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On comparing and clustering the spectral densities of several almost cyclostationary processes

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Abstract In time series analysis, comparing spectral densities of several processes with almost periodic spectra is an interested problem. The contribution of this work is to give a technique to compare and to cluster the spectral densities of some independent almost periodically correlated (cyclostationary) processes. This approach is based on the limiting distribution for the periodogram and the discrete Fourier transform. The real world examples and simulation results indicate that the approach well acts.

1. Introduction

Comparing spectral densities of several processes is an important topic that has many applications in signal processing, engineering, physics and other fields. The researchers like to explore if stochastic mechanism of some observed time series are similar or not. Many references have considered the clustering, classification and comparison of two or several time series. For example, the Refs. [1–25] studied these subjects for stationary time series. But the stationarity assumption is not satisfied in many situations, specially for the periodically

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rhythmic processes. Cyclostationary and almost cyclostationary (CS and ACS) time series can be naturally applied in these situations. The ACS time series are non-stationary processes with almost periodic mean and auto-correlation functions. These processes included CS and stationary processes. In view of frequency domain, the spectral square of ACS processes is supported with the main diagonal and parallel lines to the main diagonal, $T_j(x) = x \pm \alpha_j, j = 1, 2, \cdots$, in spectral square $[0, 2\pi) \times [0, 2\pi)$. Different problems of ACS time series have been studied in previous researches [26–50]. Some statisticians considered the comparison of two CS time series [51–53]. Ref. [54] proposed an approach to detect the cycles in the movement in motion of a walking person using Fourier transform techniques and auto-correlation of the smoothed spatio-temporal curvature function. Ref. [55] reviewed and reported past researches that studied the time series data clustering in various fields. Ref. [56], using the log spectrum, introduced a new distance measure, called cepstral distance measure, to compare and cluster of some time series. Ref. [57] combined k-means clustering technique and discriminant analysis to introduce an iterative algorithm to classify the cyclic time series. Ref. [58] studied a new spectral clustering method, called transfer spectral clustering (TSC). Ref. [59] proposed an autoregressive tree-based ensemble approach, called multivariate autoregressive forest, to classify some multivariate processes.

In this research, we will apply the asymptotic distribution of the periodogaram and discrete Fourier transform of ACS processes to construct an approach to compare and classify several ACS processes. Section 2 is devoted to notations and preliminaries. The technique to compare and classify the ACS processes is presented in Section 3. The ability of the introduced technique is investigated by means of extensive Monte Carlo simulations, and real world problems, in Sections 4 and 5, respectively.

2. Notations and preliminaries

A second order time series $\{X_t : t \in Z\}$ is an ACS time series if for every $\tau \in$ Z, its mean function, $\mu(t) = E(X_t)$, and its auto-covariance function, $B(t, \tau) = cov(X_t, X_{t+\tau})$, are almost periodic functions at t [50,60].

As Ref. [50], assume that the following assumptions are satisfied.

(A1) $\{X_t : t \in Z\}$ is a real-valued ACS time series with mean of zero.

By *(A1)*, we have

$$B(t,\tau) \sum_{\omega \in W_t} a(\omega,\tau) e^{i\omega t},$$

where

$$a(\omega,\tau) = \lim_{n\to\infty} \left(\frac{1}{n} \sum_{j=1}^{n} B(j,\tau) e^{-i\omega t} \right),$$

and for fixed τ. Also as [60,61] indicated, the set $W_\tau = \{\omega \in [0, 2\pi) : a(\omega,\tau) \neq 0\}$ is countable.

(A2) The set $W = \bigcup_{\tau \in Z} W_\tau$, is finite and the spectral square of X_t is supported with the main diagonal and parallel lines to the main diagonal, $T_j(x) = x \pm \alpha_j, j = 1, 2, \cdots$, in spectral square $[0, 2\pi) \times [0, 2\pi)$.

Thus we have

$$B(t,\tau) = \sum_{\omega \in W} a(\omega,\tau) e^{i\omega t},$$

and the set $S = \bigcup_{\omega \in W} \left\{ (v,\gamma) \in [0, 2\pi)^2 : \gamma = v - \omega \right\}$ supports the spectral measure of X_t.

Moreover, the coefficients

$$a(\omega,\tau) = \int_0^{2\pi} e^{i\xi\tau} r_\omega(d\xi),$$

are the Fourier transforms of the measures $r_\omega\left(\hat{\mathrm{A}}\cdot\right)$.

(A3) r_0 is an absolute continuous measure with respect to the Lebesgue measure.

The Ref. [61] indicated by considering this assumption and $\sum_{\tau=-\infty}^{\infty} |a(\omega,\tau)| < \infty$, for any $\omega \in W$, there exist a spectral density function $f_\omega\left(\hat{\mathrm{A}}\cdot\right)$ such that

$$f_\omega(v) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} a(\omega,\tau) e^{-iv\tau}.$$

Therefore, we can represent an ACS time series, with finite number of cyclic frequencies, as

$$X_t = \int_0^{2\pi} e^{-itx} \zeta(dx), t \in \mathbb{Z},$$

where ζ is a random spectral measure on $[0, 2\pi)$ with the following property:

$$E\left(\zeta(d\theta)\, \bar{\zeta}\left(d\theta'\right) \right) = 0, \left(\theta, \theta'\right) \notin S.$$

The spectral distribution matrix and the spectral density matrix of ζ, are given by

$$\boldsymbol{F}(d\lambda) = \left[F_{k,j}(d\lambda) \right]_{j,k=1,\cdots,m},$$

and

$$\boldsymbol{f}(\lambda) = \frac{d\boldsymbol{F}}{d\lambda} = \left[f_{k,j}(\lambda) \right]_{j,k=1,\cdots,m},$$

respectively, where

$$F_{k,j}(d\lambda) = E\left(\zeta(d\lambda + \alpha_k)\, \bar{\zeta}(d\lambda + \alpha_j) \right), k, j = 1, \cdots, m,$$

and $f_{k,j}$ is the spectral density function of $F_{k,j}$.

Assume a sample $X_0, \cdots, X_{N-1}$, from ACS process $\{X_t : t \in Z\}$. The discrete Fourier transform (DFT) and periodogram of the observations $X_0, \cdots, X_{N-1}$, are respectively given by

$$d_X(\lambda) = N^{-1/2} \sum_{t=0}^{N-1} X_t e^{-it\lambda}, \lambda \in [0, 2\pi),$$

and

$$I_X(\lambda) = |d_X(\lambda)|^2, \lambda \in [0, 2\pi).$$

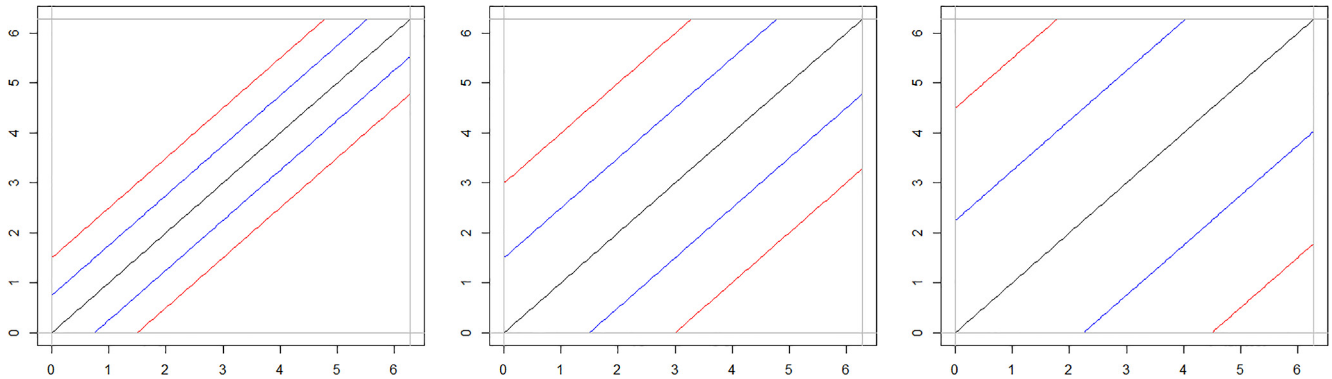

Fig. 1 The spectral square of the process, Left: $\omega_i = 0.5$, Middle: $\omega_i = 1$, and Right:$\omega_i = 2$.

The study the asymptotic properties of periodogram and DFT of ACS time series, see the Refs. [41–42,49,50].

3. Methodology

Suppose $\{X_t^{(1)}, t = 1, \cdots, n_1\}$, $\{X_t^{(2)}, t = 1, \cdots, n_2\}, \cdots$, $\{X_t^{(l)}, t = 1, \cdots, n_l\}$, are l independent ACS processes with m spectral cycles.

Commonly, the researchers explore to test the null hypothesis $H_0 : X_t^{(1)} \equiv X_t^{(2)} \equiv \cdots \equiv X_t^{(l)}$, that is equivalent to $H_0 : \boldsymbol{f}_1 = \boldsymbol{f}_2 = \cdots = \boldsymbol{f}_l$, such that $\boldsymbol{f}_1, \cdots, \text{and} \boldsymbol{f}_l$ are the spectral density matrices respectively corresponding to $X_t^{(1)}$, ..., and $X_t^{(l)}$. If the null hypothesis H_0 is not accepted then it can be concluded that at least two time series of the l time series have different rhythms, and if H_0 is accepted consequently the stochastic behaviours of all time series are similar.

The periodogram for ACS time series is introduced [50] by

$$\boldsymbol{I}_X^n(\lambda) = \boldsymbol{d}_X^n(\lambda)\boldsymbol{d}_X^{n*}(\lambda),$$

such that

$$\boldsymbol{d}_X^n(\lambda) = (d_X(T_1(\lambda)), d_X(T_2(\lambda)) \cdots, d_X(T_m(\lambda)))', \lambda \in [0, 2\pi),$$

Table 1 The values of $\hat{\alpha}$ and $\hat{\pi}$ for the introduced approach.

ω_1	ω_2	ω_3	(n_1, n_2, n_3) (100, 50, 75)	(150, 75, 100)	(200, 150, 100)	(500, 250, 300)
0.75	0.75	0.75	0.052	0.050	0.050	0.049
		1.5	0.736	0.845	0.909	0.976
		2.25	0.727	0.828	0.925	0.978
	1.5	0.75	0.706	0.837	0.907	0.999
		1.5	0.760	0.864	0.959	0.994
		2.25	0.766	0.866	0.955	0.987
	2.25	0.75	0.718	0.812	0.903	0.970
		1.5	0.758	0.889	0.969	0.985
		2.25	0.800	0.883	0.945	0.986
1.5	0.75	0.75	0.772	0.864	0.969	1.000
		1.5	0.727	0.842	0.939	0.972
		2.25	0.760	0.861	0.946	0.989
	1.5	0.75	0.741	0.842	0.917	0.996
		1.5	0.051	0.051	0.050	0.050
		2.25	0.725	0.845	0.906	0.970
	2.25	0.75	0.782	0.897	0.963	0.983
		1.5	0.717	0.835	0.919	0.993
		2.25	0.790	0.880	0.943	0.997
2.25	0.75	0.75	0.791	0.888	0.952	0.984
		1.5	0.777	0.857	0.963	0.999
		2	0.729	0.847	0.904	0.994
	1.5	0.75	0.779	0.870	0.965	0.989
		1.5	0.797	0.871	0.959	0.989
		2.25	0.718	0.835	0.933	0.978
	2.25	0.75	0.718	0.844	0.904	0.976
		1.5	0.724	0.844	0.933	0.973
		2.25	0.052	0.051	0.049	0.049

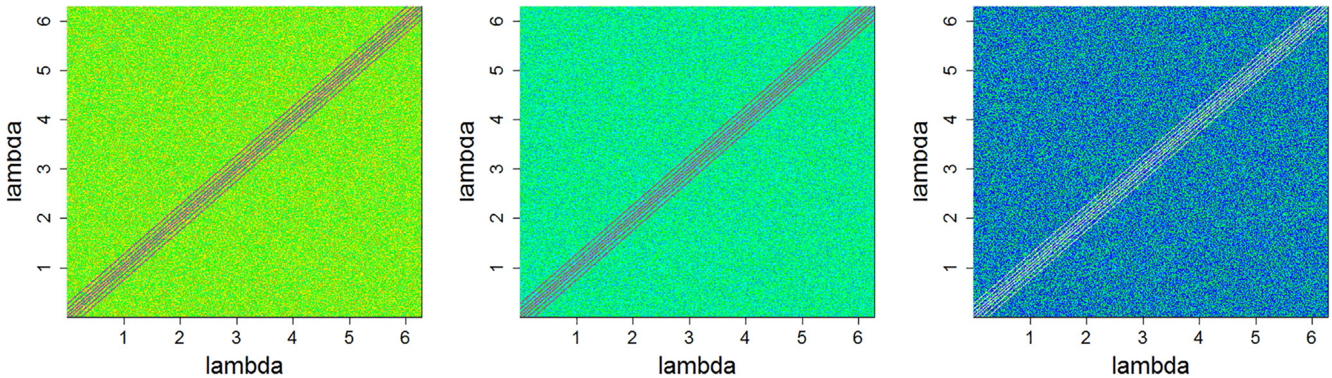

Fig. 2 Spectral frequency square (Left: Part 1, Middle: Part 2, Right: Part 3).

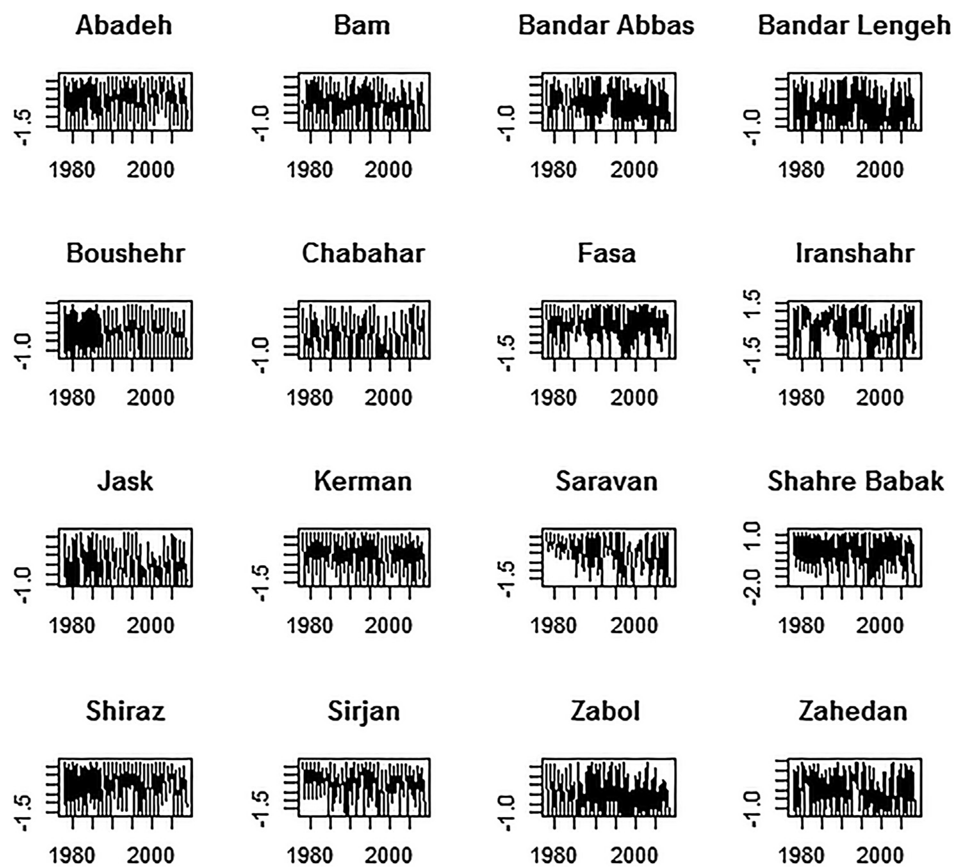

Fig. 3 Seasonal RDI time series of 16 Iranian synoptic stations (1980–2010).

Table 2 Testing the equality of different parts.

Test Statistic	P-Value
$\chi^{2*} = 4.193$	0.651

where $\boldsymbol{d}_X^{n*}(\lambda)$ is the complex conjugate transpose of $\boldsymbol{d}_X^n(\lambda)$.

Lemma 3.1. *Let* $\{X_t, t \in \mathbb{Z}\}$ *is an ACS time series with corresponding spectral density* $\boldsymbol{f}(\lambda), \lambda \in [0, 2\pi)$. *Assume the frequencies* $\lambda_1 < \cdots < \lambda_J \in 0, 2\pi)$. *Then*

(i) $\boldsymbol{f}(\lambda)$ can be asymptotically estimated by $\widehat{\boldsymbol{f}}(\lambda) := \frac{\boldsymbol{I}_X^m(\lambda)}{2\pi}$.

(ii) $\boldsymbol{d}_X^m(\lambda_j), j = 1, \cdots, J$, have the asymptotic and independent m-variate complex normal distributions, $N_m^c(0, 2\pi \boldsymbol{f}(\lambda_j))$.

(iii) $\boldsymbol{I}_X^m(\lambda_j), j = 1, \cdots, J$, have the asymptotic and independent complex Wishart distributions, $W_m^c(1, 2\pi \boldsymbol{f}(\lambda_j))$.

Proof: (*[50]*.). □

Let $Y_j^{(k)} = Re(\boldsymbol{d}_{X^{(k)}}^n(\lambda_j)), j = 1, \cdots, J, k = 1, 2, \cdots, l$, and $Z_j^{(k)} = Im(\boldsymbol{d}_{X^{(k)}}^n(\lambda_j)), j = 1, \cdots, J, k = 1, 2, \cdots, l$, such that $\boldsymbol{d}_{X^{(k)}}^m(\lambda_j)$, is the $\boldsymbol{d}_X^T(\lambda_j)$ corresponding to k^{th} time series. As a result of Lemma 3.1, it can be concluded that for $k = 1, 2, \cdots, l,$ the asymptotic distribution of $W_j^{(k)} = \left(Y_j^{(k)}, Z_j^{(k)}\right)'$ is $N_{2m}\left(0, \Sigma_j^{(k)}\right)$, such that $\Sigma_j^{(k)} = \begin{bmatrix} V_{Y_jY_j}^{(k)} & V_{Y_jZ_j}^{(k)} \\ V_{Z_jY_j}^{(k)} & V_{Z_jZ_j}^{(k)} \end{bmatrix}$, $V_{AB} = COV(A, B)$. Consequently, the asymptotic distribution of $U^{(k)} = \sum_{j=1}^{J} W_j^{(k)}$ is $N_{2m}\left(0, \Sigma^{(k)}\right)$, such that $\Sigma^{(k)} = \Sigma_1^{(k)} + \cdots + \Sigma_J^{(k)}$.

3.1. Test of hypothesis

As previous discussion, usually, the researchers want to test the null hypothesis $H_0 : \boldsymbol{f}_1 = \boldsymbol{f}_2 = \cdots = \boldsymbol{f}_l$, that is equivalent to $H_0 : \Sigma^{(1)} = \Sigma^{(2)} = \cdots = \Sigma^{(l)}$. It can be concluded that the

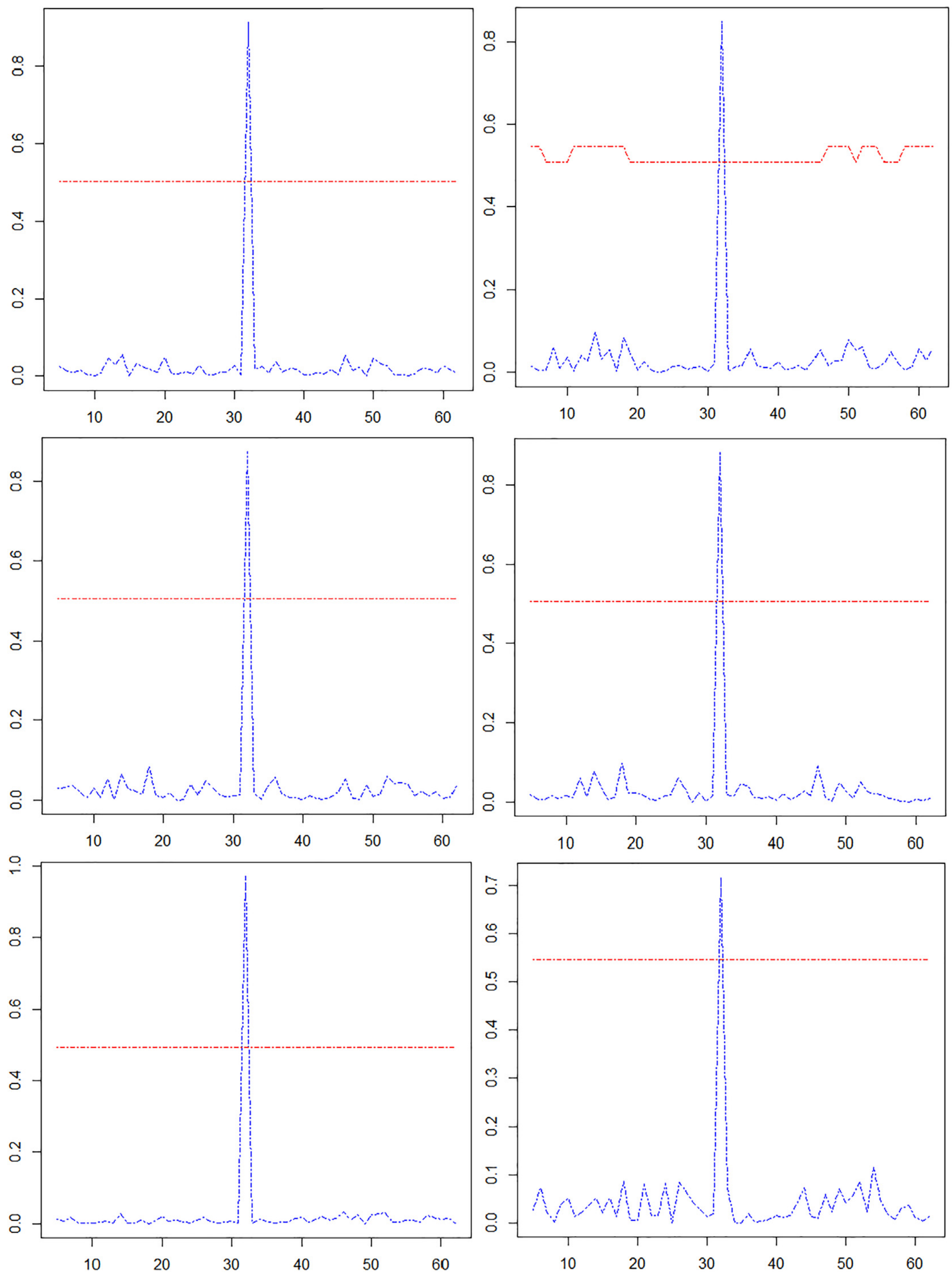

Fig. 4 Periodogram for seasonal RDI time series of Iranian synoptic stations (1980–2010). Left: Top: Abadeh, Middle: Bandar Abbas, Bottom: Boushehr. Right: Top: Bam, Middle: Bandar Lengeh, Bottom: Chabahar. Left: Top: Fasa, Middle: Jask, Bottom: Saravan. Right: Top: Iranshahr, Middle: Kerman, Bottom: Shahre Babak. Left: Top: Shiraz, Bottom: Zabol. Right: Top: Sirjan, Bottom: Zahedan.

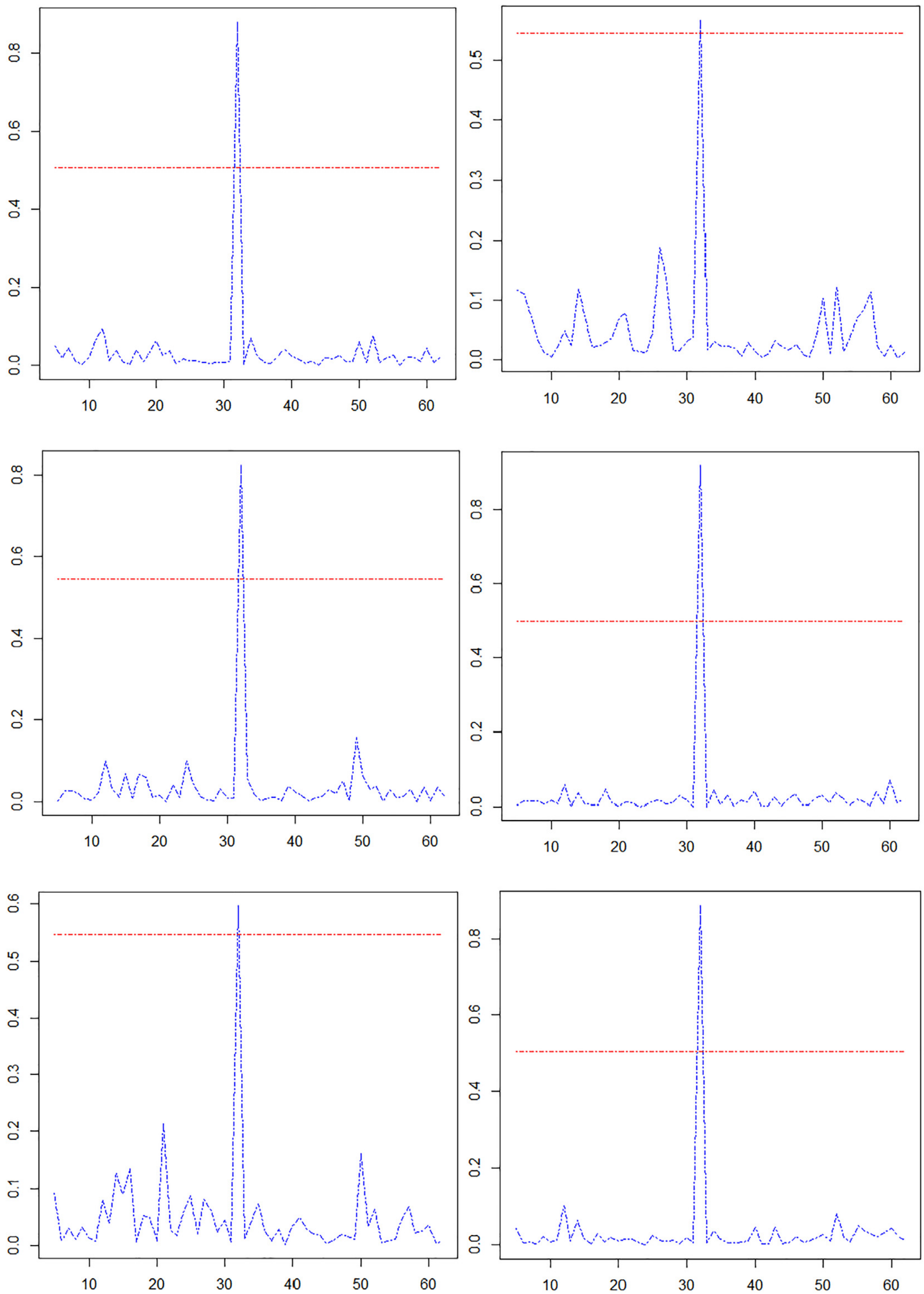

Fig. 4 (*continued*)

asymptotic distribution of $U = U^{(1)} + U^{(2)} + \cdots + U^{(l)}$ is $N_{2m}(0, \Sigma)$, such that $\Sigma = \Sigma^{(1)} + \Sigma^{(2)} + \cdots + \Sigma^{(l)}$.

Therefore the asymptotic distribution of statistic

$$\chi^2 = (U)^{'}(\Sigma)^{-1}(U),$$

is chi-square with *2 m* degrees of freedom, $\chi^2(2m)$.

The asymptotic distribution of statistic χ^2 can be applied to establish test of hypothesis about the null hypothesis H_0. As can be seen, the statistic χ^2 is related to the unknown parameter Σ.

Remark 3.1. To produce a sample of size N_i for the discrete Fourier transform of i^{th} process, different bootstrap approaches can be applied. In this research we used the

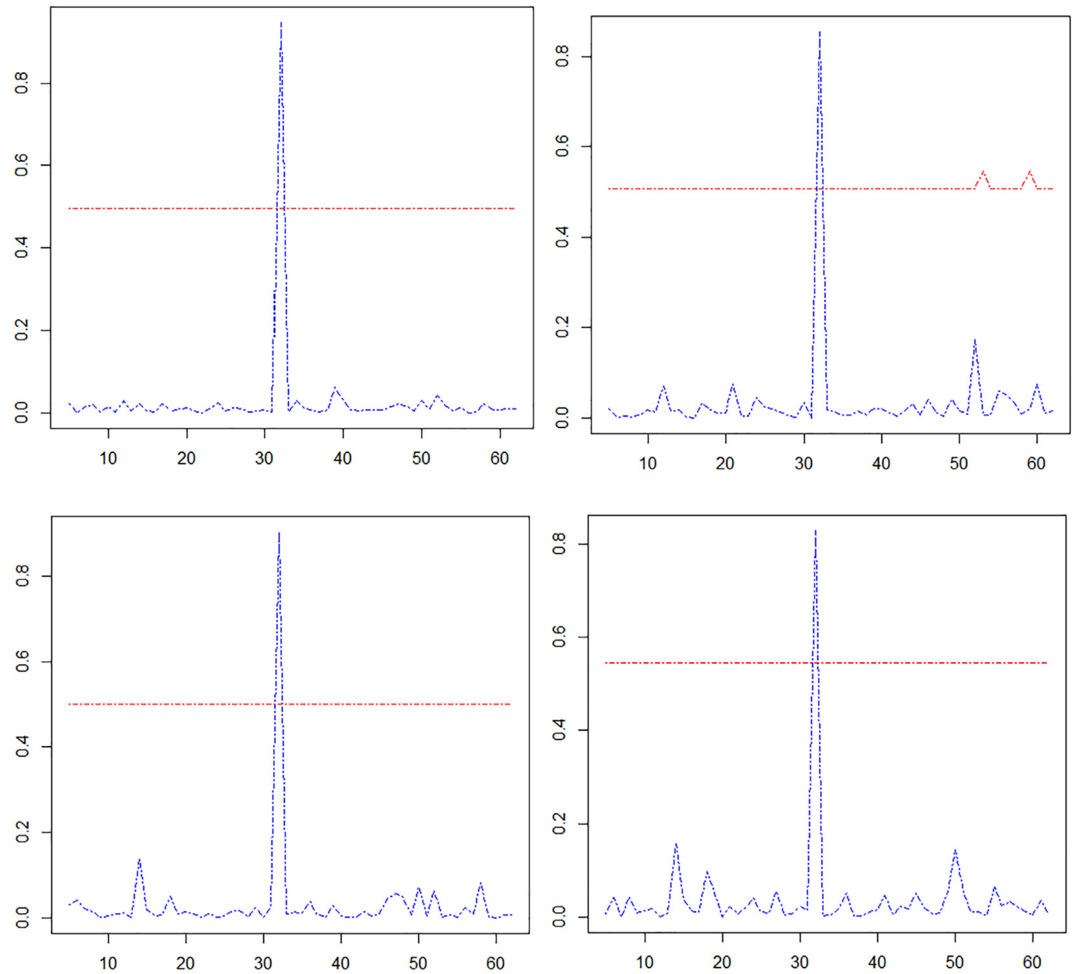

Fig. 4 (*continued*)

moving block bootstrap (MBB) (more details are given in [62]).

Let $\boldsymbol{S}=\frac{(N_1-1)\mathbf{S}^{(1)}+(N_2-1)\mathbf{S}^{(2)}+\cdots+(N_l-1)\mathbf{S}^{(l)}}{N_1+N_2+\cdots+N_l-l}$, as the pooled covariance matrix of $\mathbf{S}^{(1)},\cdots,\mathbf{S}^{(l)}$, such that $\mathbf{S}^{(k)}=\mathbf{S}_1^{(k)}+\cdots+\mathbf{S}_J^{(k)},\mathbf{S}_j^{(k)}=\begin{bmatrix}\widehat{\boldsymbol{V}}_{Y_jY_j}^{(k)} & \widehat{\boldsymbol{V}}_{Y_jZ_j}^{(k)}\\ \widehat{\boldsymbol{V}}_{Z_jY_j}^{(k)} & \widehat{\boldsymbol{V}}_{Z_jZ_j}^{(k)}\end{bmatrix}$, and $\widehat{V}_{AB}=\widehat{COV}(A,B)$. If the null hypothesis $H_0:\Sigma^{(1)}=\Sigma^{(2)}=\cdots=\Sigma^{(l)}$, be true, then the covariance matrix Σ can be consistently estimated by $\boldsymbol{S}$, and consequently by using the Weak Law of Large Numbers, the asymptotic distribution of the test statistic

$$\chi^{2*}=(U)'(\mathbf{S})^{-1}(U),$$

is $\chi^2(2m)$. Therefore, for a given size α, H_0 is rejected if $\chi^{2*}>\chi^2_\alpha(2m)$.

3.2. Clustering and classifying the processes

In Section 3.1, the rejection of H_0 is equivalent to that at least two time series significantly differ from each other. Multiple comparisons should be applied to compare each pair of observed ACS time series.

Therefore, to classify and cluster the ACS processes, the following can be applied:

Step 1: First, we test the null hypothesis $H_0:\boldsymbol{f}_1=\boldsymbol{f}_2=\cdots=\boldsymbol{f}_l$.

If H_0 is not rejected, consequently the stochastic behaviours of all processes are similar and we have only one cluster contained all of time series $X_t^{(1)}$, …, and $X_t^{(l)}$, and the clustering procedure is stopped.

If the null hypothesis H_0 is rejected then it can be concluded that at least two time series of the l time series have different rhythms, and we will continue next steps.

Step 2: For $,i,j=1,\cdots,l$, we test the null hypothesis $H_0:\boldsymbol{f}_i=\boldsymbol{f}_j$ and compute the P-values.
Step 3: If the comparison test of the i^{th} and j^{th} processes is not significant (P-value greater than 0.05), then there is no significant difference between two processes and they fall into one cluster. If else (P-value < 0.05), these two processes significantly differ from each other and consequently they fall into separate clusters.
Step 4: The APC time series are clustered based on all comparisons.

4. Simulation study

To analyze the accuracy of proposed method, we generated $(n_1, n_2, n_3) = \{(100, 50, 75), (150, 75, 100), (200, 150, 100), (500, 250, 300)\}$ observations from the ACS processes

$$X_t^{(i)} = (1 + \cos(\omega_i t))\, Y_t^{(i)}, \omega \in (0, \infty), \quad i = 1, 2, 3,$$

where

$$Y_t^{(i)} = Z_t^{(i)} + 0.5 Z_{t-1}^{(i)},$$

and $Z_t^{(i)}, i = 1, 2, 3$, are independent sequences of IIDN(0,1).

The spectral supports of $X_t^{(i)}$ is as following lines:

$$T_1(x) = x, T_2(x) = x + \omega_i, T_3(x) = x - \omega_i, T_4(x) = x - 2\omega_i, T_5(x) = x + 2\omega_i.$$

Fig. 1 indicates the spectral plane$[0, 2\pi)^2$, for $\omega_i = \{0.75, 1.5, 2.25\}$.

First, we estimated the Type I error probability ($\widehat{\alpha}$) and power ($\widehat{\pi}$) based on 1000 replications and 1000 iterations, as the percent of iterations that the assumption H_0 is rejected. Then we graph Q–Q plot for the test statistic χ^{2*} based on the computed values of the simulation runs.

Table1 report the values of $\widehat{\alpha}$ (in rows: 1th, 14th and 27th) and $\widehat{\pi}$ (other rows). The results indicates that the values of $\widehat{\alpha}$ is very close to the considered size ($\alpha = 0.05$), especially when (n_1, n_2, n_3) grows. Also the power studies show that the proposed method excellently discriminate H_0 from H_1.

5. Real data

This section is devoted to illustrate the ability of introduced in practical cases.

Example 1. The dataset includes the first difference of centered moving average filter 2 × 12 moving average (MA) applied for logarithm of industrial production index (IPI) in Poland (2005 = 100%) since January 1995 untile December 2009, [43].

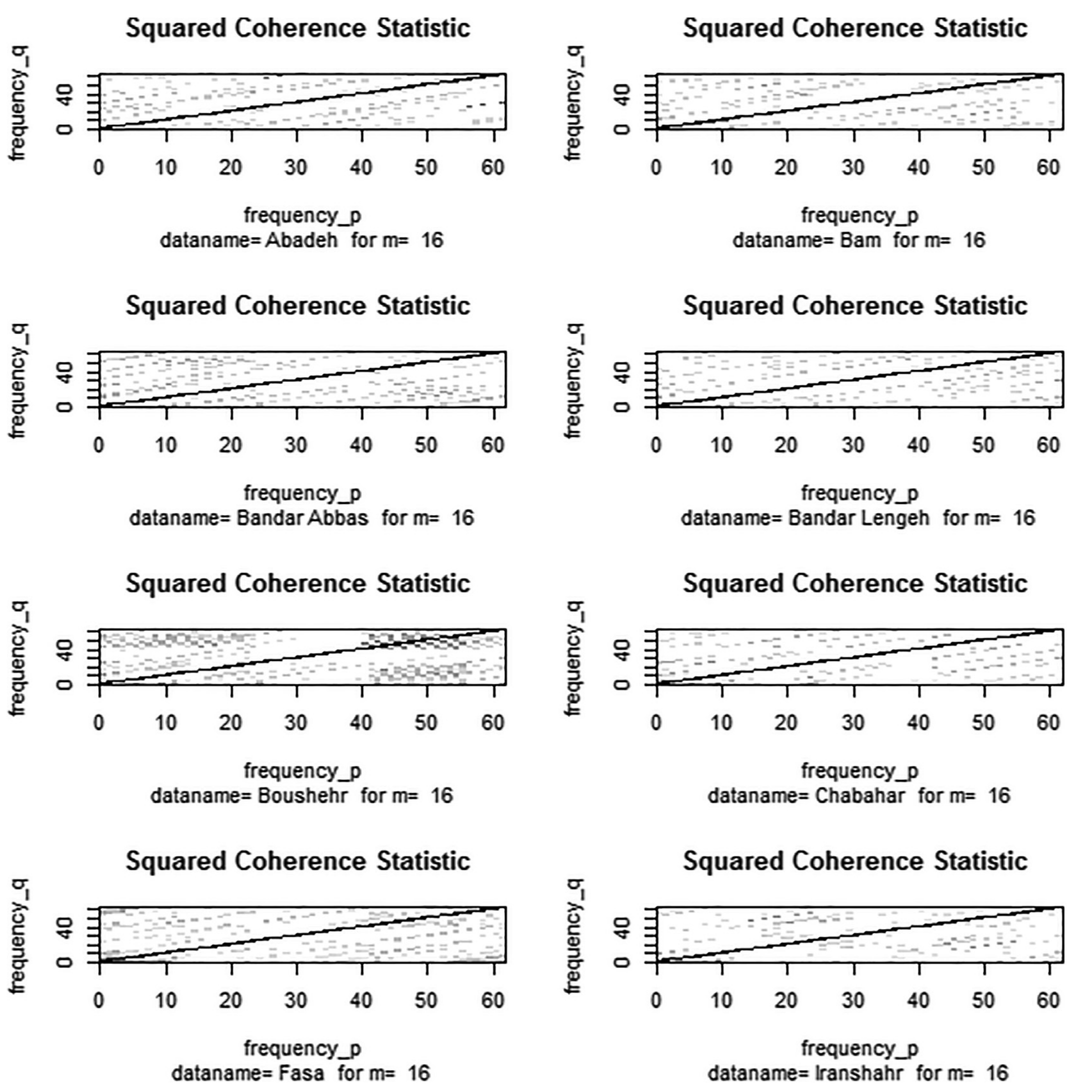

Fig. 5 Spectral frequency square for seasonal RDI time series of 16 Iranian synoptic stations (1980–2010).

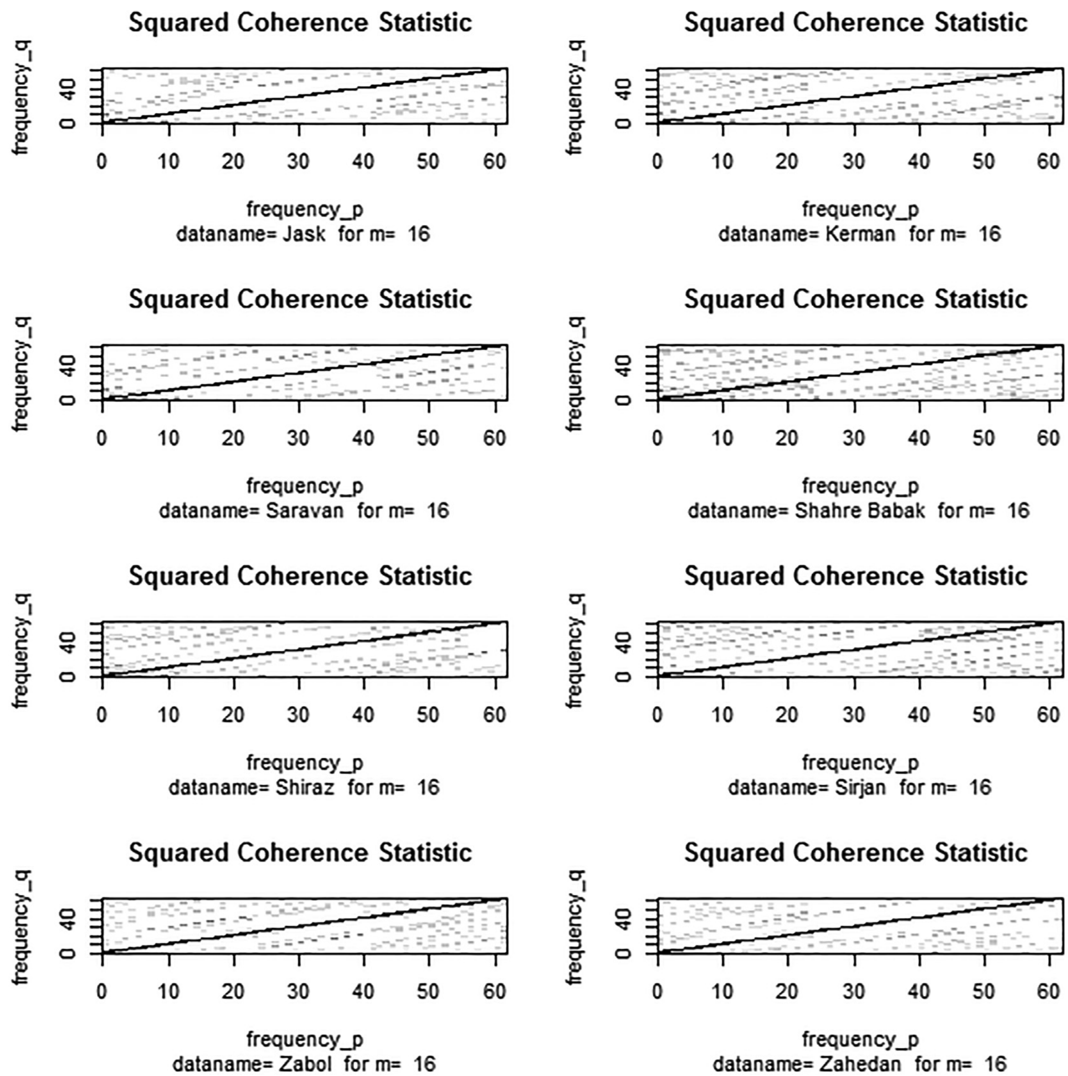

Fig. 5 (*continued*)

Table 3 Testing the equality of seasonal RDI time series of 16 Iranian synoptic stations (1980–2010).

Test Statistic	P-Value
$\chi^{2*} = 22.545$	0.004

We split this dataset in three parts with equal sizes. The spectral frequency squares of these parts are given in Fig. 2. The results detect ACS time series with spectra on the lines $T_j(x) = x \pm \alpha, \alpha \in \{0.062, 0.153, 0.258\}$. This result verifies the given result in [43]. Then the introduced technique is used to test the the hypothesis $\Sigma^{(1)} = \Sigma^{(2)} = \Sigma^{(3)}$ (or equivalently, $f_1 = f_2 = f_3$). Table 2 summarizes the results. As can be seen, since the P-value is more than 0.05, thus the null hypothesis cannot be rejected and consequently the stochastic behaviours of all processes are similar.

Example 2. In agricultural and environmental sciences, the drought is an important subject. Droughts have many effects on crop, urban water supply, degradation and desertification.

Table 4 Multiple comparisons between seasonal RDI time series of 16 Iranian synoptic stations (1980–2010).

Cluster 1	Cluster 2	Cluster 3
Abadeh	Bam	Chabahar
Boushehr	Bandar Abbas	Iranshahr
Fasa	Bandar Lengeh	Saravan
Kerman	Jask	
Shahre Babak	Zabol	
Shiraz	Zahedan	
Sirjan		

To evaluate the rate of drought, researchers introduced many different drought indices [63–65]. In this work, the datasets contain the seasonal reconnaissance drought index (RDI) values from 16 Iranian synoptic stations from 1980 to 2010 (Fig. 3). The periodogram and the spectral frequency squares of these datasets are given in Figs. 4 and 5, respectively. As can be seen in Fig. 4, the periodogram of all datasets detect PC processes with periods 4 ($T = 124/31 = 4$, $p < 0.05$). This result also confirms by the spectral frequency squares of these datasets, where the spectral squares are supported by the lines $T_j(x) = x \pm \frac{2\pi j}{4}, j = 0, \cdots, 3$.

Now, the introduced technique is used to test the the hypothesis $\Sigma^{(1)} = \cdots = \Sigma^{(16)}$ (or equivalently, $f_1 = \cdots = f_{16}$). Table 3 summarizes the results. As can be seen, since the P-value is < 0.05, thus the null hypothesis is rejected and consequently at least the stochastic behaviours of two processes are different.

Now, we are interested in clustering these stations. To response this question, the proposed approach has been applied. Table 4 summarized the result for the multiple comparisons. As Table 4 indicates, these stations can be classified in 2 clusters; Cluster 1: Abadeh, Boushehr, Fasa, Kerman, Shahre Babak, Shiraz, Sirjan; Cluster 2: Bam, Bandar Abbas, Bandar Lengeh, Jask, Zabol, Zahedan; Cluster 3: Chabahar, Iranshahr, Saravan.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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