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Complex traveling-wave and solitons solutions to the Klein-Gordon-Zakharov equations

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ABSTRACT

This paper studies complex solutions and solitons solutions to the Klein-Gordon-Zakharov equations (KGZEs). Solitons solutions including bright, dark, W-shape bright, breather also trigonometric function solutions and singular solutions of KGZEs are obtained by three integration algorithm. From the spatio-temporal and 3-D and 2-D contour plot, it is observed that obtained solutions move without any deformation that implies the steady state of solutions. Furthermore, these solutions will be helpful to explain the interactions in hight frequency plasma and solitary wave theory.

1. Introduction

Majority of physical phenomena appearing such as in fluid dynamics, plasma, chemistry, nonlinear fibers optics are described by nonlinear differential equations (NLDEs). Specific solutions of nonlinear ordinary differential equations (NODEs) are widely used in science, engineering and in other fields of technology. Nevertheless, exact solutions of nonlinear evolution equations, namely; the Kolmogorov-Petrovskii-Piskunov equation[1], the (3 + 1) dimensional Jimbo-Miwa equation [2], the two-dimensional Korteweg-de Vries-Burgers equations [3], the fractional Lane-Emden-type equations [4], the (2 + 1)-dimensional Yu-Toda-Sasa-Fukuyama equation [5], the (2 + 1)-dimensions Hirota-Satsuma-Ito equation [6,7], the three-Component Coupled modified KdV System [8] have been constructed. These solutions hold a significant place in nonlinear science. In the literature, a lot of effort have been proposed to build exact traveling-wave solutions of the NODEs. Some of relevant methods may be listed as auxiliary equation method, Sine-Gordon expansion method, F-expansion method, Jacobi elliptic function method, modified direct algebraic equation method, the tanh method. Interested reader may look at the references in [9–20] for the details of these studies. In the references [9–20], authors investigated the solitons, exact solutions and several interesting properties of KGZEs and some other evolution equations. In [2], Ma obtained most of the methods described in the references between [9–20] by means of method of transformed rational function. In [8], Ma also acquired some important results in computing limiting behaviours of solutions incorporating features of solitons and analytical solutions.

Solitary wave solutions of KGZEs [21] were obtained by generalized Kudryashov method. As a result of this, a lot of attention in various branches such as biology, plasma physics, optic fibers have been focused to the KGZ model. KGZEs describe interactions between Langmiur wave and ion acoustic wave in hight frequency plasma [22]. Many research work have been done to build solitary wave solutions, topological solitons, bifurcation analysis, trigonometric functions solutions, Jacobi elliptic function solutions to the KGZEs [23–27].

We investigate complex traveling-wave solutions and solitons solutions to the KGZEs [13].

$$\phi_{tt} - \phi_{xx} + \phi + \alpha\psi\phi = 0, \tag{1}$$

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$$\psi_{tt} - \psi_{xx} = \beta(|\phi|^2)_{xx}, \tag{2}$$

$\phi(x, t)$ is a complex function and $\psi(x, t)$ a real function, t represents the time, x is the distance along the direction of propagation. The parameters α and β are non zero valued.

In order to perform these, our paper consists of following sections. In Section 2, by applying the traveling-wave hypothesis, we obtain the nonlinear differential equations. In Section 3, three integration technique are used to investigated complex and solitons solutions to (1)-(2). Thereafter, in Section 4 graphical illustration of the obtained results is presented. The last section will deal with summary of the work.

2. Traveling-wave solutions to Klein-Gordon-Zhakharov equation

To investigate exact solutions to the KGZEs (1)-(2), we assume the complex function $\phi(x, t)$ as:

$$\phi(x, t) = u(x, t) + iv(x, t), \tag{3}$$

where $u(x, t)$ and $v(x, t)$ are two real functions, that will be determined. Let us notice that

$$|\phi|^2 = u^2 + v^2, \quad \phi_{xx} = u_{xx} + iv_{xx}, \quad \phi_{tt} = u_{tt} + iv_{tt}. \tag{4}$$

Inserting (3) and (4) into (1–2), we get

$$u_{tt} + iv_{tt} - (u_{xx} + iv_{xx}) + (u + iv) + \alpha\psi(u + iv) = 0, \tag{5}$$

and

$$\psi_{tt} - \psi_{xx} - \beta(u^2 + v^2)_{xx} = 0. \tag{6}$$

Splitting the real and imaginary, we obtain

$$\begin{aligned} &u_{tt} - u_{xx} + u + \alpha\psi u = 0,\\ &v_{tt} - v_{xx} + v + \alpha\psi v = 0,\\ &\psi_{tt} - \psi_{xx} - \beta(u^2 + v^2)_{xx} = 0. \end{aligned} \tag{7}$$

To seek the exact solutions of (7), we assume $u(x, t) = v(x, t)$. Consequently (7) turns to be

$$\begin{aligned} &u_{tt} - u_{xx} + u + \alpha\psi u = 0,\\ &\psi_{tt} - \psi_{xx} - 2\beta(u^2)_{xx} = 0. \end{aligned} \tag{8}$$

To obtain the ordinary differential equation (ODE) to Eq. (8), we apply the traveling-wave transformation as in the form:

$$u(x, t) = \varphi(\xi), \quad \xi = \kappa(x - ct), \tag{9}$$

It is revealed that

$$\psi(x, t) = \frac{\kappa^2(1 - c^2)\varphi''}{\alpha\varphi} - \frac{1}{\alpha}, \tag{10}$$

Suppose that

$$\psi(x, t) = w(\xi) = w(\kappa(x - ct)). \tag{11}$$

Next, inserting (11) into (8), and integrating it twice with respect to ξ, we get

$$w(\xi) = \frac{2\beta}{(c^2 - 1)}\varphi^2 + a_0. \tag{12}$$

where a_0 is an integration constant. Equating (12) and (10) gives

$$\varphi'' + \frac{(1 + \alpha a_0)}{\kappa^2(c^2 - 1)}\varphi + \frac{2\alpha\beta}{\kappa^2(c^2 - 1)^2}\varphi^3 = 0. \tag{13}$$

To unearth new exact solution and solitary waves solution to (13), the next section will present three interesting analytical methods.

2.1. *On solving the KGZE by the auxiliary equation method*

The analytical solution of the KGZE in this section can be expressed as follows [24,28,29]

$$\varphi(\xi) = A_0 + \sum_{j=1}^{N} A_j(g(\xi))^j, \tag{14}$$

where $A_j (j = 1, 2, 3, ...,N)$ are reals parameters to determine, while $g(\xi)$ satisfies the following ordinary differential equation

$$g_\xi = \sqrt{2(C_0 + C_1 g + C_2 g^2 + C_3 g^3 + C_4 g^4)}, \tag{15}$$

$$g_{\xi\xi} = C_1 + 2C_2 g + 3C_3 g^2 + 4C_4 g^3, \tag{16}$$

and $g_\xi = \frac{\partial g}{\partial \xi}$, $C_i(i = (0, 1, 2, 3, 4)), A_0, A_i, i = (1, 2, ...,n)$, are reals constants to be determined later. Using the homogeneous balance principle between higher order nonlinear term (φ^3) and higher derivative term φ'', we get $N = 1$. Therefore, (14) reads as follows:

$$\varphi(\xi) = A_0 + A_1 g(\xi), \tag{17}$$

Inserting (16) and (15) into (13) yields the system of equation in terms of $(g(\xi))^j$. Solving the obtained system of equations with the help of MAPLE, we obtain:

- Set 1: $A_0 = 0,\ A_1 = A_1,\ C_2 = -\frac{1}{2}\frac{\alpha a_0 + 1}{\kappa^2(c^2 - 1)},\ C_4 = -\frac{1}{2}\frac{\alpha\beta A_1^2}{\kappa^2(c - 1)^2(c + 1)^2},\ a_0 = a_0.$

C ase 1: For $C_0 = C_1 = C_3 = 0$, and $C_2 < 0,\ C_4 > 0$. From Set 1, complex trigonometric function solutions and singular function solutions to (1)-(2) are obtained:

$$\phi_{A,11}(x, t) = A_1(1 + i)\sqrt{\frac{-C_2}{C_4}}\sec(\sqrt{-2C_2}\kappa(x - ct)), \tag{18}$$

$$\psi_{A,11}(x, t) = \frac{2\beta}{(c^2 - 1)}\left\{A_1\sqrt{\frac{-C_2}{C_4}}\sec(\sqrt{-2C_2}\kappa(x - ct))\right\}^2 + a_0, \tag{19}$$

$$\phi_{A,12}(x, t) = A_1(1 + i)\sqrt{\frac{-C_2}{C_4}}\csc(\sqrt{-2C_2}\kappa(x - ct)), \tag{20}$$

$$\psi_{A,12}(x, t) = \frac{2\beta}{(c^2 - 1)}\left\{A_1\sqrt{\frac{-C_2}{C_4}}\csc(\sqrt{-2C_2}\kappa(x - ct))\right\}^2 + a_0, \tag{21}$$

C ase 2: For $C_0 = \frac{C_2^2}{4C_4}$, and $C_2 > 0,\ C_4 > 0$. From Set 2, complex trigonometric function solutions to (1)-(2) are obtained:

$$\phi_{A,13}(x, t) = A_1(1 + i)\sqrt{\frac{C_2}{2C_4}}\tan(\sqrt{C_2}\kappa(x - ct)), \tag{22}$$

$$\psi_{A,13}(x, t) = \frac{2\beta}{(c^2 - 1)}\left\{A_1\sqrt{\frac{C_2}{2C_4}}\tan(\sqrt{C_2}\kappa(x - ct))\right\}^2 + a_0. \tag{23}$$

- Set 2: $A_0 = A_0,\ A_1 = A_1,\ C_2 = -\frac{2\alpha\beta A_0^2}{\kappa^2(c - 1)^2(c + 1)^2},\ C_3 = -\frac{2A_0\alpha\beta A_1}{\kappa^2(c - 1)^2(c + 1)^2},\ C_4 = -\frac{1}{2}\frac{\alpha\beta A_1^2}{\kappa^2(c - 1)^2(c + 1)^2},\quad a_0 = -\frac{2\alpha\beta A_0^2 + c^2 - 1}{\alpha(c - 1)(c + 1)}.$

Case 3: For $C_0 = C_1 = 0\ C_2 > 0\ C_4 > 0$, it is revealed from set 3 that:

$$\phi_{A,14}(x, t) = A_0(1 + i) + A_1(1 + i)\frac{C_2 sech^2(\sqrt{2C_2}\frac{\kappa(x - ct)}{2})}{2\sqrt{C_2C_4}\tanh(\sqrt{2C_2}\frac{\kappa(x - ct)}{2}) - C_3}, \tag{24}$$

$$\psi_{A,14}(x, t) = \frac{2\beta}{(c^2 - 1)}\left\{A_0 + A_1\frac{C_2 sech^2(\sqrt{2C_2}\frac{\kappa(x - ct)}{2})}{2\sqrt{C_2C_4}\tanh(\sqrt{2C_2}\frac{\kappa(x - ct)}{2}) - C_3}\right\}^2 + a_0, \tag{25}$$

Case 4: For $C_0 = C_1 = 0$, and $C_2 > 0,\ C_3^2 - 4C_2C_4 > 0$, using set 4 it is revealed that

$$\phi_{A,15}(x,t) = A_0(1+i) + A_1(1+i)\frac{2C_2 sech(\sqrt{2C_2}(x-vt))}{\sqrt{C_3^2-4C_2C_4} - C_3 sech(\sqrt{2C_2}\kappa(x-ct))}, \quad (26)$$

$$\psi_{A,15}(x,t) = \frac{2\beta}{(c^2-1)}\left\{A_0 + A_1\frac{2C_2 sech(\sqrt{2C_2}\kappa(x-ct))}{\sqrt{C_3^2-4C_2C_4} - C_3 sech(\sqrt{2C_2}\kappa(x-ct))}\right\}^2 + a_0, \quad (27)$$

Case 5: For $C_0 = C_1 = 0$, and $C_2 > 0$, it is obtained that

$$\phi_{A,16}(x,t) = A_0(1+i) + A_1(1+i)\frac{C_2C_3 sech^2(\sqrt{2C_2}\frac{\kappa(x-ct)}{2})}{C_2C_4(1-\tanh(\sqrt{2C_2}\frac{\kappa(x-ct)}{2}))^2 - C_3^2}. \quad (28)$$

$$\psi_{A,16}(x,t) = \frac{2\beta}{(c^2-1)}\left\{A_0 + A_1\frac{C_2C_3 sech^2(\sqrt{2C_2}\frac{\kappa(x-ct)}{2})}{C_2C_4(1-\tanh(\sqrt{2C_2}\frac{\kappa(x-ct)}{2}))^2 - C_3^2}\right\}^2 + a_0. \quad (29)$$

Figs. 1–6 depicted analytical solutions 3-D and 2-D of Eqs. (28) and (29). This illustrated the stability of the bright and dark solitons which are candidates for data transmission in thousand kilometers. The other obtained solutions are periodic and hyperbolic function solutions. It is also observed that the results depicted depend on the free parameters of the GKZ equation (see Figs. 1 and 2). We also remark that solitons solution Eqs. (28 and 29) can either have one-bright or multi-bright (one dark or multi dark) intensity profiles which depend on the free parameter (α, see Figs. (2 and 3) or β) see Figs. (1 and 2) of the GKZ equation. In the other observing Figs. 5 and 6, when the value of the free parameter β) increase, the one-bright soliton period increase (see Fig. 5). However, the value of free parameter (α is considered as small. In addition to these, when the value of free parameter (α of GKZ equation is increasing, the opposite phenomenon occurs (the one bright soliton periodicity is reduced (see Fig. 6.)

2.2. On solving the Klein-Gordon-Zhakharov equation by the Sine-Gordon expansion approach

To investigate complex traveling wave and soliton solutions to (1)-(2), the sine-Gordon expansion approach will be considered in the following expression [23,24]:

$$\varphi(\xi) = \sum_{j=1}^{n} \cos(s)^{j-1}(B_j\sin(s) + A_j\cos(s)) + A_0. \quad (30)$$

and

$$\sin(s(\xi)) = sech(\xi), \quad or \quad \cos(s(\xi)) = tanh(\xi), \quad (31)$$

$$\sin(s(\xi)) = icsch(\xi), \quad or \quad \cos(s(\xi)) = coth(\xi). \quad (32)$$

Employing the balance principle between the higher nonlinear term and the higher derivative term, it is recovered to (13) that the integer $N = 1$.

Thereafter, substituting the valued of integer N, into (36), turns to

$$\varphi(\xi) = A_0 + B_1\sin(s) + A_1\cos(s) \quad (33)$$

Substituting (39) into (13), it is obtained the set system of equation. Solving the set of system of algebraic equations with help of MAPLE, we obtain the following results.

- Set 1: $A_0 = 0,\ A_1 = \pm\sqrt{-\frac{1}{\alpha\beta}}(c^2-1)\kappa,\ B_1 = 0,\ a_0 = \frac{2\kappa^2(c^2-1)-1}{\alpha}$.
- Set 2: $A_0 = 0,\ A_1 = 0,\ B_1 = \pm\sqrt{\frac{1}{\alpha\beta}}(c^2-1)\kappa,\ a_0 = -\frac{\kappa^2(c^2-1)+1}{\alpha}$.

To use (38) and considering the obtained results above, it is revealed the complex solitons solutions to (1)-(2) as

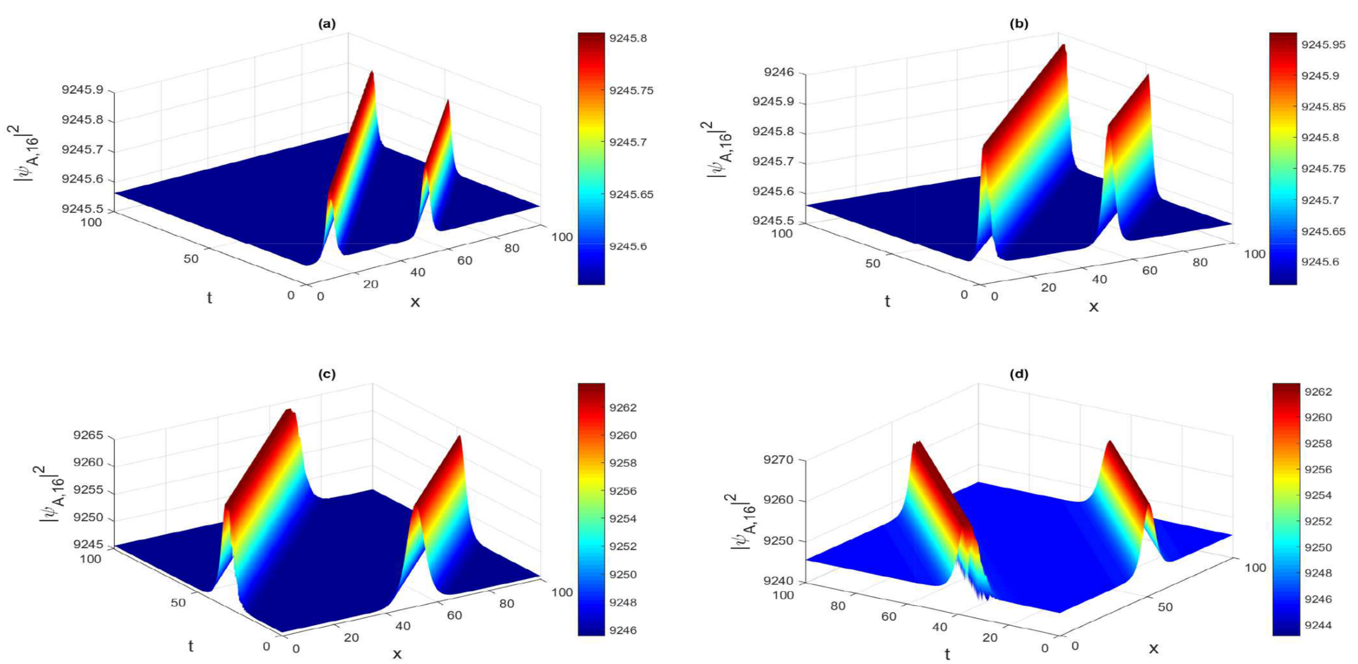

Fig. 1. Spatio-temporal plot of $|\phi_{A,16}|^2$ (28) for (a) [$c = 1.0005$, $C_2 = 0, 919$, $C_3 = 0.00356$, $C_4 = 0.0035$, $\kappa = -1.8007$, $a_0 = 96.24$, $A_0 = 0.00814$, $A_1 = 0.9120$, $\beta = -1.007$, $\alpha = 0.0104$, $\kappa = 1$], (b) [$c = 1.0005$, $C_2 = 0, 919$, $C_3 = 0.00356$, $C_4 = 0.0035$, $\kappa = -1.8007$, $a_0 = 96.24$, $A_0 = 0.00814$, $A_1 = 0.9120$, $\beta = -1.017$, $\alpha = 0.104$, $\kappa = 1$], , (c) [$c = 1.0005$, $C_2 = 0, 919$, $C_3 = 0.00356$, $C_4 = 0.0035$, $\kappa = -1.8007$, $a_0 = 96.24$, $A_0 = 0.00814$, $A_1 = 0.9120$, $\beta = -1.027$, $\alpha = 0.204$, $\kappa = 1$], (d) [$c = 1.0005$, $C_2 = 0, 919$, $C_3 = 0.00356$, $C_4 = 0.0035$, $\kappa = -1.8007$, $a_0 = 96.24$, $A_0 = 0.00814$, $A_1 = 0.9120$, $\beta = -1.047$, $\alpha = 0.304$, $\kappa = 1$], respectively.

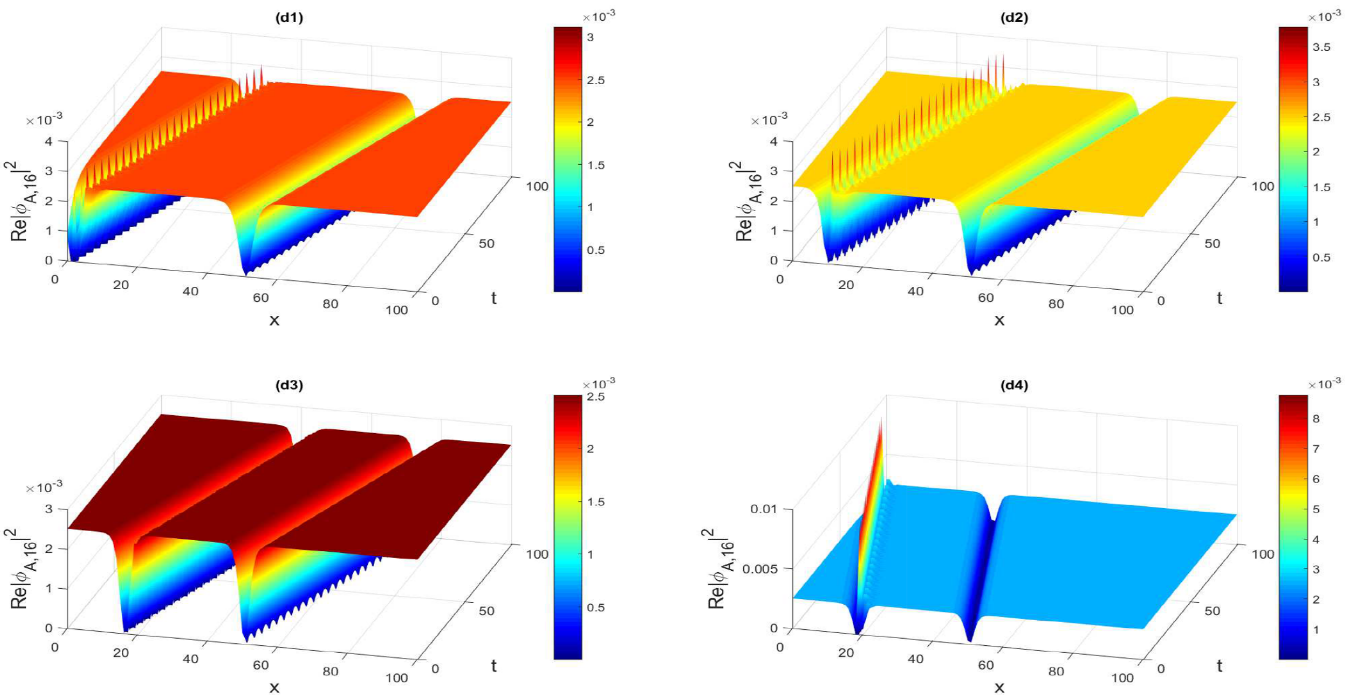

Fig. 2. Spatio-temporal plot of $|\Re\phi_{A,16}|^2$ (29) for (d1) [$c = 0.24$, $C_2 = 274.5035$, $C_3 = 4.9970$, $C_4 = 2.2741$, $\kappa = 0.0401$, $A_0 = 0.0501$, $A_1 = 0.9120$, $\beta = -5.2024$, $\alpha = 10.0106$], (d2) [$c = 0.24$, $C_2 = 474.5035$, $C_3 = 5.9970$, $C_4 = 2.2741$, $\kappa = 0.0401$, $A_0 = 0.0501$, $A_1 = 0.9120$, $\beta = -5.2024$, $\alpha = 10.0106$], (d3) [$c = 0.24$, $C_2 = 365.5035$, $C_3 = 6.06$, $C_4 = 3.031$, $\kappa = 0.0401$, $A_0 = 0.0501$, $A_1 = 0.9120$, $\beta = -5.2024$, $\alpha = 20.0106$], (d4) [$c = -0.2$, $C_2 = 4400.7$, $C_3 = 8.02$, $C_4 = 3.65$, $\kappa = 0.0401$, $A_0 = 0.0501$, $A_1 = 0.9120$, $\beta = -5.2024$, $\alpha = 25.0106$], respectively.

$$\phi_{B,11}(x,t) = \pm(1+i)\sqrt{-\frac{1}{\alpha\beta}}(c^2-1)\kappa\tanh(\kappa(x-ct)), \tag{34}$$

$$\psi_{B,11}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\sqrt{-\frac{1}{\alpha\beta}}(c^2-1)\kappa\tanh(\kappa(x-ct))\right\}^2 + \frac{2\kappa^2(c^2-1)-1}{\alpha}. \tag{35}$$

$$\phi_{B,12}(x,t) = \pm(1+i)\sqrt{-\frac{1}{\alpha\beta}}(c^2-1)\kappa\coth(\kappa(x-ct)), \tag{36}$$

$$\psi_{B,12}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\sqrt{-\frac{1}{\alpha\beta}}(c^2-1)\kappa\coth(\kappa(x-ct))\right\}^2 + \frac{2\kappa^2(c^2-1)-1}{\alpha}. \tag{37}$$

$$\phi_{B,13}(x,t) = \pm(1+i)\sqrt{\frac{1}{\alpha\beta}}(c^2-1)\kappa\, sech(\kappa(x-ct)), \tag{38}$$

$$\psi_{B,13}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\sqrt{\frac{1}{\alpha\beta}}(c^2-1)\kappa\, sech(\kappa(x-ct))\right\}^2 - \frac{\kappa^2(c^2-1)+1}{\alpha} \tag{39}$$

$$\phi_{B,14}(x,t) = \pm(1+i)\sqrt{\frac{1}{\alpha\beta}}(c^2-1)\kappa\, i\, csch(\kappa(x-ct)), \tag{40}$$

$$\psi_{B,14}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\sqrt{\frac{1}{\alpha\beta}}(c^2-1)\kappa\, i\, csch(\kappa(x-ct))\right\}^2 - \frac{\kappa^2(c^2-1)+1}{\alpha} \tag{41}$$

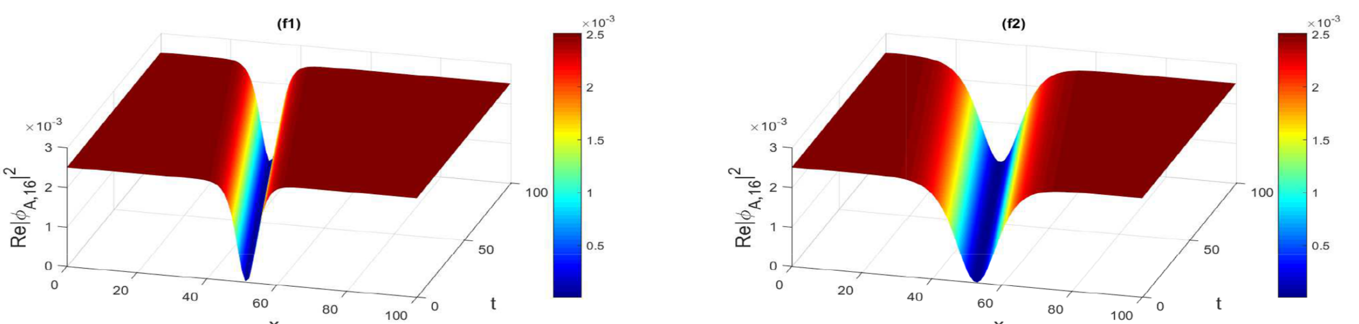

Fig. 3. Spatio-temporal plot of $|\Re\phi_{A,16}|^2$ (28) for (f1) [$c = -0.2$, $C_2 = 88.30$, $C_3 = 1.60$, $C_4 = 7.31$, $\kappa = 0.0401$, $A_0 = 0.05$, $A_1 = 0.912$, $\beta = -5.2024$, $\alpha = 5.01$], (f2) [$c = 0.24$, $C_2 = 21.88$, $C_3 = 398.34$, $C_4 = 1.81$, $\kappa = 0.0401$, $A_0 = 0.0501$, $A_1 = 0.9120$, $\beta = -1.2$, $\alpha = 10.0106$], respectively.

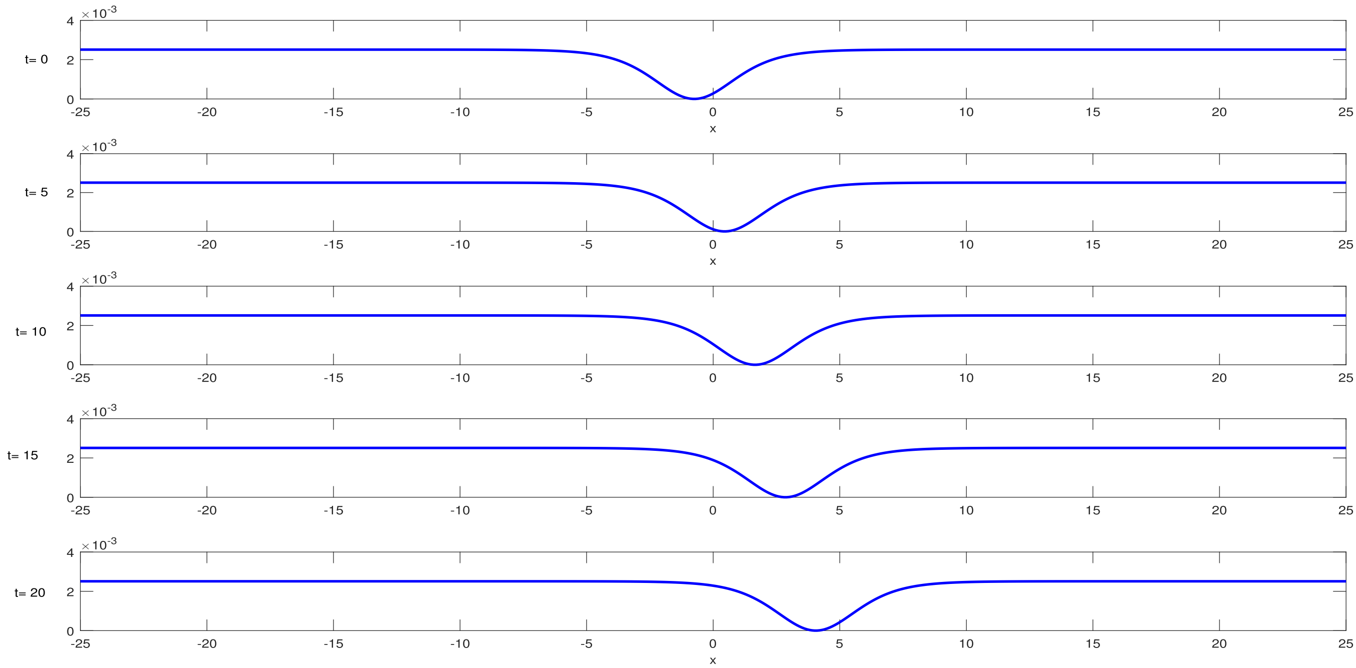

Fig. 4. Spatio-temporal plot of $|\Re\phi_{A,16}|^2$ (29) $c = 0.24$, $C_2 = 1.23$, $C_3 = 2.250$, $C_4 = 1.024$, $\varkappa = 0.0401$, $A_0 = 0.0501$, $A_1 = 0.912$, $\beta = -70.20$, $\alpha = 5.01$ for $-25 \leqslant x \leqslant 25$ and $t = 0$, $t = 5$, $t = 10$, $t = 15$, $t = 20$ respectively.

2.3. ***On solving the Klein-Gordon-Zhakharov equation by the extended rational sine-cosine method.***

We adopt the efficient technique, namely; the extended rational method. The marvel of this method reside in the fact that it leads to different form of solutions obtained by adopting diverse integration technique such as tanh-function method, the extended tanh-function method, the sech-function method [31–37] and so on. In reality, this integration scheme summarize many other analytical method. That is why, in this paper, it became very useful to assume two cases. The obtained results by handling the ODE Eq. (8) with this efficient technique will help to complete the obtained previous results by adopting the auxiliary equation method and the sine-Gordon expansion method. To do so, we first suppose that (13) has the following expression as solution [42,43].

$$\varphi(\xi) = \frac{A_0 \sin(\mu\xi)}{A_2 + A_1 \cos(\mu\xi)}, \quad \cos(\mu\xi) \neq -\frac{A_2}{A_1}, \tag{42}$$

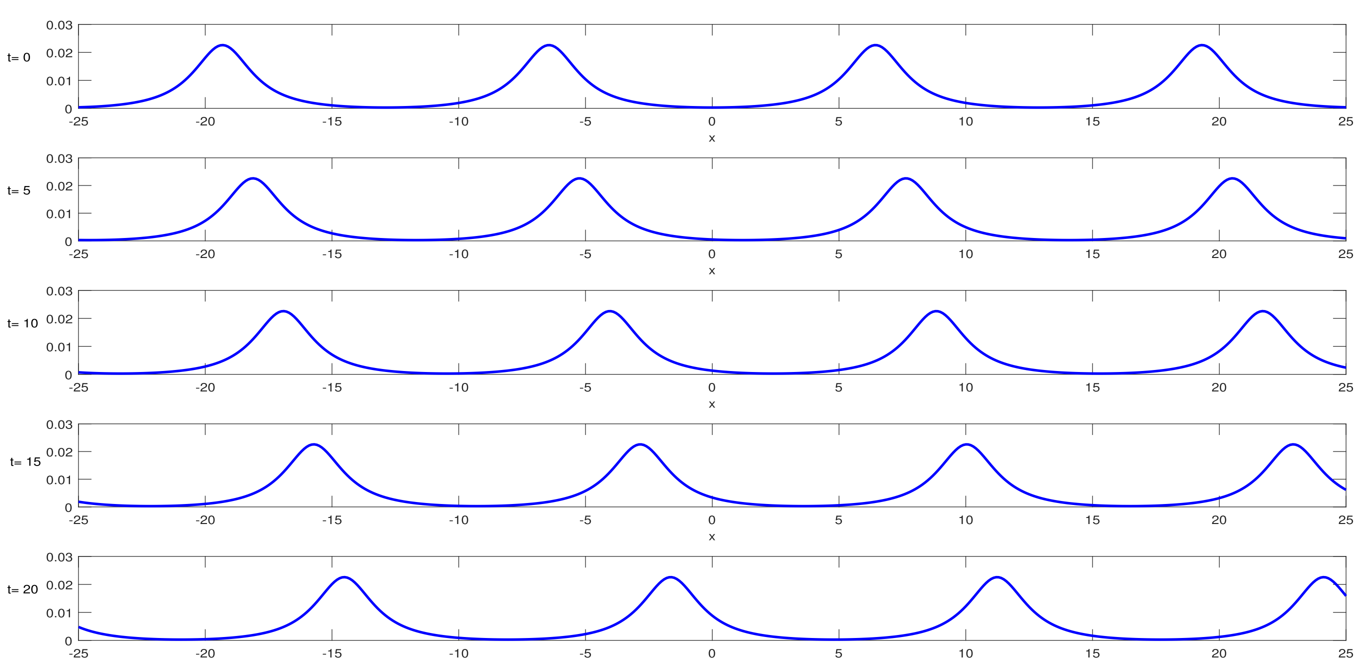

Fig. 5. Spatio-temporal plot of $|\Re\phi_{A,16}|^2$ (29) $c = 1.24$, $C_2 = 1.23$, $C_3 = 2.250$, $C_4 = 1.024$, $\varkappa = 0.04$, $A_0 = 0.05$, $A_1 = 0.05$, $\beta = 40.20$, $\alpha = 4.01$ for $-25 \leqslant x \leqslant 25$ and $t = 0$, $t = 5$, $t = 10$, $t = 15$, $t = 20$ respectively.

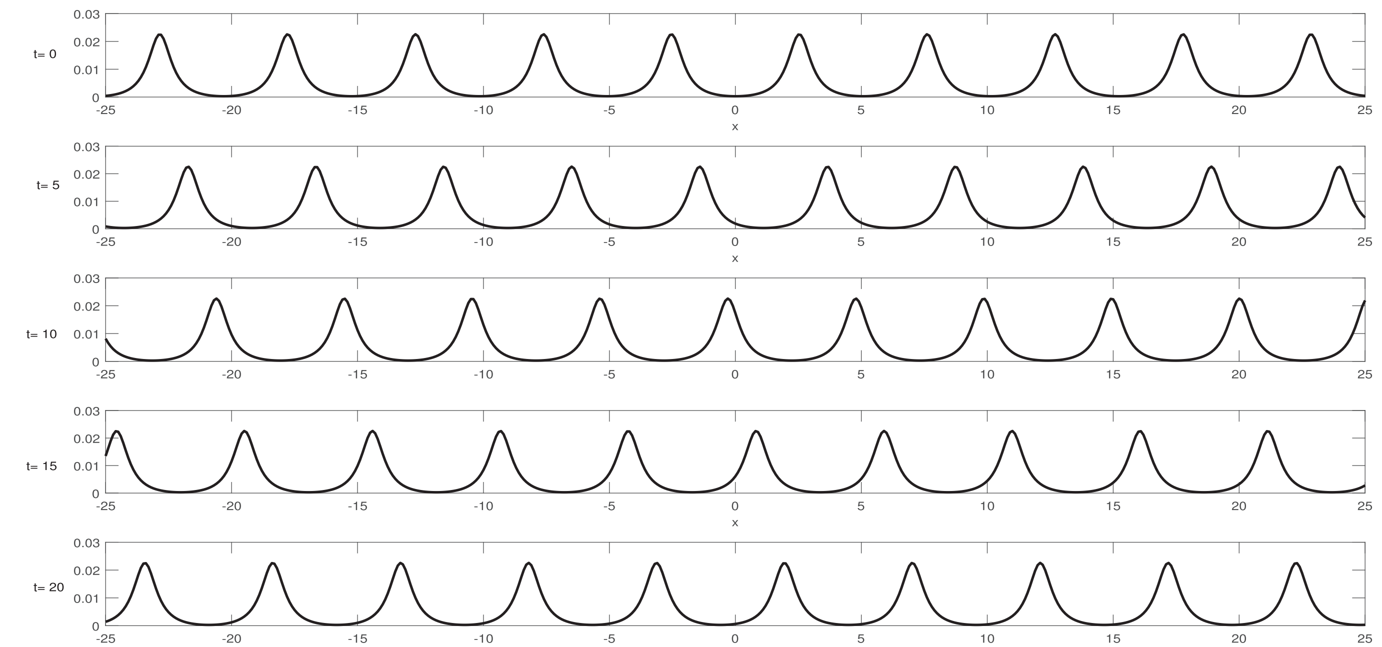

Fig. 6. Spatio-temporal plot of $|\Re\phi_{A,16}|^2$ (29) $c = 1.24$, $C_2 = 1.43$, $C_3 = 4.250$, $C_4 = 1.508$, $\kappa = 0.04$, $A_0 = 0.05$, $A_1 = 0.05$, $\beta = 2.20$, $\alpha = 20.01$ for $-25 \leqslant x \leqslant 25$ and $t = 0$, $t = 5$, $t = 10$, $t = 15$, $t = 20$ respectively.

$$\varphi(\xi) = \frac{A_0\cos(\mu\xi)}{A_2 + A_1\sin(\mu\xi)}, \quad \sin(\mu\xi) \neq -\frac{A_2}{A_1}, \tag{43}$$

where A_0, A_1 and A_2 are real parameters to determine. Substituting Eq. (42) into (13) is obtained the set of results

Set 1: $A_0 = \pm\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}A_2,\ A_1 = A_2,\ A_2 = A_2,\ \mu = \pm\sqrt{\frac{2(\alpha a_0+1)}{\kappa^2(1-c^2)}},$

Set 2: $A_0 = \pm\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}A_2,\ A_1 = -A_2,\ A_2 = A_2,\ \mu = \pm\sqrt{\frac{2(\alpha a_0+1)}{\kappa^2(1-c^2)}}.$

From set 1, it is recovered

$$\phi_{C,11}(x,t) = \pm(1+i)\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(1+\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)\right)} \tag{44}$$

$$\psi_{C,11}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(1+\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)\right)}\right\}^2 + a_0, \tag{45}$$

To use set 2, it is obtained

$$\phi_{C,12}(x,t) = \pm(1+i)\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(-1+\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{2\kappa^2(1-c^2)}}(x-ct)\right)\right)} \tag{46}$$

$$\psi_{C,12}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(-1+\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)\right)}\right\}^2 + a_0, \tag{47}$$

Next, we use (43), it is obtained that

$$\phi_{C,13}(x,t) = \pm(1+i)\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cot\left(\sqrt{\frac{(\alpha a_0+1)}{2(1-c^2)}}(x-ct)\right), \tag{48}$$

$$\psi_{C,13}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cot\left(\sqrt{\frac{(\alpha a_0+1)}{2(1-c^2)}}(x-ct)\right)\right\}^2 + a_0, \tag{49}$$

$$\phi_{C,14}(x,t) = \pm(1+i)\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(1\pm\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)\right)} \tag{50}$$

$$\psi_{C,14}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(1\pm\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)\right)}\right\}^2 + a_0, \tag{51}$$

$$\phi_{C,15}(x,t) = \pm(1+i)\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(-1\pm\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{2\kappa^2(1-c^2)}}(x-ct)\right)\right)} \tag{52}$$

$$\psi_{C,15}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cos\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)}{\left(-1\pm\sin\left(\sqrt{\frac{2(\alpha a_0+1)}{(1-c^2)}}(x-ct)\right)\right)}\right\}^2 + a_0, \tag{53}$$

2.4. On solving the Klein-Gordon-Zhakharov equation by the extended rational sinh-cosh method.

Assume the solution of (13) as follow:

$$\varphi(\xi) = \frac{A_0\sinh(\mu\xi)}{A_2 + A_1\cosh(\mu\xi)}, \quad \cosh(\mu\xi) \neq -\frac{A_2}{A_1}, \tag{54}$$

$$\varphi(\xi) = \frac{A_0\cosh(\mu\xi)}{A_2 + A_1\sinh(\mu\xi)}, \quad \sinh(\mu\xi) \neq -\frac{A_2}{A_1}, \tag{55}$$

where A_0, A_1 and A_2 are real parameters to determine.

Using the same procedure in section (C), it is revealed the set system of algebraic equations below.

Solving the above system of algebraic equations with aid of the MAPLE, we attempt.

Set 1: $A_0 = \pm\sqrt{-\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}A_1,\ A_1 = \pm A_2,\ A_2 = A_2,\ \mu = \pm\sqrt{\frac{2(\alpha a_0+1)}{\kappa^2(c^2-1)}}$,

From set 1, it is recovered

$$\phi_{D,11}(x,t) = \pm(1+i)\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\sinh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)}{\left(\pm1+\cosh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)\right)} \quad (56)$$

$$\psi_{D,11}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\sinh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)}{\left(\pm1+\cosh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)\right)}\right\}^2 + a_0, \quad (57)$$

From (55), it is obtained.

Set 2: $A_0 = \pm\sqrt{-\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}A_1,\ A_1 = iA_2,\ A_2 = A_2,\ \mu = \pm\sqrt{\frac{2(\alpha a_0+1)}{\kappa^2(c^2-1)}}$,

From set 2, it is recovered

$$\phi_{D,12}(x,t) = \pm(1+i)\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cosh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)}{\left(i+\sinh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)\right)} \quad (58)$$

$$\psi_{D,12}(x,t) = \frac{2\beta}{(c^2-1)}\left\{\frac{\sqrt{\frac{(c^2-1)(\alpha a_0+1)}{2\alpha\beta}}\cosh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)}{\left(i+\sinh\left(\sqrt{\frac{2(\alpha a_0+1)}{(c^2-1)}}(x-ct)\right)\right)}\right\}^2 + a_0, \quad (59)$$

3. Conclusions

In this paper, complex traveling-wave solutions and solitons solutions of the Klein-Gordon-Zakharov equations have been obtained by employing three schemes of integration. More recently, some authors studied the model and have pointed out bell-type, kink-type, singular, periodic waves solutions, topological and non-topological by using tanh method, hyperbolic function structure, extended hyperbolic functions method, bifurcation method and sine-cosine method [38–41]. From these studies, the obtained results are of paramount importance in the field of solitary waves and give meaningful explanations of complex systems. On the other hand, for example, in [39], authors obtained bright, dark soliton solutions and rational solutions by extended hyperbolic function method. Unfortunately, they have not obtained solutions (Eq. (24), Eq. (25) and Eq. (26)) which are the combined bright and dark soliton solutions. Equally, in Ref. ([38]), the authors used $sech^p(x)$ and $\tanh^p(x)$ functions, as a results only bright and dark solutions have been obtained compare to our work. On the same way in Ref. [40], they have used three integration schemes such as sine-cosine method, extended tanh method, rational sinh-cosh method and rational exponential functions method, they obtained bright, dark and hyperbolic function solutions. Regarding this, we can safely make the mistake that our results are the subject of a summary of the previous works and set out fresh solutions. In addition, some new solutions have erred by using the auxiliary equation method. To summarize, we have adopted the traveling-wave transformation to construct the nonlinear ordinary differential equation of the KGZ equations model. By using the auxiliary equations, it is revealed dark (Fig. 3 and 4), double bright (Fig. 1), double dark soliton (Fig. 2) and multi-bright solitons (5 and 6) compare to the previous works [38–41]. Beside, the sine-Gordon expansion revealed complex trigonometric function solutions and singular function as solutions. We note that some of the obtained results such as (24–29) are new compare to the standard integration method summarized in Table 1.2 by Li [30] and Refs. [38–40]. Nevertheless, the auxiliary equations is independent of the integrability of NODE and we can also obtained the periodic Jacobi elliptic function solutions that we omit to present them inhere. Furthermore, from the sine-Gordon expansion, dark, bright solitons and complex singular solutions are revealed. The rational sine-cosine and sinh-cosh yield to some new complex trigonometric function solutions. Moreover, the obtained results are new compared to the works [11,12,21,32–35]. These solutions will be useful in solitary waves theory and all satisfied the KGZE. In a future work, it will be added some perturbation terms to KGZEs to investigate solitons solutions and rogue waves.

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Conflict of interest

The authors have no conflict of interest.

CRediT authorship contribution statement

Alphonse Houwe: Conceptualization, Methodology, Software. **Souleymanou Abbagari:** Data curation, Writing - original draft. **Yakada Salathiel:** Visualization, Investigation. **Mustafa Inc:** Supervision. **Serge Y. Doka:** Software, Validation. **Kofane Timoléon Crépin:** Writing - review & editing.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttps://doi.org/10.1016/j.rinp.2020.103127.

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