Hindawi Publishing Corporation
Advances in Mathematical Physics
Volume 2014, Article ID 590574, 5 pages
http://dx.doi.org/10.1155/2014/590574

Research Article

Local Fractional Poisson and Laplace Equations with Applications to Electrostatics in Fractal Domain

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Received 19 March 2014; Accepted 1 April 2014; Published 16 April 2014

Academic Editor: Carlo Cattani

From the local fractional calculus viewpoint, Poisson and Laplace equations were presented in this paper. Their applications to the electrostatics in fractal media are discussed and their local forms in the Cantor-type cylindrical coordinates are also obtained.

1. Introduction

Poisson and Laplace equations had successfully played an important role in electrodynamics [1–3]. Mathematically, they are two-order partial differential equations and exist in the spaces of different kinds [4, 5]. Their solutions were studied by different. There are approximate and numerical methods for them, such as the finite difference method [6], the finite element method [7], the random walk method [8], the quadrilateral quadrature element [9], and the complex polynomial method [10].

Since Mandelbrot [10] described the fractals, the fractional calculus [11–13] and local fractional calculus [14–16] were applied to the real world problem based on them. For example, Engheta discussed the fractional-order electromagnetic theory [17]. Tarasov studied the fractal distribution of charges [18]. Calcagni et al. suggested the electric charge in multiscale space and times [19]. In [20], the local fractional approach for Maxwell's equations was considered. Local fractional calculus [20–25] has been successfully applied to describe dynamical systems with the nondifferentiable functions. For example, the Maxwell theory on Cantor sets was studied in [20]. The Heisenberg uncertainty relation was discussed by using the local fractional Fourier analysis [21]. The system of Navier-Stokes equations arising in fractal flows was reported in [22]. The local fractional nonhomogeneous heat equations arising in fractal heat flow were presented in [23]. The fractal forest gap within the local fractional derivative was investigated in [24].

In the present paper, the local fractional Poisson and Laplace equations within the nondifferentiable functions arising in electrostatics in fractal domain and in the Cantor-type cylindrical coordinates [25] based upon the local fractional Maxwell equations [20] will be derived from the fractional vector calculus.

The outline of the paper is depicted below. Section 2 introduces the local fractional Maxwell equations. Section 3 discusses the local fractional Poisson and Laplace equations arising in electrostatics in fractal media. In Section 4, the local fractional Poisson and Laplace equations in the Cantor-type cylindrical coordinates are presented. Finally, Section 5 is devoted to the conclusions.

2. Local Fractional Maxwell's Equations

In this section, the local fractional Maxwell's equations are introduced and the concepts of the local fractional vector calculus are reviewed. We first introduce the local fractional vector calculus and its theorems [14, 20–23].

The local fractional line integral of the function $\mathbf{u}$ along a fractal line l^{α} was defined as [14, 20]

$$\int_{l^{(\alpha)}} \mathbf{u}\left(x_P, y_P, z_P\right) \cdot d\mathbf{l}^{(\alpha)} = \lim_{N\to\infty} \sum_{P=1}^{N} \mathbf{u}\left(x_P, y_P, z_P\right) \cdot \Delta\mathbf{l}_P^{(\alpha)}, \tag{1}$$

where the quantity $\Delta\mathbf{l}_P^{(\alpha)}$ is elements of line, $|\Delta\mathbf{l}_P^{(\alpha)}| \to 0$ as $N \to \infty$, and $\alpha \in (0, 1]$.

The local fractional surface integral was defined as [14, 20–23]

$$\iint \mathbf{u}\left(r_P\right) \cdot d\mathbf{S}^{(\beta)} = \lim_{N\to\infty} \sum_{P=1}^{N} \mathbf{u}\left(r_P\right) \cdot \mathbf{n}_P \Delta S_P^{(\beta)}, \tag{2}$$

where the quantity $\Delta S_P^{(\beta)}$ is elements of surface, the quantity $\mathbf{n}_P$ is N elements of area with a unit normal local fractional vector, and $\Delta S_P^{(\beta)} \to 0$ as $N \to \infty$ for $\beta = 2\alpha$.

The local fractional volume integral of the function $\mathbf{u}$ was defined as [14, 20–23]

$$\iiint \mathbf{u}\left(r_P\right) dV^{(\gamma)} = \lim_{N\to\infty} \sum_{P=1}^{N} \mathbf{u}\left(r_P\right) \Delta V_P^{(\gamma)}, \tag{3}$$

where the quantity $\Delta V_P^{(\gamma)}$ is the elements of volume, $\Delta V_P^{(\gamma)} \to 0$ as $N \to \infty$, and $\gamma = 3\alpha$.

The local fractional Stokes' theorem of the fractal field states that [13, 20]

$$\oint_{l^{(\alpha)}} \mathbf{u} \cdot d\mathbf{l}^{(\alpha)} = \iint_{S^{(\beta)}} \left(\nabla^{\alpha} \times \mathbf{u}\right) \cdot d\mathbf{S}^{(\beta)}. \tag{4}$$

The electric Gauss law for the fractal electric field was suggested as [20]

$$\oiint_{S^{(\beta)}} D \cdot d\mathbf{S}^{(\beta)} = \iiint_{V^{(\gamma)}} \rho \, dV^{(\gamma)}, \tag{5}$$

which leads to

$$\nabla^{\alpha} \cdot D = \rho, \tag{6}$$

where the quantity ρ denotes the free charges density and the quantity D is the fractal electric displacement.

The Ampere law in the fractal magnetic field was presented as [20]

$$\oint_{l^{(\alpha)}} H \cdot d\mathbf{l}^{(\alpha)} = \iint_{S^{(\beta)}} \left(J_a + \frac{\partial^{\alpha} D}{\partial t^{\alpha}}\right) \cdot d\mathbf{S}^{(\beta)}, \tag{7}$$

which leads to

$$\nabla^{\alpha} \times H = J_a + \frac{\partial^{\alpha} D}{\partial t^{\alpha}}, \tag{8}$$

where the quantity H is the fractal magnetic field strength and the quantity J_a denotes the fractal conductive current.

The Faraday law in the fractal electric field reads as [20]

$$\oint_{l^{(\alpha)}} E \cdot d\mathbf{l}^{(\alpha)} + \frac{\partial^{\alpha}}{\partial t^{\alpha}} \iint_{S^{(\beta)}} B \cdot d\mathbf{S}^{(\beta)} = 0, \tag{9}$$

which leads to

$$\nabla^{\alpha} \times E = -\frac{\partial^{\alpha} B}{\partial t^{\alpha}}, \tag{10}$$

where the constitutive relationships in fractal electric field are

$$D = \varepsilon_f E \tag{11}$$

with the fractal dielectric permittivity ε_f and the fractal dielectric field E.

The magnetic Gauss law for the fractal magnetic field was written as [20]

$$\oiint_{S^{(\beta)}} B \cdot d\mathbf{S}^{(\beta)} = 0, \tag{12}$$

which leads to

$$\nabla^{\alpha} \cdot B = 0, \tag{13}$$

where the constitutive relationships in fractal magnetic field are

$$H = \mu_f B \tag{14}$$

with the fractal magnetic permeability μ_f and the fractal magnetic field B.

3. Local Fractional Poisson and Laplace Equations in Fractal Media

In this section, we derive the local fractional Poisson and Laplace equations arising in electrostatics in fractal media.

In view of (11), from (6) we have

$$\nabla^{\alpha} \cdot \left(\varepsilon_f E\right) = \rho, \tag{15}$$

so that

$$\varepsilon_f \left(\nabla^{\alpha} \cdot E\right) = \rho, \tag{16}$$

where ρ denotes the free charges density in fractal homogeneous medium, ε_f denotes the fractal dielectric permittivity, and E denotes the fractal dielectric field.

Hence, the local fractional differential form of Gauss's law in local fractional divergence operator reads as

$$\nabla^{\alpha} \cdot E = \frac{\rho}{\varepsilon_f}. \tag{17}$$

If the electrostatics in fractal domain is described by the expression

$$\nabla^{\alpha} \times E = -\frac{\partial^{\alpha} B}{\partial t^{\alpha}} = 0, \tag{18}$$

then the fractal electric field within the local fractional gradient is

$$E = -\nabla^{\alpha} \psi, \tag{19}$$

where the quantity ψ is a nondifferentiable term and E is the fractal dielectric field.

In view of (16) and (19), we obtain

$$\varepsilon_f \left[\nabla^{\alpha} \cdot \left(-\nabla^{\alpha} \psi\right)\right] = \rho, \tag{20}$$

which leads to

$$-\varepsilon_f \left[\nabla^{\alpha} \cdot \nabla^{\alpha} \psi\right] = \rho, \tag{21}$$

where the quantity ψ is a nondifferentiable term and ε_f denotes the fractal dielectric permittivity.

From (21) we arrive at

$$\nabla^{\alpha} \cdot \nabla^{\alpha} \psi = -\frac{\rho}{\varepsilon_f}. \tag{22}$$

Let us define the local fractional operator

$$\nabla^{\alpha} \cdot \nabla^{\alpha} = \nabla^{2\alpha}. \tag{23}$$

Making use of (22) and (23), we have

$$\nabla^{2\alpha} \psi = -\frac{\rho}{\varepsilon_f}. \tag{24}$$

In the Cantorian coordinates, from (24), the local fractional Poisson equation arising in electrostatics in fractal domain can be written as

$$\frac{\partial^{2\alpha}}{\partial x^{2\alpha}} \psi(x, y, z) + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} \psi(x, y, z) + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}} \psi(x, y, z) = -\frac{\rho(x, y, z)}{\varepsilon_f}, \tag{25}$$

where both $\psi(x, y, z)$ and $\rho(x, y, z)$ are nondifferentiable functions; the local fractional operator $\nabla^{2\alpha}$ in the Cantorian coordinates was written as [14]

$$\nabla^{2\alpha} = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}. \tag{26}$$

In the Cantorian coordinates, from (25), the local fractional Laplace equation arising in electrostatics in fractal domain is

$$\frac{\partial^{2\alpha}}{\partial x^{2\alpha}} \psi(x, y, z) + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} \psi(x, y, z) + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}} \psi(x, y, z) = 0, \tag{27}$$

where the quantity $\psi(x, y, z)$ is a nondifferentiable function.

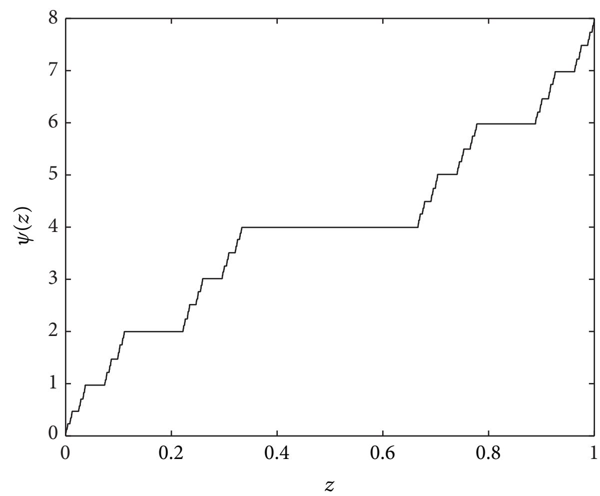

Figure 1: Plot of (31) with parameters $\alpha = \ln 2 / \ln 3$ and $b = 1$.

From (25) the local fractional Laplace equation arising in electrostatics in fractal domain with two variables can be written as

$$\frac{\partial^{2\alpha}}{\partial x^{2\alpha}} \psi(x, y) + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} \psi(x, y) = -\frac{\rho(x, y)}{\varepsilon_f}, \tag{28}$$

where both $\psi(x, y)$ and $\rho(x, y)$ are nondifferentiable functions.

From (27) the local fractional Laplace equation arising in electrostatics in fractal domain with two variables can be written as

$$\frac{\partial^{2\alpha}}{\partial x^{2\alpha}} \psi(x, y) + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} \psi(x, y) = 0, \tag{29}$$

where the quantity $\psi(x, y)$ is a nondifferentiable function.

For the boundary conditions on the fractal potential

$$\psi(0) = 0, \qquad \psi(b) = 8, \tag{30}$$

we have the local fractional Laplace's equation

$$\frac{\partial^{2\alpha}}{\partial z^{2\alpha}} \psi(z) = 0, \tag{31}$$

which leads to the nondifferentiable solution given y

$$\psi(z) = 8\left(\frac{z}{b}\right)^{\alpha} \tag{32}$$

and its graph is shown in Figure 1.

We notice that the local fractional Poisson's equation shows the potential behavior in the fractal regions with nondifferentiable functions where there is the free charge, while local fractional Laplace's equation governs the nondifferentiable potential behavior in fractal regions where there is no free charge.

4. Local Fractional Poisson and Laplace Equations in the Cantor-Type Cylindrical Coordinates

In this section, the local fractional Poisson and Laplace equations in the Cantor-type cylindrical coordinates are considered. We first start with the Cantor-type cylindrical coordinate method.

We now consider the Cantor-type cylindrical coordinates given by [14, 25]

$$\begin{aligned} x^{\alpha} &= R^{\alpha}\cos_{\alpha}\theta^{\alpha}, \\ y^{\alpha} &= R^{\alpha}\sin_{\alpha}\theta^{\alpha}, \\ z^{\alpha} &= z^{\alpha} \end{aligned} \quad (33)$$

with $R \in (0, +\infty)$, $z \in (-\infty, +\infty)$, $\theta \in (0, \pi]$, and $x^{2\alpha} + y^{2\alpha} = R^{2\alpha}$.

From (33) we have [25]

$$\nabla^{\alpha}\phi(R,\theta,z) = \mathbf{e}_R^{\alpha}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\phi + \mathbf{e}_{\theta}^{\alpha}\frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial\theta^{\alpha}}\phi + \mathbf{e}_z^{\alpha}\frac{\partial^{\alpha}}{\partial z^{\alpha}}\phi, \quad (34)$$

$$\nabla^{2\alpha}\phi(R,\theta,z) = \frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\phi + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial\theta^{2\alpha}}\phi + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\phi + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}\phi, \quad (35)$$

where

$$\begin{aligned} \mathbf{e}_R^{\alpha} &= \cos_{\alpha}\theta^{\alpha}\mathbf{e}_1^{\alpha} + \sin_{\alpha}\theta^{\alpha}\mathbf{e}_2^{\alpha}, \\ \mathbf{e}_{\theta}^{\alpha} &= -\sin_{\alpha}\theta^{\alpha}\mathbf{e}_1^{\alpha} + \cos_{\alpha}\theta^{\alpha}\mathbf{e}_2^{\alpha}, \\ \mathbf{e}_z^{\alpha} &= \mathbf{e}_3^{\alpha}, \end{aligned} \quad (36)$$

and the local fractional vector suggested by

$$\mathbf{r} = R^{\alpha}\cos_{\alpha}\theta^{\alpha}\mathbf{e}_1^{\alpha} + R^{\alpha}\sin_{\alpha}\theta^{\alpha}\mathbf{e}_2^{\alpha} + z^{\alpha}\mathbf{e}_3^{\alpha} = r_R\mathbf{e}_R^{\alpha} + r_{\theta}\mathbf{e}_{\theta}^{\alpha} + r_z\mathbf{e}_z^{\alpha}. \quad (37)$$

In view of (35), the local fractional Poisson equation in the Cantor-type cylindrical coordinates is written as

$$\begin{aligned} &\frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\psi(R,\theta,z) + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial\theta^{2\alpha}}\psi(R,\theta,z) + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\psi(R,\theta,z) \\ &\quad + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}\psi(R,\theta,z) = -\frac{1}{\varepsilon_f}\rho(R,\theta,z), \end{aligned} \quad (38)$$

where both $\psi(R,\theta,z)$ and $\rho(R,\theta,z)$ are nondifferentiable functions.

From (35), the local fractional Laplace equation in the Cantor-type cylindrical coordinates is

$$\begin{aligned} &\frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\psi(R,\theta,z) + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial\theta^{2\alpha}}\psi(R,\theta,z) \\ &\quad + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\psi(R,\theta,z) + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}\psi(R,\theta,z) = 0, \end{aligned} \quad (39)$$

where the quantity $\psi(R,\theta,z)$ is a nondifferentiable function.

We now consider the Cantor-type circle coordinates given by [14]

$$\begin{aligned} x^{\alpha} &= R^{\alpha}\cos_{\alpha}\theta^{\alpha}, \\ y^{\alpha} &= R^{\alpha}\sin_{\alpha}\theta^{\alpha} \end{aligned} \quad (40)$$

with $R \in (0, +\infty)$, $\theta \in (0, 2\pi]$, and $x^{2\alpha} + y^{2\alpha} = R^{2\alpha}$.

Making use of (37), we obtain

$$\nabla^{\alpha}\phi(R,\theta) = \mathbf{e}_R^{\alpha}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\phi + \mathbf{e}_{\theta}^{\alpha}\frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial\theta^{\alpha}}\phi, \quad (41)$$

$$\nabla^{2\alpha}\phi(R,\theta) = \frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\phi + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial\theta^{2\alpha}}\phi + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\phi, \quad (42)$$

where [14]

$$\begin{aligned} \mathbf{e}_R^{\alpha} &= \cos_{\alpha}\theta^{\alpha}\mathbf{e}_1^{\alpha} + \sin_{\alpha}\theta^{\alpha}\mathbf{e}_2^{\alpha}, \\ \mathbf{e}_{\theta}^{\alpha} &= -\sin_{\alpha}\theta^{\alpha}\mathbf{e}_1^{\alpha} + \cos_{\alpha}\theta^{\alpha}\mathbf{e}_2^{\alpha}, \end{aligned} \quad (43)$$

and the local fractional vector is suggested by [14]

$$\mathbf{r} = R^{\alpha}\cos_{\alpha}\theta^{\alpha}\mathbf{e}_1^{\alpha} + R^{\alpha}\sin_{\alpha}\theta^{\alpha}\mathbf{e}_2^{\alpha} = r_R\mathbf{e}_R^{\alpha} + r_{\theta}\mathbf{e}_{\theta}^{\alpha}. \quad (44)$$

From (28) and (42) the local fractional Poisson equation in fractal domain with two variables can be written as

$$\begin{aligned} &\frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\psi(R,\theta) + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial\theta^{2\alpha}}\psi(R,\theta) + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\psi(R,\theta) \\ &\quad = -\frac{1}{\varepsilon_f}\rho(R,\theta), \end{aligned} \quad (45)$$

where both $\psi(R,\theta)$ and $\rho(R,\theta)$ are nondifferentiable functions.

From (29) and (42) the local fractional Laplace equation in fractal domain with two variables can be written as

$$\frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\psi(R,\theta) + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial\theta^{2\alpha}}\psi(R,\theta) + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}}{\partial R^{\alpha}}\psi(R,\theta) = 0, \quad (46)$$

where the quantity $\psi(R,\theta)$ is a nondifferentiable function.

5. Conclusions

In this work we derived the local fractional Poisson and Laplace equations arising in electrostatics in fractal domain from local fractional vector calculus. The local fractional Poisson and Laplace equations in the Cantor-type cylindrical coordinates were also discussed. The nondifferentiable solution for local fractional Laplace equation was also given.

Conflict of Interests

The authors declare that they have no conflict of interests regarding this paper.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (no. 41371345 and no. 41201335).

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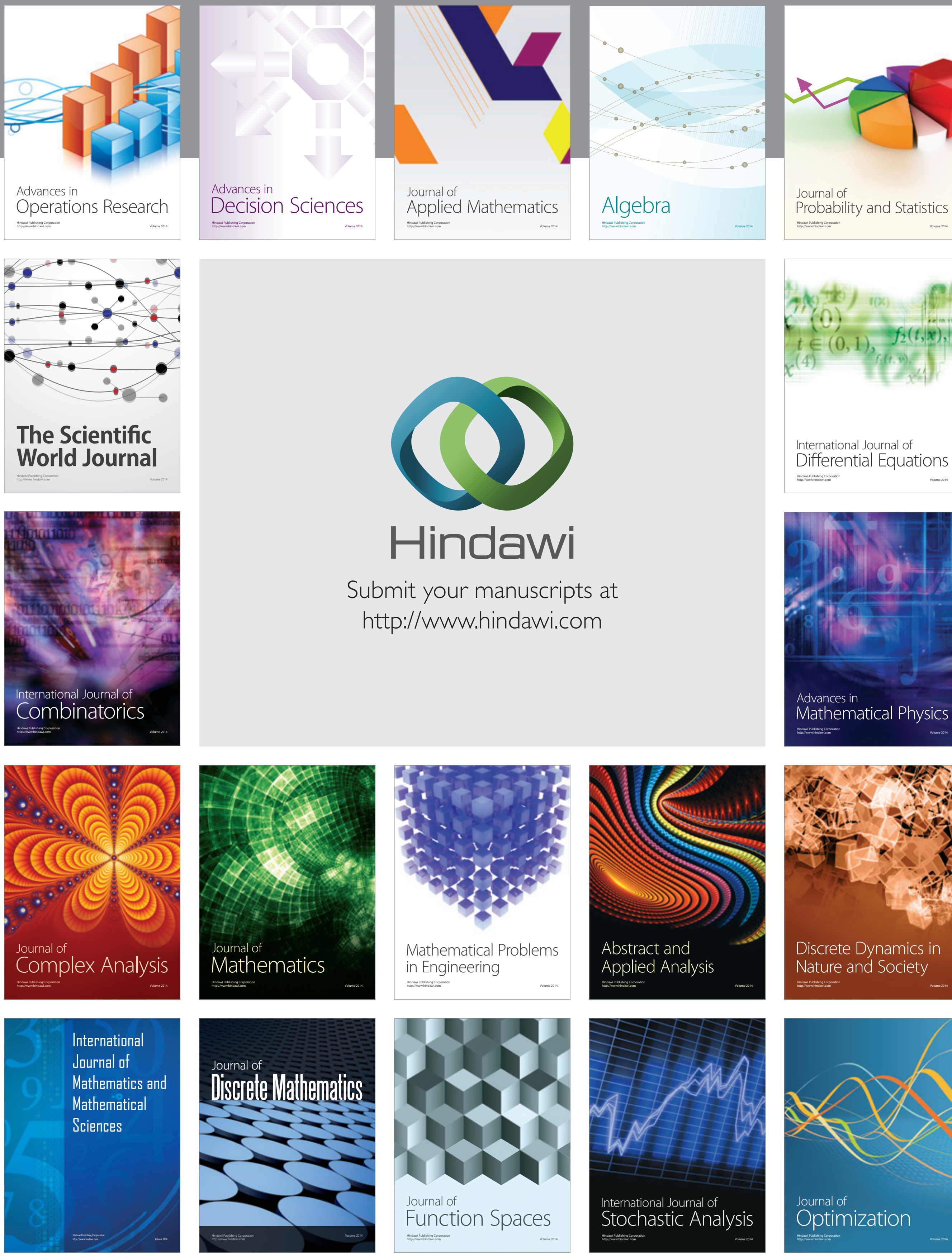

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