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Different Types of Progressive Wave Solutions via the 2D-Chiral Nonlinear Schrödinger Equation

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A versatile integration tool, namely the protracted (or extended) Fan sub-equation (PFS-E) technique, is devoted to retrieving a variety of solutions for different models in many fields of the sciences. This essay presents the dynamics of progressive wave solutions via the 2D-chiral nonlinear Schrödinger (2D-CNLS) equation. The solutions acquired comprise dark optical solitons, periodic solitons, singular dark (bright) solitons, and singular periodic solutions. By comparing the results gained in this work with other literature, it can be noticed that the PFS-E method is an useful technique for finding solutions to other similar problems. Furthermore, some new types of solutions are revealed that will help us better understand the dynamic behaviors of the 2D-CNLS model.

Keywords: 2D-CNLS equation, PFS-E algorithm, solitons, analytical solutions, waves structures

1. INTRODUCTION

The attainment of analytical solutions for different models described by NLPDEs plays a major role in applied mathematics, fluid mechanics, fluid dynamics, plasma and solid-state physics, nonlinear optics, and chemistry. Among these solutions, the optical solitons, which have significant applications in modern communication systems, and have attracted particular attention from physicists as well as mathematicians [1–5]. Optical solitons can propagate over extremely large distances without shape change when a balance between the linear dispersion and nonlinear effects is achieved. There are many types of solitons, including bright, dark, anti-dark, and singular solitons, amongst many others. Bright solitons exist on a zero-intensity background, while dark solitons arise as an intensity dip in an infinitely extended constant background. Moreover, dark solitons are less influenced by the perturbations, which means that dark solitons could be more preferable than bright ones in optical communication systems. The anti-dark solitons have profiles similar to those of the bright ones but exist on a nonzero background like the dark ones [6, 7]. Many effective methods have been presented to solve these equations [8–26].

The PFS-E method [27, 28] is a direct and concise method to solve nonlinear evolution equations. It is employed to find and study the wave solutions of the 2D-CNLS equation. The predominating equation is described by [29–32]:

$$i\,\Phi_t + \mu_1\left(\Phi_{xx}+\Phi_{yy}\right) + i\left(\mu_2\left(\Phi\Phi_x^* + \Phi^*\Phi_x\right) + \mu_3\left(\Phi\Phi_y^* + \Phi^*\Phi_y\right)\right)\Phi = 0, \tag{1}$$

where $\Phi = \Phi(x, y, t)$ refers to the complex-valued function, μ_1 is the second-order dispersion coefficient, and μ_2, μ_3 are the self-steepening coefficients. The 2D-CNLS equation has been established by a one-dimensional reduction of the structure that defines the fractional quantum Hall effect (it is a quantum-mechanical version of the Hall effect existing in 2D electron systems related to strong magnetic fields and low temperatures). An extraordinary characteristic of Equation (1) is the nonlinearity of the current density, which informs the new execution for the SPM and self-focusing through the current [29–32]. This equation cannot pass the Painlevè test of integrability and is not invariant under the Galilean transformation [32].

Bulut et al. [30] discussed Equation (1) in 1D and 2D and found bright and dark soliton solutions via the extended sinh-Gordon equation method. Nishino et al. [33] solved Equation (1) in 1D only and introduced two categories of wave solutions like bright and dark soliton trains. Very recently, Raza and Javid [32] investigated the singular and dark solitons for the 2D-CNLS equation by two different approaches, namely the extended direct algebraic and trial equation methods. To the best of our knowledge, no studies have found optical wave solutions for (1) via the extended Fan sub-equation method.

The paper is organized as follows. Different solutions for the 2D-CNLS equation are evaluated in section 2. The physical interpretation of the solutions is discussed in section 3. The main deductions are presented in section 4.

2. MATHEMATICAL ANALYSIS

In this section, we use the PFS-E technique to find more forms of exact solutions for Equation (1) by considering a more general transformation stated in [34, 35]. The PFS-E method includes an algebraic strategy to find different analytical solutions for NLPDEs that can be expressed as a polynomial in the variable that satisfies the general Riccati equation. The most significant achievement of this approach is that it offers all the solutions that can be found by the use of other methods such as processes using the Riccati equation, an elliptic equation of the first kind, an auxiliary ordinary equation, or the generalized Riccati equation as mapping equation.

Let the wave profile be defined as

$$\Phi(x, y, t) = e^{i\psi}\,\Omega(\xi), \tag{2}$$

while the amplitude

$$\xi = \alpha_1 x + \beta_1 y - \lambda t, \tag{3}$$

and

$$\psi = \alpha_0 x + \beta_0 y + \lambda_0 t + \eta_0. \tag{4}$$

Inserting (2) into (1) and separating its real and imaginary parts, we get

$$\delta_1\Omega - \delta_2\Omega^3 - \delta_3\Omega'' = 0, \tag{5}$$

where

$$\begin{aligned}\delta_1 &= \mu_1\left(\alpha_0^2+\beta_0^2\right)+\lambda_0,\\ \delta_2 &= 2\left(\alpha_0\mu_2+\beta_0\mu_3\right),\\ \delta_3 &= \mu_1\left(\alpha_1^2+\beta_1^2\right),\end{aligned} \tag{6}$$

and

$$\left(2\mu_1\left(\alpha_0\alpha_1+\beta_0\beta_1\right)-\lambda\right)\frac{\partial\Omega}{\partial\xi} = 0. \tag{7}$$

From (7), we have

$$\lambda = 2\mu_1\left(\alpha_0\alpha_1+\beta_0\beta_1\right). \tag{8}$$

From the homogeneous balance condition on (5), the general solution can be written as

$$\Omega = a_0 + a_1\phi(\xi), \tag{9}$$

where ϕ is given by the following auxiliary equation,

$$\left(\frac{d\phi(\xi)}{d\xi}\right)^2 = \zeta_0+\zeta_1\phi(\xi)+\zeta_2\phi^2(\xi)+\zeta_3\phi^3(\xi)+\zeta_4\phi^4(\xi), \tag{10}$$

where $\zeta_i (i = 0, 1, 2, 3, 4)$ are free parameters.
Plugging (9) and (10) into (5) and setting the coefficients of ϕ^j, $j = 0, 1, \cdots, 4$ identical zero, we get

$$\begin{aligned}a_0^3(-\delta_2)+a_0\delta_1-\frac{1}{2}a_1\delta_3\zeta_1 &= 0,\\ -3a_1a_0^2\delta_2+a_1\delta_1-a_1\delta_3\zeta_2 &= 0,\\ -3a_0a_1^2\delta_2-\frac{3}{2}a_1\delta_3\zeta_3 &= 0,\\ -a_1^3\delta_2-2a_1\delta_3\zeta_4 &= 0.\end{aligned} \tag{11}$$

Here, suitable values are selected for $\zeta_i, (i = 0, 1, 2, 3, 4)$.

$$\begin{aligned}\zeta_0 &= \zeta_0,\\ \zeta_1 &= -\frac{2\left(a_0^3\delta_1+3a_0\delta_3\right)}{3a_1\delta_2},\\ \zeta_2 &= -\frac{a_0^2\delta_1+\delta_3}{\delta_2},\\ \zeta_3 &= -\frac{2a_0a_1\delta_1}{3\delta_2},\\ \zeta_4 &= -\frac{a_1^2\delta_1}{6\delta_2},\end{aligned} \tag{12}$$

which give

$$a_0 = \frac{\sqrt{\delta_1 - \delta_3\zeta_2}}{\sqrt{3}\sqrt{\delta_2}},$$
$$a_1 = \frac{\sqrt{2}\sqrt{-\delta_3}\sqrt{\zeta_4}}{\sqrt{\delta_2}}. \tag{13}$$

Different cases [34, 35] can be introduced to obtain the following solutions.

Case I.

If $\zeta_0 = \vartheta_3^2, \zeta_1 = 2\vartheta_1\vartheta_3, \zeta_2 = 2\vartheta_2\vartheta_3 + \vartheta_1^2, \zeta_3 = 2\vartheta_1\vartheta_2, \zeta_4 = \vartheta_2^2$, we get the solution of (1) in the form $\Phi_\eta^I, (\eta = 1, 3, 5, 10, 13, 20, 24)$. A variety of significant solitons are obtained below.

Type I: when $\vartheta_1^2 - 4\vartheta_2\vartheta_3 > 0, \ \vartheta_1\vartheta_2 \neq 0, \ \vartheta_2\vartheta_3 \neq 0$. A set of dark optical solitons is acquired as

$$\Phi_1^I(\xi) = \Bigg[a_0 + a_1\Bigg(-\frac{\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\tanh\left(\frac{1}{2}\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) + \vartheta_1}{2\vartheta_2}\Bigg)\Bigg] \times e^{i\psi}. \tag{14}$$

A variety of bright-dark optical soliton is gained as

$$\Phi_3^I(\xi) = \Bigg[a_0 - \frac{a_1}{2\vartheta_2}\Bigg(\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\Bigg(i\,\mathrm{sech}\left(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) + \tanh\left(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right)\Bigg) + \vartheta_1\Bigg)\Bigg] \times e^{i\psi}. \tag{15}$$

A set of singular dark optical solitons is obtained as

$$\Phi_5^I(\xi) = \Bigg[a_0 - \frac{a_1}{2\vartheta_2}\Bigg(\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\Bigg(\tanh\left(\frac{1}{4}\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) + \coth\left(\frac{1}{4}\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right)\Bigg) + \vartheta_1\Bigg)\Bigg] \times e^{i\psi}. \tag{16}$$

The family of solitons is obtained as

$$\Phi_{10}^I(\xi) = \Bigg[a_0 + a_1\Bigg(2\cosh\left(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right)\Bigg(\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\sinh\left(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) - \Bigg(\vartheta_1\cosh\left(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) \pm i\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\Bigg)\Bigg)^{-1}\Bigg)\Bigg] \times e^{i\psi}. \tag{17}$$

Type II: when $\vartheta_1^2 - 4\vartheta_2\vartheta_3 < 0, \ \vartheta_1\vartheta_2 \neq 0, \ \vartheta_2\vartheta_3 \neq 0$. The following collections of periodic solitons are given by

$$\Phi_{13}^I(\xi) = \Bigg[a_0 + a_1\Bigg(-\frac{\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\tan\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) - \vartheta_1}{2\vartheta_2}\Bigg)\Bigg] \times e^{i\psi}, \tag{18}$$

$$\Phi_{20}^I(\xi) = \Bigg[a_0 + a_1\Bigg(-\frac{2\vartheta_3\cos\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)}{\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\sin\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) + \vartheta_1\cos\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)}\Bigg)\Bigg] \times e^{i\psi}, \tag{19}$$

$$\Phi_{24}^I(\xi) = \Bigg[a_0 + a_1\Bigg(\Bigg(4r\sin\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)\cos\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)\Bigg)\Bigg(2\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\cos^2\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) - 2\vartheta_1\sin\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)\cos\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) - \sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\Bigg)^{-1}\Bigg)\Bigg] \times e^{i\psi}. \tag{20}$$

Case II.

If $\zeta_0 = \vartheta_3^2, \zeta_1 = 2\vartheta_1\vartheta_3, \zeta_2 = 0, \zeta_3 = 2\vartheta_1\vartheta_2, \zeta_4 = \vartheta_2^2$, we get the solution of (1) in the form $\Phi_\eta^{II}, (\eta = 1, 5)$. A collection of dark optical solitons is given by

$$\Phi_1^{II}(\xi) = \left[a_0 + a_1\left(-\frac{\sqrt{-6\vartheta_2\vartheta_3}\tanh\left(\frac{1}{2}\xi\sqrt{-6\vartheta_2\vartheta_3}\right) + \sqrt{-2\vartheta_2\vartheta_3}}{2\vartheta_2}\right)\right] \times e^{i\psi}. \tag{21}$$

Also, a different shape of singular dark optical soliton is acquired

$$\Phi_5^{II}(\xi) = \left[a_0 + a_1\left(-\frac{\sqrt{-6qr}\left(\tanh\left(\frac{1}{4}\xi\sqrt{-6qr}\right) + \coth\left(\frac{1}{4}\xi\sqrt{-6qr}\right)\right) + 2\sqrt{-2qr}}{4q}\right)\right] \times e^{i\psi}, \tag{22}$$

Case III.

If $\zeta_0 = \zeta_1 = 0$, we get the solution of (1) in the form $\Phi_\eta^{III}, (\eta = 1, 2, 3, 4, 6, 8, 9)$.

Type I: $\zeta_2 = 1, \zeta_3 = \frac{-2\lambda_3}{\lambda_1}, \zeta_4 = \frac{\lambda_3^2-\lambda_2^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

$$\Phi_1^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{sech}(\xi)}{\lambda_2 \operatorname{sech}(\xi) + \lambda_3}\right)\right] \times e^{i\psi}. \tag{23}$$

Type II: $\zeta_2 = 1, \zeta_3 = \frac{-2\lambda_3}{\lambda_1}, \zeta_4 = \frac{\lambda_3^2+\lambda_2^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

$$\Phi_2^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}(\xi)}{\lambda_2 \operatorname{csch}(\xi) + \lambda_3}\right)\right] \times e^{i\psi}. \tag{24}$$

In particular, if we take $\lambda_2 = 0$ in the above Equations (23), (24), we get

$$\Phi_1^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{sech}(\xi)}{\lambda_3}\right)\right] \times e^{i\psi}. \tag{25}$$

$$\Phi_2^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}(\xi)}{\lambda_3}\right)\right] \times e^{i\psi}. \tag{26}$$

Type III: $\zeta_2 = 4, \zeta_3 = -\frac{4(2\lambda_2+\lambda_4)}{\lambda_1}, \zeta_4 = \frac{4\lambda_2^2+4\lambda_4\lambda_2+\lambda_3^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are arbitrary constants.

$$\Phi_3^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{sech}^2(\xi)}{\lambda_2 \tanh(\xi) + \lambda_3 + \lambda_4 \operatorname{sech}^2(\xi)}\right)\right] \times e^{i\psi}. \tag{27}$$

Type IV: $\zeta_2 = 4, \zeta_3 = \frac{4(\lambda_4-2\lambda_2)}{\lambda_1}, \zeta_4 = \frac{4\lambda_2^2-4\lambda_4\lambda_2+\lambda_3^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are arbitrary constants.

$$\Phi_4^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}^2(\xi)}{\lambda_2 \coth(\xi) + \lambda_3 + \lambda_4 \operatorname{csch}^2(\xi)}\right)\right] \times e^{i\psi}. \tag{28}$$

In particular, if we consider $\lambda_2 = \lambda_4$, we have another solution as

$$\Phi_4^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}^2(\xi)}{\lambda_2 \coth(\xi) + \lambda_3 + \lambda_2 \operatorname{csch}^2(\xi)}\right)\right] \times e^{i\psi}. \tag{29}$$

Type V: $\zeta_2 = -1, \zeta_3 = \frac{2\lambda_3}{\lambda_1}, \zeta_4 = \frac{\lambda_3^2-\lambda_2^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

$$\Phi_6^{III}(\xi) = \left[a_0 + a_1\left(-\frac{\lambda_1(\sinh(\lambda_1\xi) + \cosh(\lambda_1\xi))(\sinh(\lambda_1\xi) + \cosh(\lambda_1\xi) + \lambda_2)}{\lambda_3}\right)\right] \times e^{i\psi}. \tag{30}$$

Type VI: $\zeta_2 = 4, \zeta_3 = \frac{-2\lambda_3}{\lambda_1}, \zeta_4 = \frac{\lambda_3^2-\lambda_2^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3$ are arbitrary constants.

$$\Phi_8^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \csc(\xi)}{\lambda_2 \csc(\xi) + \lambda_3}\right)\right] \times e^{i\psi}. \tag{31}$$

Type VII: $\zeta_2 = -4, \zeta_3 = \frac{4(2\lambda_2+\lambda_4)}{\lambda_1}, \zeta_4 = -\frac{4\lambda_2^2+4\lambda_4\lambda_2-\lambda_3^2}{\lambda_1^2}$, where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are arbitrary constants.

$$\Phi_9^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \sec^2(\xi)}{\lambda_2 \tan(\xi) + \lambda_3 + \lambda_4 \sec^2(\xi)}\right)\right] \times e^{i\psi}. \tag{32}$$

Case IV.

If $\zeta_1 = \zeta_3 = 0$, we get the solution of (1) in the form $\Phi_\eta^{IV}, (\eta = 3, 13)$.

For $\zeta_0 = \frac{1}{4}, \zeta_2 = \frac{1-2m^2}{2}, \zeta_4 = \frac{1}{4}$, the solution of (1) of the form

$$\Phi_3^{IV}(\xi) = \left[a_0 + a_1(cn\xi)\right] \times e^{i\psi}, \tag{33}$$

leads to the bright optical soliton when $m \to 1$,

$$\Phi_3^{IV}(\xi) = \left[a_0 + a_1 \operatorname{sech}(\xi)\right] \times e^{i\psi}, \tag{34}$$

and the singular periodic solutions when $m \to 0$,

$$\Phi_3^{IV}(\xi) = \left[a_0 + a_1 \cos(\xi)\right] \times e^{i\psi}, \tag{35}$$

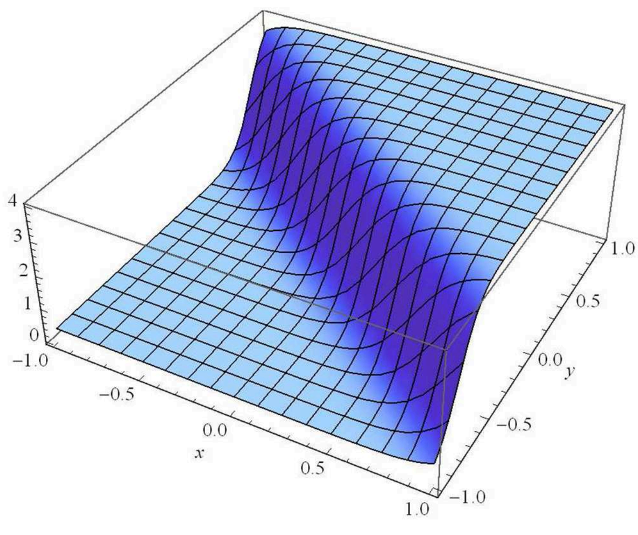

FIGURE 1 | $|\Phi(x,y,t)|_1^{II}$: ϑ_1=1, ϑ_2= −1, ϑ_3 = 1, μ_1 = 1, μ_2 = 1, μ_3 = 1, α_0 = 1, β_0 = 2, λ_0 = 3, α_1 = 4, β_1 = 5, λ = 6, η_0 = 1, t = 0.01.

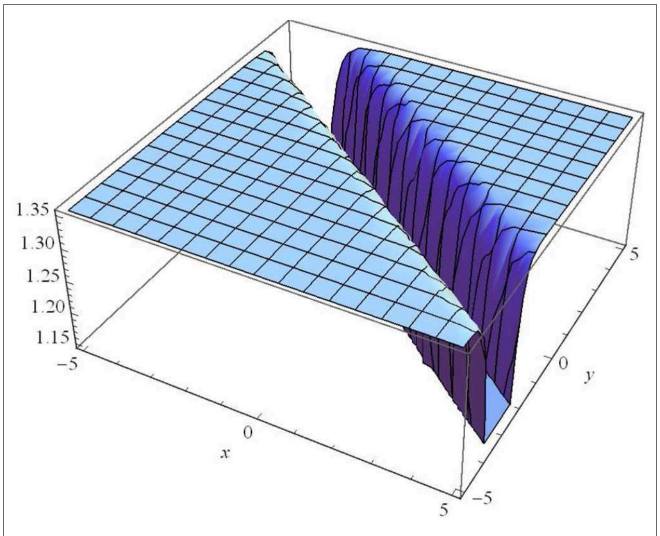

FIGURE 3 | $|\Phi(x,y,t)|_1^{III}$: λ_1 = 1, λ_2 = 2, λ_3 = 3, μ_1 = 1, μ_2 = 1, μ_3 = 1, α_0 = 1, β_0 = 2, λ_0 = 3, α_1 = 4, β_1=5, λ = 6, η_0 = 1, t = 1.

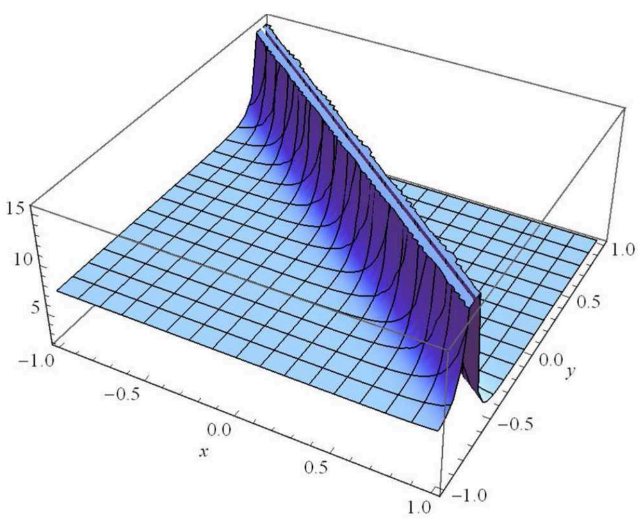

FIGURE 2 | $|\Phi(x,y,t)|_5^{II}$: ϑ_1=1, ϑ_2 = −1, ϑ_3 = 1, μ_1 = 1, μ_2 = 1, μ_3 = 1, α_0 = 1, β_0 = 2, λ_0 = 3, α_1 = 4, β_1 = 5, λ = 6, η_0 = 1, t = 0.1.

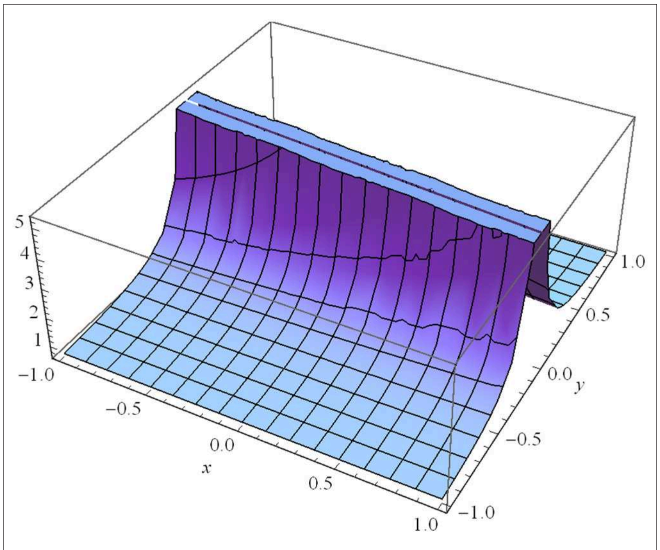

FIGURE 4 | $|\Phi(x,y,t)|_{13}^{IV}$ for $m \to 1$: μ_1 = 1, μ_2 = 1, μ_3 = 2, α_0 = 1, β_0 = 2, λ_0 = 3, α_1 = 0.1, β_1 = 1, λ = 0.5, η_0 = 1, t = 0.1.

For $\zeta_0 = \frac{1}{4}, \zeta_2 = \frac{1-2m^2}{2}, \zeta_4 = \frac{1}{4}$, the solution of (1) of the form

$$\Phi_{13}^{IV}(\xi) = \left[a_0 + a_1(ns\xi \pm cs\xi)\right] \times e^{i\psi}, \qquad (36)$$

leads to a collection of singular dark solutions when $m \to 1$,

$$\Phi_{13}^{IV}(\xi) = \left[a_0 + a_1(\coth(\xi) + \operatorname{csch}(\xi))\right] \times e^{i\psi}, \qquad (37)$$

and singular periodic solutions when $m \to 0$,

$$\Phi_{13}^{IV}(\xi) = \left[a_0 + a_1(\cot(\xi) + \csc(\xi))\right] \times e^{i\psi}. \qquad (38)$$

3. RESULTS AND DISCUSSION

In this part, the physical aspects of the solutions obtained are discussed by means of graphical 3D representations. **Figures 1–4** show different categories of background for soliton solutions classified into dark or singular soliton solutions.

Figure 1 illustrates $|\Phi(x,y,t)|_1^I$ established in Case I (Type I) for $\vartheta_1=1$, $\vartheta_2=-1$, $\vartheta_3=1$, $\mu_1=1$, $\mu_2=1$, $\mu_3=1$, $\alpha_0=1$, $\beta_0=2$, $\lambda_0=3$, $\alpha_1=4$, $\beta_1=5$, $\lambda=6$, $\eta_0=1$, $t=0.01$.

Moreover, **Figure 2** illustrates $|\Phi(x,y,t)|_5^{II}$ found in Case II for $\vartheta_1=1$, $\vartheta_2=-1$, $\vartheta_3=1$, $\mu_1=1$, $\mu_2=1$, $\mu_3=1$, $\alpha_0=1$, $\beta_0=2$, $\lambda_0=3$, $\alpha_1=4$, $\beta_1=5$, $\lambda=6$, $\eta_0=1$, $t=0.1$.

Similarly, **Figure 3** illustrates $|\Phi(x,y,t)|_1^{III}$ found in Case III (Type I) for $\lambda_1=1$, $\lambda_2=2$, $\lambda_3=3$, $\mu_1=1$, $\mu_2=1$, $\mu_3=1$, $\alpha_0=1$, $\beta_0=2$, $\lambda_0=3$, $\alpha_1=4$, $\beta_1=5$, $\lambda=6$, $\eta_0=1$, $t=1$. Similarly, **Figure 4** expresses $|\Phi(x,y,t)|_{13}^{IV}$ observed in Case IV ($m\to 1$) for $\mu_1=1$, $\mu_2=1$, $\mu_3=2$, $\alpha_0=1$, $\beta_0=2$, $\lambda_0=3$, $\alpha_1=0.1$, $\beta_1=1$, $\lambda=0.5$, $\eta_0=1$, $t=0.1$.

Figure 1 represents the absolute value of the complex wave solution given by Equation (14). We observe that this solution is a dark (or kink) soliton wave propagating along the y-axis. The kink wave is an essential aspect of numerous physical phenomena containing self-reinforcing, impulsive systems, and reaction-diffusion-advection. It is clear that there is a transmission of the dark soliton with invariant amplitude (without any gain or loss) in the homogeneous medium of motion. Due to the homogeneous (constant) coefficients of Equation (1), we cannot provide a possible way to control the propagation of the dark solitons in this medium. **Figures 2–4** represent the absolute value of the complex wave solutions given by Equations (22), (23), and (37), respectively. We observe that these solutions are singular solitons that can be linked to a solitary wave when its center becomes an imaginary position. Furthermore, It is clear that their intensity gets Stronger, and consequently, they are not stable. These solutions have a cusp, which may lead to the formation of Rogue waves.

4. CONCLUSIONS

Herein, a large set of new analytical solutions with different wave structures of the 2D-CNLS equation has been reproduced with the aid of the PFS-E technique. As a positive result, a wide variety of unprecedented exact solutions were gained in an easy manner. Our study presents whether the suggested approach is trustworthy in treatment NLPDEs to promote a variety of exact solutions. Finally, we have plotted some 3D graphs of these solutions and have shown that these graphs can be controlled by adjusting the parameters. According to our knowledge, the obtained solutions are likely to provide a useful supplement to the existing literature on nonlinear optics.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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