta(y,t,\gamma S_c,\gamma\alpha)).\end{aligned} \tag{3.31}$$

3.4. Fractional optimality of concentration via ABC derivative

Generating Eq. (2.11) for the fractionalized form, we imposed Eqs. (2.17) and (2.18) on Eq. (2.11), we arrive at

$$S_c\left(\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\right)\bar{\phi}(y,q)=\frac{\partial^2\bar{\phi}(y,q)}{\partial y^2}. \tag{3.32}$$

The second order partial differential equation say (3.32) can have the incomplete solution as investigated in Eq. (3.32)

$$\bar{\phi}(y,q)=c_1e^{-y\sqrt{S_c\left(\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\right)}}+c_2e^{y\sqrt{S_c\left(\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\right)}}. \tag{3.33}$$

With the help of boundary conditions for concentration field, c_1 and c_2 can be traced out in the following equation:

$$\bar{\phi}(y,q)=\frac{1}{q}e^{-y\sqrt{S_c\left(\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\right)}}. \tag{3.34}$$

The above solution (3.34) can written in more appropriate form

$$\bar{\phi}(y,q)=\frac{1}{q^{1-\alpha}}\frac{e^{-y\sqrt{\frac{\gamma_1Scq^\alpha}{q^\alpha+\gamma_1\alpha}}}}{q^\alpha}. \tag{3.35}$$

Inverting an exponential equation through Laplace transform subject to the an Appendix (A6)–(A10), we get final solution for concentration field as:

$$\phi(y,t)=h(t,\alpha)\times(\chi(y,t,\gamma_1,\gamma_1\alpha,0)), \tag{3.36}$$

where γ_1 is letting parameter can be taken as $\gamma_1=\frac{1}{1-\alpha}$.

3.5. Fractional optimality of temperature via ABC derivative

Generating Eq. (2.10) for the fractionalized form, we imposed Eqs. (2.17) and (2.18) on Eq. (2.10), we arrive at

$$\begin{aligned}\left(\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\right)\bar{\theta}(y,q)&=\frac{1}{P_r}\left(\frac{\partial^2\bar{\theta}(y,q)}{\partial y^2}\right)\\&+\frac{D_f.S_c}{P_r}\left(\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\right)\bar{\phi}(y,q).\end{aligned} \tag{3.37}$$

The second order partial differential equation say (3.37) can have the incomplete solution as investigated in Eq. (3.37)

$$\bar{\theta}(y,q)=c_1e^{y\sqrt{\frac{\gamma_1Prq^\alpha}{q^\alpha+\alpha\gamma_1}}}+c_2e^{-y\sqrt{\frac{\gamma_1Prq^\alpha}{q^\alpha+\alpha\gamma_1}}}-\frac{A_1}{q}e^{y\sqrt{\frac{\gamma_1Scq^\alpha}{q^\alpha+\alpha\gamma_1}}}. \tag{3.38}$$

With the help of boundary conditions for concentration field, constants c_1 and c_2 can be traced out in the following equation:

$$\bar{\theta}(y,q)=\frac{e^{-y\sqrt{\frac{\gamma_1Prq^\alpha}{q^\alpha+\alpha\gamma_1}}}}{q}+\frac{A_1}{q}\times\left(e^{-y\sqrt{\frac{\gamma_1Prq^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1Scq^\alpha}{q^\alpha+\alpha\gamma_1}}}\right), \tag{3.39}$$

$$\bar{\theta}(y,q)=\frac{1}{q^{1-\alpha}}\frac{e^{-y\sqrt{\frac{\gamma_1Prq^\alpha}{q^\alpha+\alpha\gamma_1}}}}{q^\alpha}+\frac{A_1}{q^{1-\alpha}}\times\left(\frac{e^{-y\sqrt{\frac{\gamma_1Prq^\alpha}{q^\alpha+\alpha\gamma_1}}}}{q^\alpha}-\frac{e^{y\sqrt{\frac{\gamma_1Scq^\alpha}{q^\alpha+\alpha\gamma_1}}}}{q^\alpha}\right). \tag{3.40}$$

Inverting an exponential equation through Laplace transform subject to the an Appendix (A6)–(A10), we get final solution for temperature as:

$$\begin{aligned}\theta(y,t) = {} & h(t,\alpha)\times(\chi(y,t,\gamma_1 P_r,\gamma_1\alpha,0)) \\ & +A_1 h(t,\alpha)\times(\chi(y,t,\gamma_1 P_r,\gamma_1\alpha,0)) \\ & -A_1 h(t,\alpha)\times(\chi(y,t,\gamma_1 S_c,\gamma_1\alpha,0)).\end{aligned} \tag{3.41}$$

3.6. Fractional optimality of velocity field via ABC derivative

Generating Eq. (2.9) for the fractionalized form, we imposed Eqs. (2.17) and (2.18) on Eq. (2.9), we arrive at

$$\frac{q^\alpha}{q^\alpha-\alpha q^\alpha+\alpha}\bar{u}\ (y,q)=\frac{\partial^2\bar{u}(y,q)}{\partial y^2}+G_r\bar{\theta}(y,q)+G_m\bar{\phi}(y,q). \tag{3.42}$$

By substituting $\bar{\theta}$ and $\bar{\phi}$ into (3.42), after some minor simplification, we get:

$$\begin{aligned}\bar{u}(y,q) = {} & c_1 e^{y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}+c_2 e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-\frac{B_1(q^\alpha+\alpha\gamma_1)}{q^\alpha.q}e^{y\sqrt{\frac{\gamma_1 P_r q^\alpha}{q^\alpha+\alpha\gamma_1}}} \\ & -\frac{B_2(q^\alpha+\alpha\gamma_1)}{q^\alpha.q}e^{y\sqrt{\frac{\gamma_1 P_r q^\alpha}{q^\alpha+\alpha\gamma_1}}}+\frac{B_3(q^\alpha+\alpha\gamma_1)}{q^\alpha.q}e^{y\sqrt{\frac{\gamma_1 S_c q^\alpha}{q^\alpha+\alpha\gamma_1}}} \\ & -\frac{B_4(q^\alpha+\alpha\gamma_1)}{q^\alpha.q}e^{y\sqrt{\frac{\gamma_1 S_c q^\alpha}{q^\alpha+\alpha\gamma_1}}}.\end{aligned} \tag{3.43}$$

With the help of boundary conditions for concentration field, constants c_1 and c_2 can be traced out:

$$\begin{aligned}\bar{u}(y,q) = {} & \left(\frac{q}{q^2+w^2}\right)e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}+\frac{B_1}{q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 P_r q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right) \\ & +\frac{B_1\alpha\gamma_1}{q^\alpha.q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 P_r q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right)+\frac{B_2}{q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 P_r q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right) \\ & +\frac{B_2\alpha\gamma_1}{q^\alpha.q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 P_r q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right)-\frac{B_3}{q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 S_c q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right) \\ & -\frac{B_3\alpha\gamma_1}{q^\alpha.q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 S_c q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right)+\frac{B_4}{q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 S_c q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right) \\ & +\frac{B_4\alpha\gamma_1}{q^\alpha.q}\left(e^{-y\sqrt{\frac{\gamma_1 q^\alpha}{q^\alpha+\alpha\gamma_1}}}-e^{y\sqrt{\frac{\gamma_1 S_c q^\alpha}{q^\alpha+\alpha\gamma_1}}}\right).\end{aligned} \tag{3.44}$$

Inverting an exponential equation through Laplace transform subject to the an Appendix (A6)–(A10), we get final solution for temperature as:

$$\begin{aligned}u(y,t) = {} & \cos(wt)\times\frac{t^{1-\alpha}}{\Gamma(\alpha)}*\chi(y,t,\gamma_1,\gamma_1\alpha,0)+B_1 h(t,\alpha) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,P_r\gamma_1,\gamma_1\alpha,0))+B_1\alpha\gamma_1 h(t,0) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,P_r\gamma_1,\gamma_1\alpha,0))+B_2 h(t,\alpha) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,P_r\gamma_1,\gamma_1\alpha,0))+B_2\alpha\gamma_1 h(t,0) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,P_r\gamma_1,\gamma_1\alpha,0))+B_3 h(t,\alpha) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,S_c\gamma_1,\gamma_1\alpha,0))+B_3\alpha\gamma_1 h(t,0) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,S_c\gamma_1,\gamma_1\alpha,0))+B_4 h(t,\alpha) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,S_c\gamma_1,\gamma_1\alpha,0))+B_4\alpha\gamma_1 h(t,0) \\ & *(\chi(y,t,\gamma_1,\gamma_1\alpha,0)-\chi(y,t,S_c\gamma_1,\gamma_1\alpha,0)).\end{aligned} \tag{3.45}$$

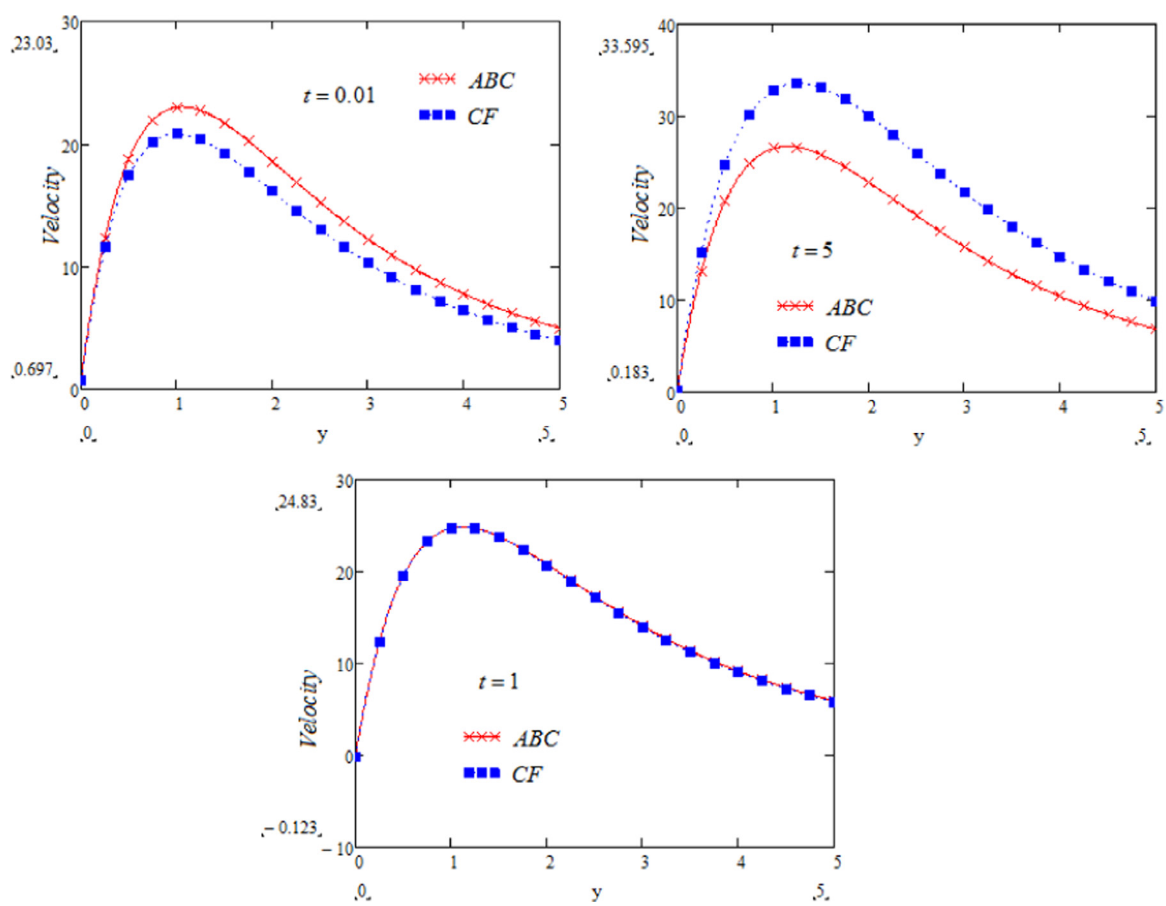

Fig. 1 Comparison of velocities with variation of time for *ABC* and *CF*.

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

4. Results and discussion

The investigation of Dufour effect on viscous fluid through a channel has been discussed by using Atangana-Baleanu fractional time derivative (ABC) and Caputo-Fabrizio fractional time derivative (CF) with suitable initial and boundary value problem. The influence of fractional parameter $\alpha, G_m, G_r, D_f, S_c$ and P_r have been analyzed through graphically with different differential operator. i.e. CF and ABC.

The behavior of velocity profile with variation of time for ABC and CF discussed in Fig. 1. The velocity profile achieved by ABC approach is greater than the velocity obtained with CF approach for small time. Whereas for large value of time, the response of velocities behavior are totally reverse. The velocity achieved by CF is larger than the velocity achieved by ABC. Another observation is consider for $t = 1$, both velocities are behave alike. The significant role of fractional parameter (α) for CF and ABC as shown in Figs. 2 and 3. Clearly, fluid velocity increases with the increase in α for small and large time. As time increases, the boundary layer thickness become greater and velocity is highest in the vicinity of plate. While making comparison velocity for Atangana-Baleanu model is larger because it has nonlocal kernel. The non-integer fractional order derivative reduce to classical model by $\alpha \to 1$.

The influence of G_m on velocity field are shown in Figs. 4 and 5. The magnification in buoyancy forces leads to reduce the viscous force which help to enhance the velocity field with increase of G_m. For large and small value of time, the behavior of velocities are same in G_m. The velocity field of Atangana-Baleanu is larger than the velocity for other fractional operators. Figs. 6 and 7 shows the behavior of G_r on velocity field. An increase in G_r , resultant velocity increases. It related to thermal buoyancy effect, as buoyancy effect rises due to which velocity of fluid increases. A comparative study is carried out for fluid velocity profile for all models. Figs. 8 and 9 analyzed the influence of D_f on fluid velocity. As increase in D_f causes the velocity enhance. Figs. 10 and 11 shows the effect of Schmidt number S_c on velocity profile. The resultant velocity reduce with increase in S_c. It is observed that fluid velocity for ABC fractional model is greater than CF model.

The resultant fluid velocity reduce with enhance the value of P_r. The impact of P_r on fluid velocity analyzed in Figs. 10 and 11. The thermal conductivity and thickness of boundary layer shrink with increase the value of P_r and viscosity of fluid respectively (Figs. 12 and 13). The influence of α, D_f, P_r and S_c on temperature profile are analyzed in Figs. 14–17 by ABC approach. The temperature reduce with increase in α discussed in Fig. 14. The increasing behavior of temperature with large

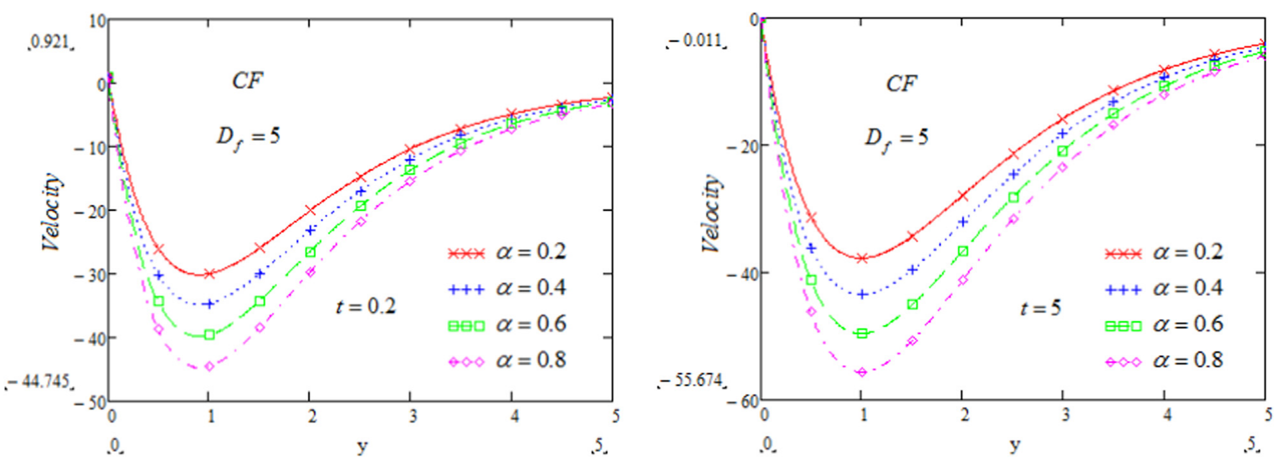

Fig. 2 Profile velocity of α for CF with other parameters are $G_m = 5, G_r = 10, P_r = 0.71$.

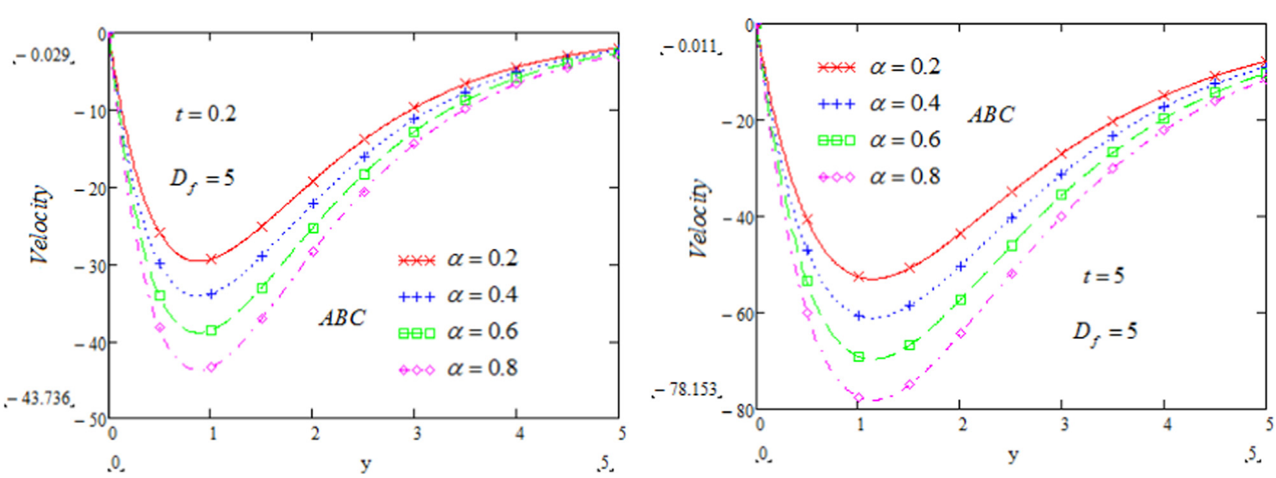

Fig. 3 Profile velocity of α for ABC with other parameters are $G_m = 5, G_r = 10, P_r = 0.71$.

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

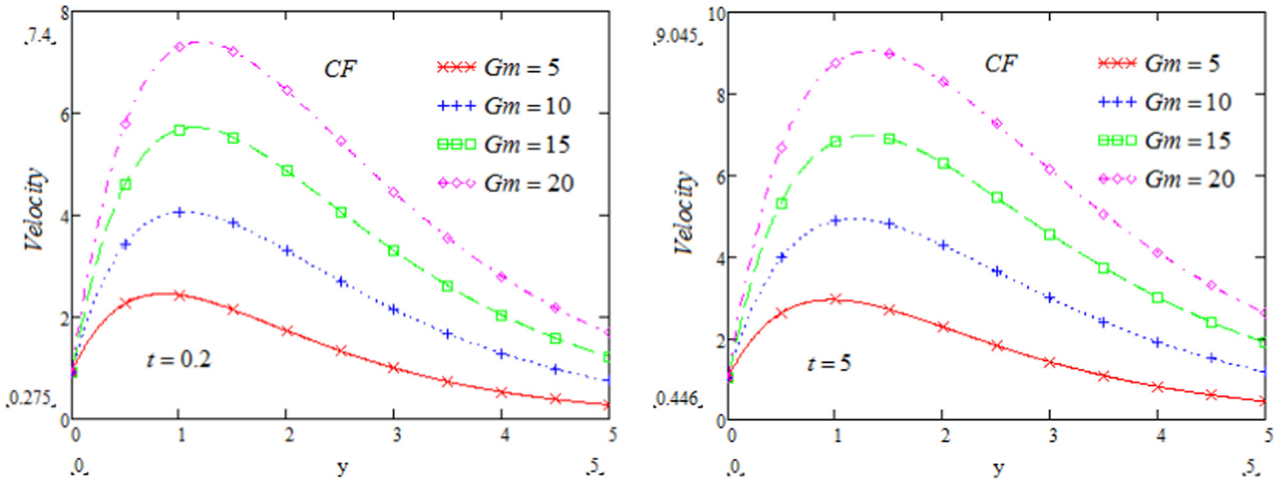

Fig. 4 Profile velocity of G_m for CF with other parameters are $D_f = 5, G_r = 2, P_r = 2$.

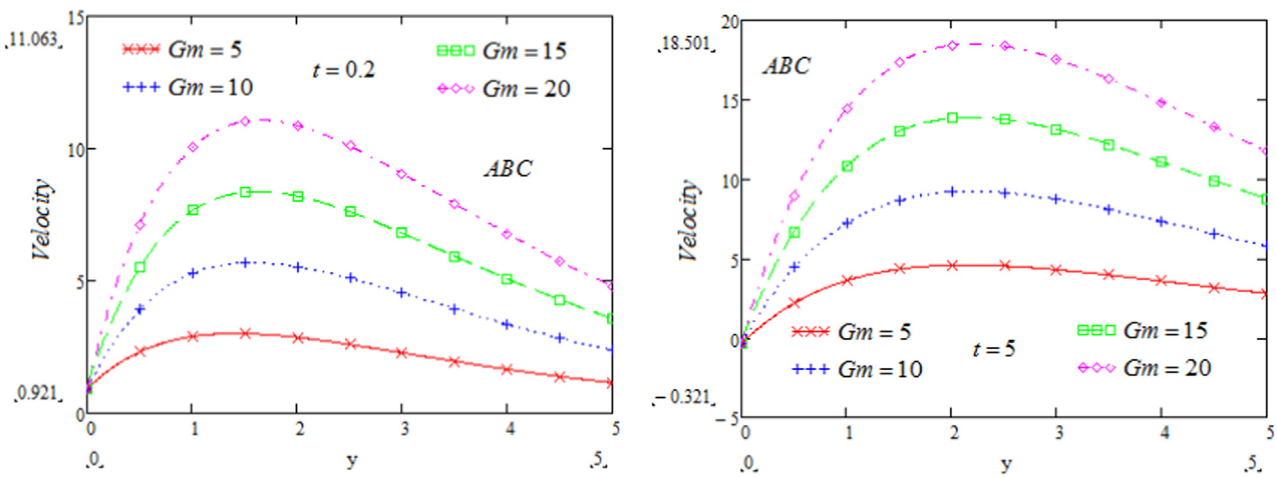

Fig. 5 Profile velocity of G_m for ABC with other parameters are $D_f = 5, G_r = 2, P_r = 2$.

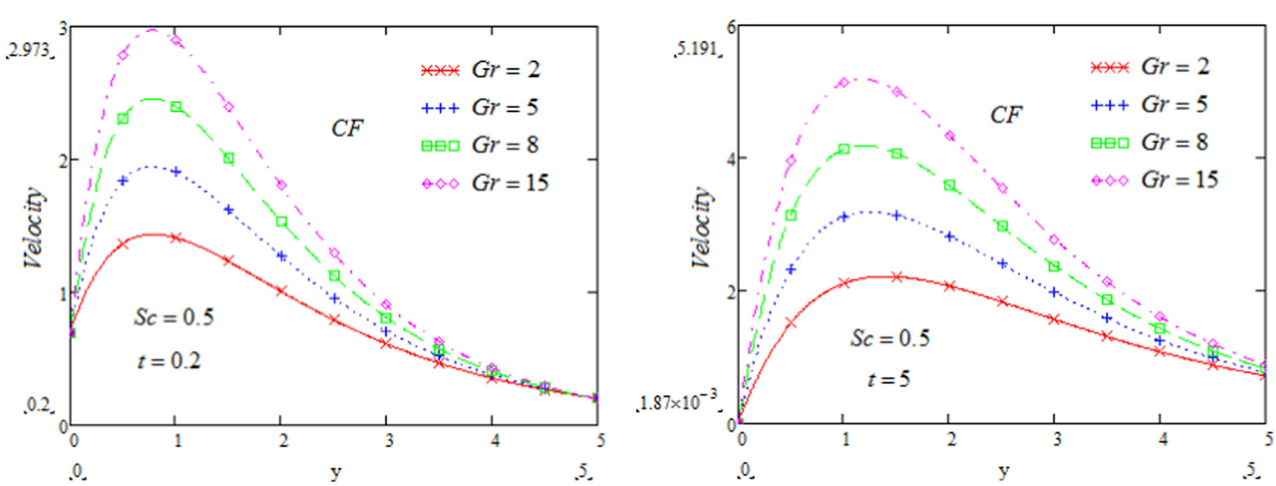

Fig. 6 Profile velocity of G_r for CF with other parameters are $D_f = 2, G_m = 10, P_r = 0.5$.

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

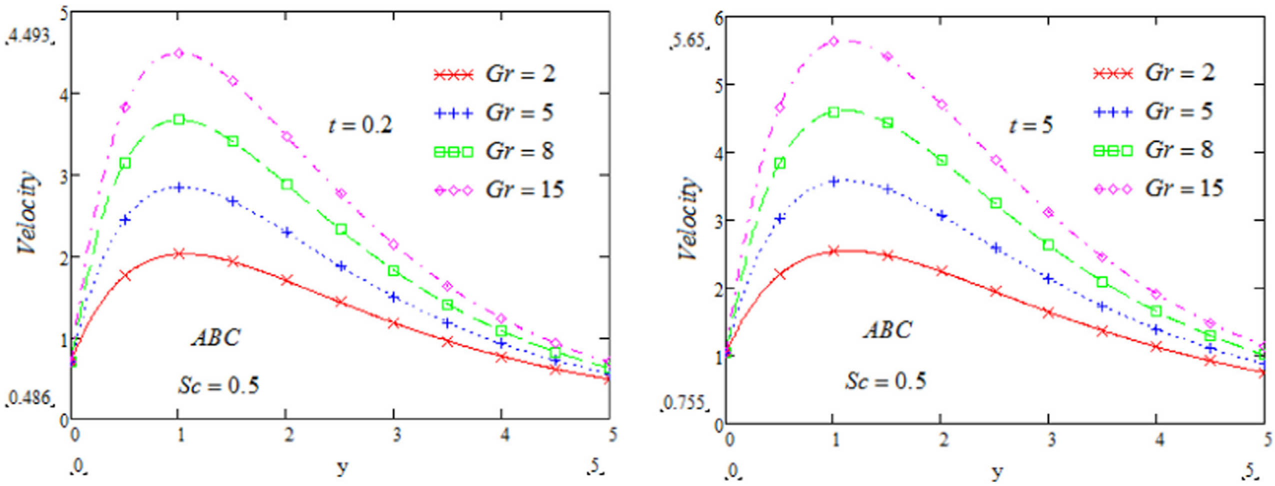

Fig. 7 Profile velocity of G_r for ABC with other parameters are $D_f = 2, G_m = 10, P_r = 0.5$.

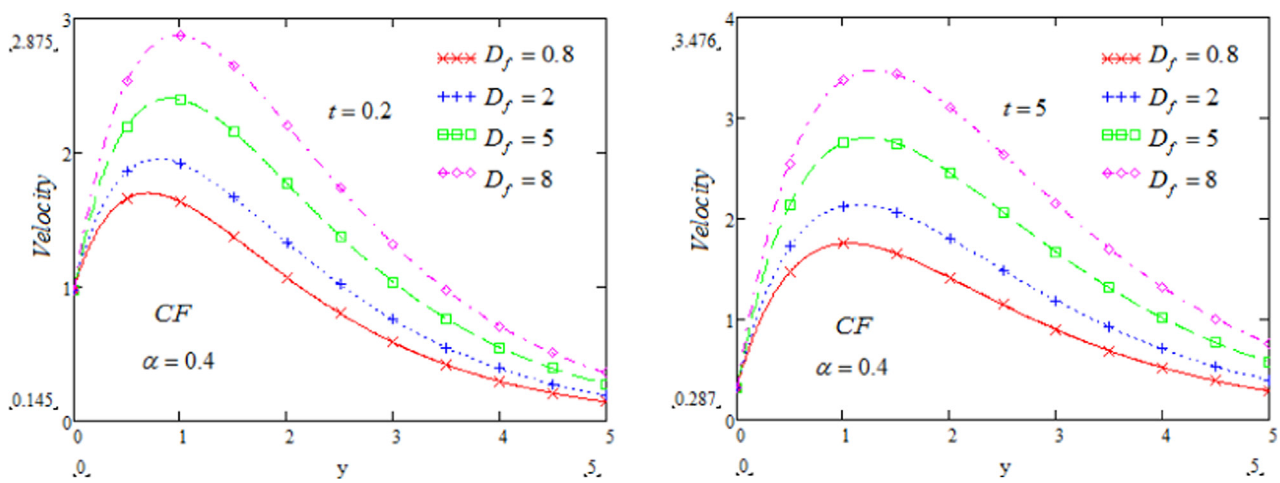

Fig. 8 Profile velocity of D_f for CF with other parameters are $S_c = 0.8, G_m = 8, P_r = 2$.

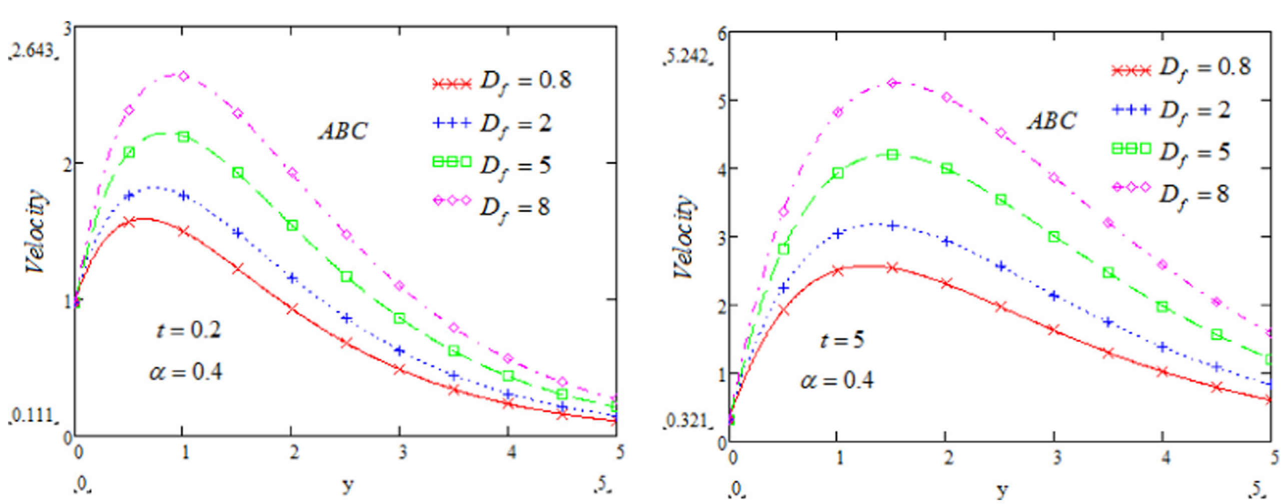

Fig. 9 Profile velocity of D_f for ABC with other parameters are $S_c = 0.8, G_m = 8, P_r = 2$.

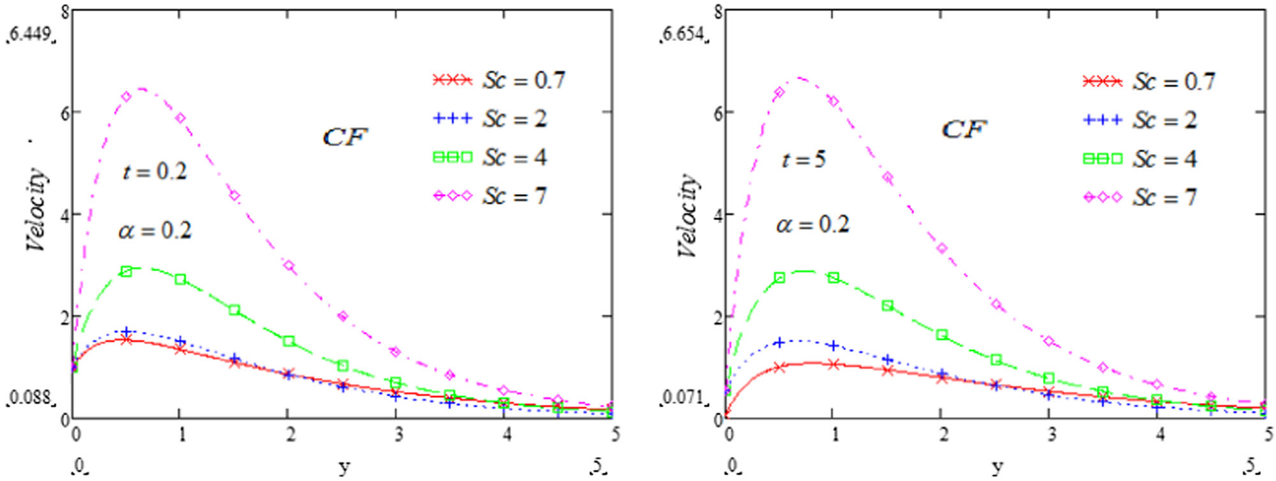

Fig. 10 Profile velocity of S_c for CF with other parameters are $G_r = 5, G_m = 10, D_f = 0.8$.

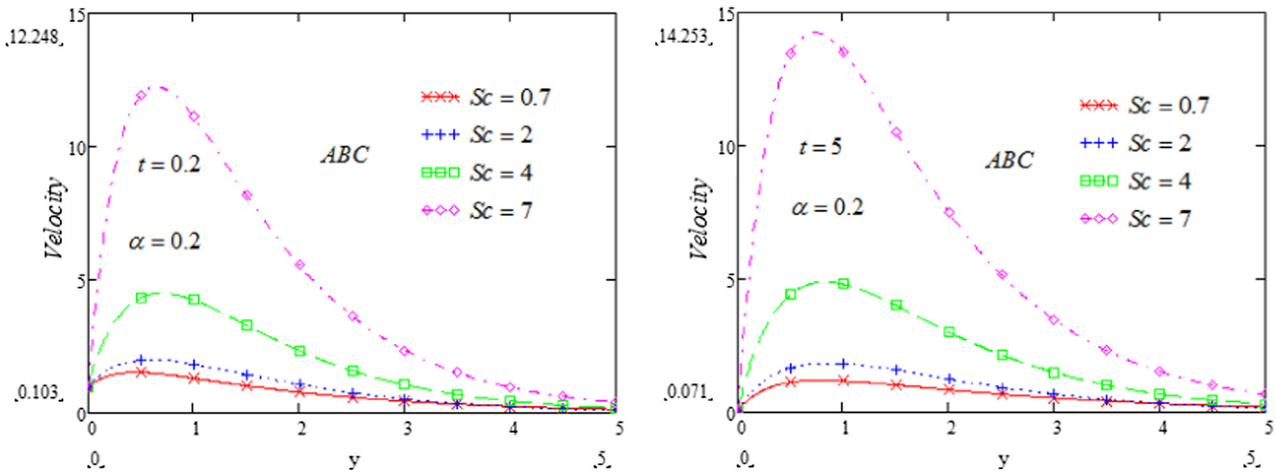

Fig. 11 Profile velocity of S_c for ABC with other parameters are $G_r = 5, G_m = 10, D_f = 0.8$.

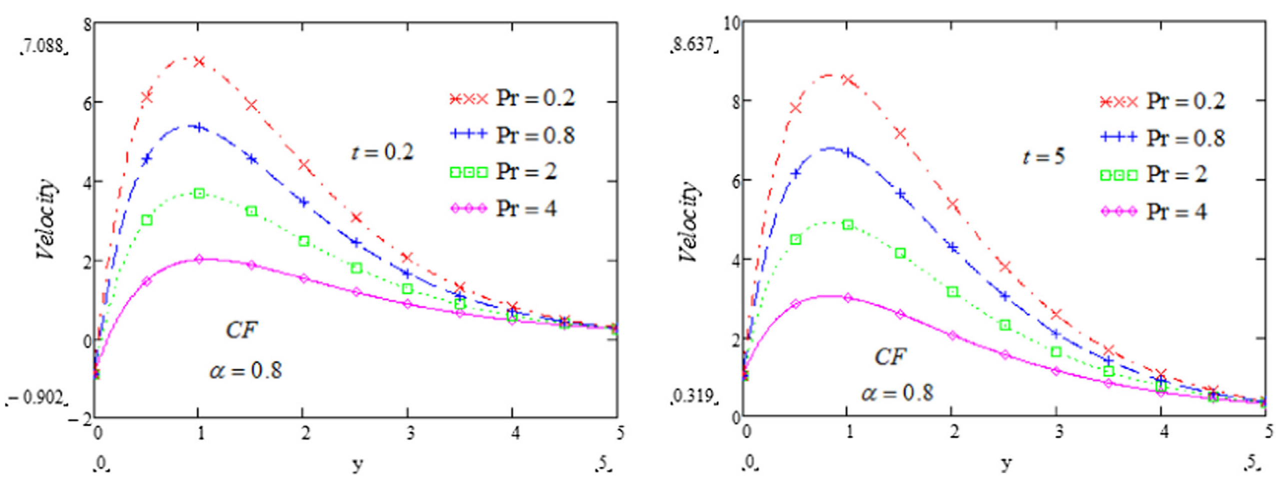

Fig. 12 Profile velocity of P_r for CF with other parameters are $S_c = 0.5, G_m = 5, D_f = 0.8$.

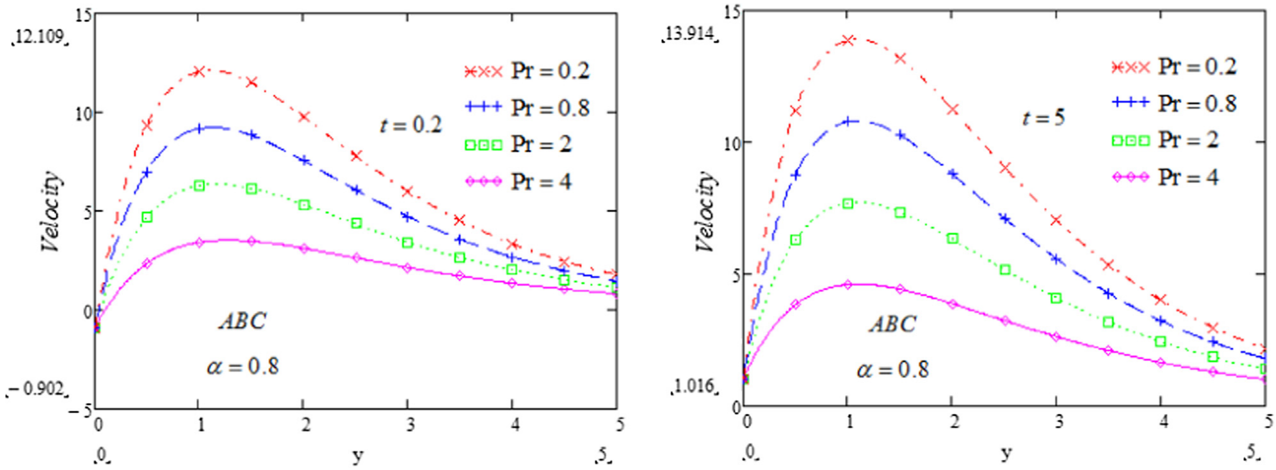

Fig. 13 Profile velocity of P_r for ABC with other parameters are $S_c = 0.5, G_m = 5, D_f = 0.8$.

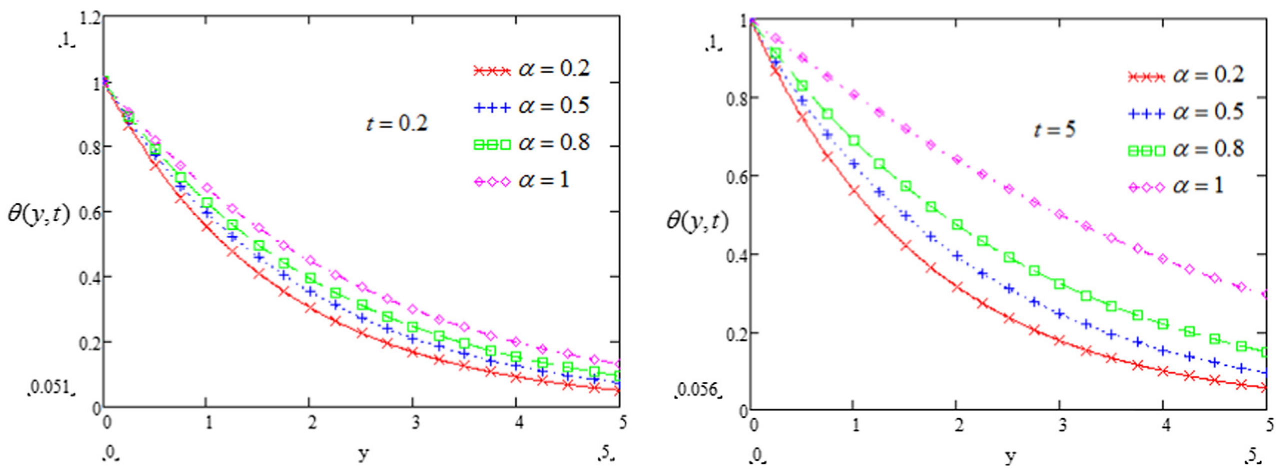

Fig. 14 Profile temperature for α with variation of time and other parameters are $G_r = 5, G_m = 10, D_f = 2, P_r = 0.8$.

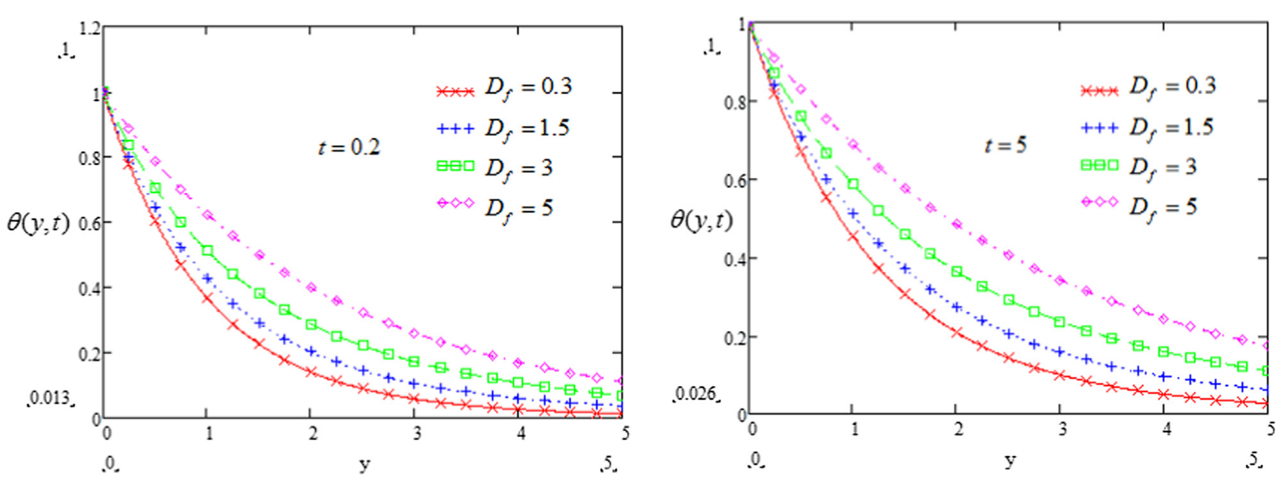

Fig. 15 Profile temperature for D_f with variation of time and other parameters are $\alpha = 0.4, G_r = 10, S_c = 1.5, P_r = 3$.

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

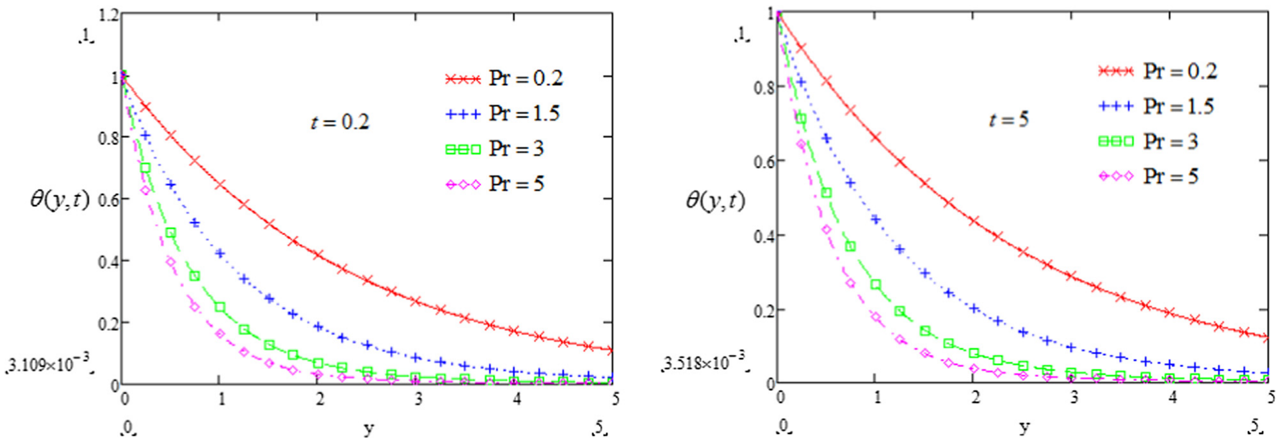

Fig. 16 Profile temperature for P_r with variation of time and other parameters are $\alpha = 0.6, G_m = 5, S_c = 0.4, P_r = 2$.

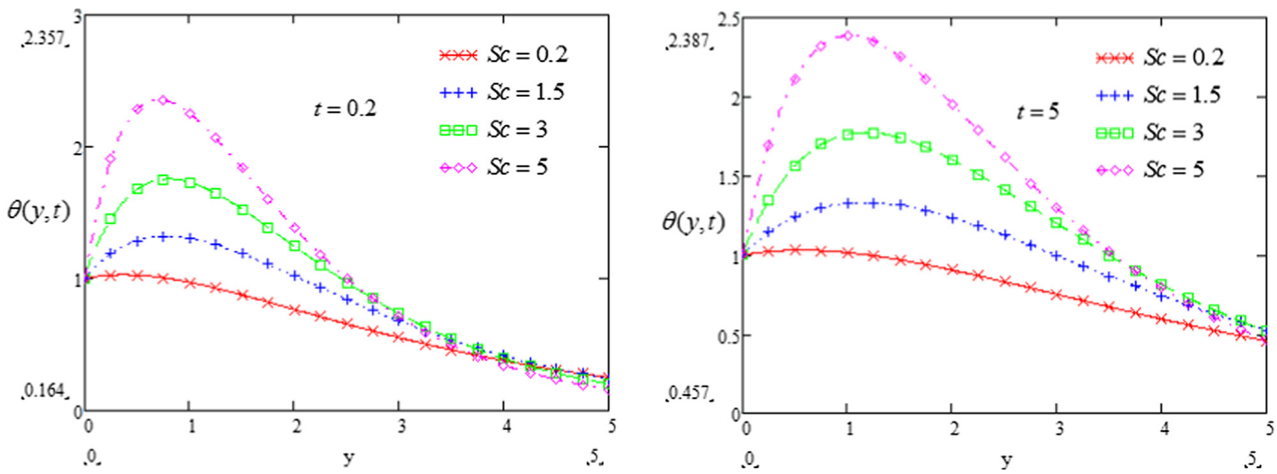

Fig. 17 Profile temperature for S_c with variation of time and other parameters are $\alpha = 0.8, G_m = 10, S_c = 2, P_r = 0.8$.

value of D_f shown in Fig. 15. The effect of P_r on temperature discussed in Fig. 16. As P_r increase, temperature decrease. In Fig. 17, the value of S_c increase with increase in temperature.

5. Conclusion

The comparative study of viscous fluid along vertical plate with Dufour effect has been analyzed in view of non-local and non-singular kernel. Caputo-Fabrizio (CF) and Atangana-Baleanu (ABC) approach are used to compare the fluid velocity behavior. The key points have extracted from this article.

- The velocity profile shows opposite behavior for different values of time while $t = 1$ the behavior of ABC and CF approach are same.
- The magnitude of the velocity increase as increase in fractional parameter.
- Velocity enhance with increase the value of G_r and G_m.
- Increase the value of D_f and S_c, the resultant velocity also increase.
- Temperature reduce for increasing values of α and P_r.
- ABC fractional derivative is more considerable as compared to CF.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A.

$$\delta(y,t,a,b) = L^{-1}\left(\frac{1}{q}exp\left(-y\sqrt{\frac{aq}{q+b}}\right)\right) = 1 - \frac{2a}{\pi}\int_0^\infty \frac{\sin(ys)}{s(a+s^2)}e^{\left(\frac{-bts^2}{a+s^2}\right)}ds \quad \text{(A1)}$$

$$\Xi(y,q,a,b,c) = \frac{1}{q-c}\exp\left(-y\sqrt{\frac{aq}{q+b}}\right) \quad \text{(A2)}$$

$$\Xi(y,t,a,b,c) = exp(ct - y\sqrt{\frac{ac}{a+c}}) - \frac{2a}{\pi}\int_0^\infty \frac{\sin(y\tau)}{\tau(a+\tau^2)}exp\left(-\frac{\tau^2 bt}{a+\tau^2}\right)d\tau - \frac{2ac}{\pi}\int_0^\infty \frac{\sin(y\tau)}{\tau(ac+(a+c)\tau^2)}exp\left(-\frac{\tau^2 bt}{a+\tau^2}\right)d\tau \quad \text{(A3)}$$

$$\vartheta(y,q,m_0,m_1) = \frac{1}{q^2}\times exp\left(-y\sqrt{\frac{m_0 q}{q+m_1}}\right) \quad \text{(A4)}$$

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

$$\vartheta(y,t,m_0,m_1)=\int_0^{\infty}\Xi(y,s,m_0,m_1,0)ds \tag{A5}$$

$$\bar{\chi}_1(y,q,a,b,c)=\frac{1}{q^{\alpha}+a}exp\left(-y\sqrt{\frac{q^{\alpha}+b}{q^{\alpha}+c}}\right) \tag{A6}$$

$$\chi_1(y,t,a,b,c)=\int_0^{\infty}\chi(y,t,a,b,c)g(u,t)du \tag{A7}$$

$$\chi(y,t,a,b,c)=e^{-at-y}-\frac{y\sqrt{b-c}}{2\sqrt{\pi}}\int_0^{\infty}\int_0^{t}\frac{e^{-at}}{\sqrt{t}}\times exp(at - ct-\frac{y^2}{4u}-u)I_1(2\sqrt{(b-c)ut})dtdu \tag{A8}$$

$$h(t,\varsigma)=\frac{1}{t^{\varsigma}\Gamma(1-\varsigma)},g(u,t)=t^{-1}\aleph(0,-\varsigma,-ut^{-\varsigma}) \tag{A9}$$

$$\aleph(k,-\varsigma,t)=\Sigma\frac{t^n}{n!\Gamma(k-n\varsigma)} \tag{A10}$$

References

[1] B.K. Jha, A.O. Ajibade, Diffusion-thermo effects on free convective heat and mass transfer flow in a vertical channel with symmetric boundary conditions, J. Heat Transf. 133 (2011), 052502.
[2] L. Florio, A. Harnoy, Combination technique for improving natural convection cooling in electronics, Int. J. Sci. 46 (2007) 76–92.
[3] M. Fossa, G. Tanda, Frost formation in vertical channels under natural convection, Int. J. Multiph. Flow 36 (2010) 210–220.
[4] J.C. Umavathi, M.A. Sheremet, Mixed convection flow of an electrically conducting fluid in a vertical channel using Robin boundary conditions with heat source/sink, Eur. J. Mech. B/Fluids 55 (2016) 132–145.
[5] J. Umavathi, M. Sheremet, S. Mohiuddin, Combined effect of variable viscosity and thermal conductivity on mixed convection flow of a viscous fluid in a vertical channel in the presence of first order chemical reaction, Eur. J. Mech. B/Fluids (2016).
[6] M.A. Ismael, I. Pop, A.J. Chamkha, Mixed convection in a lid-driven square cavity with partial slip, Int. J. Sci. 82 (2014) 47–61.
[7] M. Narahari, T. Sowmya, P. Rajashekhar, Ramped temperature and Dufour effects on transient MHD natural convection flow past an infinite vertical plate in a porous medium, Eur. Phys. J. Plus 130 (2015) 251.
[8] D. Kalidas, Magnetohydrodynamics free convection flow of a radiating and chemically reacting fluid past an impulsively moving plate with ramped wall temperature, J. Appl. Mech. 79 (2012), 061017.
[9] A.J. Chamkha, A.-R.A. Khaled, A.A. Khaled, Hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid–saturated porous medium, Int. J. Numer. Methods Heat Fluid Flow 10 (455) (2000) 477.
[10] A.J. Chamkha, Double-diffusive convection in a porous enclosure with cooperating temperature and concentration gradients and heat generation or absorption effects, Numer. Heat Transf. Part A Appl. 41 (2002) 65–87.
[11] Z. Ismail, A. Hussanan, I. Khan, S. Shafie, MHD and radiation effects on natural convection flow in a porous medium past an infinite inclined plate with ramped wall temperature: an exact analysis, Int. J. Appl. Math. Stat. 45 (2013) 77–86.
[12] J.C. Umavathi, J.P. Kumar, M.A. Sheremet, Heat and mass transfer in a vertical double passage channel filled with electrically conducting fluid, Phys. A 465 (2017) 195–216, https://doi.org/10.1016/j.physa.2016.07.073.
[13] A.J. Chamkha, R.A. Mohamed, S.E. Ahmed, Unsteady MHD natural convection from a heated vertical porous plate in a micropolar fluid with Joule heating, chemical reaction and radiation effects, Meccanica 46 (2011) 399–411.
[14] A. O Ajibade, B.K. Jha, Free convection heat and mass transfer flow in a vertical channel with the Dufour effect, Proc. Inst. Mech. Eng. Part E J. Mech. Eng. 224 (2010) 91–101.
[15] P.S. Reddy, A.J. Chamkha, Soret and Dufour effects on MHD convective flow of Al2O3–water and TiO2–water nanofluids past a stretching sheet in porous media with heat generation/absorption, Adv. Powder Technol. 27 (2016) 1207–1218, https://doi.org/10.1016/j.apt.2016.04.005.
[16] M. Narahari, Transient free convection flow between long vertical parallel plates with ramped wall temperature at one boundary in the presence of thermal radiation and constant mass diffusion, Meccanica 47 (2012) 1961–1976.
[17] M. Narahari, S. Tippa, R. Pendyala, M. Nayan, Ramped temperature effect on unsteady MHD natural convection flow past an infinite inclined plate in the presence of radiation, heat source and chemical reaction, in: Recent Advances in Applied and Theoretical Mechanics, Hikari Ltd, Rousse, Bulgaria, 2015, ISBN 978-1-61804-304-7.
[18] A. Siyal, K.A. Abro, M.A. Solangi, Thermodynamics of magnetohydrodynamic Brinkman fluid in porous medium, J. Therm. Anal. Calorim. 136 (2019) 2295–2304.
[19] A. Atangana, On the new fractional derivative and application to nonlinear fishers reaction–diffusion equation, Appl. Math. Comput 273 (2016) 948–956.
[20] S. Iyad, T. Abdeljawad, F. Jarad, Monotonicity analysis for nabla h-discrete fractional Atangana-Baleanu differences, Chaos, Solit. Fract. 117 (2018) 50–59.
[21] M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel, Progr. Fract. Differ. Appl. 1 (2015) 1–13.
[22] T. Abdeljawad, M.B. Riaz, S.T. Saeed, A. Atangana, Role of Local and Non-Local Kernels on MHD Flow of Oldroyd-B Fluid under Effect of Ramped Wall Temperature and Velocity (submitted for publication).
[23] M.B. Riaz, A. Atangana, T. Saeed, MHD free convection flow over a vertical plate with ramped wall temperature and chemical reaction in view of non-singular kernel. (Book Chapter) Publisher: Wiley, ISBN: 9781119654162.
[24] M.A. Imran, I. Khan, M. Ahmad, N.A. Shah, M. Nazar, Heat and mass transport of differential type fluid with non-integer order time-fractional Caputo derivatives, J. Mol. Liq. 229 (2017) 67–75.
[25] K. Das, R.P. Sharma, A. Sarkar, Heat and mass transfer of a second grade magnetohydrodynamic fluid over a convectively heated stretching sheet, J. Comput. Des. Eng. 3 (2016) 330–336.
[26] G.S. Seth, B. Kumbhakar, S. Sarkar, Soret and Hall effects on unsteady MHD free convection flow of radiating and chemically reactive fluid past a moving vertical plate with ramped temperature in rotating system, Int. J. Eng., Sci. Technol. 7 (2015) 94–108.
[27] N.A. Shah, A. Hajizadeh, M. Zeb, S. Ahmad, Y. Mahsud, I.L. Animasaun, Effect of magnetic field on double convection flow of viscous fluid over a moving vertical plate with constant temperature and general concentration by using new trend of fractional derivative, Open J. Math. Sci. 2 (2018) 253–265.
[28] J.R. Pattnaik, G.C. Dash, S. Singh, Radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature, Ain Shams Eng. J. 8 (2017) 67–75.

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012

[29] N.A. Shahid, study of heat and mass transfer in a fractional MHD flow over an infinite oscillating plate, SpringerPlus 4 (2015) 640.

[30] M.B. Riaz, S.T. Saeed, Comprehensive analysis of integer order, Caputo-Fabrizio and Atangana-Baleanu fractional time derivative for MHD Oldroyd-B fluid with slip effect and time dependent bounday conditions. Discr. Continuous Dynam. Syst., 2020 (in press).

[31] M.A. Imran, M. Aleem, M.B. Riaz, R. Ali, I. Khan, A comprehensive report on convective flow of fractional (ABC) and (CF) MHD viscous fluid subject to generalized boundary conditions, Chaos, Solit. Fract. 118 (2018) 274–289.

[32] C. Ravichandrana, K. Logeswaria, F. Jarad, New results on existence in the framework of Atangana-Baleanu derivative for fractional integro-differential equations, Chaos, Solit. Fract. 125 (2019) 194–200.

[33] S.A. Khan, K. Shah, G. Zaman, F. Jarad, Existence theory and numerical solutions to smoking model under Caputo-Fabrizio fractional derivative, Chaos 29 (2019).

[34] F. Jarada, T. Abdeljawadb, Z. Hammouchc, On a class of ordinary differential equations in the frame of Atangana-Baleanu fractional derivative Chaos, Solit. Fract. 117 (2018) 16–20.

[35] A. Akgül, A novel method for a fractional derivative with non-local and non-singular kernel, Chaos, Solit. Fract., vol. 114, pp. 478–482.

[36] E.K. Akgüla, Solutions of the linear and nonlinear differential equations within the generalized fractional derivatives, Chaos: An Interdiscip. J. Nonlinear Sci., 29 (2), 023108.

Please cite this article in press as: S.T. Saeed et al., A mathematical study of natural convection flow through a channel with non-singular kernels: An application to transport phenomena, Alexandria Eng. J. (2020), https://doi.org/10.1016/j.aej.2020.02.012