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Application of the method of transition boundary to the half-planes with mixed boundary conditions

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ABSTRACT

The scattering problems of waves by soft-hard and hard-soft half-planes are investigated by the method of transition boundary. The functional values at the transition boundaries are found to be identical to the ones for the soft and hard half-screens. It is shown that the factorization process should be applied by also taking into account the boundary conditions besides the principle of reciprocity. The exact diffracted field expressions are obtained for both of the half-planes.

1. Introduction

The method of transition boundary (MTB) was recently introduced by Umul [1]. The method proposes five steps for the solution of a diffraction problem. It was shown in [1] that MTB was leading to the exact solutions of scattering scenarios for planar objects. The technique is based on five steps and the diffracted field is obtained as a result of the factorization of a function, the values of which are known at the transition boundaries. With this feature, MTB is the counterpart of the Wiener-Hopf [2,3] and double integral solution [4] methods in the real domain. Thus there is no need for the Fourier transform of integral equations into the complex plane. In [1], the diffraction problems of waves by a soft (total field is equal to zero on the surface), hard (normal derivative of the total field is equal to zero on the surface) and impedance half-planes with an impedance sheet junction. In these problems, the total field was satisfying only a single boundary condition on the planar surfaces of the scatterers.

The aim of this paper is to apply MTB to the problems with mixed boundary conditions. Such problems consist of soft-hard and hard-soft half-planes. The exact solutions of these problems can be found in the literature [5–9]. The difference in the procedure of factorization will be the consideration of the boundary conditions besides the principle of reciprocity, since the functional values at the transition boundaries are the same with the ones for the soft and hard half-planes.

Time factor of $\exp(j\omega t)$ is suppressed throughout the paper. ω is the angular frequency.

2. Diffraction by a soft-hard half-plane

A half-plane, located at $x > 0$, $y = 0$ and $z \in (-\infty, \infty)$, is taken into account. The plane wave

$$u_i = u_0 e^{jk(x\cos\varphi_0 + y\sin\varphi_0)} \tag{1}$$

is illuminating the surface. u_0 is the complex amplitude and k wave-number. φ_0 shows the angle of incidence. The boundary conditions of

$$u_T|_{\varphi=0} = 0 \tag{2}$$

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and

$$\left.\frac{\partial u_T}{\partial \varphi}\right|_{\varphi=0} = 0 \tag{3}$$

are satisfied on the surface of the half-plane. u_T is the total field. The cylindrical coordinates are given by (ρ,φ,z). The procedure of MTB will be applied in order to find the diffracted fields. The high-frequency asymptotic expression of the diffracted wave can be written as

$$u_d = u_0 \frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}} \frac{f(\varphi, \varphi_0)}{\cos\varphi + \cos\varphi_0} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{4}$$

where f is a function to be determined.

According to MTB, first of all the initial field must be found. This field component is the one which exists in space when the geometry that causes diffraction is excluded. In this case, the initial field is directly equal to the incident wave. The second step is the construction of the total geometric optics (GO) field, which can be represented by

$$u_{TGO} = u_i U(-\xi_-) - u_r U(-\xi_+) \tag{5}$$

for the problem under consideration. u_r is the reflected wave and has the expression of

$$u_r = u_0 e^{jk(x\cos\varphi_0 - y\sin\varphi_0)} \tag{6}$$

and $\xi_\pm$ can be defined by

$$\xi_\pm = -\sqrt{2k\rho}\cos\frac{\varphi \pm \varphi_0}{2}, \tag{7}$$

which is the detour parameter. The third step is the evaluation of the scattered GO wave by subtracting the initial field from the total GO wave. Thus the scattered GO field becomes

$$u_{SGO} = -u_r U(-\xi_+) - u_i U(\xi_-) \tag{8}$$

for our problem.

The fourth step is the determination of the values of $f(\varphi,\varphi_0)$ at the transition boundaries by using a relation between the diffracted and GO fields [1]. The relations of

$$f(\pi - \varphi_0, \varphi_0) = \sin\varphi_0 \tag{9}$$

and

$$f(\pi + \varphi_0, \varphi_0) = \sin\varphi_0 \tag{10}$$

at the reflection and shadow boundaries respectively. It is interesting to note that Eqs. (9) and (10) are the same ones, obtained for a soft half-plane [1].

The final step is the determination of the function $f(\varphi,\varphi_0)$ by a factorization, which also takes into account reciprocity and boundary conditions. Eq. (9) can be written as

$$f(\pi - \varphi_0, \varphi_0) = \sin\varphi_0 + 1 - 1, \tag{11}$$

which leads to the expression of

$$f(\pi - \varphi_0, \varphi_0) = \left(\cos\frac{\varphi_0}{2} + \sin\frac{\varphi_0}{2}\right)^2 - 1. \tag{12}$$

Eq. (13) can be arranged as

$$f(\pi - \varphi_0, \varphi_0) = \left(\cos\frac{\varphi_0}{2} + \sin\frac{\varphi_0}{2} - 1\right)\left(\cos\frac{\varphi_0}{2} + \sin\frac{\varphi_0}{2} + 1\right) \tag{13}$$

that leads to the relation of

$$f(\pi - \varphi_0, \varphi_0) = 2\left(\frac{1}{2}\sin\frac{\varphi_0}{2} - \frac{1}{2} + \frac{1}{2}\cos\frac{\varphi_0}{2}\right)\left(\cos\frac{\varphi_0}{2} + \sin\frac{\varphi_0}{2} + 1\right) \tag{14}$$

by arranging the terms, in the first bracket. Eq. (14) becomes

$$f(\pi - \varphi_0, \varphi_0) = 2\left(\sin\frac{\varphi_0}{4}\cos\frac{\varphi_0}{4} - \sin^2\frac{\varphi_0}{4}\right)\left(\cos\frac{\varphi_0}{2} + \sin\frac{\varphi_0}{2} + 1\right) \tag{15}$$

after further trigonometric manipulations. Eq. (15) can be rewritten as

$$f(\pi - \varphi_0, \varphi_0) = 2\left(\cos\frac{\varphi_0}{4} - \sin\frac{\varphi_0}{4}\right)\sin\frac{\varphi_0}{4}\left(\cos\frac{\varphi_0}{2} + \sin\frac{\varphi_0}{2} + 1\right), \tag{16}$$

which leads to the expression of

$$f(\pi-\varphi_0,\varphi_0)=2\sqrt{2}\sin\frac{\pi-\varphi_0}{4}\sin\frac{\varphi_0}{4}\left(1+\cos\frac{\pi-\varphi_0}{2}+\cos\frac{\varphi_0}{2}\right). \tag{17}$$

Eq. (10) can be arranged as

$$f(\pi+\varphi_0,\varphi_0)=1-1+\sin\varphi_0 \tag{18}$$

that yields

$$f(\pi+\varphi_0,\varphi_0)=1-\left(\cos\frac{\varphi_0}{2}-\sin\frac{\varphi_0}{2}\right)^2. \tag{19}$$

Eq. (19) can be written by

$$f(\pi+\varphi_0,\varphi_0)=\left(1-\cos\frac{\varphi_0}{2}+\sin\frac{\varphi_0}{2}\right)\left(1-\sin\frac{\varphi_0}{2}+\cos\frac{\varphi_0}{2}\right), \tag{20}$$

which gives the expression of

$$f(\pi+\varphi_0,\varphi_0)=2\left(\sin^2\frac{\varphi_0}{4}+\sin\frac{\varphi_0}{4}\cos\frac{\varphi_0}{4}\right)\left(1-\sin\frac{\varphi_0}{2}+\cos\frac{\varphi_0}{2}\right). \tag{21}$$

Eq. (21) leads to

$$f(\pi+\varphi_0,\varphi_0)=2\left(\sin^2\frac{\varphi_0}{4}+2\cos\frac{\varphi_0}{4}\right)\sin\frac{\varphi_0}{4}\left(1-\sin\frac{\varphi_0}{2}+\cos\frac{\varphi_0}{2}\right) \tag{22}$$

that gives the equation of

$$f(\pi+\varphi_0,\varphi_0)=2\sqrt{2}\sin\frac{\pi+\varphi_0}{4}\sin\frac{\varphi_0}{4}\left(1+\cos\frac{\pi+\varphi_0}{2}+\cos\frac{\varphi_0}{2}\right) \tag{23}$$

after some trigonometric manipulations. It is apparent that Eqs. (17) and (23) are in the same form and the function $f(\varphi,\varphi_0)$ can directly be determined as

$$f(\varphi,\varphi_0)=2\sqrt{2}\sin\frac{\varphi}{4}\sin\frac{\varphi_0}{4}\left(1+\cos\frac{\varphi}{2}+\cos\frac{\varphi_0}{2}\right) \tag{24}$$

by putting φ instead of $\pi\pm\varphi_0$. The diffracted field becomes

$$u_d=u_0\frac{2e^{-j\frac{\pi}{4}}}{\sqrt{\pi}}\frac{\sin\frac{\varphi}{4}\sin\frac{\varphi_0}{4}}{\cos\varphi+\cos\varphi_0}\left(1+\cos\frac{\varphi}{2}+\cos\frac{\varphi_0}{2}\right)\frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{25}$$

when Eq. (24) is used in Eq. (4). Eq. (25) is the exact diffracted field expression by a soft-hard half-plane [5].

3. Diffraction by a hard-soft half-plane

The location of the half-plane and the incident wave are the same with the previous problem. The total GO field can be written as

$$u_{TGO}=u_iU(-\xi_-)+u_rU(-\xi_+) \tag{26}$$

in this case. Thus the scattered GO field is found to be

$$u_{SGO}=u_rU(-\xi_+)-u_iU(\xi_-) \tag{27}$$

by subtracting the incident wave from the total GO field. The diffracted field expression, in Eq. (4), is taken into consideration. The relation of

$$f(\pi-\varphi_0,\varphi_0)=-\sin\varphi_0 \tag{28}$$

and

$$f(\pi+\varphi_0,\varphi_0)=\sin\varphi_0 \tag{29}$$

at the transition boundaries. These are the same expressions, obtained for a hard half-plane [1]. Eq. (28) can be represented by

$$f(\pi-\varphi_0,\varphi_0)=1-1-\sin\varphi_0, \tag{30}$$

which leads to the expression of

$$f(\pi-\varphi_0,\varphi_0)=1-\left(\sin\frac{\varphi_0}{2}+\cos\frac{\varphi_0}{2}\right)^2 \tag{31}$$

when a trigonometric relation is used. Eq. (31) directly yields

$$f(\pi-\varphi_0,\varphi_0)=\left(1+\cos\frac{\varphi_0}{2}+\sin\frac{\varphi_0}{2}\right)\left(1-\cos\frac{\varphi_0}{2}-\sin\frac{\varphi_0}{2}\right) \tag{32}$$

that can also be expressed by

$$f(\pi-\varphi_0,\varphi_0)=2\left(\cos^2\frac{\varphi_0}{4}+\sin\frac{\varphi_0}{4}\cos\frac{\varphi_0}{4}\right)\left(1-\cos\frac{\varphi_0}{2}-\sin\frac{\varphi_0}{2}\right). \tag{33}$$

Eq. (33) can be written as

$$f(\pi-\varphi_0,\varphi_0)=2\left(\cos\frac{\varphi_0}{4}+\sin\frac{\varphi_0}{4}\right)\cos\frac{\varphi_0}{4}\left(1-\cos\frac{\varphi_0}{2}-\sin\frac{\varphi_0}{2}\right), \tag{34}$$

which gives

$$f(\pi-\varphi_0,\varphi_0)=2\sqrt{2}\cos\frac{\pi-\varphi_0}{4}\cos\frac{\varphi_0}{4}\left(1-\cos\frac{\pi-\varphi_0}{2}-\cos\frac{\varphi_0}{2}\right) \tag{35}$$

by using the sine and cosine of π/4. As a second step, Eq. (29) is taken into account. This equation can be rewritten as

$$f(\pi+\varphi_0,\varphi_0)=1-1+\sin\varphi_0, \tag{36}$$

which yields the equation of

$$f(\pi+\varphi_0,\varphi_0)=1-\left(\sin\frac{\varphi_0}{2}-\cos\frac{\varphi_0}{2}\right)^2. \tag{37}$$

Further arrangement of Eq. (38) gives

$$f(\pi+\varphi_0,\varphi_0)=\left(1+\cos\frac{\varphi_0}{2}-\sin\frac{\varphi_0}{2}\right)\left(1+\sin\frac{\varphi_0}{2}-\cos\frac{\varphi_0}{2}\right) \tag{38}$$

that can also be expressed as

$$f(\pi+\varphi_0,\varphi_0)=2\left(\cos^2\frac{\varphi_0}{4}-\sin\frac{\varphi_0}{4}\cos\frac{\varphi_0}{4}\right)\left(1+\sin\frac{\varphi_0}{2}-\cos\frac{\varphi_0}{2}\right) \tag{39}$$

by using some trigonometric properties. Eq. (39) can be rewritten by

$$f(\pi+\varphi_0,\varphi_0)=2\left(\cos\frac{\varphi_0}{4}-\sin\frac{\varphi_0}{4}\right)\cos\frac{\varphi_0}{4}\left(1+\sin\frac{\varphi_0}{2}-\cos\frac{\varphi_0}{2}\right), \tag{40}$$

which directly leads to the expression of

$$f(\pi+\varphi_0,\varphi_0)=2\sqrt{2}\cos\frac{\pi+\varphi_0}{4}\cos\frac{\varphi_0}{4}\left(1-\cos\frac{\pi+\varphi_0}{2}-\cos\frac{\varphi_0}{2}\right) \tag{41}$$

when the sine and cosine of π/2 is used. The function $f(\varphi,\varphi_0)$ will be determined as

$$f(\varphi,\varphi_0)=2\sqrt{2}\cos\frac{\varphi}{4}\cos\frac{\varphi_0}{4}\left(1-\cos\frac{\varphi}{2}-\cos\frac{\varphi_0}{2}\right) \tag{42}$$

if Eqs. (35) and (41) are compared. The diffracted field reads

$$u_d=u_0\frac{2e^{-j\frac{\pi}{4}}}{\sqrt{\pi}}\frac{\cos\frac{\varphi}{4}\cos\frac{\varphi_0}{4}}{\cos\varphi+\cos\varphi_0}\left(1-\cos\frac{\varphi}{2}-\cos\frac{\varphi_0}{2}\right)\frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{43}$$

when Eq. (42) is used in Eq. (4). It is apparent that Eq. (43) is the exact diffracted wave by a hard-soft half-plane [5].

4. Conclusions

In this paper, we obtained the solutions of the diffraction problem of waves by soft-hard and hard-soft half-planes with MTB. The main difficulty in planar surfaces with mixed boundary conditions is the functional values, obtained for $f(\varphi,\varphi_0)$ at the transition regions. This function takes the same values with the ones, obtained in the problems with single boundary conditions. Our analysis showed that the sinusoidal functions, obtained as a result of the factorization process, contain angle values, divided by four. It is shown in this study that MTB leads to the exact diffracted field expressions for mixed boundary value problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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