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Research article

Certain k-fractional calculus operators and image formulas of k-Struve function

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Abstract: In this article, the Saigo's k-fractional order integral and derivative operators involving k-hypergeometric function in the kernel are applied to the k-Struve function; outcome are expressed in the term of k-Wright function, which are used to present image formulas of integral transforms including beta transform. Also special cases related to fractional calculus operators and Struve functions are considered.

Keywords: extended Bessel-Maitland function; extended beta function; integral transform; Riemann-Liouville fractional calculus operators
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1. Introduction and preliminaries

In recent years, the fractional calculus has become a significant instrument for the modeling analysis and assumed a important role in different fields, for example, material science, science, mechanics, power, science, economy and control theory. In addition, research on fractional differential equations (ordinary or partial) and other analogous topics is very active and extensive around the world. One may refer to the recent papers [1–7] on the subject. In continuous, a series of research publications in respect to the generalized classical fractional calculus operators, Mubeen and Habibullah [8] were bring-out k-fractional order integral of the Riemann-Liouville version and its application, Dorrego [9] was introduced an alternative definition for the k-Riemann-Liouville fractional derivative.

In recent case, Gupta and Parihar [10] introduced the following Saigo k-fractional integral and derivative operators involving the k-hypergeometric function for $x \in \mathbb{R}^{+}$, $\omega, \xi, \gamma \in \mathbb{C}$ with $\Re(\omega) >$

$0, k > 0$, we have

$$\left(I_{0+,k}^{\omega,\xi,\gamma} f\right)(x) = \frac{x^{\frac{-\omega-\xi}{k}}}{k\Gamma_k(\omega)} \int_0^x (x-t)^{\frac{\omega}{k}-1}$$

$$\times {}_2F_{1,k}\left((\omega+\xi,k),(-\gamma,k);(\omega,k);\left(1-\tfrac{t}{x}\right)\right) f(t)\,dt, \tag{1.1}$$

$$\left(I_{-,k}^{\omega,\xi,\gamma} f\right)(x) = \frac{1}{k\Gamma_k(\omega)} \int_x^\infty (t-x)^{\frac{\omega}{k}-1} t^{\frac{-\omega-\xi}{k}}$$

$$\times {}_2F_{1,k}\left((\omega+\xi,k),(-\gamma,k);(\omega,k);\left(1-\frac{x}{t}\right)\right) f(t)\,dt, \tag{1.2}$$

$$\left(D_{0+,k}^{\omega,\xi,\gamma} f\right)(x) = \left(\frac{d}{dx}\right)^r \left(I_{0+,k}^{-\omega+r,-\xi-r,\omega+\gamma-r} f\right)(x),\ r = [\Re(\omega)+1], \tag{1.3}$$

$$= \left(\frac{d}{dx}\right)^r \frac{x^{\frac{\omega+\xi}{k}}}{k\Gamma_k(-\omega+r)} \int_0^x (x-t)^{\frac{\omega}{k}+r-1}$$

$$(\times)\ {}_2F_{1,k}\left((-\omega-\xi,k),(-\gamma-\omega+r,k);(-\omega+r,k);\left(1-\tfrac{t}{x}\right)\right) f(t)\,dt,$$

$$\left(D_{-,k}^{\omega,\xi,\gamma} f\right)(x) = \left(-\frac{d}{dx}\right)^r \left(I_{-,k}^{-\omega+r,-\xi-r,\omega+\gamma} f\right)(x),\ r = [\Re(\omega)+1], \tag{1.4}$$

$$= \left(-\frac{d}{dx}\right)^r \frac{1}{k\Gamma_k(-\omega+r)} \int_x^\infty (t-x)^{-\frac{\omega+r}{k}-1} t^{\frac{\omega+\xi}{k}}$$

$$(\times)\ {}_2F_{1,k}\left((-\omega-\xi,k),(-\gamma-\omega,k);(-\omega+r,k);\left(1-\tfrac{x}{t}\right)\right) f(t)\,dt,$$

where $[\Re(\omega)]$ is the integer part of $\Re(\omega)$ and ${}_2F_{1,k}((\omega,k),(\xi,k);(\gamma,k);x)$ defined by [11] for $x \in \mathbb{C}, |x| < 1, \Re(\gamma) > \Re(\xi) > 0$ as:

$${}_2F_{1,k}((\omega,k),(\xi,k);(\gamma,k)\ ;x) = \sum_{r=0}^\infty \frac{(\omega)_{r,k}(\xi)_{r,k}\, x^r}{(\gamma)_{r,k}\, r!}. \tag{1.5}$$

The k-hypergeometric function F_k is defined by Mubeen and Habibullah [11] in a power series form as:

$$F_k((\xi,k);(\gamma,k);x) = \sum_{r=0}^\infty \frac{\xi_{r,k} x^r}{(\gamma)_{r,k}\, r!},\ k \in \mathbb{R}^+, \xi, \gamma \in \mathbb{C}, \Re(\xi) > 0, \Re(\gamma) > 0. \tag{1.6}$$

and its integral representation can be obtained as follows:

$${}_1F_1((\xi,k);(\gamma,k);x) = \frac{\Gamma_k(\gamma)}{k\Gamma_k(\xi)\Gamma_k(\gamma-\xi)} \int_0^1 t^{\frac{\xi}{k}-1}(1-t)^{\frac{\gamma-\xi}{k}-1} e^{xt} dt, \tag{1.7}$$

Also, if $\Re(\gamma) > \Re(\xi) > 0, k > 0, m \geq 0, m \in \mathbb{R}^+$ and $|x| < 1$, then

$${}_{m+1}F_{m,k}\left[\begin{array}{c} (\omega,k),\left(\frac{\xi}{m},k\right),\left(\frac{\xi+k}{m},k\right),\cdots,\left(\frac{\gamma+(m-1)k}{m},k\right); \\ \left(\frac{\gamma}{m},k\right),\ \left(\frac{\gamma+k}{m},k\right),\cdots,\left(\frac{\gamma+(m-1)k}{m},k\right); \end{array} x\right]$$

$$= \frac{\Gamma_k(\gamma)}{k\Gamma_k(\xi)\Gamma_k(\gamma-\xi)} \int_0^1 t^{\frac{\xi}{k}-1}(1-t)^{\frac{\gamma-\xi}{k}-1}(1-kxt)^{-\frac{\omega}{k}} dt. \tag{1.8}$$

and if $\Re(\gamma) > \Re(\xi) > 0$ and $|x| < 1$, then

$${}_2F_{1,k}((\omega,k),(\xi,k);(\gamma,k);x) = \frac{\Gamma_k(\gamma)}{k\Gamma_k(\xi)\Gamma_k(\gamma-\xi)} \int_0^1 t^{\frac{\xi}{k}-1}(1-t)^{\frac{\gamma-\xi}{k}-1}(1-kxt)^{-\frac{\omega}{k}} dt. \tag{1.9}$$

Remark 1.1. *For $k = 1$, Eqs. (1.1) to (1.4) reduces in to Saigo's fractional order integral and derivative operators stated in [12].*

Now, we recollect few notable formulas for the fractional integral and derivative operators (1.1), (1.2), (1.3) and (1.4) as in the leading Lemma (see [10]).

Lemma 1.2. *Let $\omega,\xi,\gamma,\vartheta \in \mathbb{C}$ and $\Re(\omega) > 0$, $k \in \mathbb{R}^+(0,\infty)$ such that $\Re(\vartheta) > \max[0, \Re(\xi-\gamma)]$, then*

$$\left(I^{\omega,\xi,\gamma}_{0+,k} t^{\frac{\vartheta}{k}-1}\right)(x) = \sum_{r=0}^{\infty} k^r \frac{\Gamma_k(\vartheta)\Gamma_k(\vartheta-\xi+\gamma)}{\Gamma_k(\vartheta-\xi)\Gamma_k(\vartheta+\omega+\gamma)} x^{\frac{\vartheta-\xi}{k}-1}. \tag{1.10}$$

Lemma 1.3. *Let $\omega,\xi,\gamma,\vartheta \in \mathbb{C}$ and $\Re(\omega) > 0$, $k \in \mathbb{R}^+(0,\infty)$ such that $\Re(\vartheta) > \max[\Re(-\xi), \Re(-\gamma)]$, then*

$$\left(I^{\omega,\xi,\gamma}_{-,k} t^{-\frac{\vartheta}{k}}\right)(x) = \sum_{r=0}^{\infty} k^r \frac{\Gamma_k(\vartheta+\xi)\Gamma_k(\vartheta+\gamma)}{\Gamma_k(\vartheta)\Gamma_k(\vartheta+\omega+\xi+\gamma)} x^{\frac{-\vartheta-\xi}{k}}. \tag{1.11}$$

Lemma 1.4. *Let $\omega,\xi,\gamma,\vartheta \in \mathbb{C}$, $r = (\Re(\omega)) + 1, k \in \mathbb{R}^+(0,\infty)$ such that $Re(\vartheta) > \max[0, \Re(-\omega-\xi-\gamma)]$, then*

$$\left(D^{\omega,\xi,\gamma}_{0+,k} t^{\frac{\vartheta}{k}-1}\right)(x) = \sum_{r=0}^{\infty} \frac{\Gamma_k(\vartheta)\Gamma_k(\vartheta+\xi+\gamma+\omega)}{\Gamma_k(\vartheta+\gamma)\Gamma_k(\vartheta+\xi+r-rk)} x^{\frac{\vartheta+\xi+r}{k}-r-1}. \tag{1.12}$$

Lemma 1.5. *Let $\omega,\xi,\gamma,\vartheta \in \mathbb{C}$, $r = (\Re(\omega)) + 1$, $k \in \mathbb{R}^+(0,\infty)$ such that $Re(\vartheta) > \max[\Re(-\omega-\gamma), \Re(\xi-rk+r)]$, then*

$$\left(D^{\omega,\xi,\gamma}_{-,k} t^{-\frac{\vartheta}{k}}\right)(x) = \sum_{r=0}^{\infty} \frac{\Gamma_k(\vartheta-\xi-r+rk)\Gamma_k(\vartheta+\omega+\gamma)}{\Gamma_k(\vartheta)\Gamma_k(\vartheta-\xi+\gamma)} x^{\frac{-\vartheta+\xi+r}{k}-r}. \tag{1.13}$$

***k*-Struve function:** The generalized k-Struve function defined by Nisar et al. [13] as:

$$S^k_{v,c}(z) = \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^{2r+\frac{v}{k}+1}, \tag{1.14}$$

where $k \in \mathbb{R}^+$; $v > -1$ and $c \in \mathbb{R}$ and $\Gamma_k(z)$ is the k-gamma function defined in Díaz and Pariguan [14] as:

$$\Gamma_k(z) = \int_0^{\infty} t^{z-1} e^{-\frac{t^k}{k}} dt,\ z \in \mathbb{C}. \tag{1.15}$$

By inspection the following relation holds:

$$\Gamma_k(z+k) = z\,\Gamma_k(z), \tag{1.16}$$

and

$$\Gamma_k(z) = k^{(z/k)-1}\Gamma\left(\frac{z}{k}\right). \tag{1.17}$$

If $k \to 1$ and $c = 1$, reduces to yield the well-known Struve function of order v defined by Baricz [15] as

$$H_v(z) = \sum_{r=0}^{\infty} \frac{(-1)^r}{\Gamma\left(r+v+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^{2r+v+1}. \tag{1.18}$$

For further detail about Struve function and its properties (see [16–21]). Also Díaz et al. [22, 23] introduced the k-gamma function, k-beta function and Pochhammer k-symbols, Mubeen and Rehman [24] have studied extension of k-gamma and Pochhammer k-symbol, Mubeen and Habibullah [11] introduced k-fractional integration with its application and an integral representation of k-hypergeometric functions ${}_{m+1}F_{m,k}$ within Pochhammer k-symbols, k-gamma and k-beta functions.

***k*-Wright function:** Gehlot and Prajapati [25] introduced the generalized k-Wright function ${}_p\Psi_q^k(z)$ defined for $k \in \mathbb{R}^+; z, a_i, b_j \in \mathbb{C}, A_i, B_j \in \mathbb{R}\left(A_i, B_j \neq 0\right)$ where $i = 1, 2, \cdots, p$; $j = 1, 2, \cdots, q$ and $(a_i + A_i r), \left(b_j + B_j r\right) \in \mathbb{C}\backslash k\mathbb{Z}^-$

$${}_p\Psi_q^k(z) = {}_p\Psi_q^k\left[\begin{array}{c} (a_i, A_i)_{1,p} \\ \left(b_j, B_j\right)_{1,q} \end{array}\middle| z\right] = \sum_{r=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(a_i + A_i r)}{\prod_{j=1}^{q} \Gamma_k\left(b_j + B_j r\right)} \frac{z^r}{r!}, \tag{1.19}$$

satisfies the following condition

$$\sum_{j=1}^{q} \frac{B_j}{k} - \sum_{i=1}^{p} \frac{A_i}{k} > -1. \tag{1.20}$$

2. Saigo *k*-fractional integration in term of *k*-Wright function

Here, we present formulas for the Saigo k-fractional integrals (1.1) and (1.2) associated with the generalized k-Struve function (1.14), which are verbalized in terms of the k-Wright function in (1.19).

Theorem 2.1. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, $k \in \mathbb{R}^+$ *such that* $\Re(\omega) > 0$, $\Re(\vartheta) > \max[0, \Re(\xi - \gamma)]$. *If condition (1.20) satisfied and* $I_{0+,k}^{\omega,\xi,\gamma}$ *be the left sided operator of the generalized k-fractional integration involving k-hypergeometric function, thereupon the subsequent result true:*

$$\left(I_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1} S_{v,c}^{k}\left[t^{\frac{\varsigma}{k}}\right]\right)\right)(x) = \sqrt{k} x^{\frac{\vartheta+\varsigma-\xi}{k}+\frac{v\varsigma}{k^2}-1} (1/2)^{\frac{v}{k}+1}$$

$$\times {}_3\Psi_4^k\left[\begin{array}{c} \left(\vartheta+\varsigma+\frac{v\varsigma}{k}, 2\varsigma\right), \left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi+\gamma, 2\varsigma\right), (k,k) \\ \left(v+\frac{3k}{2}, k\right), \left(\frac{3k}{2}, k\right), \left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi, 2\varsigma\right), \left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\omega+\gamma, 2\varsigma\right) \end{array}\middle| \frac{-ckx^{\frac{2\varsigma}{k}}}{4}\right]. \tag{2.1}$$

Proof. By applying (1.14) on the left side of (2.1), we have

$$= I^{\omega,\xi,\gamma}_{0+,k}\left(t^{\frac{\vartheta}{k}-1}\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{t^{\frac{\varsigma}{k}}}{2}\right)^{2r+\frac{v}{k}+1}\right)(x),$$

$$= \sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1} I^{\omega,\xi,\gamma}_{0+,k}\left(t^{\frac{\vartheta+(2r+1)\varsigma+\frac{v\varsigma}{k}}{k}-1}\right), \tag{2.2}$$

which upon Lemma (1.2), yields

$$= x^{\frac{\vartheta+\varsigma-\xi+\frac{v\varsigma}{k}}{k}-1}\sum_{r=0}^{\infty}\frac{1}{\Gamma_k\left(v+\frac{3k}{2}+rk\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{\frac{v}{k}+1}$$

$$\times\frac{\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi+\gamma+2r\varsigma\right)}{\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\omega+\gamma+2r\varsigma\right)}\left(\frac{-ckx^{\frac{2\varsigma}{k}}}{4}\right)^r, \tag{2.3}$$

On using $\Gamma(r+1) = k^{-r}\Gamma_k(rk+k)$, we get

$$= x^{\frac{\vartheta+\varsigma-\xi+\frac{v\varsigma}{k}}{k}-1}\sum_{r=0}^{\infty}\frac{k^{\frac{1}{2}}\Gamma_k(rk+k)}{\Gamma_k\left(v+\frac{3k}{2}+rk\right)\Gamma_k\left(\frac{3k}{2}+rk\right)r!}\left(\frac{1}{2}\right)^{\frac{v}{k}+1}$$

$$\times\frac{\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi+\gamma+2r\varsigma\right)}{\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\omega+\gamma+2r\varsigma\right)}\left(\frac{-ckx^{\frac{2\varsigma}{k}}}{4}\right)^r, \tag{2.4}$$

Using the definition of (1.19) in the right-hand side of (2.4), we arrive at the result (2.1). □

Theorem 2.2. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma \in \mathbb{C}, \Re(v) > -1,\ k \in \mathbb{R}^+$ *such that* $\Re(\omega) > 0,\ \Re(\omega+\vartheta) > \max[-\Re(\xi), -\Re(\gamma)]$. *If condition (1.20) satisfied and* $I^{\omega,\xi,\gamma}_{0+,k}$ *be the right sided operator of the generalized k-fractional integration involving k-hypergeometric function, hence the leading result true:*

$$\left(I^{\omega,\xi,\gamma}_{-,k}\left(t^{\frac{-\omega-\vartheta}{k}}S^k_{v,c}\left[t^{-\frac{\varsigma}{k}}\right]\right)\right)(x) = x^{\frac{-\omega-\vartheta-\varsigma-\xi}{k}-\frac{v\varsigma}{k^2}-1}(1/2)^{\frac{v}{k}+1}\sqrt{k}$$

$$\times\, {}_3\Psi^k_4\left[\begin{matrix}\left(\omega+\vartheta+\varsigma+\xi+\frac{v\varsigma}{k},2\varsigma\right),\left(\omega+\vartheta+\varsigma+\frac{v\varsigma}{k}+\gamma,2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\omega+\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(\vartheta+\varsigma+2\omega+\xi+\gamma+\frac{v\varsigma}{k},2\varsigma\right)\end{matrix}\middle|\frac{-ckx^{\frac{-2\varsigma}{k}}}{4}\right]. \tag{2.5}$$

Proof. The proof is parallel to that of Theorem 2.1. Therefore, we omit the details. □

The results given in (2.1) and (2.5), being very general, can yield a huge number of specific cases by allotting some suited values to the involved parameters. Now, we demonstrate some Corollaries as below. If we take $k = 1$ and $c = 1$ in (2.1) and (2.5), we obtain the following two formulas in Corollaries 2.3 and 2.4.

Corollary 2.3. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,$ *such that* $\Re(\omega)>0$ *and* $\Re(\vartheta)>\max[0,\Re(\xi-\gamma)],$ *then the subsequent result true:*

$$\left(I_{0+}^{\omega,\xi,\gamma}\left(t^{\vartheta-1}H_v\left[t^{\varsigma}\right]\right)\right)(x)=x^{\vartheta+(v+1)\varsigma-\xi-1}\,(1/2)^{v+1}$$

$$\times\,{}_3\Psi_4\left[\begin{matrix}(\vartheta+(v+1)\varsigma,2\varsigma),(\vartheta+(v+1)\varsigma-\xi+\gamma,2\varsigma),(1,1)\\ \left(v+\frac{3}{2},1\right),\left(\frac{3}{2},1\right),(\vartheta+(v+1)\varsigma-\xi,2\varsigma),(\vartheta+(v+1)\varsigma+\omega+\gamma,2\varsigma)\end{matrix}\left|\frac{-x^{2\varsigma}}{4}\right.\right].\tag{2.6}$$

Corollary 2.4. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,$ *such that* $\Re(\omega)>0$ *and* $\Re(\omega+\vartheta)>\max[-\Re(\xi),-\Re(\gamma)],$ *then the following result true:*

$$\left(I_{-}^{\omega,\xi,\gamma}\left(t^{-\omega-\vartheta}H_v\left[t^{-\varsigma}\right]\right)\right)(x)=x^{-\omega-\vartheta-(v+1)\varsigma-\xi-1}\,(1/2)^{v+1}$$

$$\times\,{}_3\Psi_4\left[\begin{matrix}(\omega+\vartheta+(v+1)\varsigma+\xi,2\varsigma),(\omega+\vartheta+(v+1)\varsigma+\gamma,2\varsigma),(1,1)\\ \left(v+\frac{3}{2},1\right),\left(\frac{3}{2},1\right),(\omega+(v+1)\varsigma+\vartheta,2\varsigma),(\vartheta+(v+1)\varsigma+2\omega+\xi+\gamma,2\varsigma)\end{matrix}\left|\frac{-x^{-2\varsigma}}{4}\right.\right].\tag{2.7}$$

If we substitute $\xi=-\omega$ in Eqs. (2.1) and (2.5), Saigo k-fractional integral operators reduce to k-Riemann-Liouville integral operators as follows:

Corollary 2.5. *Let* $\omega,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,$ $k\in\mathbb{R}^+$ *such that* $\Re(\omega)>0,$ *then the pursuing result true:*

$$\left(I_{0+,k}^{\omega,}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[t^{\frac{\varsigma}{k}}\right]\right)\right)(x)=\sqrt{k}\,x^{\frac{\vartheta+\varsigma+\omega}{k}+\frac{v\varsigma}{k^2}-1}\,(1/2)^{\frac{v}{k}+1}$$

$$\times\,{}_2\Psi_3^k\left[\begin{matrix}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\omega,2\varsigma\right)\end{matrix}\left|\frac{-ckx^{\frac{2\varsigma}{k}}}{4}\right.\right].\tag{2.8}$$

Corollary 2.6. *Let* $\omega,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,$ $k\in\mathbb{R}^+$ *such that* $\Re(\omega)>0,$ *then the following result true:*

$$\left(I_{-,k}^{\omega}\left(t^{\frac{-\omega-\vartheta}{k}}S_{v,c}^{k}\left[t^{-\frac{\varsigma}{k}}\right]\right)\right)(x)=x^{\frac{-\vartheta-\varsigma}{k}-\frac{v\varsigma}{k^2}-1}\,(1/2)^{\frac{v}{k}+1}\,\sqrt{k}$$

$$\times\,{}_2\Psi_3^k\left[\begin{matrix}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\omega+\gamma+\frac{v\varsigma}{k},2\varsigma\right)\end{matrix}\left|\frac{-ckx^{\frac{-2\varsigma}{k}}}{4}\right.\right].\tag{2.9}$$

3. Saigo k-fractional differentiation in term of k-Wright function

At this point, we present formulas for the Saigo k-fractional derivative (1.1) and (1.2) associated with the generalized k -Struve function (1.14), which are suggested in terms of the k-Wright function in (1.19).

Theorem 3.1. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,$ $r=(\Re(\omega))+1,$ $k\in\mathbb{R}^+$ *be such that* $\Re(\omega)>0,$ $\Re(\vartheta)>\max[0,\Re(-\omega-\xi-\gamma)],$ *If condition (1.20) is satisfied and* $D_{0+,k}^{\omega,\xi,\gamma}$ *be the left sided operator of the generalized k-fractional differentiation involving k-Gauss hypergeometric function, and so succeeding result true:*

$$\left(D_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[t^{\frac{\varsigma}{k}}\right]\right)\right)(x)=x^{\frac{\vartheta+\xi+\varsigma}{k}+\frac{v\varsigma}{k^2}-1}\,(1/2)^{\frac{v}{k}+1}\,\sqrt{k}$$

$$\times {}_3\Psi_4^k\left[\begin{matrix}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\xi+\gamma+\omega,\ 2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\gamma,2\varsigma\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\xi,2\varsigma-k+1\right)\end{matrix}\middle|\frac{-cx^{\frac{2\varsigma+1}{k}-1}}{4}\right]. \tag{3.1}$$

Proof. By applying Eq. (1.14) in the left-side of (3.1), we get

$$= D_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1}\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{t^{\frac{\varsigma}{k}}}{2}\right)^{2r+\frac{v}{k}+1}\right)(x),$$

$$= \sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1} D_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta+(2r+1)\varsigma+\frac{v\varsigma}{k}}{k}-1}\right), \tag{3.2}$$

Using Lemma (2.5), in the above equation can be written as

$$= x^{\frac{\vartheta+\xi+\varsigma}{k}+\frac{v\varsigma}{k^2}-1}\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}$$

$$\times\sum_{r=0}^{\infty}\frac{\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\xi+\gamma+\omega+2r\varsigma\right)}{\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\gamma+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\xi+2r\varsigma+r-rk\right)}x^{\frac{2r\varsigma+r}{k}-r}, \tag{3.3}$$

On using $\Gamma(r+1)=k^{-r}\Gamma_k(rk+k)$, we get

$$= x^{\frac{\vartheta+\xi+\varsigma}{k}+\frac{v\varsigma}{k^2}-1}\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)}\frac{\Gamma_k(rk+k)}{\Gamma_k(rk+\frac{3k}{2})}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}\sqrt{k}$$

$$\times\sum_{r=0}^{\infty}\frac{\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\xi+\gamma+\omega+2r\varsigma\right)}{\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\gamma+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\xi+2r\varsigma+r-rk\right)}x^{\frac{2r\varsigma+r}{k}-r}, \tag{3.4}$$

Using the definition of (1.19) in the right-hand side of (3.4), we arrive at the result (3.1). □

Theorem 3.2. *Let $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1$, and $k\in\mathbb{R}^+$ be such that $\Re(\omega)>0$, $\Re(\vartheta)>\max\left[\Re(-\omega-\gamma),\Re(\xi-rk+r)\right]$, where $(r=[\Re(\omega+1)])$ and $D_{-,k}^{\omega,\xi,\gamma}$ be the left sided operator of the generalized k-fractional differentiation then the succeeding formula preserves true:*

$$\left(D_{-,k}^{\omega,\xi,\gamma}\left(t^{\frac{\omega-\vartheta}{k}}w_{v,c}^{k}\left[at^{-\frac{\varsigma}{k}}\right]\right)\right)(x)=x^{\frac{\omega-\vartheta-\varsigma+\xi-\frac{v\varsigma}{k}}{k}-1}(1/2)^{\frac{v}{k}+1}\sqrt{k}$$

$$\times {}_3\Psi_4^k\left[\begin{matrix}\left(\vartheta+\varsigma-\omega-\xi+\frac{v\varsigma}{k},2\varsigma+k-1\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\gamma,\ 2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta-\omega+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(\vartheta+\varsigma-\omega-\xi+\gamma+\frac{v\varsigma}{k},2\varsigma\right)\end{matrix}\middle|\frac{-cx^{\frac{-2\varsigma+1}{k}-1}}{4}\right]. \tag{3.5}$$

Proof. The proof is similar of Theorem 3.1. Therefore, we omit the details. □

If we take $k=1$, $c=1$ in (3.1) and (3.5), we obtain the following two formulas as:

Corollary 3.3. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1,$ *such that* $\Re(\omega) > 0,$ *and* $\Re(\vartheta) > \max[0, \Re(-\omega - \xi - \gamma)]$ *, then the following result true:*

$$\left(D_{0+}^{\omega,\xi,\gamma}\left(t^{\vartheta-1}H_v\left[t^{\varsigma}\right]\right)\right)(x) = x^{\vartheta+\xi+(v+1)\varsigma-1}(1/2)^{v+1}$$

$$\times {}_3\Psi_4\left[\begin{array}{c}(\vartheta+(v+1)\varsigma, 2\varsigma), (\vartheta+(v+1)\varsigma+\xi+\gamma+\omega, 2\varsigma), (1,1)\\ \left(v+\frac{3}{2}, 1\right), \left(\frac{3}{2}, 1\right), (\vartheta+(v+1)\varsigma+\gamma, 2\varsigma), (\vartheta+(v+1)\varsigma+\xi, 2\varsigma), (\vartheta+(v+1)\varsigma+\xi, 2\varsigma)\end{array}\left|\frac{-x^{2\varsigma}}{4}\right.\right]. \tag{3.6}$$

Corollary 3.4. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$ *such that* $\Re(\omega) > 0,$ $\Re(\vartheta) > \max[\Re(-\omega-\gamma), \Re(\xi - rk + r)]$ *, then the following result true:*

$$\left(D_{-}^{\omega,\xi,\gamma}\left(t^{\omega-\vartheta}H_v\left[at^{-\varsigma}\right]\right)\right)(x) = x^{\omega-\vartheta-(v+1)\varsigma+\xi-1}(1/2)^{v+1}$$

$$\times {}_3\Psi_4\left[\begin{array}{c}(\vartheta+(v+1)\varsigma-\omega-\xi, 2\varsigma), (\vartheta+(v+1)\varsigma+\gamma, 2\varsigma), (1,1)\\ \left(v+\frac{3}{2}, 1\right), \left(\frac{3}{2}, 1\right), (\vartheta+(v+1)\varsigma-\omega, 2\varsigma), (\vartheta+(v+1)\varsigma-\omega-\xi+\gamma, 2\varsigma)\end{array}\left|\frac{-x^{-2\varsigma}}{4}\right.\right]. \tag{3.7}$$

If we substitute $\xi = -\omega$ in Eqs. (3.5) and (3.8), Saigo k-fractional derivative operators reduce to k-Riemann-Liouville derivative operators as follows:

Corollary 3.5. *Let* $\omega, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1, k \in \mathbb{R}^+$ *be such that* $\Re(\omega) > 0,$ *then leading result true:*

$$\left(D_{0+,k}^{\omega}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[t^{\frac{\varsigma}{k}}\right]\right)\right)(x) = x^{\frac{\vartheta-\omega+\varsigma}{k}+\frac{v\varsigma}{k^2}-1}(1/2)^{\frac{v}{k}+1}\sqrt{k}$$

$$\times {}_2\Psi_3^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k}, 2\varsigma\right), (k,k)\\ \left(v+\frac{3k}{2}, k\right), \left(\frac{3k}{2}, k\right), \left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\omega, 2\varsigma-k+1\right)\end{array}\left|\frac{-cx^{\frac{2\varsigma+1}{k}-1}}{4}\right.\right]. \tag{3.8}$$

Corollary 3.6. *Let* $\omega, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1,$ *and* $k \in \mathbb{R}^+$*be such that* $\Re(\omega) > 0,$ *then the succeeding formula holds true:*

$$\left(D_{-,k}^{\omega}\left(t^{\frac{\omega-\vartheta}{k}}w_{v,c}^{k}\left[at^{-\frac{\varsigma}{k}}\right]\right)\right)(x) = x^{\frac{-\vartheta-\varsigma-\frac{v\varsigma}{k}}{k}-1}(1/2)^{\frac{v}{k}+1}\sqrt{k}$$

$$\times {}_2\Psi_3^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k}, 2\varsigma+k-1\right), (k,k)\\ \left(v+\frac{3k}{2}, k\right), \left(\frac{3k}{2}, k\right), \left(\vartheta-\omega+\varsigma+\frac{v\varsigma}{k}, 2\varsigma\right)\end{array}\left|\frac{-cx^{\frac{-2\varsigma+1}{k}-1}}{4}\right.\right]. \tag{3.9}$$

4. Image formulas associated with integral transforms

In this segment, we establish some theorems associated with the results obtained in previous sections pertaining to the integral transform.

***k*-Beta function:** The k-beta function [22] is defined as

$$B_k(g,h) = \frac{1}{k}\int_0^1 t^{\frac{g}{k}-1}(1-t)^{\frac{h}{k}-1}dt, \quad g > 0, h > 0. \tag{4.1}$$

and they have the following important identities

$$B_k(g,h) = \frac{1}{k}B\left(\frac{g}{k}, \frac{h}{k}\right) = \frac{\Gamma_k(g)\Gamma_k(h)}{\Gamma_k(g+h)}. \tag{4.2}$$

Now, we are defined k-beta function defined in the form:

$$B_k\left(f(t);g,h\right)=\frac{1}{k}\int_0^1 t^{\frac{g}{k}-1}\left(1-t\right)^{\frac{h}{k}-1}f(t)dt,\qquad g>0,h>0. \tag{4.3}$$

Theorem 4.1. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,\ k\in\mathbb{R}^{+}$ *such that* $\Re(\omega)>0$, $\Re(\vartheta)>\max\left[0,\Re(\xi-\gamma)\right]$, *then the leading fractional order integral holds true:*

$$B_k\left(\left(I_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[(zt)^{\frac{\varsigma}{k}}\right]\right)\right)(x)\,;\,g,\,h\right)=x^{\frac{\vartheta+\varsigma-\xi}{k}+\frac{v\varsigma}{k^2}-1}\left(1/2\right)^{\frac{v}{k}+1}\Gamma_k(h)\ \sqrt{k}$$

$$\times{}_4\Psi_5^k\left[\begin{matrix}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi+\gamma,2\varsigma\right),\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}-\xi,\,2\varsigma\right),\end{matrix}\right.$$

$$\left.\begin{matrix}\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\omega+\gamma,2\varsigma\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{matrix}\middle|\frac{-ckx^{\frac{2\varsigma}{k}}}{4}\right] \tag{4.4}$$

Proof. Let ℓ be the left-hand side of (4.4) and using (4.3), we have

$$\ell=\frac{1}{k}\int_0^1 z^{\frac{g}{k}-1}\left(1-z\right)^{\frac{h}{k}-1}\left(I_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[(zt)^{\frac{\varsigma}{k}}\right]\right)\right)(x)dz, \tag{4.5}$$

which, using (1.14) and changing the order of integration and summation, which is valid under the conditions of Theorem 2.1, yields

$$\ell=\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}\left(I_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta+\varsigma+2r\varsigma+v\varsigma/k}{k}-1}\right)\right)(x)$$

$$\times\frac{1}{k}\int_0^1 z^{\frac{g+\varsigma+2n\varsigma+v\varsigma/k}{k}-1}\left(1-z\right)^{\frac{h}{k}-1}dz, \tag{4.6}$$

which upon Lemma (1.2) and Eq. (4.2) in (4.6), we get

$$\ell=x^{\frac{\vartheta+\varsigma-\xi}{k}+\frac{v\varsigma}{k^2}-1}\sum_{r=0}^{\infty}\frac{(-ck)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}$$

$$\times\frac{\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)\ -\xi+\gamma+2r\varsigma\right)}{\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)-\xi+2r\varsigma\right)\Gamma_k\left(\vartheta+\varsigma+(v\varsigma/k)+\omega+\gamma+2r\varsigma\right)}$$

$$\times\frac{\Gamma_k\left(g+\varsigma+2r\varsigma+v\varsigma/k\right)\Gamma_k(h)}{\Gamma_k\left(g+h+\varsigma+2r\varsigma+v\varsigma/k\right)}\left(x\right)^{\frac{2r\varsigma}{k}}, \tag{4.7}$$

Using the definition of (1.19) in the right-hand side of (4.7), we arrive at the result (4.4). □

Theorem 4.2. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,\ k\in\mathbb{R}^{+}$ *such that* $\Re(\omega)>0$, $\Re(\omega+\vartheta)>\max\left[-\Re(\xi),-\Re(\gamma)\right]$, *then the following fractional integral holds true:*

$$B_k\left(\left(I_{-,k}^{\omega,\xi,\gamma}\left(t^{\frac{-\omega-\vartheta}{k}}S_{v,c}^{k}\left[(z/t)^{\frac{\varsigma}{k}}\right]\right)\right)(x)\,;\,g,h\right)=x^{\frac{-\omega-\vartheta-\varsigma-\xi}{k}-\frac{v\varsigma}{k^2}}\left(1/2\right)^{\frac{v}{k}+1}\ \sqrt{k}\Gamma_k(h)$$

$$\times {}_4\Psi_5^k\left[\begin{array}{c}\left(\omega+\vartheta+\varsigma+\gamma+\frac{v\varsigma}{k},2\varsigma\right),\left(\omega+\vartheta+\varsigma+\frac{v\varsigma}{k}+\xi,\,2\varsigma\right),\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\omega+\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\end{array}\right.$$

$$\left.\begin{array}{c}\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(\vartheta+\varsigma+2\omega+\gamma+\xi+\frac{v\varsigma}{k},2\varsigma\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{array}\middle|\frac{-ckx^{\frac{-2\varsigma}{k}}}{4}\right]. \tag{4.8}$$

Proof. The proof is similar of Theorem 4.1. Therefore we omit the details. □

Theorem 4.3. *Let* $\omega,\xi,\gamma,\vartheta,\varsigma\in\mathbb{C},\Re(v)>-1,\ k\in\mathbb{R}^+$ *be such that* $\Re(\omega)>0$, $\Re(\vartheta)>\max\left[0,\Re(-\omega-\xi-\gamma)\right]$, *then the following fractional derivative holds true:*

$$B_k\left(\left(D_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{\,k}\left[(zt)^{\frac{\varsigma}{k}}\right]\right)\right)(x);\ g,\ h\right)=x^{\frac{\vartheta+\varsigma+\xi}{k}+\frac{v\varsigma}{k^2}-1}\,(1/2)^{\frac{v}{k}+1}\ \sqrt{k}\Gamma_k(h)$$

$$\times {}_4\Psi_5^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\xi+\gamma+\omega,\,2\varsigma\right),\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\gamma,\,2\varsigma\right),\end{array}\right.$$

$$\left.\begin{array}{c}\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\xi,\,2\varsigma-k+1\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{array}\middle|\frac{-cx^{\frac{2\varsigma+1}{k}-1}}{4}\right]. \tag{4.9}$$

Proof. Let $\Im$ be the left-hand side of (4.9) and using the definition of Beta transform, we have

$$\Im=\frac{1}{k}\int_0^1 z^{\frac{g}{k}-1}(1-z)^{\frac{h}{k}-1}\left(D_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{\,k}\left[(zt)^{\frac{\varsigma}{k}}\right]\right)\right)(x)dz, \tag{4.10}$$

which, using (1.14) and changing the order of integration and summation, which is reasonable under the conditions of Theorem 3, yields

$$\Im=\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}\left(D_{0+,k}^{\omega,\xi,\gamma}\left(t^{\frac{\vartheta+\varsigma+2r\varsigma+v\varsigma/k}{k}-1}\right)\right)(x)$$

$$\times\frac{1}{k}\int_0^1 z^{\frac{g+\varsigma+2n\varsigma+v\varsigma/k}{k}-1}(1-z)^{\frac{h}{k}-1}\,dz, \tag{4.11}$$

which upon Lemma (1.4) and Eq. (4.2) in (4.11), we get

$$\Im=x^{\frac{\vartheta+\varsigma+\xi}{k}+\frac{v\varsigma}{k^2}-1}\sum_{r=0}^{\infty}\frac{(-ck)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}$$

$$\times\frac{\Gamma_k(\vartheta+\varsigma+(v\varsigma/k)+2r\varsigma)\Gamma_k(\vartheta+\varsigma+(v\varsigma/k)+\xi+\gamma+\omega+2r\varsigma)}{\Gamma_k(\vartheta+\varsigma+(v\varsigma/k)+\gamma+2r\varsigma)\Gamma_k(\vartheta+\varsigma+(v\varsigma/k)+\xi+2r\varsigma+r-rk)}$$

$$\times\frac{\Gamma_k(g+\varsigma+2r\varsigma+v\varsigma/k)\Gamma_k(h)}{\Gamma_k(g+h+\varsigma+2r\varsigma+v\varsigma/k)}(x)^{\frac{2n\varsigma+r}{k}-r}, \tag{4.12}$$

Using the definition of (1.14) in the right-hand side of (4.12), we arrive at the result (4.9). □

Theorem 4.4. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, *and* $k \in \mathbb{R}^+$ *be such that* $\Re(\omega) > 0$, $\Re(\vartheta) > \max[\Re(-\omega-\gamma), \Re(\xi - rk + r)]$, *then the following formula holds true:*

$$B_k\left(\left(D_{-,k}^{\omega,\xi,\gamma}\left(t^{\frac{\omega-\vartheta}{k}} S_{v,c}^{k}\left[(z/t)^{-\frac{\varsigma}{k}}\right]\right)\right)(x)\,;\, g, h\right) = x^{\frac{\omega-\vartheta-\varsigma-\xi}{k}-\frac{v\varsigma}{k^2}} \sqrt{k}\,(1/2)^{\frac{v}{k}+1}\,\Gamma_k(h)$$

$$\times {}_4\Psi_5^k\left[\begin{array}{c}\left(\vartheta+\varsigma-\omega+\frac{v\varsigma}{k}-\xi,\, 2\varsigma+k-1\right), \left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\gamma, 2\varsigma\right),\\ \left(v+\frac{3k}{2}, k\right), \left(\frac{3k}{2}, k\right), \left(\vartheta+\varsigma-\omega+\frac{v\varsigma}{k}, 2\varsigma\right),\end{array}\right.$$

$$\left.\begin{array}{c}\left(g+\varsigma+\frac{v\varsigma}{k}, 2\varsigma\right), (k,k)\\ \left(\vartheta+\varsigma-\omega+\frac{v\varsigma}{k}-\xi+\gamma, 2\varsigma\right), \left(g+h+\varsigma+\frac{v\varsigma}{k}, 2\varsigma\right)\end{array}\right| \left.\frac{-cx^{\frac{-2\varsigma+1}{k}-1}}{4}\right]. \tag{4.13}$$

Proof. The proof is parallel to that of Theorem 4.3. Therefore, we omit the details. □

Setting $k = 1$, $c = 1$ in (4.4), (4.7), (4.9) and (4.13), we obtain the following new formulas as:

Corollary 4.5. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, *such that* $\Re(\vartheta) > \max[0, \Re(\xi-\gamma)]$, $\Re(\omega) > 0$; *then*

$$B\left(\left(I_{0+}^{\omega,\xi,\gamma}\left(t^{\vartheta-1} H_v\left[(zt)^{\varsigma}\right]\right)\right)(x)\,;\, g,\, h\right) = x^{\vartheta-\xi+(v+1)\varsigma-1}\,(1/2)^{v+1}\,\Gamma(h)$$

$$\times {}_4\Psi_5\left[\begin{array}{c}(\vartheta+(v+1)\varsigma, 2\varsigma), (\vartheta+(v+1)\varsigma-\xi+\gamma, 2\varsigma),\\ \left(v+\frac{3}{2}, 1\right), \left(\frac{3}{2}, 1\right), (\vartheta+(v+1)\varsigma-\xi, 2\varsigma),\end{array}\right.$$

$$\left.\begin{array}{c}(g+(v+1)\varsigma, 2\varsigma), (1,1)\\ (\vartheta+(v+1)\varsigma+\omega+\gamma, 2\varsigma), (g+(v+1)\varsigma+h, 2\varsigma)\end{array}\right| \left.\frac{-x^{2\varsigma}}{4}\right]. \tag{4.14}$$

Corollary 4.6. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, *such that* $\Re(\omega+\vartheta) > \max[-\Re(\xi), -\Re(\gamma)]$, $\Re(\omega) > 0$; *then*

$$B\left(\left(I_{-,}^{\omega,\xi,\gamma}\left(t^{-\omega-\vartheta} H_v\left[(z/t)^{-\varsigma}\right]\right)\right)(x)\,;\, g, h\right) = x^{-\omega-\vartheta-\xi-(v+1)\varsigma}\,(1/2)^{v+1}\,\Gamma(h)$$

$$\times {}_4\Psi_5\left[\begin{array}{c}(\omega+\vartheta+(v+1)\varsigma+\gamma,\, 2\varsigma), (\omega+\vartheta+(v+1)\varsigma+\xi, 2\varsigma),\\ \left(v+\frac{3}{2}, 1\right), \left(\frac{3}{2}, 1\right), (\omega+(v+1)\varsigma+\vartheta, 2\varsigma),\end{array}\right.$$

$$\left.\begin{array}{c}(g+(v+1)\varsigma, 2\varsigma), (1,1)\\ (\vartheta+(v+1)\varsigma+2\omega+\gamma+\xi, 2\varsigma), (g++(v+1)\varsigma+h, 2\varsigma)\end{array}\right| \left.\frac{-x^{-2\varsigma}}{4}\right]. \tag{4.15}$$

Corollary 4.7. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, *be such that* $\Re(\vartheta) > \max[0, \Re(-\omega-\xi-\gamma)]$, $\Re(\omega) > 0$; *then*

$$B\left(\left(D_{0+}^{\omega,\xi,\gamma}\left(t^{\vartheta-1} H_v\left[(zt)^{\varsigma}\right]\right)\right)(x)\,;\, g,\, h\right) = x^{\vartheta+\xi+(v+1)\varsigma-1}\,(1/2)^{v+1}\,\Gamma(h)$$

$$\times {}_4\Psi_5\left[\begin{array}{c}(\vartheta+(v+1)\varsigma, 2\varsigma), (\vartheta+(v+1)\varsigma+\xi+\gamma+\omega,\, 2\varsigma),\\ \left(v+\frac{3}{2}, 1\right), \left(\frac{3}{2}, 1\right), (\vartheta+(v+1)\varsigma+\gamma, 2\varsigma),\end{array}\right.$$

$$\left.\begin{array}{c}(g+(v+1)\varsigma, 2\varsigma), (1,1)\\ (\vartheta+(v+1)\varsigma+\xi, 2\varsigma), (g+(v+1)\varsigma+h, 2\varsigma)\end{array}\right| \left.\frac{-x^{2\varsigma}}{4}\right]. \tag{4.16}$$

Corollary 4.8. *Let* $\omega, \xi, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, *be such that* $\Re(\omega) > 0$, $\Re(\vartheta) > \max[\Re(-\omega-\gamma), \Re(\xi)]$, *then the following formula holds true:*

$$B\left(\left(D_{-}^{\omega,\xi,\gamma}\left(t^{\omega-\vartheta}H_v\left[(z/t)^{-\varsigma}\right]\right)\right)(x);\, g,h\right) = x^{\omega-\vartheta-\varsigma-\xi-v\varsigma}\ (1/2)^{v+1}\,\Gamma(h)$$

$$\times {}_4\Psi_5\left[\begin{array}{c}(\vartheta+(v+1)\varsigma-\omega-\xi,\ 2\varsigma),(\vartheta+(v+1)\varsigma+\gamma,2\varsigma),\\ \left(v+\frac{3}{2},1\right),\left(\frac{3}{2},1\right),(\vartheta+(v+1)\varsigma-\omega,2\varsigma),\end{array}\right.$$

$$\left.\begin{array}{c}(g+(v+1)\varsigma,2\varsigma),(1,1)\\ (\vartheta+(v+1)\varsigma-\omega-\xi+\gamma,2\varsigma),(g+(v+1)\varsigma+h,2\varsigma)\end{array}\middle|\frac{-x^{-2\varsigma}}{4}\right]. \tag{4.17}$$

Similarly, If we put $\xi = -\omega$ in Eqs. (4.4), (4.7), (4.9) and (4.13), Saigo k-fractional calculus operators reduce to k-Riemann-Liouville calculus operators as follows:

Corollary 4.9. *Let* $\omega, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1, k \in \mathbb{R}^+$ *such that* $\Re(\omega) > 0$, *then*

$$B_k\left(\left(I_{0+,k}^{\omega}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[(zt)^{\frac{\varsigma}{k}}\right]\right)\right)(x);\ g,\ h\right) = x^{\frac{\vartheta+\varsigma+\omega}{k}+\frac{v\varsigma}{k^2}-1}\,(1/2)^{\frac{v}{k}+1}\,\Gamma_k(h)\ \sqrt{k}$$

$$\times\, {}_3\Psi_4^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\omega,2\varsigma\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{array}\middle|\frac{-ckx^{\frac{2\varsigma}{k}}}{4}\right]. \tag{4.18}$$

Corollary 4.10. *Let* $\omega, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1, k \in \mathbb{R}^+$ *such that* $\Re(\omega) > 0$, *then*

$$B_k\left(\left(I_{-,k}^{\omega}\left(t^{\frac{-\omega-\vartheta}{k}}S_{v,c}^{k}\left[(z/t)^{-\frac{\varsigma}{k}}\right]\right)\right)(x);g,h\right) = x^{\frac{-\vartheta-\varsigma}{k}-\frac{v\varsigma}{k^2}}\,(1/2)^{\frac{v}{k}+1}\ \sqrt{k}\Gamma_k(h)$$

$$\times\, {}_3\Psi_4^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\omega+\gamma+\frac{v\varsigma}{k},2\varsigma\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{array}\middle|\frac{-ckx^{\frac{-2\varsigma}{k}}}{4}\right]. \tag{4.19}$$

Corollary 4.11. *Let* $\omega, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1, k \in \mathbb{R}^+$ *be such that* $\Re(\omega) > 0$, *then*

$$B_k\left(\left(D_{0+,k}^{\omega}\left(t^{\frac{\vartheta}{k}-1}S_{v,c}^{k}\left[(zt)^{\frac{\varsigma}{k}}\right]\right)\right)(x);\ g,\ h\right) = x^{\frac{\vartheta+\varsigma-\omega}{k}+\frac{v\varsigma}{k^2}-1}\ (1/2)^{\frac{v}{k}+1}\ \sqrt{k}\Gamma_k(h)$$

$$\times\, {}_3\Psi_4^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma+\frac{v\varsigma}{k}+\xi,2\varsigma-k+1\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{array}\middle|\frac{-cx^{\frac{2\varsigma+1}{k}-1}}{4}\right]. \tag{4.20}$$

Corollary 4.12. *Let* $\omega, \gamma, \vartheta, \varsigma \in \mathbb{C}, \Re(v) > -1$, *and* $k \in \mathbb{R}^+$*be such that* $\Re(\omega) > 0$, *then*

$$B_k\left(\left(D_{-,k}^{\omega}\left(t^{\frac{\omega-\vartheta}{k}}S_{v,c}^{k}\left[(z/t)^{-\frac{\varsigma}{k}}\right]\right)\right)(x);\ g,h\right) = x^{\frac{2\omega-\vartheta-\varsigma}{k}-\frac{v\varsigma}{k^2}}\ \sqrt{k}\ (1/2)^{\frac{v}{k}+1}\,\Gamma_k(h)$$

$$\times\, {}_3\Psi_4^k\left[\begin{array}{c}\left(\vartheta+\varsigma+\frac{v\varsigma}{k},\ 2\varsigma+k-1\right),\left(g+\varsigma+\frac{v\varsigma}{k},2\varsigma\right),(k,k)\\ \left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right),\left(\vartheta+\varsigma-\omega+\frac{v\varsigma}{k},2\varsigma\right),\left(g+h+\varsigma+\frac{v\varsigma}{k},2\varsigma\right)\end{array}\middle|\frac{-cx^{\frac{-2\varsigma+1}{k}-1}}{4}\right]. \tag{4.21}$$

5. Conclusions

The generalized k-fractional calculus operators have advantage that it generalizes Saigo's fractional integral and derivative operators, therefore, many authors called this a general operator. So, we conclude this paper by emphasizing that many other interesting image formulas can be derived as the specific cases of our leading results Theorems 2.1, 2.2, 3.1 and 3.2, involving familiar k-fractional integral and derivative operators as above said. Some special cases of k-fractional calculus involving k-Struve function have been explored in the literature by a authors [26] with different arguments. Therefore, results existing in this article are easily regenerate in terms of a comparable type of novel interesting integrals with diverse arguments after various suitable parametric replacements.

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Conflict of interest

The authors declare no conflict of interest in this paper.

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