Advances in Difference Equations
a SpringerOpen Journal

Laplace decomposition for solving nonlinear system of fractional order partial differential equations

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Abstract

In the present article a modified decomposition method is implemented to solve systems of partial differential equations of fractional-order derivatives. The derivatives of fractional-order are expressed in terms of Caputo operator. The validity of the proposed method is analyzed through illustrative examples. The solution graphs have shown a close contact between the exact and LADM solutions. It is observed that the solutions of fractional-order problems converge towards the solution of an integer-order problem, which confirmed the reliability of the suggested technique. Due to better accuracy and straightforward implementation, the extension of the present method can be made to solve other fractional-order problems.

Keywords: Caputo operator; Adomian decomposition method; Laplace transformation; Fractional systems of partial differential equations

1 Introduction

In the last decade, scientists and engineers have paid much attention towards nonlinear equations, as the nonlinearity exists everywhere in most of the physical problems. The nonlinear partial differential equations of fractional order (FPDEs) are the special case of nonlinear equations that have many applications in science and technology, including chemistry, biology, physics, vibration, acoustic, signals processing, electromagnetic, polymeric materials, and fluid dynamics, super conductivity, optics, and quantum mechanics [1–4]. Due to frequent appearance of FPDEs in different disciplines of engineering and science, the researchers have added a lot of research contribution to both theory of mathematical science and technology [5–9].

In mathematical analysis, the most frequent developing area is fractional-order calculus. In fact, many physical phenomena are dependent on time instant as well as at the earlier history of time are modeled by using fractional-order ordinary differential equations (FODEs), and thus FDEs have attained their importance in many fields of applied sciences. Due to the importance of FDEs, many researchers have made their focus on the analytical solution as well as the numerical solutions of FDEs [10–12]. In this regard numerous techniques have been discussed for the solutions of FPDEs such as homotopy

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analysis method (HAM), homotopy perturbation technique (HPM), Laplace transformation, variational technique with Pade approximation, corrected Fourier series, natural decomposition method [13–16], and fractional complex transformation [17]. The optimal q-HAM is discussed in [18] to solve FPDEs.

Numerical solution of FPDEs has been discussed in [19] efficiently by using Bernoulli wavelets and collocation method. Auxiliary Laplace parameter method has been discussed for the solution of FDEs [20]. Exact solution of Kodomtsev–Petviashvili (KP) equation is obtained in [21] by using simple equation method. Modified variational iteration technique is developed in [2] for the result of nonlinear PDEs. The solution of linear and nonlinear FPDEs has been studied in [22] by using iterative Laplace transform method. Biological nonlinear phenomena like shallow water waves and multicellular biological dynamics can be modeled in terms of nonlinear PDEs of integer order [23]. Numerous FPDEs do not have exact or analytical solution, so numerical methods are used as an alternative. In this regard, numerical solutions of FPDEs have been obtained in [24] by using tau approximation. Discrete HAM is suggested to solve linear and nonlinear FPDEs [25–27].

Laplace–Adomian decomposition method (LADM) is one of the effective and straightforward techniques to solve nonlinear FPDEs. LADM possesses the combined behavior of Laplace transformation and Adomian decomposition method (ADM). It is observed that the suggested method requires no predefined declaration size like RK4. LADM requires fewer number of parameters, no discretization and linearization as compared to other analytical techniques [28]. LADM is also compared with ADM to analyze the solution of FPDEs given in [29]. The solution of Kundu–Eckhaus equation is discussed in [30] via LADM. Multistep LADM is implemented to solve FPDEs in [31, 32]. LADM is also used for the solution of fractional Navier–Stokes, Smoke models, and third-order dispersive PDEs [33–35].

In the current study, we implement LADM for the solution of some nonlinear system of FPDEs. The desired degree of accuracy is achieved. The procedure of the suggested technique is very simple and straightforward. The accuracy is calculated in terms of absolute error. The results have shown that the present method has the desired accuracy as compared to other analytical techniques.

2 Definitions and preliminaries

Definition 2.1 The Riemann–Liouville definition of fractional integral of a function g with order $\beta \geq 0$ can be expressed as [36, 37]

$$I_{\xi}^{\beta} g(\xi) = \begin{cases} g(\xi) & \text{if } \beta = 0, \\ \frac{1}{\Gamma(\beta)} \int_0^{\xi} (\xi - \upsilon)^{\beta-1} g(\upsilon) \, d\upsilon & \text{if } \beta > 0, \end{cases}$$

where Γ is denoted as

$$\Gamma(\omega) = \int_0^{\infty} e^{-\xi} \xi^{\omega-1} \, d\xi \quad \omega \in \mathbb{C}.$$

Definition 2.2 The Caputo definition of fractional derivative of order β is described as [36, 37]

$$D_\xi^\beta g(\xi) = \frac{1}{\Gamma(m-\beta)} \int_0^\xi (\xi - \tau)^{m-\beta-1} g^{(m)}(\tau) \, d\tau,$$

for $m - 1 < \beta \le m, m \in \mathbb{N}, \xi > 0, g \in \mathbb{C}_\tau, \tau \ge -1$.

Lemma 2.3 *If $m - 1 < \beta \le m$ with $m \in \mathbb{N}$ and $g \in \mathbb{C}_\tau$ with $\tau \ge -1$, then* [36, 38]

$$D_\xi^\beta I_\xi^\beta g(\xi) = g(\xi),$$

$$I^\beta \xi^\lambda = \frac{\Gamma(\lambda+1)}{\Gamma(\beta+\lambda+1)} \xi^{\beta+\lambda}, \quad \beta > 0, \lambda > -1, \xi > 0,$$

$$D_\xi^\beta I_\xi^\beta g(\xi) = g(\xi) - \sum_{k=0}^m g^{(k)}(0^+) \frac{\xi^k}{k!} \quad \text{for } \xi > 0.$$

Definition 2.4 The Laplace transform $G(s)$ of $g(\tau)$ is expressed as [30]

$$G(s) = \mathcal{L}\big[g(\tau)\big] = \int_0^\infty e^{-s\tau} g(t) \, dt.$$

Definition 2.5 The Laplace transform of fractional derivative is [30]

$$\mathcal{L}\big(D_\tau^\beta g(\tau)\big) = s^\beta G(s) - \sum_{k=0}^{m-1} s^{\beta-1-k} g^{(k)}(0), \quad m - 1 < \beta < m.$$

3 LADM idea for FPDEs

Consider the following FPDE [30]:

$$D^\beta u(\xi, \tau) + Lu(\xi, \tau) + Nu(\xi, \tau) = q(\xi, \tau), \quad \xi, \tau \ge 0, m - 1 < \beta \le m. \tag{1}$$

The fractional derivative in equation (1) is expressed in the Caputo sense. The linear and nonlinear terms are denoted by L and N respectively, and $q(\xi, \tau)$ is the sources term with the initial condition

$$u(\xi, 0) = f(\xi). \tag{2}$$

Applying the Laplace transform on both sides of equation (1), we get [30]

$$s^\beta \mathcal{L}\big[u(\xi, \tau)\big] - s^{\beta-1} u(\xi, 0) = \mathcal{L}\big[q(\xi, \tau)\big] - \mathcal{L}\big[Lu(\xi, \tau) + Nu(\xi, \tau)\big],$$

$$\mathcal{L}\big[u(\xi, \tau)\big] = \frac{k(\xi)}{s} - \frac{1}{s^\beta} \mathcal{L}\big[Lu(\xi, \tau) + Nu(\xi, \tau) + q(\xi, \tau)\big]. \tag{3}$$

The ADM solution is

$$u(\xi, \tau) = \sum_{j=0}^\infty u_j(\xi, \tau). \tag{4}$$

The nonlinear term in the problem is expressed as

$$Nu(\xi, \tau) = \sum_{j=0}^{\infty} A_j, \tag{5}$$

where

$$A_j = \frac{1}{j!} \left[\frac{d^j}{d\lambda^j} \left[N \sum_{j=0}^{\infty} (\lambda^j u_j) \right] \right]_{\lambda=0}, \quad j = 0, 1, 2, \ldots, \tag{6}$$

are called Adomian polynomials.

Substituting equations (4) and (5) in equation (3), we get

$$\mathcal{L} \left[\sum_{j=0}^{\infty} u(\xi, \tau) \right] = \frac{f(x)}{s} - \frac{1}{s^\beta} \mathcal{L} \left[L \sum_{j=0}^{\infty} u_j(\xi, \tau) + \sum_{j=0}^{\infty} A_j + q(\xi, \tau) \right].$$

Applying the decomposition method, we get

$$\mathcal{L} \left[u_0(\xi, \tau) \right] = \frac{f(\xi)}{s} \tag{7}$$

and

$$\mathcal{L} \left[u_{j+1}(\xi, \tau) \right] = -\frac{1}{s^\beta} \mathcal{L} \left[L u_j(\xi, \tau) + A_j + q(\xi, \tau) \right], \quad j \geq 1. \tag{8}$$

Using the inverse transform to equations (7) and (8), we have [30]

$$u_0(\xi, \tau) = f(\xi),$$

$$u_{j+1}(\xi, \tau) = -\mathcal{L}^{-1} \left[\frac{1}{s^\beta} \mathcal{L} \left[L u_j(\xi, \tau) + A_j \right] \right]. \tag{9}$$

4 Theorem

Here, we study the convergence analysis in the same manner as in [39] of the LADM applied to PDEs. Let us consider the Hilbert space H which may define by $H = L^2((\alpha, \beta)X[0, T])$ the set of applications

$$u : (\alpha, \beta)X[0, T] \to \quad \text{with} \int_{(\alpha, \beta)X[0,T]} u^2(\xi, s) \, ds \, d\theta < +\infty.$$

Now we consider the PDEs in the light of the above assumptions. Let us denote

$$L(u) = \frac{\partial^\gamma u}{\partial \tau^\gamma},$$

then the fractional-order of PDEs becomes, in an operator form,

$$L(u) = \varphi \frac{\partial v(\xi, \tau)}{\partial \xi} - w \frac{\partial v(\zeta, \tau)}{\partial \zeta}.$$

The LADM reaches convergence if the following two hypotheses are satisfied:

(H1) $(L(u) - L(v), u - v) \geq k\|u - v\|^2; k > 0, \forall u, v \epsilon H.$

(H2) Whatever may be $M > 0$, there exists a constant $C(M) > 0$ such that, for $u, v \epsilon H$ with $\|u\| \leq M, \|v\| \leq M$, we have $(L(u) - L(v), u - v) \leq C(M)\|u - v\|\|w\|$ for every $w \epsilon H$.

Example 1 The system of fractional-order PDEs in [40]

$$\frac{\partial^\beta u}{\partial \tau^\beta} - v\frac{\partial u}{\partial \xi} - \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \eta} = 1 - \xi + \eta + \tau,$$
$$\frac{\partial^\beta v}{\partial \tau^\beta} - u\frac{\partial v}{\partial \xi} + \frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \eta} = 1 - \xi - \eta - \tau, \quad 0 < \beta \leq 1,$$

(10)

with the initial condition

$$u(\xi, \eta, 0) = \xi + \eta - 1, \qquad v(\xi, \eta, 0) = \xi - \eta + 1.$$

(11)

Taking the Laplace transform of equation (10), we have

$$\mathcal{L}\left[\frac{\partial^\beta u}{\partial \tau^\beta}\right] = \mathcal{L}\left[v\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \eta} + 1 - \xi + \eta + \tau\right],$$
$$\mathcal{L}\left[\frac{\partial^\beta v}{\partial \tau^\beta}\right] = \mathcal{L}\left[u\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \eta} + 1 - \xi - \eta - \tau\right],$$
$$s^\beta \mathcal{L}\left[u(\xi, \eta, \tau)\right] - s^{\beta-1}\left[u(\xi, \eta, 0)\right] = \mathcal{L}\left[v\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \eta} + 1 - \xi + \eta + \tau\right],$$
$$s^\beta \mathcal{L}\left[v(\xi, \eta, \tau)\right] - s^{\beta-1}\left[v(\xi, \eta, 0)\right] = \mathcal{L}\left[u\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \eta} + 1 - \xi - \eta - \tau\right].$$

Using the inverse transformation, we get

$$u(\xi, \eta, \tau) = \mathcal{L}^{-1}\left[\frac{u(\xi, \eta, 0)}{s} + \frac{1}{s^\beta}\mathcal{L}\left[v\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \eta} + 1 - \xi + \eta + \tau\right]\right],$$
$$v(\xi, \eta, \tau) = \mathcal{L}^{-1}\left[\frac{v(\xi, \eta, 0)}{s} + \frac{1}{s^\beta}\mathcal{L}\left[u\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \eta} + 1 - \xi - \eta - \tau\right]\right],$$
$$u(\xi, \eta, \tau) = \xi + \eta - 1 + \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[v\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \eta} + 1 - \xi + \eta + \tau\right]\right],$$
$$v(\xi, \eta, \tau) = \xi - \eta + 1 + \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[u\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \eta} + 1 - \xi - \eta - \tau\right]\right].$$

Using the ADM procedure, we get

$$\sum_{j=0}^\infty u_j(\xi, \eta, \tau) = \xi + \eta - 1 + \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\sum_{j=0}^\infty A_j(v, u) + \sum_{j=0}^\infty B_j(v, u) + 1 - \xi + \eta + \tau\right]\right],$$
$$\sum_{j=0}^\infty v_j(\xi, \eta, \tau) = \xi - \eta + 1 + \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\sum_{j=0}^\infty C_j(u, v) - \sum_{j=0}^\infty D_j(u, v) + 1 - \xi - \eta - \tau\right]\right],$$

where Adomian polynomial components $A_j(v,u)$, $B_j(v,u)$, $C_j(u,v)$, and $D_j(u,v)$ are given as follows:

$$A_0(v,u) = v_0 \frac{\partial u_0}{\partial \xi},$$

$$A_1(v,u) = v_0 \frac{\partial u_1}{\partial \xi} + v_1 \frac{\partial u_0}{\partial \xi},$$

$$A_2(v,u) = v_0 \frac{\partial u_2}{\partial \xi} + v_1 \frac{\partial u_1}{\partial \xi} + v_2 \frac{\partial u_0}{\partial \xi},$$

$$B_0(v,u) = \frac{\partial v_0}{\partial \tau} \frac{\partial u_0}{\partial \eta},$$

$$B_1(v,u) = \frac{\partial v_0}{\partial \tau} \frac{\partial u_1}{\partial \eta} + \frac{\partial v_1}{\partial \tau} \frac{\partial u_0}{\partial \eta},$$

$$B_2(v,u) = \frac{\partial v_0}{\partial \tau} \frac{\partial u_2}{\partial \eta} + \frac{\partial v_1}{\partial \tau} \frac{\partial u_1}{\partial \eta} + \frac{\partial v_2}{\partial \tau} \frac{\partial u_0}{\partial \eta},$$

$$C_0(u,v) = u_0 \frac{\partial v_0}{\partial \xi},$$

$$C_1(u,v) = u_0 \frac{\partial v_1}{\partial \xi} + u_1 \frac{\partial v_0}{\partial \xi},$$

$$C_2(u,v) = u_0 \frac{\partial v_2}{\partial \xi} + u_1 \frac{\partial v_1}{\partial \xi} + u_2 \frac{\partial v_0}{\partial \xi}, \qquad (12)$$

$$D_0(u,v) = \frac{\partial u_0}{\partial \tau} \frac{\partial v_0}{\partial \eta},$$

$$D_1(u,v) = \frac{\partial u_0}{\partial \tau} \frac{\partial v_1}{\partial \eta} + \frac{\partial u_1}{\partial \tau} \frac{\partial v_0}{\partial \eta},$$

$$D_2(u,v) = \frac{\partial u_0}{\partial \tau} \frac{\partial v_2}{\partial \eta} + \frac{\partial u_1}{\partial \tau} \frac{\partial v_1}{\partial \eta} + \frac{\partial u_2}{\partial \tau} \frac{\partial v_0}{\partial \eta},$$

$$u_0(\xi,\eta,\tau) = \xi + \eta - 1,$$

$$v_0(\xi,\eta,\tau) = \xi - \eta + 1,$$

$$u_{j+1}(\xi,\eta,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta} \mathcal{L}\left\{ \sum_{j=0}^{\infty} A_j(v,u) + \sum_{j=0}^{\infty} B_j(v,u) + 1 - \xi + \eta + \tau \right\} \right],$$

$$v_{j+1}(\xi,\eta,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta} \mathcal{L}\left\{ \sum_{j=0}^{\infty} C_j(u,v) - \sum_{j=0}^{\infty} D_j(u,v) + 1 - \xi - \eta - \tau \right\} \right]$$
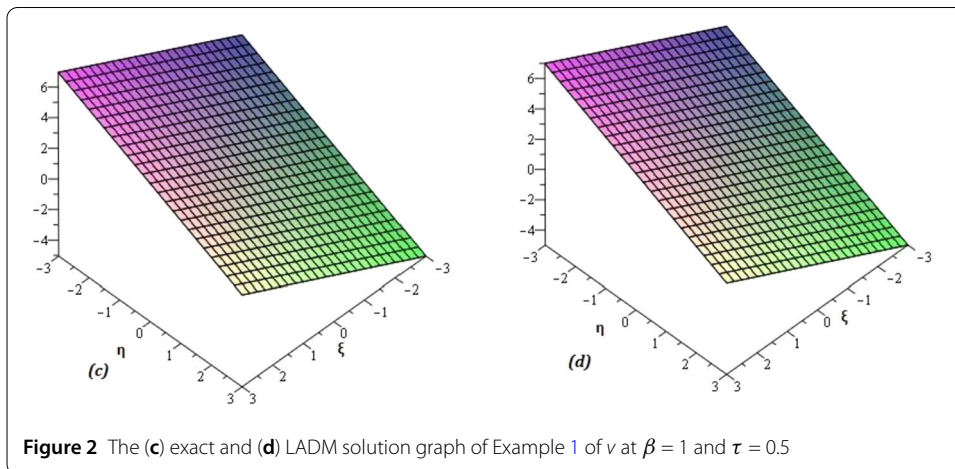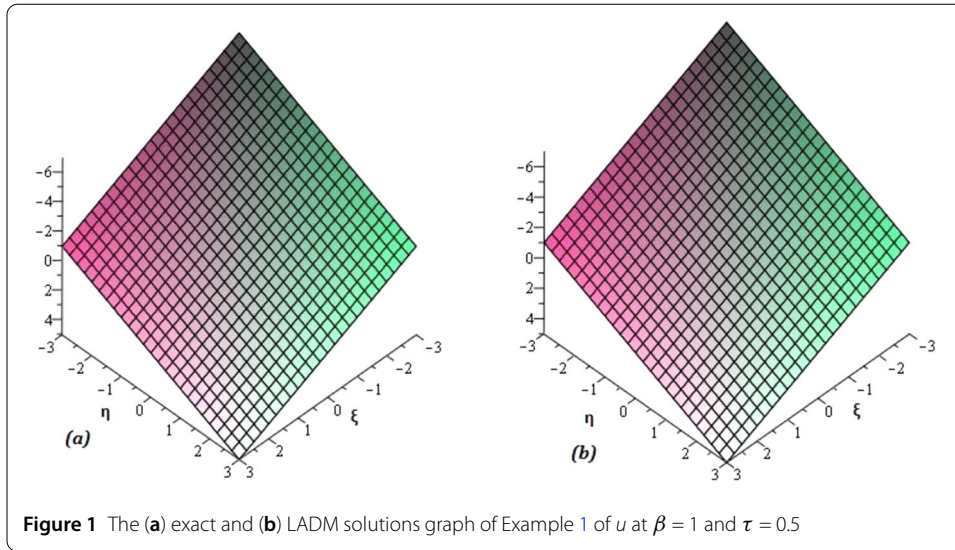
for $j = 0, 1, 2, \dots$

$$u_1(\xi,\eta,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta} \mathcal{L}\left\{ v_0 \frac{\partial u_0}{\partial \xi} + \frac{\partial v_0}{\partial \tau} \frac{\partial u_0}{\partial \eta} + 1 - \xi + \eta + \tau \right\} \right],$$

$$u_1(\xi,\eta,\tau) = \mathcal{L}^{-1}\left[\frac{2}{s^{\beta+1}} + \frac{1}{s^{\beta+2}} \right] = \frac{2\tau^\beta}{\Gamma(\beta+1)} + \frac{\tau^{\beta+1}}{\Gamma(\beta+2)},$$

$$v_1(\xi,\eta,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta} \mathcal{L}\left\{ u_0 \frac{\partial v_0}{\partial \xi} - \frac{\partial u_0}{\partial \tau} \frac{\partial v_0}{\partial \eta} + 1 - \xi - \eta - \tau \right\} \right], \qquad (13)$$

$$v_1(\xi,\eta,\tau) = \mathcal{L}^{-1}\left[\frac{-1}{s^{\beta+2}} \right] = \frac{-\tau^{\beta+1}}{\Gamma(\beta+2)}.$$

The subsequent terms are

$$u_2(\xi, \eta, \tau)$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{v_0\frac{\partial u_1}{\partial\xi} + v_1\frac{\partial u_0}{\partial\xi} + \frac{\partial v_0}{\partial\tau}\frac{\partial u_1}{\partial\eta} + \frac{\partial v_1}{\partial\tau}\frac{\partial u_0}{\partial\eta}\right\}\right]$$

$$= \frac{-\tau^{2\beta+1}}{\Gamma(2\beta+2)} - \frac{\tau^{2\beta}}{\Gamma(\beta+2)},$$

$$v_2(\xi, \eta, \tau)$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{u_0\frac{\partial v_1}{\partial\xi} + u_1\frac{\partial v_0}{\partial\xi} - \frac{\partial u_0}{\partial\tau}\frac{\partial v_1}{\partial\eta} - \frac{\partial u_1}{\partial\tau}\frac{\partial v_0}{\partial\eta}\right\}\right]$$

$$= \left(\frac{2\Gamma(\beta+2) - (\beta+1)\Gamma(\beta+1)}{\Gamma(2\beta+1)\Gamma(\beta+2)}\right)\tau^{2\beta} + \frac{\tau^{2\beta+1}}{\Gamma(2\beta+2)} - \frac{2\beta\Gamma(\beta)\tau^{2\beta-1}}{\Gamma(\beta+1)\Gamma(2\beta)},$$

$$u_3(\xi, \eta, \tau)$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{v_0\frac{\partial u_2}{\partial\xi} + v_1\frac{\partial u_1}{\partial\xi} + v_2\frac{\partial u_0}{\partial\xi} + \frac{\partial v_0}{\partial\tau}\frac{\partial u_2}{\partial\eta} + \frac{\partial v_1}{\partial\tau}\frac{\partial u_1}{\partial\eta} + \frac{\partial v_2}{\partial\tau}\frac{\partial u_0}{\partial\eta}\right\}\right] \qquad (14)$$

$$= \frac{\tau^{3\beta+1}}{\Gamma(3\beta+2)} + \left(\frac{2\Gamma(\beta+2) - (\beta+1)\Gamma(\beta+1)}{\Gamma(3\beta+1)\Gamma(\beta+2)}\right)\tau^{3\beta} + \frac{\tau^{3\beta}}{\Gamma(2\beta+2)}$$

$$- \frac{2\beta\Gamma(\beta)\tau^{3\beta-1}}{\Gamma(\beta+1)\Gamma(3\beta)} + \left(\frac{(2\Gamma(\beta+2) - (\beta+1)\Gamma(\beta+1))2\beta\Gamma(\beta)}{\Gamma(3\beta)\Gamma(2\beta+1)\Gamma(\beta+2)}\right)\tau^{3\beta-1}$$

$$- \left(\frac{2\beta\Gamma(\beta)}{\Gamma(2\beta)\Gamma(\beta+1)}\right)\tau^{3\beta-2},$$

$$v_3(\xi, \eta, \tau)$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{u_0\frac{\partial v_2}{\partial\xi} + u_1\frac{\partial v_1}{\partial\xi} + u_2\frac{\partial v_0}{\partial\xi} - \frac{\partial u_0}{\partial\tau}\frac{\partial v_2}{\partial\eta} - \frac{\partial u_1}{\partial\tau}\frac{\partial v_1}{\partial\eta} - \frac{\partial u_2}{\partial\tau}\frac{\partial v_0}{\partial\eta}\right\}\right]$$

$$= \frac{-\tau^{3\beta+1}}{\Gamma(3\beta+2)} + \left(\frac{(\beta+1)\Gamma(\beta+1)2\beta\Gamma(2\beta)}{\Gamma(3\beta)\Gamma(\beta+2)\Gamma(\beta+2)}\right)\tau^{3\beta-1}.$$

The LADM solution for Example 1 is

$$u(\xi, \eta, \tau) = u_0(\xi, \eta, \tau) + u_1(\xi, \eta, \tau) + u_2(\xi, \eta, \tau) + u_3(\xi, \eta, \tau) + \cdots,$$

$$v(\xi, \eta, \tau) = v_0(\xi, \eta, \tau) + v_1(\xi, \eta, \tau) + v_2(\xi, \eta, \tau) + v_3(\xi, \eta, \tau) + \cdots,$$

$$u(\xi, \eta, \tau) = \xi + \eta - 1 + \frac{2\tau^\beta}{\Gamma(\beta+1)} + \frac{\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{\tau^{2\beta+1}}{\Gamma(2\beta+2)} - \frac{\tau^{2\beta}}{\Gamma(2\beta+1)}$$

$$+ \frac{\tau^{3\beta+1}}{\Gamma(3\beta+2)} + \left(\frac{2\Gamma(\beta+2) - (\beta+1)\Gamma(\beta+1)}{\Gamma(3\beta+1)\Gamma(\beta+2)}\right)\tau^{3\beta} + \frac{\tau^{3\beta}}{\Gamma(2\beta+2)}$$

$$- \frac{2\beta\Gamma(\beta)\tau^{3\beta-1}}{\Gamma(\beta+1)\Gamma(3\beta)} + \left(\frac{(2\Gamma(\beta+2) - (\beta+1)\Gamma(\beta+1))2\beta\Gamma(\beta)}{\Gamma(3\beta)\Gamma(2\beta+1)\Gamma(\beta+2)}\right)\tau^{3\beta-1}$$

$$- \left(\frac{2\beta\Gamma(\beta)}{\Gamma(2\beta)\Gamma(\beta+1)}\right)\tau^{3\beta-2} + \cdots,$$

$$v(\xi, \eta, \tau) = \xi - \eta + 1 - \frac{\tau^{\beta+1}}{\Gamma(\beta+2)} + \left(\frac{2\Gamma(\beta+2) - (\beta+1)\Gamma(\beta+1)}{\Gamma(2\beta+1)\Gamma(\beta+2)}\right)\tau^{2\beta}$$

Figure 1 The (**a**) exact and (**b**) LADM solutions graph of Example 1 of u at $\beta = 1$ and $\tau = 0.5$

Figure 2 The (**c**) exact and (**d**) LADM solution graph of Example 1 of v at $\beta = 1$ and $\tau = 0.5$

$$+ \frac{\tau^{2\beta+1}}{\Gamma(2\beta + 2)} - \frac{2\beta\Gamma(\beta)\tau^{2\beta-1}}{\Gamma(\beta + 1)\Gamma(2\beta)} - \frac{\tau^{3\beta+1}}{\Gamma(3\beta + 2)}$$

$$+ \left(\frac{(\beta + 1)\Gamma(\beta + 1)2\beta\Gamma(2\beta)}{\Gamma(3\beta)\Gamma(\beta + 2)\Gamma(\beta + 2)} \right) \tau^{3\beta-1} + \cdots .$$
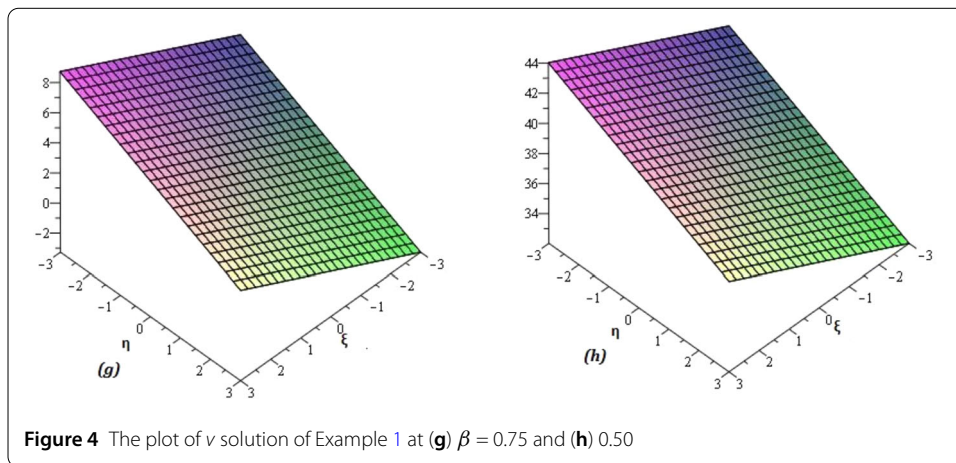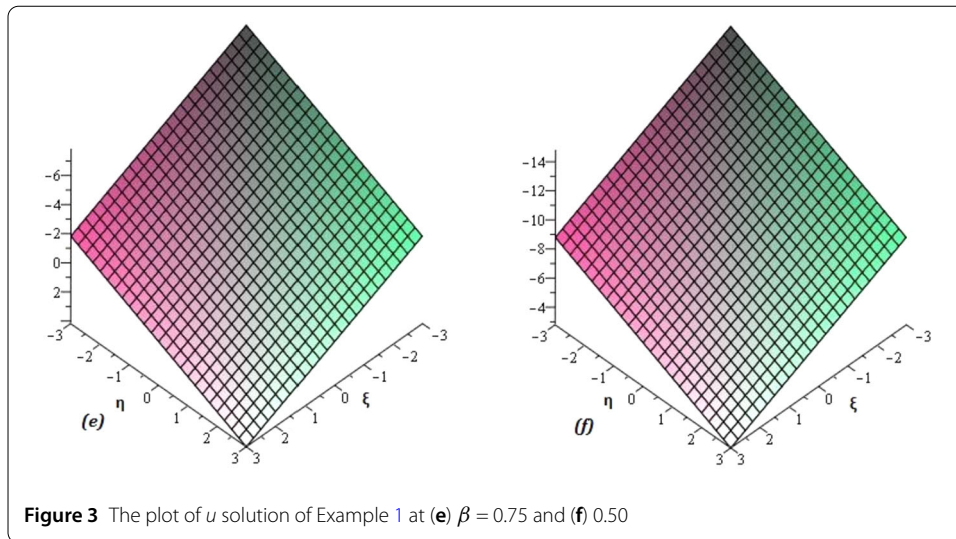
When $\beta = 1$, then LADM solution is

$$u(\xi, \eta, \tau) = \xi + \eta + \tau - 1,$$

$$v(\xi, \eta, \tau) = \xi - \eta - \tau + 1.$$

(15)

Example 2 The system of fractional-order PDEs in [40]

$$\frac{\partial^\beta u}{\partial \xi^\beta} - v\frac{\partial u}{\partial \tau} + u\frac{\partial v}{\partial \tau} = -1 + e^\xi \sin\tau,$$

$$\frac{\partial^\beta v}{\partial \xi^\beta} + \frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \xi} - \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \xi} = -1 - e^{-\xi} \sin\tau, \quad 0 < \beta \le 1,$$

(16)

Figure 3 The plot of u solution of Example 1 at (**e**) $\beta = 0.75$ and (**f**) 0.50

Figure 4 The plot of v solution of Example 1 at (**g**) $\beta = 0.75$ and (**h**) 0.50

Table 1 LADM solutions of Example 1 at different fractional order β and its corresponding absolute error (A.E) of u at $\tau = 0.5$

ξ	η	A.E ($\beta = 0.55$)	A.E ($\beta = 0.75$)	A.E [$\beta = 1$ ($j = 5$)]	A.E [$\beta = 1$ ($j = 6$)]
0.1	0.1	1.639E–02	5.828E–03	1.000E–03	2.000E–04
0.2	0.2	3.278E–02	1.165E–02	2.000E–03	4.000E–04
0.3	0.3	4.917E–02	1.748E–02	3.000E–03	6.000E–04
0.4	0.4	6.556E–02	2.331E–02	4.000E–03	8.000E–04
0.5	0.5	8.196E–02	2.914E–02	5.000E–03	1.000E–03

with the initial conditions

$$u(0, \tau) = \sin \tau, \qquad v(0, \tau) = \cos \tau. \tag{17}$$

Taking the Laplace transform of (16), we have

$$\mathcal{L}\left[\frac{\partial^\beta u}{\partial \xi^\beta}\right] = \mathcal{L}\left[v \frac{\partial u}{\partial \tau} - u \frac{\partial v}{\partial \tau} - 1 + e^\xi \sin \tau\right],$$

Table 2 LADM solutions of Example 1 at different fractional order β and its corresponding absolute error (A.E) of v at $\tau = 0.5$

ξ	η	A.E ($\beta = 0.55$)	A.E ($\beta = 0.75$)	A.E [$\beta = 1$ ($j = 5$)]	A.E [$\beta = 1$ ($j = 6$)]
0.1	0.1	1.484E–02	5.308E–03	9.949E–04	9.994E–05
0.2	0.2	2.969E–02	1.061E–02	1.989E–03	1.998E–04
0.3	0.3	4.454E–02	1.592E–02	2.984E–03	2.998E–04
0.4	0.4	5.938E–02	2.123E–02	3.979E–03	3.997E–04
0.5	0.5	7.423E–02	2.654E–02	4.974E–03	4.997E–04

$$\mathcal{L}\left[\frac{\partial^{\beta} v}{\partial \xi^{\beta}}\right] = \mathcal{L}\left[-\frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \xi} - \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \xi} - 1 - e^{-\xi}\sin\tau\right],$$

$$s^{\beta}\mathcal{L}\left[u(\xi,\tau)\right] - s^{\beta-1}\left[u(0,\tau)\right] = \mathcal{L}\left[v\frac{\partial u}{\partial \tau} - u\frac{\partial v}{\partial \tau} - 1 + e^{\xi}\sin\tau\right],$$

$$s^{\beta}\mathcal{L}\left[v(\xi,\tau)\right] - s^{\beta-1}\left[v(0,\tau)\right] = \mathcal{L}\left[-\frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \xi} - \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \xi} - 1 - e^{-\xi}\sin\tau\right].$$

Using the inverse transformation, we obtain

$$u(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{u(0,\tau)}{s} + \frac{1}{s^{\beta}}\mathcal{L}\left\{v\frac{\partial u}{\partial \tau} - u\frac{\partial v}{\partial \tau} - 1 + e^{\xi}\sin\tau\right\}\right],$$

$$v(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{v(0,\tau)}{s} + \frac{1}{s^{\beta}}\mathcal{L}\left\{-\frac{\partial u}{\partial \tau}\frac{\partial v}{\partial \xi} - \frac{\partial v}{\partial \tau}\frac{\partial u}{\partial \xi} - 1 - e^{-\xi}\sin\tau\right\}\right].$$

Using the ADM procedure, we get

$$u_0(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{u(0,\tau)}{s}\right] = \mathcal{L}^{-1}\left[\frac{\sin\tau}{s}\right] = \sin\tau,$$

$$v_0(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{v(0,\tau)}{s}\right] = \mathcal{L}^{-1}\left[\frac{\cos\tau}{s}\right] = \cos\tau,$$

$$u_{j+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{v_j\frac{\partial u_j}{\partial \tau} - u_j\frac{\partial v_j}{\partial \tau} - 1 + e^{\xi}\sin\tau\right\}\right], \quad (18)$$

$$v_{j+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{-\frac{\partial u_j}{\partial \tau}\frac{\partial v_j}{\partial \xi} - \frac{\partial v_j}{\partial \tau}\frac{\partial u_j}{\partial \xi} - 1 - e^{-\xi}\sin\tau\right\}\right],$$

$$j = 0, 1, 2, \ldots$$

for $j = 0$

$$u_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{v_0\frac{\partial u_0}{\partial \tau} - u_0\frac{\partial v_0}{\partial \tau} - 1 + e^{\xi}\sin\tau\right\}\right],$$

$$u_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{\sin\tau}{s^{\beta}(s-1)}\right] = \sin\tau\,\xi^{\beta}\sum_{k=0}^{\infty}\frac{\xi^k}{\Gamma(k+\beta+1)},$$

$$v_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{-\frac{\partial u_0}{\partial \tau}\frac{\partial v_0}{\partial \xi} - \frac{\partial v_0}{\partial \tau}\frac{\partial u_0}{\partial \xi} - 1 - e^{-\xi}\sin\tau\right\}\right], \quad (19)$$

$$v_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{-1}{s^{\beta+1}} + \frac{\cos\tau}{s^{\beta}(s+1)}\right] = \frac{-\tau^{\beta}}{\Gamma(\beta+1)} - \cos\tau\,\xi^{\beta}\sum_{k=0}^{\infty}\frac{-\xi^k}{\Gamma(k+\beta+1)},$$
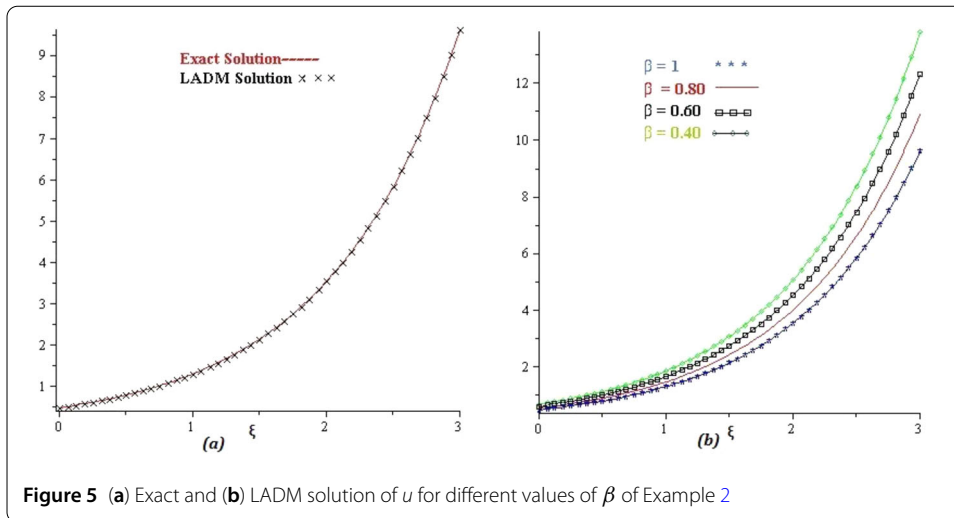
Figure 5 (**a**) Exact and (**b**) LADM solution of *u* for different values of β of Example 2
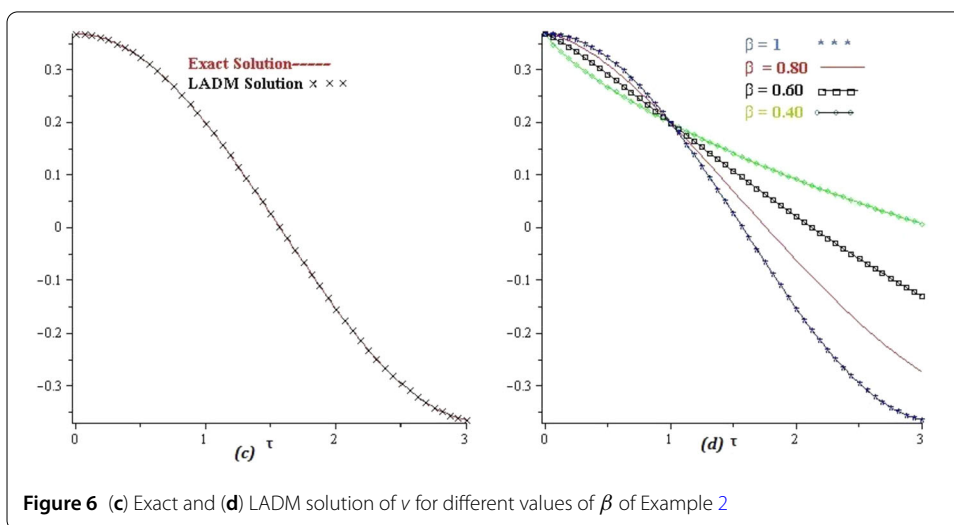

Figure 6 (**c**) Exact and (**d**) LADM solution of *v* for different values of β of Example 2

The subsequent terms are

$$
\begin{aligned}
u_2(\xi,\tau) &= \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{v_0\frac{\partial u_1}{\partial \tau} + v_1\frac{\partial u_0}{\partial \tau} - u_0\frac{\partial v_1}{\partial \tau} - u_1\frac{\partial v_0}{\partial \tau}\right\}\right]\\
&= \sum_{k=0}^{\infty}\frac{\xi^{2\beta+k}}{\Gamma(2\beta+k+1)} - \sum_{k=0}^{\infty}\frac{(-\xi)^{2\beta+k}}{\Gamma(2\beta+k+1)} - \cos\tau\,\frac{\xi^{2\beta}}{\Gamma(2\beta+1)},\\
v_2(\xi,\tau) &= \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{-\frac{\partial u_0}{\partial \tau}\frac{\partial v_1}{\partial \xi} - \frac{\partial u_1}{\partial \tau}\frac{\partial v_0}{\partial \xi} - \frac{\partial v_0}{\partial \tau}\frac{\partial u_1}{\partial \xi} - \frac{\partial v_1}{\partial \tau}\frac{\partial u_0}{\partial \xi}\right\}\right],\\
&= \cos\tau\,\frac{\xi^{2\beta-1}}{\Gamma(2\beta)} + \cos^2\tau\sum_{k=0}^{\infty}\frac{(-\xi)^{2\beta+k-1}}{\Gamma(2\beta+k)} + \sin^2\tau\sum_{k=0}^{\infty}\frac{\xi^{2\beta+k-1}}{\Gamma(2\beta+k)}.
\end{aligned}
\tag{20}
$$

The obtained result for Example 2 is as follows:

$$
u(\xi,\tau) = u_0(\xi,\tau) + u_1(\xi,\tau) + u_2(\xi,\tau) + u_3(\xi,\tau) + \cdots,
$$

$$
v(\xi,\tau) = v_0(\xi,\tau) + v_1(\xi,\tau) + v_2(\xi,\tau) + v_3(\xi,\tau) + \cdots,
$$

$$u(\xi, \tau) = \sin \tau + \sin \tau \xi^\beta \sum_{k=0}^\infty \frac{\xi^k}{\Gamma(k + \beta + 1)} + \sum_{k=0}^\infty \frac{\xi^{2\beta+k}}{\Gamma(2\beta + k + 1)}$$

$$- \sum_{k=0}^\infty \frac{(-\xi)^{2\beta+k}}{\Gamma(2\beta + k + 1)} - \cos \tau \frac{\xi^{2\beta}}{\Gamma(2\beta + 1)} \cdots,$$

$$v(\xi, \tau) = \cos \tau - \frac{\tau^\beta}{\Gamma(\beta + 1)} - \cos \tau \xi^\beta \sum_{k=0}^\infty \frac{-\xi^k}{\Gamma(k + \beta + 1)} + \cos \tau \frac{\xi^{2\beta-1}}{\Gamma(2\beta)}$$

$$+ \cos^2 \tau \sum_{k=0}^\infty \frac{(-\xi)^{2\beta+k-1}}{\Gamma(2\beta + k)} + \sin^2 \tau \sum_{k=0}^\infty \frac{\xi^{2\beta+k-1}}{\Gamma(2\beta + k)} + \cdots,$$

When $\beta = 1$, then LADM solution is

$$u(\xi, \tau) = e^\xi \sin \tau,$$
$$v(\xi, \tau) = e^{-\xi} \cos \tau. \tag{21}$$

Example 3 The system of inhomogeneous fractional-order nonlinear PDEs in [40]

$$\frac{\partial^\beta u}{\partial \tau^\beta} - \frac{\partial w}{\partial \xi} \frac{\partial v}{\partial \tau} - \frac{1}{2} \frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} = -4\xi\tau,$$

$$\frac{\partial^\beta v}{\partial \tau^\beta} - \frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} = 6\tau, \tag{22}$$

$$\frac{\partial^\beta w}{\partial \tau^\beta} - \frac{\partial^2 u}{\partial \xi^2} - \frac{\partial v}{\partial \xi} \frac{\partial w}{\partial \tau} = 4\xi\tau - 2\tau - 2, \quad 0 < \beta \le 1$$

with the initial conditions

$$u(\xi, 0) = \xi^2 + 1, \qquad v(\xi, 0) = \xi^2 - 1, \qquad w(\xi, 0) = \xi^2 - 1. \tag{23}$$

Taking the Laplace transform of (22), we get

$$\mathcal{L}\left[\frac{\partial^\beta u}{\partial \tau^\beta}\right] = \mathcal{L}\left[\frac{\partial w}{\partial \xi} \frac{\partial v}{\partial \tau} + \frac{1}{2} \frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} - 4\xi\tau\right],$$

$$\mathcal{L}\left[\frac{\partial^\beta v}{\partial \tau^\beta}\right] = \mathcal{L}\left[\frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} + 6\tau\right],$$

$$\mathcal{L}\left[\frac{\partial^\beta w}{\partial \tau^\beta}\right] = \mathcal{L}\left[\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial v}{\partial \xi} \frac{\partial w}{\partial \tau} + 4\xi\tau - 2\tau - 2\right],$$

$$s^\beta \mathcal{L}\left[u(\xi, \tau)\right] - s^{\beta-1}\left[u(\xi, 0)\right] = \mathcal{L}\left[\frac{\partial w}{\partial \xi} \frac{\partial v}{\partial \tau} + \frac{1}{2} \frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} - 4\xi\tau\right],$$

$$s^\beta \mathcal{L}\left[v(\xi, \tau)\right] - s^{\beta-1}\left[v(\xi, 0)\right] = \mathcal{L}\left[\frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} + 6\tau\right],$$

$$s^\beta \mathcal{L}\left[w(\xi, \tau)\right] - s^{\beta-1}\left[w(\xi, 0)\right] = \mathcal{L}\left[\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial v}{\partial \xi} \frac{\partial w}{\partial \tau} + 4\xi\tau - 2\tau - 2\right].$$

Using the inverse transformation, we have

$$u(\xi, \tau) = \mathcal{L}^{-1}\left[\frac{u(\xi, 0)}{s} + \frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w}{\partial \xi} \frac{\partial v}{\partial \tau} + \frac{1}{2} \frac{\partial w}{\partial \tau} \frac{\partial^2 u}{\partial \xi^2} - 4\xi\tau\right]\right],$$

$$v(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{v(\xi,0)}{s} + \frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w}{\partial \tau}\frac{\partial^2 u}{\partial \xi^2} + 6\tau\right]\right],$$

$$w(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{w(\xi,0)}{s} + \frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial v}{\partial \xi}\frac{\partial w}{\partial \tau} + 4\xi\tau - 2\tau - 2\right]\right].$$

Using the ADM procedure, we get

$$u_0(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{u(\xi,0)}{s}\right] = \mathcal{L}^{-1}\left[\frac{\xi^2+1}{s}\right] = \xi^2+1,$$

$$v_0(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{v(\xi,0)}{s}\right] = \mathcal{L}^{-1}\left[\frac{\xi^2-1}{s}\right] = \xi^2-1,$$

$$w_0(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{w(\xi,0)}{s}\right] = \mathcal{L}^{-1}\left[\frac{\xi^2-1}{s}\right] = \xi^2-1,$$

$$u_{j+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_j}{\partial \xi}\frac{\partial v_j}{\partial \tau} + \frac{1}{2}\frac{\partial w_j}{\partial \tau}\frac{\partial^2 u_j}{\partial \xi^2} - 4\xi\tau\right]\right], \tag{24}$$

$$v_{j+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_j}{\partial \tau}\frac{\partial^2 u_j}{\partial \xi^2} + 6\tau\right]\right],$$

$$w_{j+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial^2 u_j}{\partial \xi^2} + \frac{\partial v_j}{\partial \xi}\frac{\partial w_j}{\partial \tau} + 4\xi\tau - 2\tau - 2\right]\right],$$

$$j = 0,1,2,\dots$$

for $j = 0$

$$u_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_0}{\partial \xi}\frac{\partial v_0}{\partial \tau} + \frac{1}{2}\frac{\partial w_0}{\partial \tau}\frac{\partial^2 u_0}{\partial \xi^2} - 4\xi\tau\right]\right],$$

$$u_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{-4\xi}{s^{\beta+2}}\right] = \frac{-4\xi\tau^{\beta+1}}{\Gamma(\beta+2)},$$

$$v_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_0}{\partial \tau}\frac{\partial^2 u_0}{\partial \xi^2} + 6\tau\right]\right],$$

$$v_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{6}{s^{\beta+2}}\right] = \frac{6\tau^{\beta+1}}{\Gamma(\beta+2)}, \tag{25}$$

$$w_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial^2 u_0}{\partial \xi^2} + \frac{\partial v_0}{\partial \xi}\frac{\partial w_0}{\partial \tau} + 4\xi\tau - 2\tau - 2\right]\right],$$

$$w_1(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{4\xi}{s^{\beta+2}} - \frac{2}{s^{\beta+2}}\right] = \frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}.$$

The subsequent terms are

$$u_2(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_0}{\partial \xi}\frac{\partial v_1}{\partial \tau} + \frac{\partial w_1}{\partial \xi}\frac{\partial v_0}{\partial \tau} + \frac{1}{2}\frac{\partial w_0}{\partial \tau}\frac{\partial^2 u_1}{\partial \xi^2} + \frac{1}{2}\frac{\partial w_1}{\partial \tau}\frac{\partial^2 u_0}{\partial \xi^2}\right]\right],$$

$$= \frac{16\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{2\tau^\beta}{\Gamma(\beta+2)},$$

$$v_2(\xi, \tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_0}{\partial \tau}\frac{\partial^2 u_1}{\partial \xi^2} + \frac{\partial w_1}{\partial \tau}\frac{\partial^2 u_0}{\partial \xi^2}\right]\right],$$

$$= \frac{8\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{4\tau^{2\beta}}{\Gamma(\beta+2)},$$

(26)

$$w_2(\xi, \tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial^2 u_1}{\partial \xi^2} + \frac{\partial v_0}{\partial \xi}\frac{\partial w_1}{\partial \tau} + \frac{\partial v_1}{\partial \xi}\frac{\partial w_0}{\partial \tau}\right]\right],$$

$$= \frac{8\xi^2\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{4\xi\tau^{2\beta}}{\Gamma(\beta+2)},$$

$$u_3(\xi, \tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_0}{\partial \xi}\frac{\partial v_2}{\partial \tau} + \frac{\partial w_1}{\partial \xi}\frac{\partial v_1}{\partial \tau} + \frac{\partial w_2}{\partial \xi}\frac{\partial v_0}{\partial \tau}\right]\right]$$

$$+ \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{1}{2}\frac{\partial w_0}{\partial \tau}\frac{\partial^2 u_2}{\partial \xi^2} + \frac{1}{2}\frac{\partial w_1}{\partial \tau}\frac{\partial^2 u_1}{\partial \xi^2} + \frac{1}{2}\frac{\partial w_2}{\partial \tau}\frac{\partial^2 u_0}{\partial \xi^2}\right]\right],$$

$$= \left(\frac{4(\beta+1)\Gamma(2\beta+2)}{\Gamma(\beta+2)\Gamma(\beta+2)\Gamma(3\beta+2)}\right)\tau^{3\beta+1} + \left(\frac{48\xi^2(\beta)\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1}$$

$$- \left(\frac{24\xi(\beta)\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1},$$

$$v_3(\xi, \tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial w_0}{\partial \tau}\frac{\partial^2 u_2}{\partial \xi^2} + \frac{\partial w_1}{\partial \tau}\frac{\partial^2 u_1}{\partial \xi^2} + \frac{\partial w_2}{\partial \tau}\frac{\partial^2 u_0}{\partial \xi^2}\right]\right],$$

(27)

$$= \left(\frac{32\xi^2\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} - \left(\frac{16\xi\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1},$$

$$w_3(\xi, \tau) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left[\frac{\partial^2 u_2}{\partial \xi^2} + \frac{\partial v_0}{\partial \xi}\frac{\partial w_2}{\partial \tau} + \frac{\partial v_1}{\partial \xi}\frac{\partial w_1}{\partial \tau} + \frac{\partial v_2}{\partial \xi}\frac{\partial w_0}{\partial \tau}\right]\right],$$

$$= \left(\frac{32\xi^3\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} - \left(\frac{16\xi^2\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1}.$$
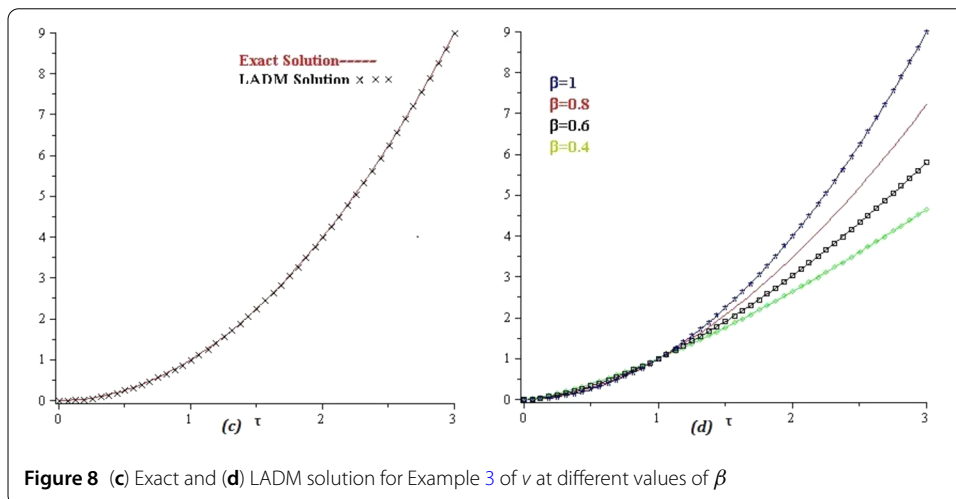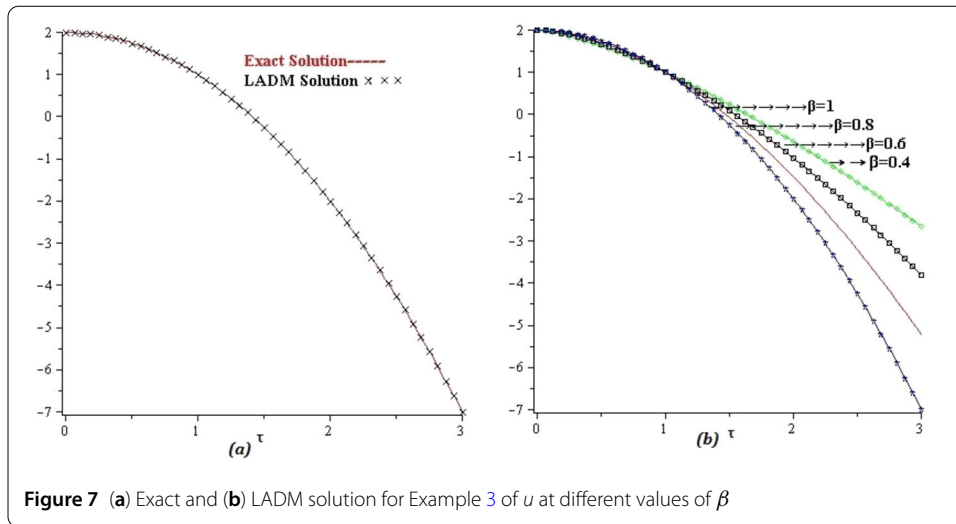
The obtained result for Example 3

$$u(\xi, \tau) = u_0(\xi, \tau) + u_1(\xi, \tau) + u_2(\xi, \tau) + u_3(\xi, \tau) + \cdots,$$

$$v(\xi, \tau) = v_0(\xi, \tau) + v_1(\xi, \tau) + v_2(\xi, \tau) + v_3(\xi, \tau) + \cdots,$$

$$w(\xi, \tau) = w_0(\xi, \tau) + w_1(\xi, \tau) + w_2(\xi, \tau) + w_3(\xi, \tau) + \cdots,$$

$$u(\xi, \tau) = \xi^2 + 1 - \frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)} + \frac{16\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{2\tau^\beta}{\Gamma(\beta+2)}$$

$$+ \left(\frac{4(\beta+1)\Gamma(2\beta+2)}{\Gamma(\beta+2)\Gamma(\beta+2)\Gamma(3\beta+2)}\right)\tau^{3\beta+1}$$

$$+ \left(\frac{48\xi^2(\beta)\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1}$$

$$- \left(\frac{24\xi(\beta)\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} + \cdots,$$

$$v(\xi, \tau) = \xi^2 - 1 + \frac{6\tau^{\beta+1}}{\Gamma(\beta+2)} + \frac{8\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{4\tau^{2\beta}}{\Gamma(\beta+2)}$$

$$+ \left(\frac{32\xi^2\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} - \left(\frac{16\xi\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} + \cdots,$$

Figure 7 (**a**) Exact and (**b**) LADM solution for Example 3 of u at different values of β

**Figure 8** (**c**) Exact and (**d**) LADM solution for Example 3 of v at different values of β

$$
w(\xi, \tau) = \xi^2 - 1 + \frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{2\tau^{\beta+1}}{\Gamma(\beta+2)} + \frac{8\xi^2\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{4\xi\tau^{2\beta}}{\Gamma(\beta+2)}
$$
$$
+ \left(\frac{32\xi^3\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} - \left(\frac{16\xi^2\beta\Gamma(2\beta)}{\Gamma(\beta+2)\Gamma(3\beta)}\right)\tau^{3\beta-1} + \cdots.
$$

When $\beta = 1$, then LADM solution is

$$
\begin{aligned}
u(\xi, \tau) &= \xi^2 - \tau^2 + 1, \\
v(\xi, \tau) &= \xi^2 + \tau^2 - 1, \\
w(\xi, \tau) &= \xi^2 - \tau^2 - 1.
\end{aligned}
\tag{28}
$$

5 Conclusion

In this research paper, a powerful analytical technique, called LADM, is applied to find the solution of some important system of fractional-order partial differential equations. The obtained results are interesting and also in good agreement towards the exact solutions.

Figure 9 (**e**) Exact and (**f**) LADM solution for Example 3 of w at different values of β
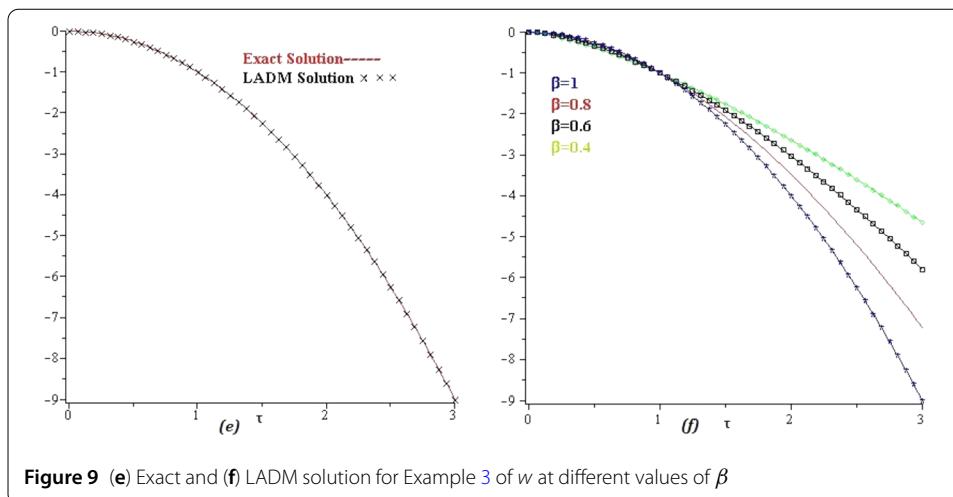
The behavior and validity of the present method is checked by taking some numerical examples. The procedure and results of LADM have shown higher accuracy of the current method as compared to other methods in literature. The convergence of fractional-order solutions towards integer-order solution can be observed from the graphs in the paper.

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