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Closed-form solutions to the solitary wave equation in an unmagnatized dusty plasma

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Solitary wave solutions

Abstract The research of unmagnetized dusty plasmas is extremely amiable as long as theoretical aspects and their applicability. They are an outstanding mechanism for generating exact solitary waves and solitons. The present article examines the KdV-Burgers type equation in an unmagnetized dusty plasma and the Kadomtsev-Petviashvili dynamical equation in unmagnetized dust plasma. We present the modified $\left(\frac{G'}{G}\right)$-expansion process to secure few exact solitary wave answers. The acquired outcomes confirm that the studied method is an outspoken and useful analytical device for NLEEs in mathematical physics.

1. Introduction

A solitary structure is a hump or dips formed a nonlinear wave of constant form. To discover it from a soliton, we remark that a soliton is a particular kind of solitary waves which conserve their appearance and speed after the interaction. It appears because of the balance between the impressions of the nonlinearity and the dispersion (meanwhile, the outcome of dissipation is negligible in connection with those of the nonlinearity and dispersion). But, meanwhile the dissipative impressions are relative to or further imperative than the dispersive impacts, one finds shock waves. The inadequate nevertheless measurable amplitude solitary wave structures are supervised through a KdV type equation, while a KdV-Burgers type equation describes the shock waves. PDEs have been studied through numerous contributors who are interested in nonlinear phenomena which survive in all disciplines including unless the systematic tasks or engineerings, such as plasma physics, fluid mechanics, hydrodynamics, fluid mechanics, biology, material science, optical fibres and many more. The investigation of travelling wave answers of some NLEEs acquired from such areas performed an essential function in the interpretation of some phenomena [1–9]. To concern travelling wave answers, numerous useful techniques have been manifested in the literature, for example, Extended direct algebraic sech

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method [10], extended modified mapping method [11], Sech-tanh method [12,13], the $exp(-\phi(\xi))$-expansion method [14–16], Modified auxiliary equation method, the (G'/G)-expansion method [17–21], Solitary wave ansatz method, the generalized exponential function, the extended mapping method [22], Residual power series method [23], the reproducing kernel algorithm [24], reproducing kernel Hilbert space method [25], the jacobi elliptic function expansion method [26], modified exp-function method [27], Lie symmetry analysis [28,29], modified extended mapping method [30], modified direct algebraic method [31], modified simple equation method [32], sine-cosine method [33], homotopy perturbation method [34], the improved Bernoulli sub-equation function method [35], the direct algebraic method [36], extended Kudryashovs method [37], modified extended mapping method [38], the sine-Gordon expansion method [39], the Weierstrass elliptic function method [40] and modified (G'/G)-expansion method, direct agebriac method, extended mapping method and Seadawy techniques [41–47].

The present paper use the modified (G'/G)-expansion method. The key understanding of the method is an extension of the basic (G'/G)-expansion method, where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$ with λ and μ being arbitrary constants and $u(\xi) = \sum_{i=-m}^{m} A_i(\frac{G'}{G} + \frac{\lambda}{2})^i$ being the traveling wave solution. The rest paper is constructed as follows. In Section 2 and 3, the new closed-form wave solutions of the solitary wave equation in an unmagnatized dusty plasma are derived by the modified $(\frac{G\prime}{G})$-expansion scheme. Finally, the conclusion is given in last section.

2. The solitary wave equation of the KdV-Burgers type equation in an unmagnatized dusty plasma

In this section, we will implement the modified $(\frac{G\prime}{G})$-expansion scheme to find various novel and more various general traveling wave answers for the DIA shock waves in unmagnetized plasma. The nonlinear governing equations for the DIA shocks in representations of the normalized variable are [48]

$$\frac{\partial(W_1\phi_1)}{\partial t} + \frac{\partial\phi_1}{\partial t} = 0, \tag{1}$$

$$\frac{\partial W_1}{\partial t} + W_i\frac{\partial W_1}{\partial x} = -\frac{\partial\Omega}{\partial x} - 3\sigma_i\phi_i\frac{\partial\phi_1}{\partial x} + \eta_i\frac{\partial^2 W_1}{\partial x^2}, \tag{2}$$

and

$$\delta\frac{\partial^2\Omega}{\partial x^2} = e^{\Omega} - \delta\phi_i + (\delta - 1), \tag{3}$$

where W_i, Ω and $\eta_i = \frac{\delta\mu_d}{\omega_{pi}\lambda^2 D\epsilon}$ in which μ_d are the ion fluid speed, the electrostatic wave potential and the kinematic viscosity respectively. Here the time and space variables are units of the ion plasma period ω_{pi}^{-1} and electon Debye length $\frac{\lambda_{D\epsilon}}{\sqrt{\delta}}$ respectively. If we expand

$$\phi_i = 1 + \epsilon\phi_i^{(1)} + \epsilon^{(2)}\phi_i^{(2)} + \epsilon^{(3)}\phi_i^{(3)} + \epsilon^{(4)}\phi_i^{(4)} + \dots \tag{4}$$

$$W_i = 1 + \epsilon n W_i^{(1)} + \epsilon^{(2)}W_i^{(2)} + \epsilon^{(3)}W_i^{(3)} + \epsilon^{(4)}W_i^{(4)} + \dots \tag{5}$$

$$\Omega = 1 + \epsilon\Omega^{(1)} + \epsilon^{(2)}\Omega^{(2)} + \epsilon^{(3)}\Omega^{(3)} + \epsilon^{(4)}\Omega^{(4)} + \dots \tag{6}$$

and present the stretched variables $x = \epsilon^{\frac{1}{2}}(z - v_0\tau)$ and $t = \epsilon^{\frac{1}{2}}\tau$. Next, we quickly get KdV-Burgers model:

$$\alpha_1 w_t + ww_x + \alpha_2 w_{xxx} - \alpha_3 w_{xx} = 0, \tag{7}$$

here $w = \Omega^{(1)}, \alpha_1 = \frac{2v_0}{\alpha}, \alpha_2 = \frac{\delta^2}{\alpha}, \alpha_3 = \frac{v_0 n_z}{\alpha}$ and $\alpha = \frac{3\delta - \delta^2 + 12\sigma}{\delta}$.

As $v_0 > 0$ and $n_0 > 0$, the sign of the coefficients $\alpha_1, \alpha_2, \alpha_3$ are determined by the sign of α. Upon using the transformation

$$w(x,t) = u(\xi); \xi = x + Vt, \tag{8}$$

where V is speed of travel, Eq. (7) is transformed to

$$\alpha_1 Vu' + uu' + \alpha_2 u''' - \alpha_3 u'' = 0, \tag{9}$$

where the prime denotes differentiation with respect to ξ Eq. (9) is integrable, therefore, integratig, we obtain

$$\alpha_1 Vu + \frac{u^2}{2} + \alpha_2 u'' - \alpha_3 u' + K = 0, \tag{10}$$

where K is an integral constant which is to be determined.

According to the modified (G'/G)-expansion method [41], applying homogeneous balance rule between u'' with u^2 gives $M = 2$. Therefore, we find:

$$w(\xi) = A_2F^2 + A_1F + A_0 + A_{-1}F^{-1} + A_{-2}F^{-2} \tag{11}$$

Using Eq. (11) and Eq. (10), we find:

- **Phase 1:**

$$K = \frac{1}{2}V^2\alpha_1^2 - \frac{18}{25}\alpha_3^2\lambda^2 + \frac{72}{25}\alpha_3^2\mu, \quad \alpha_2 = \pm\sqrt{\frac{1}{25\lambda^2 - 100\mu}}\alpha_3, A_1 = 0, A_2 = 0,$$

$$A_0 = 3\lambda^2\alpha_2 - \alpha_1 V - 12\mu\alpha_2, \quad B_1 = -\frac{3}{5}\alpha_3(\lambda^2 - 4\mu), B_2 = -\frac{3}{4}(\lambda^2 - 4\mu)^2\alpha_2.$$

Using the phase one, Eq. (11) and Eq. (10), we achieve:

$$u_{11}(\xi) = \left\{3\lambda^2\alpha_1^2\alpha_2 - 12\mu\alpha_1^2\alpha_2 - \alpha_1 V - \frac{6}{5}\alpha_1\sqrt{\lambda^2 - 4\mu} \times \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - 3(\lambda^2 - 4\mu)\alpha_1^2\alpha_2 \times \coth^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)\right\}, \tag{12}$$

$$u_{12}(\xi) = \left\{3\lambda^2\alpha_1^2\alpha_2 - 12\mu\alpha_1^2\alpha_2 - \alpha_1 V - \frac{6}{5}\alpha_1\sqrt{\lambda^2 - 4\mu} \times \tanh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi) - 3(\lambda^2 - 4\mu)\alpha_1^2\alpha_2\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)\right\}, \tag{13}$$

$$u_{13}(\xi) = \left\{3\lambda^2\alpha_1^2\alpha_2 - 12\mu\alpha_1^2\alpha_2 - \alpha_1 V - \frac{3}{5}\alpha_1(\lambda^2 - 4\mu)\xi - \frac{3}{4}(\lambda^2 - 4\mu)^2\alpha_1^2\alpha_2\xi^2\right\}, \tag{14}$$

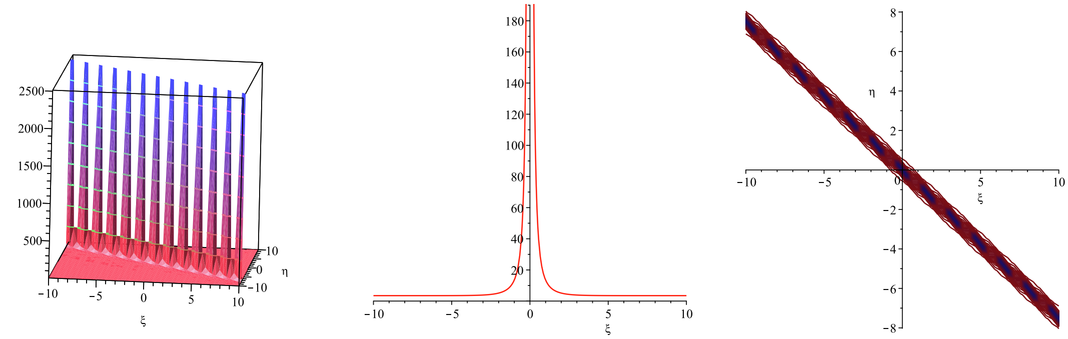

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Fig. 1 Plots of the solitary wave solution of Eq. (12) for $\lambda = 3, \mu = 1, \alpha_1 = 0.3, \alpha_3 = 0.8$, and $V = 1$: (a) 3D plot, (b) 2D plot with the time $t = 0.01$.

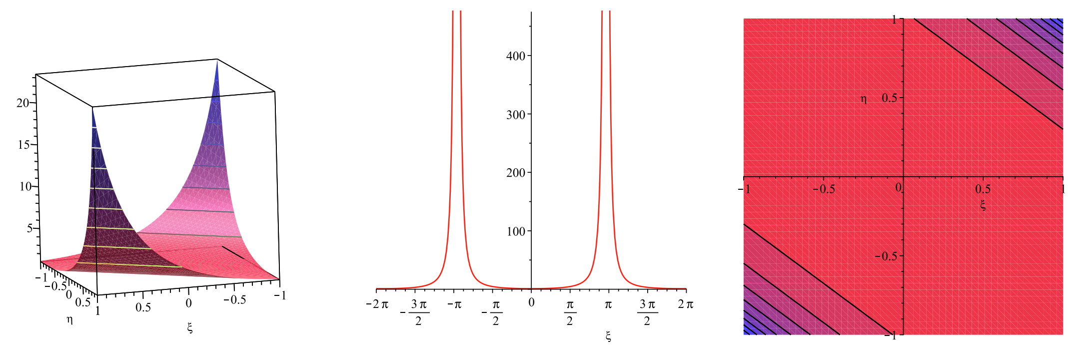

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Fig. 2 Plots of the periodic wave solution of Eq. (13) for $\lambda = 3, \mu = 1, \alpha_1 = 0.3, \alpha_3 = 0.8$, and $V = 1$: (a) Real 3D plot, (b) 2D plot with the time $t = 0.01$, (c) 3D plot surface evalution with density plot, (d) 3D plot surface evalution with contour plot.

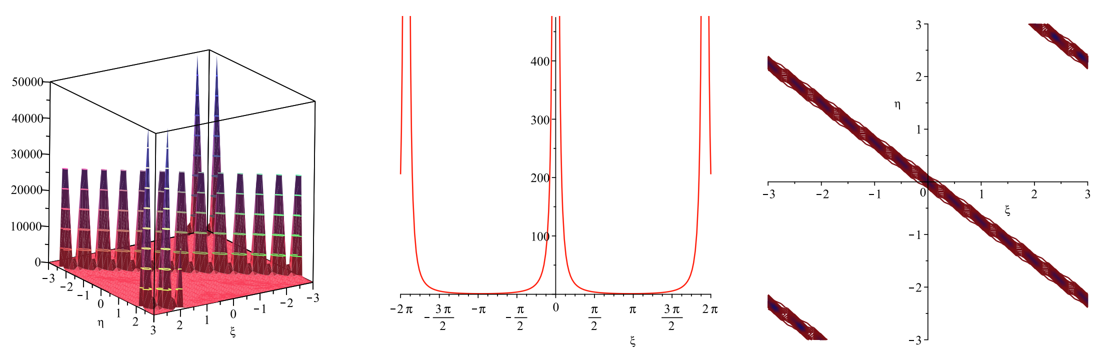

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Fig. 3 Plots of the periodic wave solution of Eq. (16) for $\lambda = 1, \mu = 1, \alpha_1 = 0.3, \alpha_3 = 0.8$, and $V = 1$: (a) Real 3D plot, (b) Real 2D plot with the time $t = 0.01$, (c) Complex 3D plot, (d) Complex 2D plot with the time $t = 0.01$.

$$u_{14}(\xi) = \left\{3\lambda^2\alpha_1^2\alpha_2 - 12\mu\alpha_1^2\alpha_2 - \alpha_1 V - \frac{6}{5}\alpha_1 \times \sqrt{4\mu - \lambda^2}cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) + 3(4\mu - \lambda^2)\alpha_1^2\alpha_2\cot^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right)\right\}, \quad (15)$$

$$u_{15}(\xi) = \left\{3\lambda^2\alpha_1^2\alpha_2 - 12\mu\alpha_1^2\alpha_2 - \alpha_1 V - \frac{6}{5}\alpha_1\sqrt{4\mu - \lambda^2} \times \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) + 3(4\mu - \lambda^2)\alpha_1^2\alpha_2 \times \tan^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right)\right\}, \quad (16)$$

Figs. 1–3 show two of the solutions including $2D, 3D$ and the contour plot surfaces with the help of the symbolic computer software like Maple.

- **Phase 2:**

$$K = \frac{1}{2}V^2\alpha_1^2 - \frac{18}{25}\alpha_3^2\lambda^2 + \frac{72}{25}\alpha_3^2\mu, \alpha_2 = \pm\sqrt{\frac{1}{25\lambda^2 - 100\mu}}\alpha_3, B_1 = 0, B_2 = 0,$$

$$A_0 = -\frac{25\alpha_2 V\alpha_1 - 3\alpha_3}{25\alpha_2}, A_1 = -\frac{12}{5}\alpha_3, A_2 = -12\alpha_2\alpha_3,$$

Similarly, we achieve:

$$u_{21}(\xi) = \left\{ -3(\lambda^2 - 4\mu)\alpha_1^2\alpha_2\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - \frac{6}{5}\alpha_1 \times \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - 25V\alpha_1\alpha_2 + 3\right\}, \quad (17)$$

$$u_{22}(\xi) = \left\{ -3(\lambda^2 - 4\mu)\alpha_1^2\alpha_2\coth^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - \frac{6}{5}\alpha_1 \times \sqrt{\lambda^2 - 4\mu}\coth(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi) - 25V\alpha_1\alpha_2 + 3\right\}, \quad (18)$$

$$u_{23}(\xi) = \left\{\frac{12\alpha_1^2\alpha_2}{\xi^2} - \frac{12\alpha_1}{5\xi} - 25V\alpha_1\alpha_2 + 3\right\}, \quad (19)$$

$$u_{24}(\xi) = \left\{ -3(4\mu - \lambda^2)\alpha_1^2\alpha_2 tan^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) - \frac{6}{5}\alpha_1\sqrt{4\mu - \lambda^2}tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) - 25V\alpha_1\alpha_2 + 3\right\}, \quad (20)$$

$$u_{24}(\xi) = \left\{ -3(4\mu - \lambda^2)\alpha_1^2\alpha_2\cot^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) - \frac{6}{5}\alpha_1\sqrt{4\mu - \lambda^2}\cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) - 25V\alpha_1\alpha_2 + 3\right\}, \quad (21)$$

Fig. 4 show the solutions including $2D$ and $3D$ plot surfaces with the help of the symbolic computer software like Maple.

- **Phase 3:**

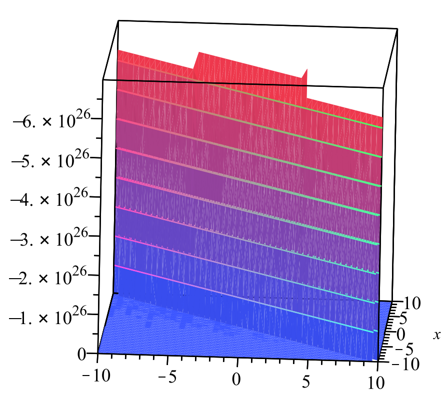
$$K = \frac{1}{2}V^2\alpha_1^2 - \frac{72}{25}\alpha_3^2\lambda^2 + \frac{288}{25}\alpha_3^2\mu, \alpha_2 = \pm\sqrt{\frac{1}{25\lambda^2 - 400\mu}}\alpha_3,$$

$$A_0 = -\frac{50\alpha_2 V\alpha_1 - 3\alpha_3}{50\alpha_2}, A_1 = -\frac{12}{5}\alpha_3, A_2 = -12\alpha_2\alpha_3,$$

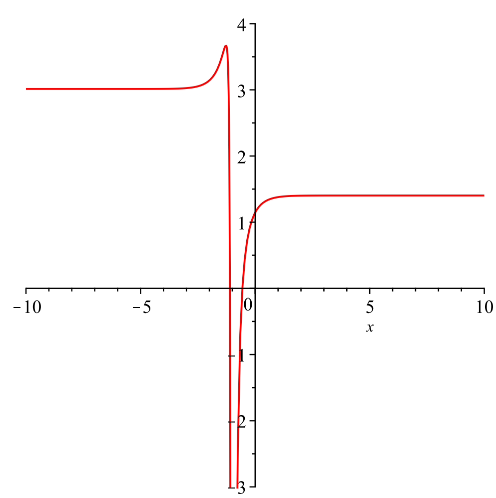
$$B_1 = -\frac{3}{5}\alpha_3(\lambda^2 - 4\mu), B_2 = -\frac{3\alpha_3(\lambda^2 - 4\mu)}{400\alpha_2}$$

Similarly, we have:

$$u_{31}(\xi) = -3(\lambda^2 - 4\mu)\alpha_1^2\alpha_2\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - \frac{6}{5}\alpha_1 \times \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - \frac{50V\alpha_1^2\alpha_2 - 3\alpha_1}{50\alpha_1\alpha_2} - \frac{6}{5}\alpha_1\sqrt{\lambda^2 - 4\mu} \times \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) - \frac{3}{100\alpha_2}\coth^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right), \quad (22)$$



a.



b.

Fig. 4 Plots of the solitary wave solution of Eq. (18) for $\lambda = 3, \mu = 1, \alpha_1 = 0.3, \alpha_3 = 0.8$, and $V = 1$: (a) $3D$ plot, (b) $2D$ plot with the time $t = 0.01$.

$$u_{32}(\xi) = -3(\lambda^2-4\mu)\alpha_1^2\alpha_2\coth^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) - \frac{6}{5}\alpha_1 \times \sqrt{\lambda^2-4\mu}\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) - \frac{50V\alpha_1^2\alpha_2-3\alpha_1}{50\alpha_1\alpha_2} - \frac{6}{5}\alpha_1\sqrt{\lambda^2-4\mu} \times \tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) - \frac{3}{100\alpha_2}\tanh^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right), \quad (23)$$

$$u_{33}(\xi) = -\frac{12\alpha_1^2\alpha_2}{\xi^2} - \frac{12\alpha_1}{5\xi} - \frac{50V\alpha_1^2\alpha_2-3\alpha_1}{50\alpha_1\alpha_2} - \frac{3}{5}\alpha_1(\lambda^2-4\mu)\xi - \frac{3(\lambda^2-4\mu)}{400\alpha_2}\xi^2, \quad (24)$$

$$u_{34}(\xi) = -3(4\mu-\lambda^2)\alpha_1^2\alpha_2\coth^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) - \frac{6}{5}\alpha_1 \times \sqrt{4\mu-\lambda^2}\coth\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) - \frac{50V\alpha_1^2\alpha_2-3\alpha_1}{50\alpha_1\alpha_2} - \frac{6}{5}\alpha_1\sqrt{4\mu-\lambda^2} \times \tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) - \frac{3}{100\alpha_2}\tanh^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right), \quad (25)$$

$$u_{35}(\xi) = -3(4\mu-\lambda^2)\alpha_1^2\alpha_2\tanh^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) - \frac{6}{5}\alpha_1\sqrt{4\mu-\lambda^2}\tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) - \frac{50V\alpha_1^2\alpha_2-3\alpha_1}{50\alpha_1\alpha_2} - \frac{6}{5}\alpha_1\sqrt{4\mu-\lambda^2} \times \coth\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) - \frac{3}{100\alpha_2}\coth^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right), \quad (26)$$

2.1. Physical explanations and disscussion

The survey of dispersion without interjecting nonlinearity damages the solitary wave as diverse Fourier harmonics start producing at various group velocities. Contrast, preceding nonlinearity without dispersion also restricts the structure of solitary waves, because the pulse energy is continuously drawn into higher pulse modes. But, if higher-order nonlinearity terms and hight derivatives are presented, solitary wave structures can be provided. Similarly to dispersion, dissipation can also perform appearance to solitary waves during connected with nonlinearity. Therefore it is fascinating to point out that the sensitive balance between the nonlinearity influence of uu_x and the dissipative impact of u_{xx} and u_{xxx} provide appearance to solitons, that after a full intercommunication including others the solitons come back preserving their identities including the same speed and shape. The KdV-Burgers model has solitary wave answers that have exponentially deteriorating wings. If two solitons of the KdV-Burgers equation collide, the solitons pass within each other and appear stable. For appropriate amounts of the parameters, solitary wave answers are introduced from the acquired exact solutions. We draw a few of the secured answers through keeping the proper values of the unknown free arbitrary parameters received. The graphical occurrences, including the 3D, 2D as well as the contour surfaces of these secured answers are displayed in Figs. 1–4 respectively. The advantages and comparison of the modified (G'/G)-expansion method over the $exp(-\Phi(\xi))$-expansion method have been presented in the following. The ordinary differential equation, as in Eq. (??) has been considered as auxiliary equation and their solutions have been used in this present paper. The chief benefit of the presented scheme over the sine-cosine method and the tanh method is that the suggested way gives much more straightforward form general and new explicit exact wave solutions, including many real free parameters. The explicit exact wave solutions have its notable application to show the interior structures of the complex physical phenomena [49–55].

Hafez et al. [56] investigated exact solutions of the KdV-Burgers type model in unmagnetized dusty plasmas through the $exp(-\Phi(\xi))$-expansion method and an obtained total of only five solutions [56]. Furthermore, in this investigation, we got fifteen exact answers of the same model through the modified $(\frac{G\prime}{G})$-expansion method. But employing the modified $(\frac{G\prime}{G})$-expansion method, we got exact solutions which are distinct from Hafez et al. [56] answers. These answers are novel and were not taken through Hafez et al. [56]. on the contrary, the auxiliary equation manipulated in this investigation is different, and so the exact answers received is also different. Furthermore, for any nonlinear evolution equation, it can be confirmed that the modified $(\frac{G\prime}{G})$-expansion method is much easier than the other schemes.

3. Dispersive solitary wave solutions of Kadomtsev-Petviashvili dynamical equation in unmagnetized dust plasma

Let us explore that unmagnetized dusty plasma concerning of

electrons, the big negatively charged dust grains, the reproduction concerning acoustic dust wave structures in a collisionless, two temperature ions, two temperature ions. The dynamics of dust acoustic solitary fluctuations in 2D in a dusty plasma holding two kinds of superthermal particles such as electrons and ions are scrutinized. Total charge neutrality at equilibrium claims that

$$n_{0e} + n_{0d}Z_{0d} = n_{0il} + n_{0ih}, \quad (27)$$

where n_{0e} is the equilibrium significance of electrons, n_{0d} is dust, n_{0il} is the lower temperature of ions, Z_{0d} is an unperturbed quantity of charges on the dust bits and n_{0ih} is the higher temperature of ions quantity densities, respectively. The ions and electrons are studied to be assigned under Maxwell-Boltzmann partition functions. Therefore, relevant dimensionless number densities concerning electrons n_e; low temperature ions n_{il} and the high temperature of ions n_{ih} as the follows: $n_e = \frac{n_{0e}}{n_{0d}Z_d}exp(\beta_1 s\Phi), n_{il} = \frac{n_{0il}}{n_{0d}Z_d}exp(-s\Phi), \quad n_{ih} =$

$\frac{n_{0ih}}{n_{0d}Z_d}exp(-\beta s\Phi)$, here $\beta_1=\frac{T_{il}}{T_e},\beta_2=\frac{T_{ih}}{T_e},\beta=\frac{\beta_1}{\beta_2}=\frac{T_{il}}{T_{ih}},s=\frac{T_{eff}}{T_{ii}}$. The resulting set of normalized 2D models of continuity, motion concerning the dust and Poisson, represent dynamics of acoustic dust wave

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d u_d)}{\partial x}+\frac{\partial(n_d v_d)}{\partial y}=0, \tag{28}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+v_d\frac{\partial u_d}{\partial y}=Z_d\frac{\partial\Phi}{\partial x}, \tag{29}$$

$$\frac{\partial v_d}{\partial t}+u_d\frac{\partial v_d}{\partial x}+v_d\frac{\partial v_d}{\partial y}=Z_d\frac{\partial\Phi}{\partial y}, \tag{30}$$

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial x^2}=Z_d n_d+n_e-n_{il}-n_{ih}, \tag{31}$$

here $n_d is dust number density, \Phi$ is electrostatic potential, Z_d is variable charge number of dust grains, u_d and v_d are velocity components of the dust bits in x and y-directions respectively. The normalized number densities of the kappa shared ions, as well as electrons, are manifested as follows $n_e=(1-\frac{T_{eff}\Phi}{T_e(k_e-\frac{3}{2})})^{\frac{1}{2}-k_e}, n_i=(1-\frac{T_{eff}\Phi}{T_i(k_i-\frac{3}{2})})^{\frac{1}{2}-k_i}$, here T_{eff} is effective temperature which given $\frac{1}{T_{eff}}=\frac{Z_{0d}n_{0d}}{\frac{n_{0e}}{T_e}+\frac{n_{0il}}{T_{il}}+\frac{n_{0ih}}{T_{ih}}}$, here T_e is the electron as well as T_i is ion plasma temperatures. To the conventional way of reductive perturbation theory, we prefer the free variables as

$$\xi=\epsilon(x-v_0t),\tau=\epsilon^3t,\eta=\epsilon^2y, \tag{32}$$

Where v_0 is the wave with the x direction as well as ϵ is a small dimensionless extension free parameter which describes the intensity of nonlinearity in the process. We can increase physical capacities which have been performed Eq. (47), Eq. (48), Eq. (49) and Eq. (50) in term of the expansion parameter ϵ as:

$$n_d=1+\epsilon^2n_{1d}+\epsilon^4n_{2d}+\ldots, \tag{33}$$

$$u_d=1+\epsilon^2u_{1d}+\epsilon^4u_{2d}+\ldots, \tag{34}$$

$$v_d=1+\epsilon^2v_{1d}+\epsilon^4v_{2d}+\ldots, \tag{35}$$

$$\Phi=\epsilon^2\Phi_1+\epsilon^4\Phi_2+\ldots, \tag{36}$$

$$Z_d=1+\epsilon^2Z_{1d}+\epsilon^4Z_{2d}+\ldots. \tag{37}$$

Also with using $\beta_1=\frac{T_{il}}{T_e},\beta_2=\frac{T_{ih}}{T_e},\beta=\frac{\beta_1}{\beta_2}=\frac{T_{il}}{T_{ih}},S=\frac{T_{eff}}{T_{ii}}$, $\delta_1=\frac{n_{0il}}{n_{0e}},\delta_2=\frac{n_{0ih}}{n_{0e}}$ one can find $S=\frac{T_{eff}}{T_{ii}}=\frac{\delta_1+\delta_2-1}{\delta_1+\delta_2\beta+\beta_1}$.

Plugging Eq. (32), Eq. (33), Eq. (34), Eq. (35), Eq. (36) and Eq. (37) converts Eq. (47), Eq. (48), Eq. (49) and Eq. (50) and Calculating like powers of ϵ, from the coefficients of lowest order we obtain:

$$n_{1d}=-\frac{\Phi_1}{v_0^2},u_{1d}=-\frac{\Phi_1}{v_0},v_0=\frac{1}{\sqrt{1+\gamma_1}}, \tag{38}$$

$$\frac{\partial v_{1d}}{\partial\xi}=-\frac{1}{v_0}\frac{\partial\Phi_1}{\partial\eta}. \tag{39}$$

Additionally, the higher orders of ϵ

$$-v_0\frac{\partial n_{2d}}{\partial\xi}+\frac{\partial n_{1d}}{\partial\tau}+\frac{\partial(u_{2d}+n_{1d}u_{1d}}{\partial\xi}+\frac{\partial v_{1d}}{\partial\eta}=0, \tag{40}$$

$$-v_0\frac{\partial u_{2d}}{\partial\xi}+\frac{\partial u_{1d}}{\partial\tau}+u_{1d}\frac{\partial u_{1d}}{\partial\xi}=Z_{1d}\frac{\partial\Phi_1}{\partial\xi}+\frac{\partial\Phi_2}{\partial\xi}, \tag{41}$$

$$\frac{\partial^2\Phi_1}{\partial\xi^2}=Z_{2d}+Z_{1d}n_{1d}+n_{2d}+\Phi_2-\frac{1}{2}(\delta_1+\delta_2\beta^2+\beta_1^2)\times\frac{S}{\delta_1+\delta_2-1}\Phi_1^2. \tag{42}$$

The KP model is procured from the above models:

$$\frac{\partial}{\partial\xi}[\frac{\partial\Phi_1}{\partial\tau}+a\Phi_1\frac{\partial\Phi_1}{\partial\xi}+b\frac{\partial^3\Phi_1}{\partial\xi^3}]+c\frac{\partial^2\Phi_1}{\partial\eta^2}=0, \tag{43}$$

here $a=\frac{v_0^2}{2}[(\delta_1+\delta_2\delta_2-\delta_1^2)\frac{\delta_1+\delta_2-1}{(\delta_1+\delta_2\beta-\beta_1)^2-2\gamma_2}]+\frac{3}{2}\gamma_1v_0-\frac{3}{2v_0},b=\frac{v_0^3}{2}$ and $c=\frac{v_0}{2}$.

Let $\Phi_1(\xi,\tau,\eta)=\Phi_1(\zeta)$ and $\zeta=k\xi+v\eta+\omega\tau$, then Eq. (43) converts

$$(\omega k+cv^2)\Phi_1(\zeta)+\frac{ak^2}{2}\Phi_1^2(\zeta)+bk^3\Phi_1''(\zeta)=0, \tag{44}$$

In accordance with the modified $(\frac{G\prime}{G})$-expansion method [41], Eq. (44) gives:

$$\Phi(\zeta)=A_2F^2+A_1F+A_0+A_{-1}F^{-1}+A_{-2}F^{-2} \tag{45}$$

From Eq. (45) and Eq. (44), we find:

- **Cluster 1:**

$$\omega=\frac{bk^3\lambda^2-4bk^3\mu-cv^2}{k},A_0=\frac{bk(\lambda^2-4\mu)}{a},A_1=0,A_2=-\frac{12bk}{a},A_{-1}=0,A_{-2}=0.$$

Using the cluster one, Eq. (45) and Eq. (44), we achieve:

$$\Phi_1(\zeta)=\left\{\frac{bk(\lambda^2-4\mu)}{a}-\frac{6bk}{a}(\lambda^2-4\mu)\tanh^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\zeta\right)\right\}, \tag{46}$$

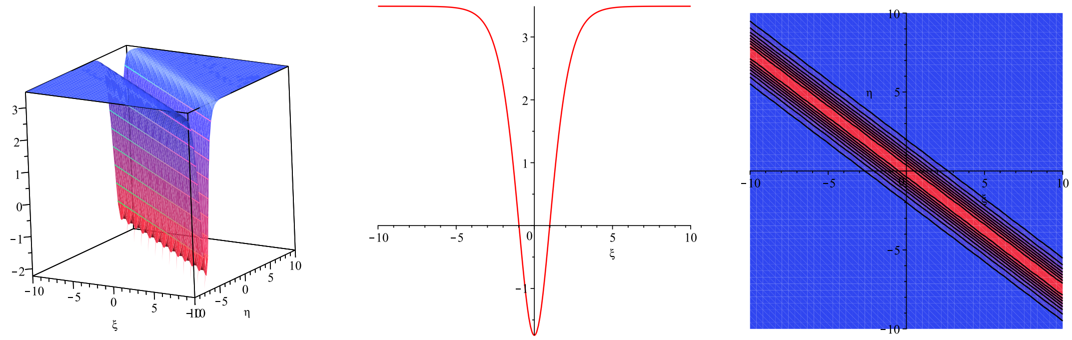
$$\Phi_1(\zeta)=\left\{\frac{bk(\lambda^2-4\mu)}{a}-\frac{6bk}{a}(\lambda^2-4\mu)\coth^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\zeta\right)\right\}, \tag{47}$$

$$\Phi_1(\zeta)=\left\{\frac{bk(\lambda^2-4\mu)}{a}-\frac{12bk}{a}\frac{1}{\zeta^2}\right\}, \tag{48}$$

$$\Phi_1(\zeta)=\left\{\frac{bk(\lambda^2-4\mu)}{a}-\frac{6bk}{a}(4\mu-\lambda^2)\tan^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\zeta\right)\right\}, \tag{49}$$

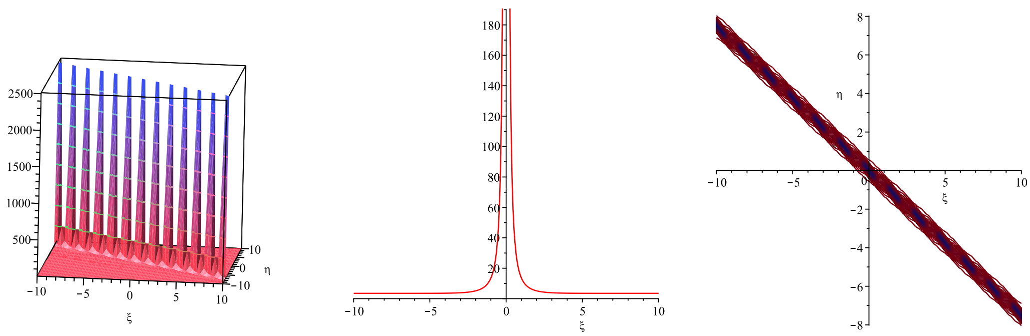
$$\Phi_1(\zeta)=\left\{\frac{bk(\lambda^2-4\mu)}{a}-\frac{6bk}{a}(4\mu-\lambda^2)\cot^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\zeta\right)\right\}, \tag{50}$$

Figs. 5–8 show two of the solutions including $2D,3D$ and the contour plot surfaces with the help of the symbolic computer software like Maple.



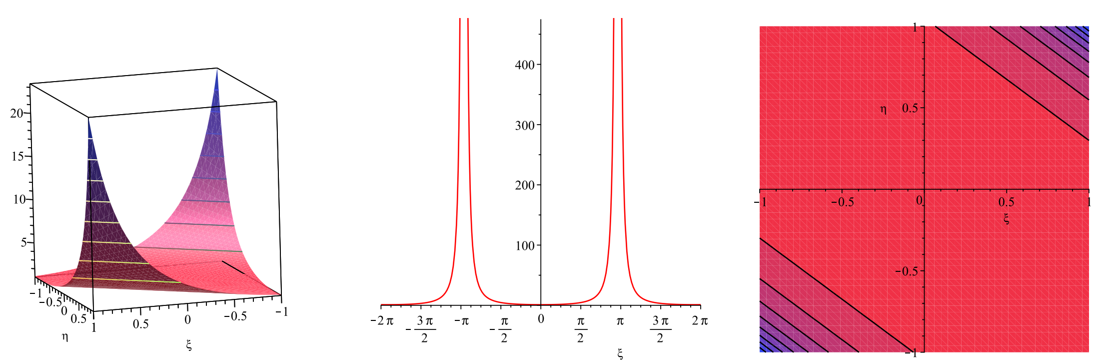
a. b. c.

Fig. 5 Plots of the periodic wave structure of Eq. (46) for $\lambda = 3, \mu = 1, a = -0.86, b = 0.5, c = 0.0, k = 0.6, \tau = 0$ and $v = 0.8$: (a)3D plot, (b) 2D plot with $\eta = 0$ and $\tau = 0$, (c) Contour surface.



a. b. c.

Fig. 6 Plots of the periodic wave structure of Eq. (47) for $\lambda = 3, \mu = 1, a = -0.86, b = 0.5, c = 0.0, k = 0.6, \tau = 0$ and $v = 0.8$: (a)3D plot, (b) 2D plot with $\eta = 0$ and $\tau = 0$, (c) Contour surface.



a. b. c.

Fig. 7 Plots of the periodic wave structure of Eq. (49) for $\lambda = 1, \mu = 1, a = -0.86, b = 0.5, c = 0.0, k = 0.6, \tau = 0$ and $v = 0.8$: (a)3D plot, (b) 2D plot with $\eta = 0$ and $\tau = 0$, (c) Contour surface.

- **Cluster 2:**

$$\omega = \frac{bk^3\lambda^2 - 4bk^3\mu + cv^2}{k}, A_0 = \frac{3bk(\lambda^2 - 4\mu)}{a},$$

$$A_1 = 0, A_2 = -\frac{12bk}{a}, A_{-1} = 0, A_{-2} = 0.$$

- **Cluster 3:**

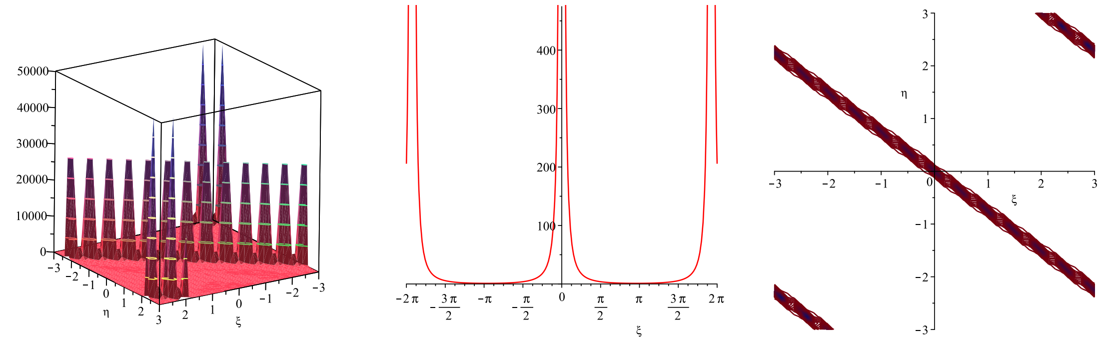
$$\omega = \frac{4bk^3\lambda^2 - 16bk^3\mu - cv^2}{k}, A_0 = -\frac{2bk(\lambda^2 - 4\mu)}{a}, A_1 = 0,$$

$$A_2 = -\frac{12bk}{a}, A_{-1} = 0, A_{-2} = -\frac{3}{4}\frac{bk(\lambda^4 - 8\lambda^2\mu + 16\mu^2)}{a}.$$

- **Cluster 4:**

$$\omega = -\frac{4bk^3\lambda^2 - 16bk^3\mu + cv^2}{k}, A_0 = -\frac{6bk(\lambda^2 - 4\mu)}{a}, A_1 = 0,$$

$$A_2 = -\frac{12bk}{a}, A_{-1} = 0, A_{-2} = -\frac{3}{4}\frac{bk(\lambda^4 - 8\lambda^2\mu + 16\mu^2)}{a}.$$



a. b. c.

Fig. 8 Plots of the periodic wave structure of Eq. (50) for $\lambda=1, \mu=1, a=-0.86, b=0.5, c=0.0, k=0.6, \tau=0$ and $v=0.8$: (a)3D plot, (b) 2D plot with $\eta=0$ and $\tau=0$, (c) Contour surface.

- **Cluster 5:**

$$\omega=\frac{bk^3\lambda^2-4bk^3\mu-cv^2}{k}, A_0=-\frac{bk(\lambda^2-4\mu)}{a}, A_1=0,$$

$$A_2=0, A_{-1}=0, A_{-2}=-\frac{3}{4}\frac{bk(\lambda^2-4\mu)^2}{a}.$$

- **Cluster 6:**

$$\omega=-\frac{bk^3\lambda^2-4bk^3\mu+cv^2}{k}, A_0=\frac{3bk(\lambda^2-4\mu)}{a}, A_1=0,$$

$$A_2=0, A_{-1}=0, A_{-2}=-\frac{3}{4}\frac{bk(\lambda^2-4\mu)^2}{a}.$$

3.1. Result and disscussion

In order to represent the performance of surface wave motion that increases or decreases from one state to another and proposes a constant at infinity, few of the secured travelling wave answers of the problem are sketched plugging choosing proper values of the unknown concerned parameters: Fig. 5 represented the dark singular solotary wave answers of Eq. (46) under $\lambda=3, \mu=1, a=-0.86, b=0.5, c=0.0, k=0.6, \tau=0$ and $v=0.8$. Fig. 6 illustrated the bright multiple periodic wave answers of Eq. (47) under $\lambda=3, \mu=1, a=-0.86$, $b=0.5, c=0.0, k=0.6, \tau=0$ and $v=0.8$. Fig. 7 depicted the bright two soliton wave answers of Eq. (49) under $\lambda=1, \mu=1, a=-0.86, b=0.5, c=0.0, k=0.6, \tau=0$ and $v=0.8$. Fig. 8 describted the bright two preiodic wave answers of Eq. (50) under $\lambda=1, \mu=1, a=-0.86, b=0.5, c=0.0$, $k=0.6, \tau=0$ and $v=0.8$. Seadawy and Rashidy [57] mentioned exact answers of the KP model in unmagnetized dusty plasma by performing the generalized extensive tanh way as well as the F-expansion way. They succeeded a total of four answers [57]. Furthermore, in this investigation, we got thirty exact answers of the same model through the modified $(\frac{G\prime}{G})$-expansion process. But employing the modified $(\frac{G\prime}{G})$-expansion method, we got exact solutions which are distinct from Seadawy and Rashidy [57] answers. These answers are novel and were not taken through Seadawy and Rashidy [57]. On the contrary, the auxiliary model manipulated in this investigation is different, and so the exact solutions received is also different. Furthermore, for any nonlinear evolution equation, it can be confirmed that the modified $(\frac{G\prime}{G})$-expansion method is much easier than the other schemes. These answers to limits are in a good agreement with those in the study of References.

4. Conclusion

In this paper, the modification of $(\frac{G\prime}{G})$-expansion scheme has been completely implemented to attain the exact answers for the nonlinear KP model and the KdV-Burgers type model in unmagnetized dusty plasmas. The KP and the KdV-Burgers type models have applications in physics. Hence, the developed (G'/G)-expansion approach is a practical and straightforward computational scheme that provides exceptional results. Finally, this procedure is significantly decreasing the length of the computational work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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