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Geometric phase for timelike spherical normal magnetic charged particles optical ferromagnetic model

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ABSTRACT

We introduce the theory of optical spherical Heisenberg ferromagnetic spin of timelike spherical normal magnetic flows of particles by the spherical frame in de Sitter space. Also, the concept of timelike spherical normal magnetic particles is investigated, which may have evolution equations. Afterward, we reveal new relationships with some integrability conditions for timelike spherical normal magnetic flows in de-Sitter space. In addition, we obtain total phases for spherical normal magnetic flows. We also acquire perturbed solutions of the nonlinear Schrödinger's equation that governs the propagation of solitons in de-Sitter space $\mathbb{S}_1^2$. Finally, we provide some numerical simulations to supplement the analytical outcomes.

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1. Introduction

Nonlinear evolution equations appear in many different disciplines including fluid mechanics, optical, solid-state and plasma physics. Exact solutions of these equations are highly important and useful for researchers. Because any general method applicable to obtain exact solutions of all evolution equations is not available, researchers developed and employed many different solution techniques, e.g. [1–8], each of which is mostly applicable only for special cases.

Recently, the concept of nonlinear transformation equation classification experience in applied differential geometry is established as geometric fluid flow. This flow can be attained over curvature or binormal flux of space curves corresponding time potential. Curvature evolution involves that a particle flows on the order of normal field comparable to its curvatures. The present description of the transformation equation involves various remarkable symmetrical partial differential equation classifications. The above-mentioned equation classifications are primarily restricted as ordinary mean curvature flows, some curvature flow, surface propagation flow and Willmore flow [9–14].

A comprehensive analysis struggle has been established to evaluate some magnetic particles and their spherical fluid flows. Comparatively, experts presented that some elastic particles are attained just as one of the solving families of the Lorentz force equation. This design a connection to some possible physical development, specifically, the Hall issue and the basic elastic concept. Additionally, magnetic particles are utilized to solve some issues [15–17].

Space curves have different uses on diverse organizations of technology just like slim vortex filament in fluid flow, twisted optical fibre, rotate designs in the magnetic string, crystal development, and fire entrance distribution, very much functions have been analyzed by means of a lot of experts [18–23]. Also, this topic is commonly studied with some solutions [24,25].

The de-Sitter space is a famous and proper model in mathematical physics, and it has been investigating under a comprehensive range of distinct perspectives. In the mathematical point of view, DSs $\mathbb{S}_1^2$ is identified to be the Lorentzian sphere in the Lorentzian spacetime with positive curvature. This feature of the DSs provides a decent pattern to explore spherical geometry. In the physical context, it has a key role in the theory of general relativity since it is one of the vacuum solutions of field equations. In the cosmological sight, solutions of formulas of Einstein by a positive cosmological constant are constructed in de Sitter metric, which models an expanding universe [26–28].

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The paper is organized as follows: Firstly, we introduce scheme of optical Heisenberg ferromagnetic spin of timelike spherical normal magnetic flows of particles by the spherical frame in DSs. Secondly, the concept of timelike spherical normal magnetic particles is investigated, which may have evolution equations. Afterwards, we reveal new relationships some integrability conditions for timelike spherical normal magnetic flows in de Sitter space. Finally, we obtain total phases for spherical normal magnetic flows.

1.1. *The geometry of the de-Sitter space $\mathbb{S}_1^2$*

In this part, we present fundamental definitions of the spherical geometry of the Lorentzian space form, which corresponds to a DSs $\mathbb{S}_1^2$. Here, we go into detail about the geometrical understanding of the DSs in order to comprehend the mathematical method that we improve to define magnetic curves in the $\mathbb{S}_1^2$.

Let $\mathbb{R}_1^{k+1}$ be a $(k+1)$-dimensional vector space equipped with the Lorentzian metric

$$h\left(,\right) = -da_1^2 + da_2^2 + \cdots + da_{k+1}^2.$$

In this case, $(\mathbb{R}_1^{k+1}, h(,))$ is named by Minkowski $(k+1)$-space. The pseudo vector product of $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_k \in \mathbb{R}_1^{k+1}$ is described to be

$$\mathbf{a}_1 \times \mathbf{a}_2 \times \cdots \times \mathbf{a}_k = \begin{bmatrix} -u_1 & u_2 & \cdots & u_{k+1} \\ a_1^1 & a_1^2 & \cdots & a_1^{k+1} \\ a_2^1 & a_2^2 & \cdots & a_2^{k+1} \\ \vdots & \vdots & \vdots & \vdots \\ a_k^1 & a_k^2 & \cdots & a_k^{k+1} \end{bmatrix},$$

where $\mathbf{a_i} = \{a_i^1, a_i^2, \cdots, a_i^{k+1}\}$ and $\{u_1, u_2, \cdots, u_{k+1}\}$ is the canonical basis of $\mathbb{R}_1^{k+1}$ [29]. A non-zero vector $\mathbf{a} \in \mathbb{R}_1^{k+1}$ is called timelike, lightlike or spacelike if $h(\mathbf{a},\mathbf{a}) < 0$, $h(\mathbf{a},\mathbf{a}) = 0$ or $h(\mathbf{a},\mathbf{a}) > 0$. Thus, one can give the norm function of the $\mathbf{a} \in \mathbb{R}_1^{k+1}$ by using the sign function as follows.

$$\|\mathbf{a}\| = \sqrt{\text{sign}\left(\mathbf{a}\right)\left(\mathbf{a},\mathbf{a}\right)},$$

where

$$\text{sign}\left(\mathbf{a}\right) = \begin{cases} 1, & \mathbf{a} \text{ is spacelike}, \\ 0, & \mathbf{a} \text{ is lightlike}, \\ -1, & \mathbf{a} \text{ is timelike}. \end{cases}$$

Let $\delta : I \to \mathbb{S}_1^2$ be a regular unit speed timelike spherical particle, that is it is arclength parametrized and sufficiently smooth. Hence, Sabban or spherical frame is defined along with the particle δ as follows.

$$\nabla_\vartheta \delta = \mathbf{T}$$
$$\nabla_\vartheta \mathbf{T} = \delta + \epsilon \mathbf{N}$$
$$\nabla_\vartheta \mathbf{N} = \epsilon \mathbf{T},$$

where ∇ is a derivative connection and $\epsilon = \det(\delta, \mathbf{T}, \mathbf{T}')$ is geodesic curvature of particle [10]. Also, pseudo vector products are

$$\delta = \mathbf{T} \times \mathbf{N}, \quad \mathbf{T} = \delta \times \mathbf{N}, \quad \mathbf{N} = \delta \times \mathbf{T}.$$

2. Timelike spherical normal magnetic curves of the de-Sitter space $\mathbb{S}_1^2$

In this division, we attempt to explore the impacts of pseudo Riemannian geometry, in particular, de-Sitter spacetime $\mathbb{S}_1^2$, on the motion of a charged particle moving on some magnetic field, which is supposed to harmonize a timelike spherical particle lying fully in $\mathbb{S}_1^2$, extracted from Lorentz equation.

Definition 2.1: Let $\delta : I \to \mathbb{S}_1^2$ *be a regular unit speed timelike spherical particle in deSitter space and G be the magnetic field on* $\mathbb{S}_1^2$. *Timelike spherical magnetic particles are established by the Lorentz force:*

$$\nabla_\vartheta \mathbf{T} = \Psi\left(\mathbf{T}\right) = \mathcal{G} \times \mathbf{N}.$$

For advance references, we recall these timelike spherical magnetic particles as an **N**-*magnetic particle.*

Proposition 2.2: *Let* $\delta : I \to \mathbb{S}_1^2$ *be a regular unit speed timelike spherical particle in deSitter space and G be the magnetic field on* $\mathbb{S}_1^2$. *Then,* **N**-*magnetic particle of Lorentz force* Ψ *with the magnetic field* $\mathcal{G}$ *is signed by*

$$\Psi(\delta) = \varkappa \mathbf{T},$$
$$\Psi(\mathbf{T}) = \varkappa \delta + \epsilon \mathbf{N},$$
$$\Psi(\mathbf{N}) = \epsilon \mathbf{T},$$
$$\mathcal{G}^{\mathbf{N}} = \epsilon \delta - \varkappa \mathbf{N},$$

where $\varkappa = h(\Psi(\mathbf{T}), \delta)$.

Let $\delta(s,t)$ is the evolution of **N**-*magnetic particle* in DSs with time. Flow of δ can easily be obtained by

$$\nabla_t \delta = \varpi_1 \mathbf{T} + \varpi_2 \mathbf{N},$$

where ϖ_1, ϖ_2 are potentials.

Firstly, we have

$$\nabla_\vartheta \nabla_t \delta = \varpi_1 \delta + \left(\frac{\partial \varpi_1}{\partial \vartheta} + \varpi_2 \epsilon\right)\mathbf{T} + \left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\mathbf{N}.$$

Lemma 2.3: *Time derivatives of spherical frame are produced by*

$$\nabla_t \delta = \varpi_1 \mathbf{T} + \varpi_2 \mathbf{N},$$
$$\nabla_t \mathbf{T} = \varpi_1 \delta + \left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\mathbf{N},$$
$$\nabla_t \mathbf{N} = \left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\mathbf{T} - \varpi_2 \delta.$$

Theorem 2.4: *Flows for Lorentz forces of δ-magnetic particle with spherical frame are presented by*

$$\nabla_t\Psi(\delta) = \varkappa\varpi_1\delta + \frac{\partial}{\partial t}\varkappa\mathbf{T} + \varkappa\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\mathbf{N},$$

$$\nabla_t\Psi(\mathbf{T}) = \left(\frac{\partial\varkappa}{\partial t} - \epsilon\varpi_2\right)\delta + \left(\varkappa\varpi_1 + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\right)\mathbf{T} + \left(\frac{\partial\epsilon}{\partial t} + \varkappa\varpi_2\right)\mathbf{N},$$

$$\nabla_t\Psi(\mathbf{N}) = \epsilon\varpi_1\delta + \frac{\partial}{\partial t}\epsilon\mathbf{T} + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\mathbf{N},$$

$$\nabla_t\mathcal{G}^{\mathbf{N}} = \left(\frac{\partial}{\partial t}\epsilon + \varkappa\varpi_2\right)\delta + \left(\epsilon\varpi_1 - \varkappa\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\right)\mathbf{T} + \left(\epsilon\varpi_2 - \frac{\partial\varkappa}{\partial t}\right)\mathbf{N},$$

where $\pi = h(\Psi(\delta), \mathbf{N})$.

3. Results and geometric presentation

3.1. Heisenberg ferromagnetic spin for $\Psi(\delta)$

Theorem 3.1: *Spherical Heisenberg ferromagnetic chain condition of $\Psi(\delta)$ are given by*

$$\varkappa\varpi_1 = \left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right),$$

$$\frac{\partial\varkappa}{\partial t} = -\varkappa^2\epsilon,$$

$$\varkappa\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) = -\pi\epsilon.$$

Proof: By definition of ferromagnetic chain, we get

$$\nabla_t\Psi(\delta) = \Psi(\delta) \times \nabla_\vartheta^2\Psi(\delta).$$

Firstly, we have

$$\Psi(\delta) \times \nabla_\vartheta^2\Psi(\delta) = \left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right)\delta - \varkappa^2\epsilon\mathbf{T} - \pi\epsilon\mathbf{N}.$$

Then, it is easy to see that

$$\varkappa\varpi_1 = \left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right),$$

$$\frac{\partial\varkappa}{\partial t} = -\varkappa^2\epsilon,$$

$$\varkappa\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) = -\pi\epsilon.$$

Total phase with Lorentz force $\Psi(\delta)$ is expressed by

$$\Phi_{\Psi(\delta)} = \iint \Psi(\delta)\cdot\nabla_\vartheta\Psi(\delta) \times \nabla_t\Psi(\delta)\,d\varrho.$$

■

Theorem 3.2: *Total phase of Lorentz force $\Psi(\delta)$ is presented by*

$$\Phi_{\Psi(\delta)} = \iint\left(-\varkappa\left(\varkappa^2\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) - \epsilon\varkappa^2\varpi_1\right)\right)d\varrho.$$

Proof: Some calculations, we have

$$\nabla_\vartheta\Psi(\delta) \times \nabla_t\Psi(\delta) = \left(\frac{\partial\varkappa}{\partial\vartheta}\varkappa\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) - \varkappa\epsilon\frac{\partial\varkappa}{\partial t}\right)\delta + \left(\varkappa^2\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) - \epsilon\varkappa^2\varpi_1\right)\mathbf{T} + \left(\varkappa\frac{\partial\varkappa}{\partial t} - \frac{\partial\varkappa}{\partial\vartheta}\varkappa\varpi_1\right)\mathbf{N}.$$

Anholonomy density of $\Psi(\delta)$ is given by

$$\rho_{\Psi(\delta)} = -\varkappa\left(\varkappa^2\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) - \epsilon\varkappa^2\varpi_1\right).$$

Also, we present

$$\Phi_{\Psi(\delta)} = \iint\left(-\varkappa\left(\varkappa^2\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) - \epsilon\varkappa^2\varpi_1\right)\right)d\varrho.$$

■

By anholonomy density with ferromagnetic model is obtained

$$\Psi(\delta) \times \nabla_\vartheta^2\Psi(\delta) = \left(\varkappa\epsilon^2\pi_2^2 - \pi\epsilon\frac{\partial\varkappa}{\partial\vartheta}\right)\delta - \left(\varkappa\epsilon\left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right) + \pi\varkappa\epsilon\right)T - \left(\frac{\partial\varkappa}{\partial\vartheta}\left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right) + \pi_2^2\epsilon\varkappa\right)\mathbf{N}.$$

By this way, we conclude

$$\rho_{\Psi(\delta)}^{\mathcal{FR}} = \varkappa\left(\varkappa\epsilon\left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right) + \pi\varkappa\epsilon\right).$$

Therefore, we can express

$$\Phi_{\Psi(\delta)}^{\mathcal{FR}} = \iint\left(\varkappa\left(\varkappa\epsilon\left(\left(\frac{\partial}{\partial\vartheta}\left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right) + \pi\epsilon^2\right) - \pi\left(1 + \frac{\partial}{\partial\vartheta}(\pi\epsilon) + \left(\epsilon + \frac{\partial\pi}{\partial\vartheta}\right)\epsilon\right)\right) + \pi\varkappa\epsilon\right)\right)d\varrho.$$

Spherical anholonomy density and spherical flow lines in the centre of the quadrupole spherical magnet with

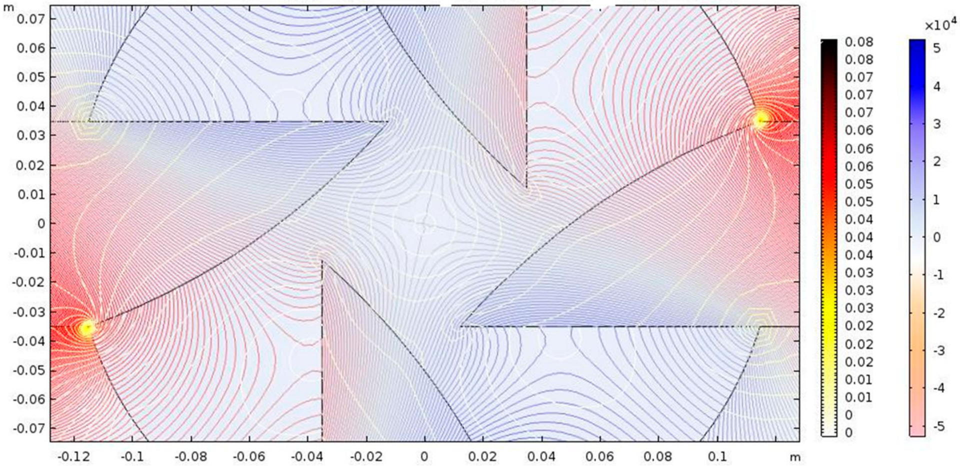

Figure 1. Anholonomy density and spherical normal magnetic flow with $\Psi(\delta)$.

Lorentz force $\Psi(\delta)$. By this force, spherical density is given by the particle-tracing algorithm in Figure 1.

3.2. Heisenberg ferromagnetic model for $\Psi(\mathbf{T})$

Theorem 3.3: *Spherical Heisenberg ferromagnetic chain condition of $\Psi(\mathbf{T})$ are given by*

$$\begin{aligned}
\frac{\partial \varkappa}{\partial t} - \epsilon\varpi_2 &= -\epsilon\left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + \frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right),\\
\varkappa\varpi_1 + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) &= \varkappa\left(\frac{\partial}{\partial\vartheta}\frac{\partial\epsilon}{\partial\vartheta} + \epsilon(\varkappa+\epsilon^2)\right) - \epsilon\left(\frac{\partial^2\varkappa}{\partial\vartheta^2} + (\varkappa+\epsilon^2)\right),\\
\frac{\partial\epsilon}{\partial t} + \varkappa\varpi_2 &= \varkappa\left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + +\frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right).
\end{aligned}$$

Proof: Ferromagnetic chain is given by

$$\nabla_t\Psi(\mathbf{T}) = \Psi(\mathbf{T}) \times \nabla_\vartheta^2\Psi(\mathbf{T}).$$

Firstly,

$$\begin{aligned}
\nabla_\vartheta^2\Psi(\mathbf{T}) = {}&\left(\frac{\partial^2\varkappa}{\partial\vartheta^2} + (\varkappa+\epsilon^2)\right)\delta\\
&+ \left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + \frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right)\mathbf{T}\\
&+ \left(\frac{\partial}{\partial\vartheta}\frac{\partial\epsilon}{\partial\vartheta} + \epsilon(\varkappa+\epsilon^2)\right)\mathbf{N}.
\end{aligned}$$

Also, we have

$$\begin{aligned}
&\Psi(\mathbf{T}) \times \nabla_\vartheta^2\Psi(\mathbf{T})\\
&= -\epsilon\left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + \frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right)\delta\\
&\quad + \left(\varkappa\left(\frac{\partial}{\partial\vartheta}\frac{\partial\epsilon}{\partial\vartheta} + \epsilon(\varkappa+\epsilon^2)\right)\right.\\
&\quad \left. -\epsilon\left(\frac{\partial^2\varkappa}{\partial\vartheta^2} + (\varkappa+\epsilon^2)\right)\right)\mathbf{T}\\
&\quad + \varkappa\left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + \frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right)\mathbf{N}.
\end{aligned}$$

Moreover, from above equation, we have

$$\begin{aligned}
\nabla_t\Psi(\mathbf{T}) = {}&\left(\frac{\partial\varkappa}{\partial t} - \epsilon\varpi_2\right)\delta + \left(\varkappa\varpi_1 + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\right)\mathbf{T}\\
&+ \left(\frac{\partial\epsilon}{\partial t} + \varkappa\varpi_2\right)\mathbf{N}.
\end{aligned}$$

By some short calculations yields

$$\begin{aligned}
\frac{\partial \varkappa}{\partial t} - \epsilon\varpi_2 &= -\epsilon\left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + \frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right),\\
\varkappa\varpi_1 + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right) &= \varkappa\left(\frac{\partial}{\partial\vartheta}\frac{\partial\epsilon}{\partial\vartheta} + \epsilon(\varkappa+\epsilon^2)\right) - \epsilon\left(\frac{\partial^2\varkappa}{\partial\vartheta^2} + (\varkappa+\epsilon^2)\right),\\
\frac{\partial\epsilon}{\partial t} + \varkappa\varpi_2 &= \varkappa\left(\frac{\partial}{\partial\vartheta}(\varkappa+\epsilon^2) + \frac{\partial\epsilon}{\partial\vartheta}\epsilon + \frac{\partial\varkappa}{\partial\vartheta}\right).
\end{aligned}$$

■

Theorem 3.4: *Total phase of Lorentz force $\Psi(\mathbf{T})$ is presented by*

$$\begin{aligned}
\Phi_{\Psi(\mathbf{T})} = \iint\Bigg(&\varkappa\left((\varkappa+\epsilon^2)\left(\frac{\partial\epsilon}{\partial t} + \varkappa\varpi_2\right)\right.\\
&\left.-\frac{\partial\epsilon}{\partial\vartheta}\left(\varkappa\varpi_1 + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\right)\right)\\
&+\epsilon\left(\frac{\partial\varkappa}{\partial\vartheta}\left(\varkappa\varpi_1 + \epsilon\left(\frac{\partial\varpi_2}{\partial\vartheta} + \varpi_1\epsilon\right)\right)\right.\\
&\left.-(\varkappa+\epsilon^2)\left(\frac{\partial\varkappa}{\partial t} - \epsilon\varpi_2\right)\right)\Bigg)\,d\varrho.
\end{aligned}$$

Proof: First, we express

$$\begin{aligned}
&\nabla_\vartheta \Psi(\mathbf{T}) \times \nabla_t \Psi(\mathbf{T}) \\
&= \left((\varkappa + \epsilon^2)\left(\frac{\partial \epsilon}{\partial t} + \varkappa \varpi_2\right) - \frac{\partial \epsilon}{\partial \vartheta}\left(\varkappa \varpi_1 + \epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\right)\right)\delta \\
&+ \left(\frac{\partial \varkappa}{\partial \vartheta}\left(\frac{\partial \epsilon}{\partial t} + \varkappa \varpi_2\right) - \frac{\partial \epsilon}{\partial \vartheta}\left(\frac{\partial \varkappa}{\partial t} - \epsilon \varpi_2\right)\right)\mathbf{T} \\
&+ \left(\frac{\partial \varkappa}{\partial \vartheta}\left(\varkappa \varpi_1 + \epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\right) - (\varkappa + \epsilon^2)\left(\frac{\partial \varkappa}{\partial t} - \epsilon \varpi_2\right)\right)\mathbf{N}.
\end{aligned}$$

Then

$$\begin{aligned}
\rho_{\Psi(\mathbf{T})} = \varkappa\Bigg(&(\varkappa + \epsilon^2)\left(\frac{\partial \epsilon}{\partial t} + \varkappa \varpi_2\right) - \frac{\partial \epsilon}{\partial \vartheta}\left(\varkappa \varpi_1 + \epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\right)\Bigg) \\
&+ \epsilon\left(\frac{\partial \varkappa}{\partial \vartheta}\left(\varkappa \varpi_1 + \epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\right) - (\varkappa + \epsilon^2)\left(\frac{\partial \varkappa}{\partial t} - \epsilon \varpi_2\right)\right).
\end{aligned}$$

From phase, we obtain

$$\begin{aligned}
\Phi_{\Psi(\mathbf{T})} = \iint \Bigg(\varkappa\Bigg(&(\varkappa + \epsilon^2)\left(\frac{\partial \epsilon}{\partial t} + \varkappa \varpi_2\right) - \frac{\partial \epsilon}{\partial \vartheta}\left(\varkappa \varpi_1 + \epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\right)\Bigg) \\
&+ \epsilon\left(\frac{\partial \varkappa}{\partial \vartheta}\left(\varkappa \varpi_1 + \epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right)\right) - (\varkappa + \epsilon^2)\left(\frac{\partial \varkappa}{\partial t} - \epsilon \varpi_2\right)\right)\Bigg) \, \mathrm{d}\varrho.
\end{aligned}$$

Using ferromagnetic spin for $\Psi(\mathbf{T})$, we get

$$\begin{aligned}
\rho^{\mathcal{FR}}_{\Psi(\mathbf{T})} = \varkappa\Bigg(&(\varkappa + \epsilon^2)\varkappa\left(\frac{\partial}{\partial \vartheta}(\varkappa + \epsilon^2) + \frac{\partial \epsilon}{\partial \vartheta}\epsilon + \frac{\partial \varkappa}{\partial \vartheta}\right) \\
&- \frac{\partial \epsilon}{\partial \vartheta}\left(\varkappa\left(\frac{\partial}{\partial \vartheta}\frac{\partial \epsilon}{\partial \vartheta} + \epsilon(\varkappa + \epsilon^2)\right) - \epsilon\left(\frac{\partial^2 \varkappa}{\partial \vartheta^2} + (\varkappa + \epsilon^2)\right)\right)\Bigg) \\
&+ \epsilon\Bigg((\varkappa + \epsilon^2)\epsilon\left(\frac{\partial}{\partial \vartheta}(\varkappa + \epsilon^2) + \frac{\partial \epsilon}{\partial \vartheta}\epsilon + \frac{\partial \varkappa}{\partial \vartheta}\right) + \frac{\partial \varkappa}{\partial \vartheta}\left(\varkappa\left(\frac{\partial}{\partial \vartheta}\frac{\partial \epsilon}{\partial \vartheta} + \epsilon(\varkappa + \epsilon^2)\right) - \epsilon\left(\frac{\partial^2 \varkappa}{\partial \vartheta^2} + (\varkappa + \epsilon^2)\right)\right)\Bigg)
\end{aligned}$$

Finally, total phase can be given by

$$\begin{aligned}
\Phi^{\mathcal{FR}}_{\Psi(\mathbf{T})} = \iint \Bigg(\varkappa\Bigg(&(\varkappa + \epsilon^2)\varkappa\left(\frac{\partial}{\partial \vartheta}(\varkappa + \epsilon^2) + \frac{\partial \epsilon}{\partial \vartheta}\epsilon + \frac{\partial \varkappa}{\partial \vartheta}\right) \\
&- \frac{\partial \epsilon}{\partial \vartheta}\left(\varkappa\left(\frac{\partial}{\partial \vartheta}\frac{\partial \epsilon}{\partial \vartheta} + \epsilon(\varkappa + \epsilon^2)\right) - \epsilon\left(\frac{\partial^2 \varkappa}{\partial \vartheta^2} + (\varkappa + \epsilon^2)\right)\right)\Bigg) \\
&+ \epsilon\Bigg((\varkappa + \epsilon^2)\epsilon\left(\frac{\partial}{\partial \vartheta}(\varkappa + \epsilon^2) + \frac{\partial \epsilon}{\partial \vartheta}\epsilon + \frac{\partial \varkappa}{\partial \vartheta}\right) + \frac{\partial \varkappa}{\partial \vartheta}\left(\varkappa\left(\frac{\partial}{\partial \vartheta}\frac{\partial \epsilon}{\partial \vartheta} + \epsilon(\varkappa + \epsilon^2)\right) - \epsilon\left(\frac{\partial^2 \varkappa}{\partial \vartheta^2} + (\varkappa + \epsilon^2)\right)\right)\Bigg)\Bigg) \, \mathrm{d}\varrho.
\end{aligned}$$

Spherical anholonomy density and spherical flow lines in the centre of the quadrupole spherical magnet with Lorentz force $\Psi(\mathbf{T})$. By this force, spherical density is given by the particle-tracing algorithm in Figure 2. ■

3.3. Heisenberg ferromagnetic model for $\Psi(\mathbf{N})$

Theorem 3.5: *Spherical Heisenberg ferromagnetic chain condition of $\Psi(\mathbf{N})$ are given by*

$$3\frac{\partial \epsilon}{\partial \vartheta}\epsilon = \varpi_1,$$
$$2\frac{\partial \epsilon}{\partial \vartheta} = \left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right).$$

Proof: From spherical frame, we get

$$\nabla^2_\vartheta \Psi(\mathbf{N}) = \left(\frac{\partial^2 \epsilon}{\partial \vartheta^2} + \epsilon + \epsilon^3\right)\mathbf{T} + 2\frac{\partial \epsilon}{\partial \vartheta}\delta + 3\frac{\partial \epsilon}{\partial \vartheta}\epsilon\mathbf{N}.$$

Since

$$\Psi(\mathbf{N}) \times \nabla^2_\vartheta \Psi(\mathbf{N}) = 3\frac{\partial \epsilon}{\partial \vartheta}\epsilon^2\delta + 2\epsilon\frac{\partial \epsilon}{\partial \vartheta}\mathbf{N}.$$

We instantly calculate

$$3\frac{\partial \epsilon}{\partial \vartheta}\epsilon = \varpi_1,$$
$$2\frac{\partial \epsilon}{\partial \vartheta} = \left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right).$$

■

Theorem 3.6: *Total phase of Lorentz force $\Psi(\mathbf{N})$ is presented by*

$$\Phi_{\Psi(\mathbf{N})} = \iint \left(-\pi\left(\left(\frac{\partial \epsilon}{\partial \vartheta} - \pi\right)(\epsilon\left(\frac{\partial \chi_2}{\partial \vartheta} + \chi_1 \epsilon\right) - \pi\chi_2) - \epsilon^2\left(\frac{\partial \epsilon}{\partial t} - \pi\chi_1\right)\right)\delta\right) \mathrm{d}\varrho.$$

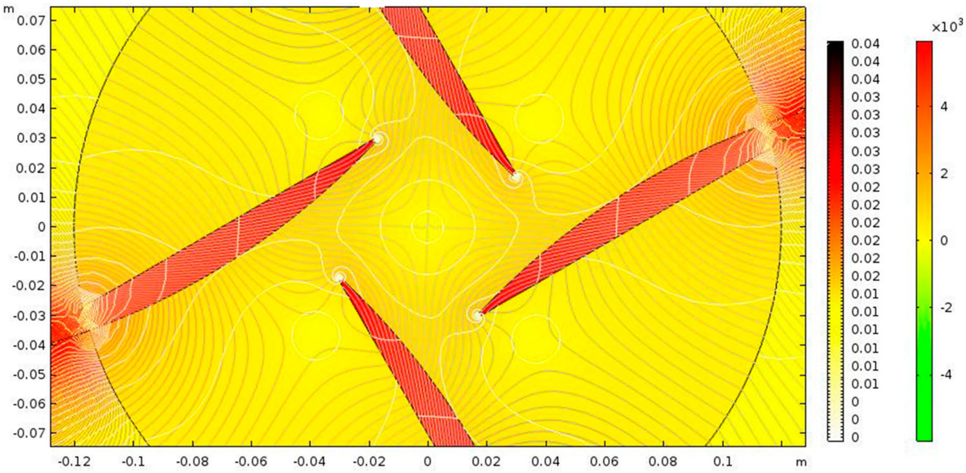

Figure 2. Anholonomy density and spherical normal magnetic flow with $\Psi(\mathbf{T})$.

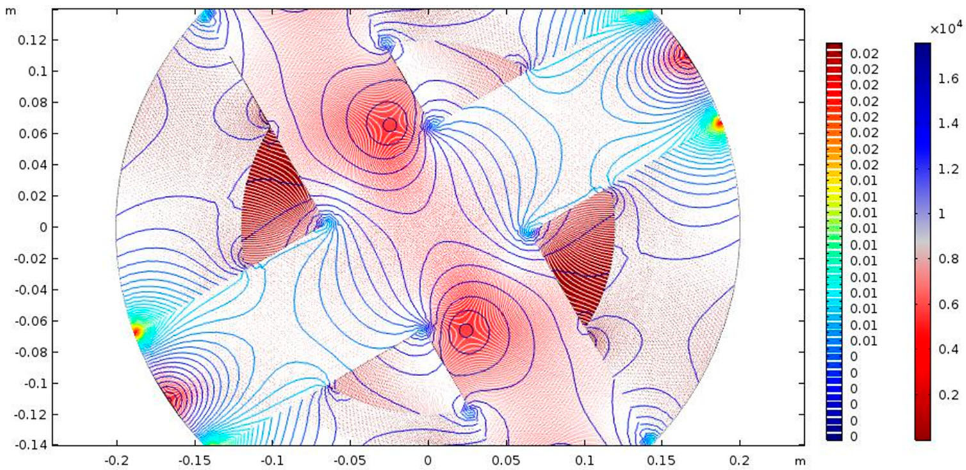

Figure 3. Anholonomy density and spherical normal magnetic flow with $\Psi(\mathbf{N})$.

Proof: By cross product, we have

$$\begin{aligned}
&\nabla_\vartheta \Psi(\mathbf{N}) \times \nabla_t \Psi(\mathbf{N}) \\
&\quad = \left(\frac{\partial \epsilon}{\partial \vartheta}\epsilon\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right) - \epsilon^2 \frac{\partial \epsilon}{\partial t}\right)\delta \\
&\qquad + \left(\epsilon^2\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right) - \epsilon^3 \varpi_1\right)\mathbf{T} \\
&\qquad + \left(\epsilon \frac{\partial \epsilon}{\partial t} - \frac{\partial \epsilon}{\partial \vartheta}\epsilon \varpi_1\right)\mathbf{N}.
\end{aligned}$$

Anholonomy density of $\Psi(\mathbf{N})$ is given by

$$\rho_{\Psi(\mathbf{N})} = -\epsilon\left(\epsilon^2\left(\frac{\partial \varpi_2}{\partial \vartheta} + \varpi_1 \epsilon\right) - \epsilon^3 \varpi_1\right).$$

Using density equation in phase, we obtain

$$\begin{aligned}
\Phi_{\Psi(\mathbf{N})} = \iint &\left(-\pi\left(\left(\frac{\partial \epsilon}{\partial \vartheta} - \pi\right)\left(\epsilon\left(\frac{\partial \chi_2}{\partial \vartheta} + \chi_1 \epsilon\right)\right.\right.\right. \\
&\left.\left.\left. - \pi \chi_2\right) - \epsilon^2\left(\frac{\partial \epsilon}{\partial t} - \pi \chi_1\right)\right)\delta\right) \,\mathrm{d}\varrho.
\end{aligned}$$

By using spherical frame, we have

$$\rho^{\mathcal{FR}}_{\Psi(\mathbf{N})} = \epsilon\left(2\epsilon \frac{\partial \epsilon}{\partial \vartheta} + 3\frac{\partial \epsilon}{\partial \vartheta}\epsilon^4\right).$$

Since, we immediately arrive at

$$\Phi^{\mathcal{FR}}_{\Psi(\mathbf{N})} = \iint \left(\epsilon\left(2\epsilon \frac{\partial \epsilon}{\partial \vartheta} + 3\frac{\partial \epsilon}{\partial \vartheta}\epsilon^4\right)\right) \mathrm{d}\varrho.$$

Spherical anholonomy density and spherical flow lines in the centre of the quadrupole spherical magnet with Lorentz force $\Psi(\mathbf{N})$. By this force, spherical density is given by the particle-tracing algorithm in Figure 3. ■

4. Rational solutions for evolution equations of Lorentz force vectors by travelling wave hypothesis approach

In this section, we consider perturbed solutions of NLSE. These solutions govern travelling soliton propagations throughout Lorenz force field vectors in an optical fibre with Figures 4 and 5. Travelling assumption is used to determine analytical solutions. In order to support analytical solutions, numerical simulations are also provided. Here we consider evolution equations of the Lorentz force $\Psi(\mathbf{N})$ field vectors. Firstly, we have

$$3\epsilon(\vartheta, t)\frac{\partial \epsilon(\vartheta, t)}{\partial \vartheta} = \varpi_1,$$

$$2\frac{\partial \epsilon(v,t)}{\partial \vartheta} = \frac{\partial \varpi_2(\vartheta,t)}{\partial \vartheta} + \varpi_1 \epsilon(\vartheta,t). \tag{1}$$

We consider the given below traveling wave transformation for Equations (1)

$$\begin{aligned} &\epsilon(\vartheta,t) = u(\phi), \\ &\varpi_2(\vartheta,t) = r(\phi), \quad \phi = \vartheta - Qt, \end{aligned} \tag{2}$$

where Q describes the speed of the wave

By placing Equation (2) into Equation (1), are obtained as follows

$$\begin{aligned} &-\varpi_1 + 3u(\phi)u'(\phi) = 0, \\ &-\varpi_1 u(\phi) + 2u'(\phi) - r'(\phi) = 0. \end{aligned} \tag{3}$$

Solving Equations (3), we obtain that

$$\begin{aligned} u(\phi) &= \sqrt{\frac{2\varpi_1\phi}{3} + 2c_1}, \\ r(\phi) &= -\frac{2}{3}\sqrt{\frac{2\varpi_1\phi}{3} + 2c_1}(-3 + \varpi_1\phi + 3c_1) + c_2. \end{aligned} \tag{4}$$

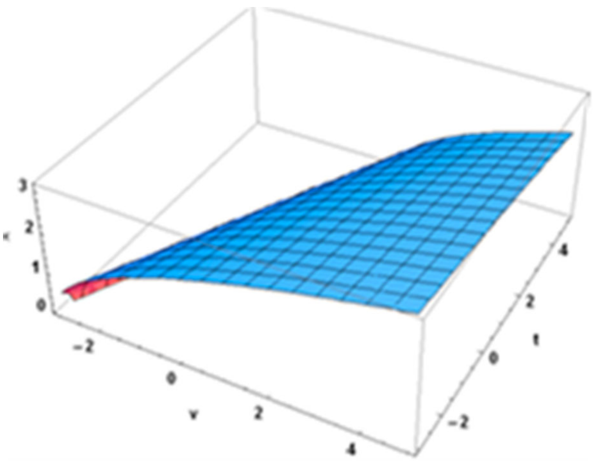

Figure 4. The 3D graphic for the $\epsilon(\vartheta,t)$ analytical solution (5) of Equations (1) for $\varpi_1 = 1.8, Q = 0.5, c_1 = 1$.

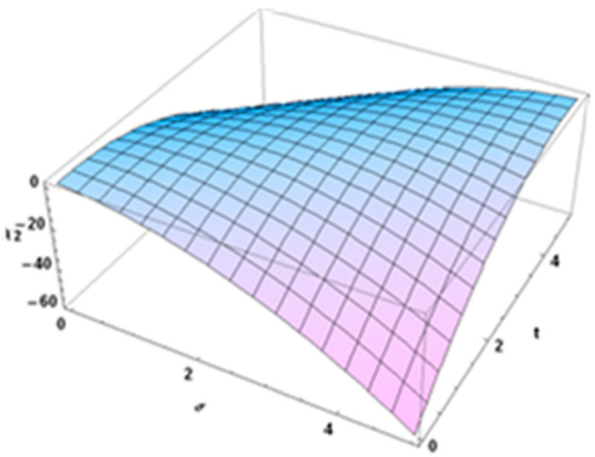

Figure 5. The 3D graphic for the $\varpi_2(\vartheta,t)$ analytical solution (5) of Equations (1) for $\varpi_1 = 1.8, Q = 0.5, c_1 = c_2 = 1$.

From Equations (4), we get the solutions of Equations (1) as follows:

$$\begin{aligned} \epsilon(\vartheta,t) &= \sqrt{\frac{2\varpi_1(\vartheta - Qt)}{3} + 2c_1}, \\ \varpi_2(\vartheta,t) &= -\frac{2}{3}\sqrt{\frac{2\varpi_1(\vartheta - Qt)}{3} + 2c_1} \\ &\quad \times (-3 + \varpi_1(\vartheta - Qt) + 3c_1) + c_2. \end{aligned} \tag{5}$$

5. Conclusion

Understanding the motion of curves is significant in several physical events including vortex filaments and Heisenberg spin chain dynamics. The connection between a certain class of the moving curves in Euclidean space with certain integrable equations.

In this paper, we obtain optical Heisenberg ferromagnetic spin of timelike spherical magnetic flows of particles by the spherical frame in de Sitter space. Also, the concept of timelike spherical magnetic particles is investigated, which may have evolution equations. Finally, we obtain total phases for spherical magnetic flows.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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