A novel method for the analytical solution of fractional Zakharov–Kuznetsov equations

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Abstract

In this article, an efficient analytical technique, called Laplace–Adomian decomposition method, is used to obtain the solution of fractional Zakharov–Kuznetsov equations. The fractional derivatives are described in terms of Caputo sense. The solution of the suggested technique is represented in a series form of Adomian components, which is convergent to the exact solution of the given problems. Furthermore, the results of the present method have shown close relations with the exact approaches of the investigated problems. Illustrative examples are discussed, showing the validity of the current method. The attractive and straightforward procedure of the present method suggests that this method can easily be extended for the solutions of other nonlinear fractional-order partial differential equations.

Keywords: Laplace transformation; Adomian decomposition method; Zakharov–Kuznetsov equations; Caputo operator

1 Introduction

Over the past decade, fractional differential equations (FDEs) have gained a lot of attention of researchers due to their ability to enhance real-world issues, used in various fields of engineering and physics. Numerous physical marvels in signal processing, chemical physics, electrochemistry of corrosion, probability and statistics, acoustics and electromagnetic are precisely modeled by DEs of fractional order [1]. Nonlinear partial differential equations (PDEs) can be considered the generalization of the differential equations of integer order [2]. In the modern age it is impossible to imagine modeling of many real world problems without using fractional partial differential equations (FPDEs). Indeed, fractional calculus can be called this century's calculus [3] because of the diversity of applications in different areas of science and technology. Many numerical and analytical techniques have been suggested for the solutions of linear and nonlinear FPDEs [3]. Some emerging analytical approximate approaches for FDEs are homotopy analysis method (HAM) [4], variational iteration method (VIM) [5], generalized fractional Taylor series method [6–9], iterative fractional power series system to solve a number of fractional integro-differential equations [10]; using the Kudryashov technique and trial solution technique, we obtained traveling wave approaches to a fractional, nonlinear Schrodinger problem [11–13]; analytical solution to solve a particular homogeneous time-invariant fractional

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original value problem [14]; Taylor power series solution method is used to obtain approximate 2D time-space fractional diffusion, wave-like, telegraph, time-fractional Phi-4 equation, and Burger models from both closed-form and supportive series alternatives [15, 16]; fractional temporal evolution of optical solitons [17], fractional generalized reaction Duffing model by generalized projective Riccati equation method [18], ternary-fractional differential transform [19], Adomian decomposition method (ADM) [20], and homotopy perturbation method (HPM) [21].

The Korteweg–de Vries (KdV) equations play a vital role in application of sciences. One of the notorious variations is the equations of Zakharov–Kuznetsov (ZK), the electrostatic-acoustic pulses are analyzed in magnetized ions. It is an ocean-based coastal waves investigation [3]. The ZK equation was initial obtained in the study of weak nonlinear acoustic ion waves that significantly attract two-dimensional ion losses. In the present work, we investigate the following fractional ZK equation:

$$D_{t_1}^{\gamma} u + \theta \left(u^{\eta} \right)_{x_1} + \psi \left(u^{\rho} \right)_{x_1 x_1 x_1} + \tau \left(u^{\kappa} \right)_{x_1 y_1 y_1} = 0,$$

where $u = u(x_1, y_1, t_1)$, γ is the parameter defining the structure of the fractional derivative ($0 < \gamma \le 1$), and θ, ψ, and τ are random constants [1]. η, ρ, and κ are integers responsible for the conduct of weak nonlinear acoustic ion vibrations in a plasma comprising cool ions and hot exothermic electrons in the systematic electric field [22].

Some successful analytical approaches for the well-known ZK equation are perturbation iteration algorithm and continuous power series technique [22]. Nonlinear fractional ZK calculation in two dimensions was discussed numerically in [23] using a new iteration Sumudu transform method (NISTM). Exact solutions for traveling waves were obtaine in [3] using the functional variable method for the first time. Another new method for G'/G-expansion was studied in [24, 25]. A modified ZK equation successfully was obtained by the homogeneous balance method [26] and the Riccati sub-equation technique was used to approximate a Zakharov–Kuznetsov fractional model [27, 28]. The first integral method was suggested in [29] for modified KdV-ZK equations. Analytical results of ZK equation were obtained in [22] using the homotopy perturbation method. Solution of ZK equation was studied in [30] using the mapping and ansatz methods. Perusing the above development, contributed by many researchers, in this paper we discuss a new analytical approach, i.e., the Laplace–Adomian decomposition technique.

George Adomian introduced a modern mathematical method to solve nonlinear differential equations in the 1980s described as Adomian decomposition method (ADM) [31]. Similarly, another powerful method found by Pierre-Simon Laplace to solve PDEs was described as the Laplace transformation technique that transforms the initial DEs into a mathematical expression [32]. The technique of Laplace–Adomian decomposition method (LADM) is the appropriate analytical approach for determining nonlinear FPDEs. LADM is a mixture of two strong techniques: ADM and Laplace transform (LT). This method is considered to be suitable for those equations that define nonlinear models. LADM has smaller parameters compared to other analytical methods, so LADM is the best method, not needing discretion and linearization [33]. A comparison for the assessment of FPDEs between the LADM and ADM is mentioned in [34]. In the modern physics, the Kundu–Eckhaus solution deals with the analytical approach of these nonlinear PDEs using LADM in [31]. Multi-step LADM was defined for nonlinear FDEs in

[35]. A fractional-order smoke scheme and Korteweg–de Vries equation evaluation have been effectively analyzed using LADM [36, 37]. In the time fractional multi-dimensional Navier–Stokes models, third-order dispersive equations and telegraph equations has been solved by Laplace–Adomian decomposition method [38–40]. In this paper, motivated by the above research, we extend LADM to solve the model of fractional ZK problems [1].

2 Preliminaries

Definition 2.1 The Laplace transformation of $q(t_1)$, $t_1 > 0$, is represented as

$$Q(s) = \mathcal{L}\big[q(t_1)\big] = \int_0^\infty e^{-st_1} q(t_1)\, dt_1.$$

Definition 2.2 The Laplace transform in a form of convolution is as follows:

$$\mathcal{L}[q_1 * q_2] = \mathcal{L}\big[q_1(t_1)\big] * \mathcal{L}\big[q_2(t_1)\big],$$

here $q_1 * q_2$ defines the convolution between q_1 and q_2,

$$(q_1 * q_2)t_1 = \int_0^\tau q_1(\tau) q_2(t_1 - \tau)\, dt_1.$$

Laplace transform is a fractional derivative

$$\mathcal{L}\big(D_{t_1}^\gamma q(t_1)\big) = s^\gamma Q(s) - \sum_{k=0}^{n-1} s^{\gamma-1-k} q^{(k)}(0), \quad n-1 < \gamma < n,$$

where $Q(s)$ is the Laplace transformation of $q(t_1)$.

Definition 2.3 (Riemann–Liouville fractional integral [41, 42])

$$I_{x_1}^\gamma g(x_1) = \begin{cases} g(x_1) & \text{if } \gamma = 0, \\ \frac{1}{\Gamma(\gamma)} \int_0^{x_1} (x_1 - \upsilon)^{\gamma-1} g(\upsilon\cdot)\, d\upsilon & \text{if } \gamma > 0, \end{cases}$$

where Γ represents the gamma function as

$$\Gamma(\rho) = \int_0^\infty e^{-x_1} x_1{}^{\rho-1}\, dx_1, \quad \rho \in \mathbb{C}.$$

Definition 2.4 The following mathematical expression is given to the Caputo of fractional derivative of order γ for $m \in \mathbb{N}$, $x_1 > 0$, $g \in \mathbb{C}_{t_1}$, $t_1 \geq -1$:

$$D^\gamma g(x_1) = \frac{\partial^\gamma g(x_1)}{\partial t_1^\gamma} = \begin{cases} I^{m-\gamma}\big[\frac{\partial^\gamma g(x_1)}{\partial t_1^\gamma}\big] & \text{if } m-1 < \gamma \leq m, m \in \mathbb{N}, \\ \frac{\partial^\gamma g(x_1)}{\partial t_1^\gamma}. \end{cases}$$

Lemma 2.5 *If* $m-1 < \gamma \leq m$ *with* $m \in \mathbb{N}$ *and* $g \in \mathbb{C}_{t_1}$ *with* $t_1 \geq -1$, *then* [43]

$$I^\gamma I^a g(x_1) = I^{\gamma+a} g(x_1), \quad a, \gamma \geq 0,$$

$$I^\gamma x_1^\lambda = \frac{\Gamma(\lambda + 1)}{\Gamma(\gamma + \lambda + 1)} x_1^{\gamma+\lambda}, \quad \gamma > 0, \lambda > -1, x_1 > 0,$$

$$I^\gamma D^\gamma g(x_1) = g(x_1) - \sum_{k=0}^{m-1} g^{(k)}(0^+) \frac{x_1^k}{k!},$$

for $x_1 > 0, m - 1 < \gamma \le m.$

Definition 2.6 Function of Mittag-Leffler $E_\gamma(b)$ for $\gamma > 0$ is defined as follows:

$$E_\gamma(b) = \sum_{m=0}^{\infty} \frac{b^m}{\Gamma(\gamma m + 1)}, \quad \gamma > 0, b \in \mathbb{C}.$$

3 The procedure of LADM

In this section, the LADM is discussed for the solution of fractional nonlinear nonhomogeneous PDEs.

$$D_{t_1}^\gamma v(x_1, y_1, t_1) + Lv(x_1, y_1, t_1) + Nv(x_1, y_1, t_1) = q(x_1, y_1, t_1),$$

$$m - 1 < \gamma < m, \tag{1}$$

where $D^\gamma = \frac{\partial^\gamma}{\partial t_1^\gamma}$ the operator of Caputo, in which L and N are operators.

With the initial guess

$$v(x_1, y_1, 0) = k(x_1), \quad 0 < \gamma \le 1, t_1 > 0, \tag{2}$$

and using the Laplace transform differentiation property

$$\mathcal{L}\big[D^\gamma v(x_1, y_1, t_1)\big] + \mathcal{L}\big[Lv(x_1, y_1, t_1) + Nv(x_1, y_1, t_1)\big] = \mathcal{L}\big[q(x_1, y_1, t_1)\big], \tag{3}$$

using the Laplace transform differentiation property, we get

$$s^\gamma \mathcal{L}\big[v(x_1, y_1, t_1)\big] - s^{\gamma-1} v(x_1, y_1, 0)$$

$$= \mathcal{L}\big[q(x_1, y_1, t_1)\big] - \mathcal{L}\big[Lv(x_1, y_1, t_1) + Nv(x_1, y_1, t_1)\big],$$

$$\mathcal{L}\big[v(x_1, y_1, t_1)\big] = \frac{k(x_1, y_1)}{s} + \frac{1}{s^\gamma} \mathcal{L}\big[q(x_1, y_1, t_1)\big]$$

$$- \frac{1}{s^\gamma} \mathcal{L}\big[Lv(x_1, y_1, t_1) + Nv(x_1, y_1, t_1)\big]. \tag{4}$$

The given infinite series represents the LADM solution of $v(x_1, y_1, t_1)$ as

$$v(x_1, y_1, t_1) = \sum_{j=0}^{\infty} v_j(x_1, y_1, t_1). \tag{5}$$

The nonlinear term in the problem can be expressed in terms of Adomian polynomial as follows:

$$Nv(x_1, y_1, t_1) = \sum_{j=0}^{\infty} A_j, \tag{6}$$

$$A_j = \frac{1}{j!}\left[\frac{d^j}{d\lambda^j}\left[N\sum_{j=0}^{\infty}(\lambda^j v_j)\right]\right]_{\lambda=0}, \quad j = 0, 1, 2, \ldots. \tag{7}$$

Substituting Eq. (5) and Eq. (6) in Eq. (4), we get

$$\mathcal{L}\left[\sum_{j=0}^{\infty} v(x_1, y_1, t_1)\right] = \frac{k(x_1, y_1)}{s} + \frac{1}{s^{\gamma}}\mathcal{L}\left[q(x_1, y_1, t_1)\right]$$

$$- \frac{1}{s^{\gamma}}\mathcal{L}\left[M\sum_{j=0}^{\infty} v_j(x_1, y_1, t_1) + \sum_{j=0}^{\infty} A_j\right]. \tag{8}$$

Applying the Laplace transform linearity, we have

$$\mathcal{L}\left[v_0(x_1, y_1, t_1)\right] = \frac{v(x_1, y_1, 0)}{s} + \frac{1}{s^{\gamma}}\mathcal{L}\left[q(x_1, y_1, t_1)\right] = k(x_1, y_1, s),$$

$$\mathcal{L}\left[v_1(x_1, y_1, t_1)\right] = -\frac{1}{s^{\gamma}}\mathcal{L}\left[Lv_0(x_1, y_1, t_1) + A_0\right].$$

Usually, we will write as follows:

$$\mathcal{L}\left[v_{j+1}(x_1, y_1, t_1)\right] = -\frac{1}{s^{\gamma}}\mathcal{L}\left[Lv_j(x_1, y_1, t_1) + A_j\right], \quad j \geq 1. \tag{9}$$

Transforming the inverse Laplace in Eq. (9)

$$v_0(x_1, y_1, t_1) = k(x_1, y_1, t_1),$$

$$v_{j+1}(x_1, y_1, t_1) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\gamma}}\mathcal{L}\left[Lv_j(x_1, y_1, t_1) + A_j\right]\right]. \tag{10}$$

4 Numerical results

Example 1 Consider the following equation of FZK $(2, 2, 2)$:

$$D_{t_1}^{\gamma} v + (v^2)_{x_1} + \frac{1}{8}(v^2)_{x_1 x_1 x_1} + \frac{1}{8}(v^2)_{x_1 y_1 y_1} = 0, \tag{11}$$

the initial condition is

$$v(x_1, y_1, 0) = \frac{4}{3}\eta \sinh^2(x_1 + y_1), \tag{12}$$

where η is an arbitrary constant.

Taking Laplace transform of Eq. (11), we obtain

$$\mathcal{L}\left[\frac{\partial^{\gamma} v}{\partial t_1^{\gamma}}\right] = \mathcal{L}\left[-(v^2)_{x_1} - \frac{1}{8}(v^2)_{x_1 x_1 x_1} - \frac{1}{8}(v^2)_{x_1 y_1 y_1}\right],$$

$$s^{\gamma}\mathcal{L}\left[v(x_1, y_1, t_1)\right] - s^{\gamma-1}\left[v(x_1, y_1, 0)\right] = \mathcal{L}\left[-(v^2)_{x_1} - \frac{1}{8}(v^2)_{x_1 x_1 x_1} - \frac{1}{8}(v^2)_{x_1 y_1 y_1}\right].$$

Applying the inverse Laplace transform, we have

$$v(x_1, y_1, t_1) = \mathcal{L}^{-1}\left(\frac{v(x_1, y_1, 0)}{s}\right) + \mathcal{L}^{-1}\left[\frac{1}{s^{\gamma}}\mathcal{L}\left[-(v^2)_{x_1} - \frac{1}{8}(v^2)_{x_1 x_1 x_1} - \frac{1}{8}(v^2)_{x_1 y_1 y_1}\right]\right],$$

$$v(x_1, y_1, t_1) = \frac{4}{3}\eta \sinh^2(x_1 + y_1) + \mathcal{L}^{-1}\left[\frac{1}{s^\gamma}\mathcal{L}\left[-(v^2)_{x_1} - \frac{1}{8}(v^2)_{x_1 x_1 x_1} - \frac{1}{8}(v^2)_{x_1 y_1 y_1}\right]\right].$$

Using the ADM procedure, we get

$$\sum_{j=0}^{\infty} v_j(x_1, y_1, t_1)$$

$$= \frac{4}{3}\eta \sinh^2(x_1 + y_1) + \mathcal{L}^{-1}\left[\frac{1}{s^\gamma}\mathcal{L}\left[-N(v)_{x_1} - \frac{1}{8}N(v)_{x_1 x_1 x_1} - \frac{1}{8}N(v)_{y_1 y_1 x_1}\right]\right],$$

where $N(v)$ is Adomian polynomial representing nonlinear terms in the above equations.

$$N(v) = v^2 = \sum_{j=0}^{\infty} A_j(v),$$

Adomian polynomials are given as follows:

$$A_0 = v_0^2,$$

$$A_1 = 2v_0 v_1,$$

$$A_2 = 2v_0 v_2 + v_1^2,$$

$$v_0(x_1, y_1, t_1) = \frac{4}{3}\eta \sinh^2(x_1 + y_1), \tag{13}$$

$$v_{j+1}(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{1}{s^\gamma}\mathcal{L}\left[-\left(\sum_{j=0}^{\infty} A_j(v)\right)_{x_1} - \frac{1}{8}\left(\sum_{j=0}^{\infty} A_j(v)\right)_{x_1 x_1 x_1}\right.\right.$$
$$\left.\left. - \frac{1}{8}\left(\sum_{j=0}^{\infty} A_j(v)\right)_{y_1 y_1 x_1}\right]\right],$$

for $j = 0, 1, 2, \ldots$.

$$v_1(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{1}{s^\gamma}\mathcal{L}\left[-(v_0^2)_{x_1} - \frac{1}{8}(v_0^2)_{x_1 x_1 x_1} - \frac{1}{8}(v_0^2)_{x_1 y_1 y_1}\right]\right],$$

$$v_1(x_1, y_1, t_1) = \left(-\frac{224}{9}\eta^2 \sinh^3(x_1 + y_1)\cosh(x_1 + y_1) - \frac{32}{3}\eta^2 \sinh(x_1 + y_1)\right.$$
$$\left. \times \cosh^3(x_1 + y_1)\right)\mathcal{L}^{-1}\left(\frac{1}{s^{\gamma+1}}\right) \tag{14}$$

$$= \left(-\frac{224}{9}\eta^2 \sinh^3(x_1 + y_1)\cos h(x_1 + y_1) - \frac{32}{3}\eta^2 \sin h(x_1 + y_1)\right.$$
$$\left. \times \cos h^3(x_1 + y_1)\right)\frac{t^\gamma}{\Gamma(\gamma + 1)}.$$

The subsequent terms are

$$v_2(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{1}{s^\gamma}\mathcal{L}\left[-(2v_0v_1)_{x_1} - \frac{1}{8}(2v_0v_1)_{x_1x_1x_1} - \frac{1}{8}(2v_0v_1)_{x_1y_1y_1}\right]\right]$$

$$= \frac{128}{27}\eta^3\left(1200\cos h^6(x_1 + y_1) - 2080\cosh^4(x_1 + y_1)\right.$$

$$\left. + 968\cosh^2(x_1 + y_1) - 79\right)\frac{t^{2\gamma}}{\Gamma(2\gamma + 1)},$$

$$v_3(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{1}{s^\gamma}\mathcal{L}\left[-\left(2v_0v_2 + v_1^2\right)_{x_1} - \frac{1}{8}\left(2v_0v_2 + v_1^2\right)_{x_1x_1x_1}\right.\right. \tag{15}$$

$$\left.\left. - \frac{1}{8}\left(2v_0v_2 + v_1^2\right)_{x_1y_1y_1}\right]\right]$$

$$= -\frac{2048}{81}\eta^4\sin h(x_1 + y_1)\cos h(x_1 + y_1)\left[88{,}400\cos h^6(x_1 + y_1)\right.$$

$$\left. - 160{,}200\cos h^4(x_1 + y_1) + 85{,}170\cos h^2(x_1 + y_1) - 11{,}903\right]$$

$$\times \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)}.$$

The LADM solution is

$$v(x_1, y_1, t_1) = v_0(x_1, y_1, t_1) + v_1(x_1, y_1, t_1) + v_2(x_1, y_1, t_1)$$

$$+ v_3(x_1, y_1, t_1) + \cdots,$$

$$v(x_1, y_1, t_1) = \frac{4}{3}\eta\sinh(x_1 + y_1) - \left(\frac{224}{9}\eta^2\sinh^3(x_1 + y_1)\cos h(x_1 + y_1)\right.$$

$$\left. + \frac{32}{3}\eta^2\sin h(x_1 + y_1)\cos h^3(x_1 + y_1)\right)\frac{t^\gamma}{\Gamma(\gamma + 1)} + \frac{128}{27}\eta^3\left(1200\right.$$

$$\times \cos h^6(x_1 + y_1) - 2080\cos h^4(x_1 + y_1) + 968\cos h^2(x_1 + y_1) - 79\right)$$

$$\times \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)} - \frac{2048}{81}\eta^4\sin h(x_1 + y_1)\cos h(x_1 + y_1)\left[88{,}400\cos h^6(x_1 + y_1)\right.$$

$$\left. - 160{,}200\cos h^4(x_1 + y_1) + 85{,}170\cos h^2(x_1 + y_1) - 11{,}903\right]\frac{t^{2\gamma}}{\Gamma(2\gamma + 1)}.$$

For $\gamma = 1$,

$$v(x_1, t_1) = \frac{4}{3}\eta\sinh^2(x_1 + y_1 - \eta t_1). \tag{16}$$

Example 2 Consider the following equation of FZK $(3, 3, 3)$:

$$D_{t_1}^\gamma v + \left(v^3\right)_{x_1} + 2\left(v^3\right)_{x_1x_1x_1} + 2\left(v^3\right)_{x_1y_1y_1} = 0, \tag{17}$$

with the initial condition

$$v(x_1, y_1, 0) = \frac{3}{2}\eta\sinh\left[\frac{1}{6}(x_1 + y_1)\right], \tag{18}$$

where η is an arbitrary constant.

Taking the Laplace transform of Eq. (17), we have

$$\mathcal{L}\left[\frac{\partial^{\gamma} v}{\partial t_1^{\gamma}}\right] = \mathcal{L}\left[-\left(v^3\right)_{x_1} - 2\left(v^3\right)_{x_1 x_1 x_1} - 2\left(v^3\right)_{x_1 y_1 y_1}\right],$$

$$s^{\gamma} \mathcal{L}\left[v(x_1, y_1, t_1)\right] - s^{\gamma-1}\left[v(x_1, y_1, 0)\right] = \mathcal{L}\left[-\left(v^3\right)_{x_1} - 2\left(v^3\right)_{x_1 x_1 x_1} - 2\left(v^3\right)_{x_1 y_1 y_1}\right].$$

Applying the inverse Laplace transform leads to

$$v(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{v(x_1, y_1, 0)}{s} + \frac{1}{s^{\gamma}}\mathcal{L}\left[-\left(v^3\right)_{x_1} - 2\left(v^3\right)_{x_1 x_1 x_1} - 2\left(v^3\right)_{x_1 y_1 y_1}\right]\right],$$

$$v(x_1, y_1, t_1) = \frac{3}{2}\eta \sinh\left[\frac{1}{6}(x_1 + y_1)\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\gamma}}\mathcal{L}\left[-\left(v^3\right)_{x_1} - 2\left(v^3\right)_{x_1 x_1 x_1} - 2\left(v^3\right)_{x_1 y_1 y_1}\right]\right].$$

Using the ADM procedure, we get

$$\sum_{j=0}^{\infty} v_j(x_1, y_1, t_1) = \frac{3}{2}\eta \sinh\left[\frac{1}{6}(x_1 + y_1)t\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\gamma}}\mathcal{L}\left[-N(v)_{x_1} - 2N(v)_{x_1 x_1 x_1}\right.\right.$$

$$\left.\left. - 2N(v)_{x_1 y_1 y_1}\right]\right],$$

where $N(v)$ is an Adomian polynomial representing nonlinear terms in the above equations.

$$N(v) = v^3 = \sum_{j=0}^{\infty} B_j(v),$$

Adomian polynomials are given as follows:

$$B_0 = v_0^3,$$

$$B_1 = 3v_0^2 v_1,$$

$$B_2 = 3v_0^2 v_2 + 3v_0^2 v_1^2,$$

$$v_0(x_1, y_1, t_1) = \frac{3}{2}\eta \sinh\left[\frac{1}{6}(x_1 + y_1)\right], \tag{19}$$

$$v_{j+1}(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{1}{s^{\gamma}}\mathcal{L}\left[-\sum_{j=0}^{\infty} B_j(v)_{x_1}\right.\right.$$

$$\left.\left. - 2\sum_{j=0}^{\infty} B_j(v)_{x_1 x_1 x_1} - 2\sum_{j=0}^{\infty} B_j(v)_{x_1 y_1 y_1}\right]\right],$$

for $j = 0, 1, 2, \ldots$.

$$v_1(x_1, y_1, t_1) = \mathcal{L}^{-1}\left[\frac{1}{s^{\gamma}}\mathcal{L}\left[-\left(v_0^3\right)_{x_1} - 2\left(v_0^3\right)_{x_1 x_1 x_1} - 2\left(v_0^3\right)_{x_1 y_1 y_1}\right]\right],$$

$$v_1(x_1, y_1, t_1) = -\left[3\eta^3 \sinh^2\left[\frac{1}{6}(x_1 + y_1)\right]\right]\cosh\left[\frac{1}{6}(x_1 + y_1)\right]$$

$$+ \frac{3}{8} \eta^3 \cosh^3 \left[\frac{1}{6}(x_1 + y_1) \right] \Bigg], \tag{20}$$

$$\mathcal{L}^{-1} \left(\frac{1}{s^{\gamma+1}} \right) = - \left[3\eta^3 \sinh^2 \left[\frac{1}{6}(x_1 + y_1) \right] \cosh \left[\frac{1}{6}(x_1 + y_1) \right] \right.$$

$$\left. + \frac{3}{8} \eta^3 \cosh^3 \left[\frac{1}{6}(x_1 + y_1) \right] \right] \frac{t^\gamma}{\Gamma(\gamma + 1)}.$$

The subsequent terms are

$$v_2(x_1, y_1, t_1) = \mathcal{L}^{-1} \left[\frac{1}{s^\gamma} \mathcal{L} \left[-\left(3v_0^2 v_1 \right)_{x_1} - 2\left(3v_0^2 v_1 \right)_{x_1 x_1 x_1} - 2\left(3v_0^2 v_1 \right)_{x_1 y_1 y_1} \right] \right]$$

$$= \frac{3}{32} \eta^5 \sinh \left[\frac{1}{6}(x_1 + y_1) \right] \left[765 \cosh^4 \left[\frac{1}{6}(x_1 + y_1) \right] \right.$$

$$\left. - 729 \cosh^2 \left[\frac{1}{6}(x_1 + y_1) \right] + 91 \right] \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)},$$

$$v_3(x_1, y_1, t_1) = \mathcal{L}^{-1} \left[\frac{1}{s^\gamma} \mathcal{L} \left[-\left(3v_0^2 v_2 + 3v_0^2 v_1^2 \right)_{x_1} \right. \right.$$

$$\left. - 2\left(3v_0^2 v_2 + 3v_0^2 v_1^2 \right)_{x_1 x_1 x_1} - 2\left(3v_0^2 v_2 + 3v_0^2 v_1^2 \right)_{x_1 y_1 y_1} \right] \right] \tag{21}$$

$$= -\frac{3}{128} \cosh \left[\frac{1}{6}(x_1 + y_1) \right] \left[171{,}738 \cosh^6 \left[\frac{1}{6}(x_1 + y_1) \right] \right.$$

$$\left. - 349{,}884 \cosh^4 \left[\frac{1}{6}(x_1 + y_1) \right] + 215{,}496 \cosh^2 \left[\frac{1}{6}(x_1 + y_1) \right] \right.$$

$$\left. - 36{,}907 \right] \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)}.$$

The LADM solution is

$$v(x_1, y_1, t_1) = v_0(x_1, y_1, t_1) + v_1(x_1, y_1, t_1) + v_2(x_1, y_1, t_1)$$

$$+ v_3(x_1, y_1, t_1) + \cdots,$$

$$v(x_1, y_1, t_1) = \frac{3}{2} \eta \sinh \left[\frac{1}{6}(x_1 + y_1) \right] - \left[3\eta^3 \sinh^2 \left[\frac{1}{6}(x_1 + y_1) \right] \cosh \left[\frac{1}{6}(x_1 + y_1) \right] \right.$$

$$\left. + \frac{3}{8} \eta^3 \cosh^3 \left[\frac{1}{6}(x_1 + y_1) \right] \right] \frac{t^\gamma}{\Gamma(\gamma + 1)} + \frac{3}{32} \eta^5 \sinh \left[\frac{1}{6}(x_1 + y_1) \right]$$

$$\times \left[765 \cosh^4 \left[\frac{1}{6}(x_1 + y_1) \right] - 729 \cosh^2 \left[\frac{1}{6}(x_1 + y_1) \right] + 91 \right] \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)}$$

$$- \frac{3}{128} \cosh \left[\frac{1}{6}(x_1 + y_1) \right] \left[171{,}738 \cosh^6 \left[\frac{1}{6}(x_1 + y_1) \right] - 349{,}884 \right.$$

$$\times \cosh^4 \left[\frac{1}{6}(x_1 + y_1) \right] + 215{,}496 \cosh^2 \left[\frac{1}{6}(x_1 + y_1) \right] - 36{,}907 \right]$$

$$\times \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)} + \cdots$$

Figure 1 The (**a**) exact and (**b**) LADM solutions of $v(x_1, y_1, t_1)$ of Example 1, *at* $\gamma = 1$

Figure 2 The LADM solution of $v(x_1, y_1, t_1)$ of Example 1, at (**c**) $\gamma = 0.75$ and (**d**) $\gamma = 0.55$
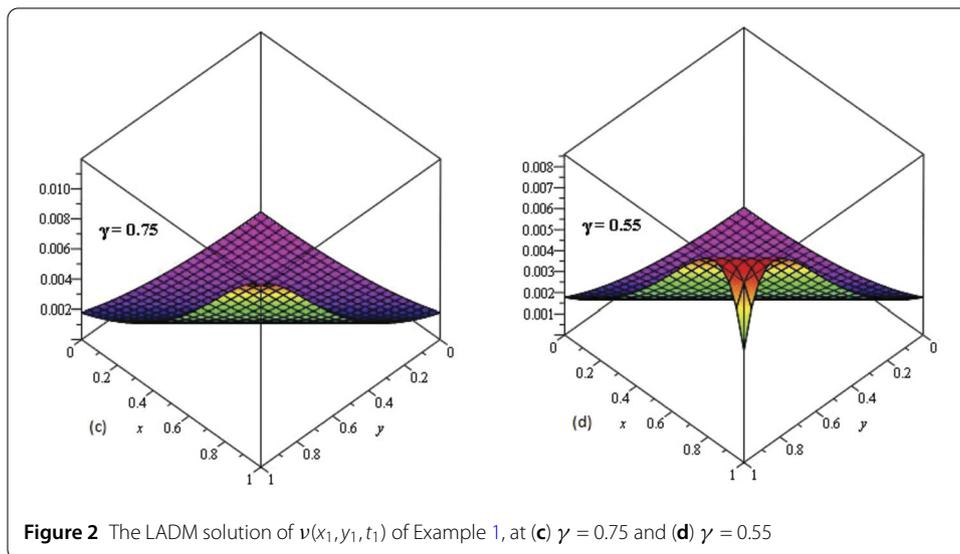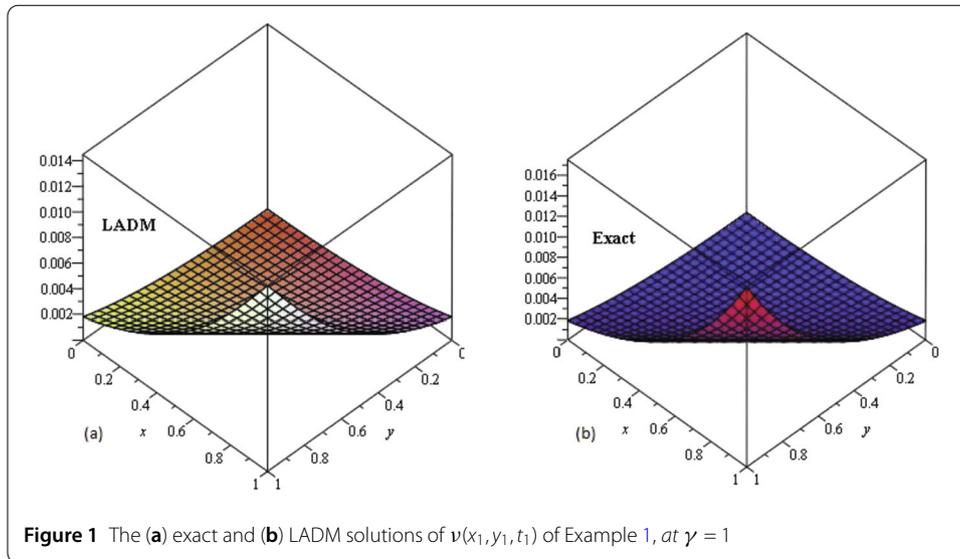
For $\gamma = 1$,

$$v(x_1, t_1) = \frac{3}{2} \eta \sinh \left[\frac{1}{6}(x_1 + y_1 - \eta t_1) \right].$$

(22)

5 Result and discussion

Figure 1 (a) and (b) represent the exact and LADM solutions of Example 1 respectively. Both of the graphs are almost identical and confirm the strong agreement of both exact and LADM solutions. In Fig. 2, the plots c and d show the LADM solutions of Example 1 at fractional order $\gamma = 0.75, 0.55$. The fractional-order solutions analyze different fractional views of Example 1. In Fig. 3, the fractional-order solutions at fractional-order $\gamma = 0.25, 0.50, 0.75, 0.9, 1$ are plotted for $x_1 = y_1 = 1$ see Table 1. Thus analysis has absorbed the asymptotic behaviour of the effect of ascending varying the fractional order. It is in-
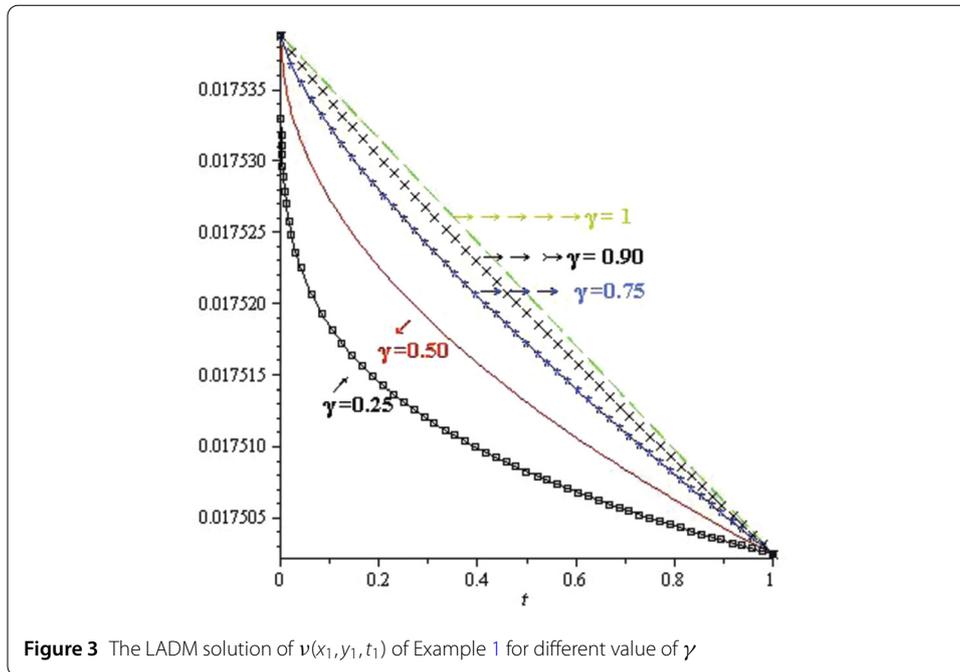
Figure 3 The LADM solution of $v(x_1, y_1, t_1)$ of Example 1 for different value of γ

Table 1 The LADM solution for a distinct value of γ when $\eta = 0.001$ and Example 1 Absolute Error (AE)

LADM							
x_1	y_1	t_1	$\gamma = 0.55$	$\gamma = 0.75$	$\gamma = 1$	Exact($\gamma = 1$)	AE($\gamma = 1$)
0.1	0.1	0.1	5.326967E-5	5.357024E-5	5.380024E-5	5.399349E-5	1.9325198E-7
		0.3	5.263797E-5	5.296756E-5	5.330817E-5	5.388407E-5	5.7590300E-7
		0.5	5.219367E-5	5.247366E-5	5.282125E-5	5.377477E-5	9.5351584E-7
0.6	0.6	0.1	2.965070E-3	2.991825E-3	3.013434E-3	3.037236E-3	2.3801556E-5
		0.3	2.909727E-3	2.937942E-3	2.967189E-3	2.967189E-3	6.8589592E-5
		0.5	2.851320E-3	2.875999E-3	2.903139E-3	3.033594E-3	1.3045515E-4
1	1	0.1	1.540570E-2	1.644689E-2	1.69445E-02	1.753518E-2	5.9062712E-4
		0.3	1.036347E-2	1.443664E-2	1.58793E-02	1.752790E-2	1.6485437E-3
		0.5	2.595877E-3	1.084553E-2	1.44731E-02	1.752063E-2	3.0474747E-3

Table 2 The LADM solution for a distinct value of γ when $\eta = 0.001$ and Example 2 absolute error (AE)

LADM							
x_1	y_1	t_1	$\gamma = 0.50$	$\gamma = 0.75$	$\gamma = 1$	Exact($\gamma = 1$)	AE($\gamma = 1$)
0.1	0.1	0.1	5.000913E-5	5.000918E-5	5.000922E-5	4.998424E-5	2.497599E-8
		0.3	5.000903E-5	5.000909E-5	5.000914E-5	4.993421E-5	7.492792E-8
		0.5	5.000896E-5	5.000901E-5	5.000907E-5	4.988419E-5	1.248797E-7
0.6	0.6	0.1	3.020038E-4	3.020039E-4	3.020039E-4	3.019785E-4	2.544951E-8
		0.3	3.020037E-4	3.020037E-4	3.020038E-4	3.019274E-4	7.634800E-8
		0.6	3.020036E-4	3.020036E-4	3.020037E-4	3.018764E-4	1.272462E-8
1	1	0.1	5.09310E-04	5.09310E-04	5.09310E-04	5.092844E-4	2.632114E-8
		0.3	5.09110E-04	5.09310E-04	5.09310E-04	5.092316E-4	7.896253E-8
		0.6	5.09310E-04	5.09310E-04	5.09310E-04	5.091788E-4	1.316031E-7

vestigated that as the fractional-order approaches integer-order, the solution of fractional-order problems converges to the solution of integer-order problem. The procedure and analysis can be represented in Figs. 4 and 5 (see also Table 2) for Example 2.

Figure 4 The (**a**) exact and (**b**) LADM solutions of $v(x_1, y_1, t_1)$ of Example 2, *at* $\gamma = 1$

Figure 5 The LADM solution of $v(x_1, y_1, t_1)$ of Example 2, *at* (**c**) $\gamma = 0.75$ and (**d**) Absolute Error
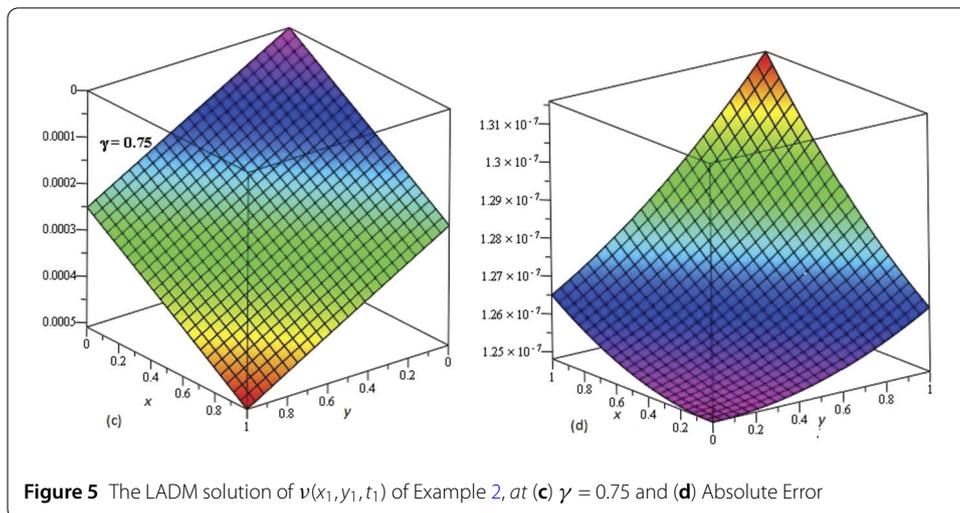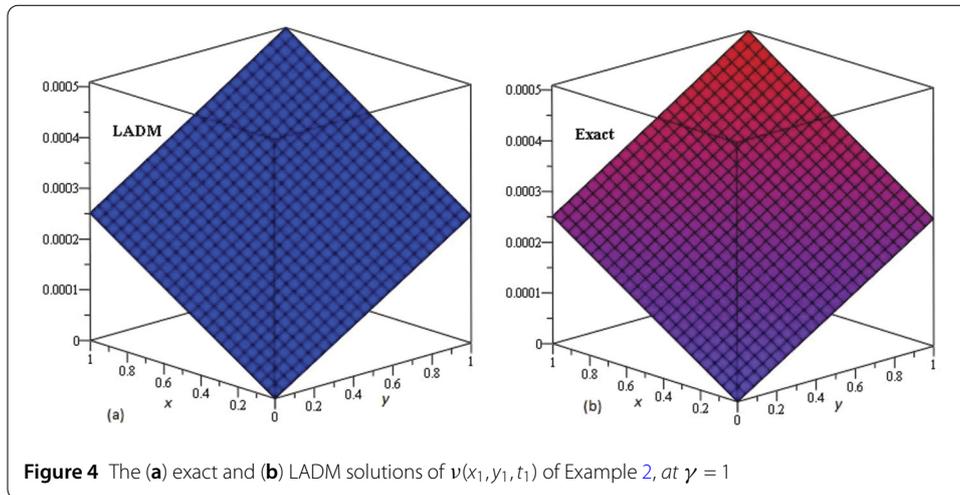
6 Conclusion

We have investigated the analytical solution of fractional Zakharov–Kuznetsov equations using the Laplace–Adomian decomposition method. The procedure of the proposed method is found to be reliable as compared to other analytical methods because of its small number of calculations. The procedure is much understandable to the readers because it consists of the direct implementation of the Laplace transform to the given problem and then applying the Adomian decomposition method. The inverse Laplace transform is then applied to obtain the analytical solution for the given problem. Illustrative examples are also presented to support the theoretical procedure of the suggested method. For this purpose, we plot four graphs, namely Figs. 1, 2, 3, and 4, to show the agreement of the obtained results and exact solutions for the problems. The display of the figures has confirmed that the results obtained by the present method are in good agreement with the exact solution of Example 1 and 2 in the paper. Moreover, the plot of absolute error has been drawn and discussed in the paper. It shows that even considering two terms of the series solution of the presented method, it provided sufficient accuracy to the solution for the problem. Thus, the proposed method is considered to be a suitable analytical technique to solve

the fractional partial differential equations and a system of fractional partial differential equations.

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