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Research article

Positivity preserving interpolation by using rational quartic spline

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Abstract: In this study, a new scheme for positivity preserving interpolation is proposed by using C^1 rational quartic spline of (quartic/quadratic) with three parameters. The sufficient condition for the positivity rational quartic interpolant is derived on one parameter meanwhile the other two are free parameters for shape modification. These conditions will guarantee to provide positive interpolating curve everywhere. We tested the proposed positive preserving scheme with four positive data and compared the results with other established schemes. Based on the graphical and numerical results, we found that the proposed scheme is better than existing schemes, since it has extra free parameter to control the positive interpolating curve.

Keywords: Positivity; rational quartic spline; shape-preserving; C^1 continuity; positive data

Mathematics Subject Classification: 65D05, 65D07, 65D10, 65D17, 65D18

1. Introduction

Shape-preserving interpolation is an important part in the application of engineering and sciences. The technique for finding the smoothness visual or graphical representation of data is not new; for example, it can be found in astronomy and meteorology. The main idea for the positivity preservation is to derive positive results in the entire given interval and if possible those rational splines should have free parameters. The free parameter provides the extra degree of freedom to the user in manipulating the final shape of the interpolating curves. Thus, shape-preserving interpolation research is focused on developing a new rational spline with the extra free parameters. Positivity preserving is about visualizing positive data sets, for instance, rainfall distribution at a certain location. The resulting interpolating curve must preserve the positivity of data sets everywhere and any negativity is meaningless.

Many researchers have contributed to the field of shape-preserving especially on the positivity preserving interpolation. A brief review is provided here. Asim and Brodlie [1] have preserved the shape-preserving for the positivity and constrained data by inserting one or more knots where are required to inserting. Goodman [2] have been surveyed the alternatives of the shape-preserving for interpolating curve. Sarfraz [3,4] have been used the rational function for preserve the shape-preserving interpolation.

Delbourgo & Gregory [5] give the clear concept of the shape-preserving interpolation by using a rational cubic spline. The amount of gas discharge during experiments by Brodlie and Butt [6], Brodlie et al. [7] and Sarfraz et al. [8] are the examples of positive data. Peng et al. [9] discussed the preserving of positivity interpolation of data on rectangular grids. Their scheme extended the rational cubic spline of Sarfraz [4] with two parameters to construct the bivariate rational interpolating function with C^1 continuity. The main strategy of this scheme was by converting the bivariate interpolating function into bi-cubic. The sufficient condition for the positivity derived and used the lower bound have been discussed in Chan et al. [10].

Brodlie and Butt [6] and Butt and Brodlie [11] constructed the piecewise cubic Hermite interpolant with C^1 continuity to preserve the positivity and monotonicity respectively. In their scheme, needs to insert one or two extra knots is needed to be in the interval where the shape violation exists. It will increase the total number of data points and increases the required computation. No derivation is required. Brodlie et al [7] extended the idea of Butt and Brodlie [11] to construct the positivity preserving of positive surface data arranged over a rectangular grid.

Karim and Pang [12] have proposed shape-preserving using rational cubic spline with three parameters (cubic/quadratic). Karim and Pang [13] have extended the function of rational cubic spline into C^1 continuity interpolating curve for shape -preserving. Karim and Saaban [14] has been discussed shape-preserving interpolation with C^1 continuity by using rational cubic ball function with one parameter and its application in image interpolation. This scheme has been extended the function of the univariate rational cubic ball to the bivariate function.

Hussain et al. [15] extended the application of the rational quartic spline given by Wang and Tan [16] to preserve the positivity of data. The data-dependent sufficient condition for the rational interpolant to satisfy the shape-preserving properties are derived. These schemes were not successful in producing positive and convex interpolating curves. The main difference between Wang and Tan [16] and Hussain et al. [15] scheme with the proposed scheme in this study is that we constructed the rational quartic spline with quadratic denominator with three parameters compared to their scheme which used two parameters to achieve the positive curve interpolation everywhere.

The main contribution of this study can be summarized as follow:

a. The proposed scheme does not require the modification of the first derivative if the shape-preserving is violated as shown in the work of Brodlie and Butt [6].
b. The proposed scheme is guaranteed to produce the positivity preserving interpolation on all intervals, while the schemes of Hussain et al. [15] and Sarfraz et al. [17] cannot not producing positive preserving interpolation on all intervals.
c. The proposed scheme is better than existing schemes, since it has extra free parameters to control the positive interpolating curve and many of the existing schemes are special cases of our proposed scheme and have similar results to existing schemes for various values of the shape parameters.

This paper is organized as follows: section 2 is devoted to the new rational quartic spline interpolation including construction and derivative estimation as in Harim et al. [18]. The sufficient condition for the positivity preserving is discussed in section 3. The numerical and graphical results for positivity preserving is discussed in section 4. The conclusion is given in the final section.

2. Rational quartic spline interpolant

This section discusses the construction rational quartic spline with three parameters including the definition from Harim et al. [18].

2.1. Construction rational quartic spline

Given scalar data set $\{(x_I, f_i), i = 1,2,..n\}$ where $x_1 < x_2 <\ldots< x_n$ and the first derivative d_i, at the respective point. Then the rational quartic spline with three shape parameters $\alpha_i > 0, \beta_i > 0$ and $\gamma_i \geq 0$ on the interval is given by ([18]):

$$\mathrm{S(x)} = \frac{\mathrm{P_i(\theta)}}{\mathrm{Q_i(\theta)}} \tag{1}$$

where

$$P_i(\theta) = (1-\theta)^4\alpha_i f_i + (1-\theta)^3\theta A_i + (1-\theta)^2\theta^2 B_i + (1-\theta)\theta^3 C_i + \theta^4\beta_i f_{i+1}$$

$$Q_i(\theta) = \alpha_i(1-\theta)^2 + \gamma_i(1-\theta)\theta + \beta_i\theta^2$$

with $h_i = x_{i+1} - x_i$, $\Delta_i = \frac{f_{i+1}-f_i}{h_i}$and $\theta = \frac{x-x_i}{h_i}$.

The rational quartic spline in (1) is C^1 continuity at the knots $x_i, i = 1,2,\ldots,n-1$and satisfy the following conditions:

$$\begin{array}{cc} S(x_i) = f_i & S(x_{i+1}) = f_{i+1} \\ S^{(1)}(x_i) = d_i & S^{(1)}(x_{i+1}) = d_{i+1} \end{array} \tag{2}$$

From condition in (2) and after some derivations, the unknown $\mathrm{A_i}, \mathrm{B_i}$ and C_i are given as in

Harim et al. [10].

$$\begin{aligned} A_i &= (2\alpha_i + \gamma_i)f_i + \alpha_i h_i d_i \\ B_i &= (\alpha_i + \gamma_i)f_{i+1} + (\beta_i + \gamma_i)f_i \\ C_i &= (2\beta_i + \gamma_i)f_{i+1} - \beta_i h_i d_{i+1} \end{aligned} \tag{3}$$

where $s(x) \in C^{(1)}[x_0, x_n]$.

2.2. Derivative estimation

There are many alternative methods to estimate the values of first derivative such as Arithmetic Mean Method (AMM) by Gregory & Delbourgo [18], Geometric Mean Method (GMM) by Gregory & Delbourgo [19] and Harmonic Mean Method (HMM) by Gregory & Delbourgo [5]. For this study, we choose the Arithmetic Mean Method (AMM) because it is simple and suitable for all types of data sets. The derivative is given as follow Harim et al. [18].

Suppose a 2D set of data $\{(x_i, f_i), i = 1,1,2,\ldots,n\}$ is considered that $x_1 < x_2 <\ldots< x_n$. The first derivatives, d_i at the point $x_i, i = 1,2,\ldots,n$. Let $h_i = x_{i+1} - x_i$ and $\Delta_i = \frac{f_{i+1} - f_i}{h_i}$.

At the end points of x_1 and x_n.

$$d_1 = \Delta_1 + (\Delta_1 - \Delta_2)\left(\frac{h_1}{h_1 + h_2}\right) \tag{4}$$

$$\mathrm{d_n} = \Delta_{\mathrm{n}-1} + (\Delta_{\mathrm{n}-1} - \Delta_{\mathrm{n}-2})\left(\frac{\mathrm{h_{n-1}}}{\mathrm{h_{n-1}} + \mathrm{h_{n-2}}}\right) \tag{5}$$

At the center point, $x_i = 2,3,..,n-1$ the value of d_i are given as

$$d_i = \left(\frac{h_{i-1}\Delta_i + h_i\Delta_{i-1}}{h_{i-1} + h_i}\right) \tag{6}$$

3. Positivity preserving interpolation

In this section, an effective and automatic method to generate the positive interpolating curve is described. The new rational quartic spline with three parameters in Eq (1) is used for this purpose. The data under consideration are positive. The sufficient condition is described in the following theorem.

In this study, given that given that $\{(x_i, f_i), i = 1,1,2,\ldots,n\}$ with $x_1 < x_2 <\ldots< x_n$is positive to avoid dividing by zero such that

$$f_i > 0 \quad i = 1,2,..,n \tag{7}$$

The rational quartic spline, $S(x) > 0$ if and only if for both polynomials $P_i(\theta) > 0$ and $Q_i(\theta) > 0$. Since $Q_i(\theta) > 0$ for $\alpha_i > 0, \beta_i > 0 \quad \gamma_i \geq 0 \quad i = 1,2,\ldots,n-1$. Therefore, the sufficient condition for the positivity of $S(x)$ can be derived from Equation (7). The main objective this paper is to find the best value of the parameter when $\alpha_i > 0, \beta_i > 0 \quad i = 1,2,\ldots,n-1$ that will ensure that the rational quartic spline in Eq (1) is positive to interpolate of the positive interpolating curve on the whole domain in given data sets. We have adapted the method from Hussain et al. [15].

The following theorem gives sufficient conditions for the positivity preserving of the proposed scheme. It is data-dependent and has two free parameters to achieve the final interpolating curve of the positive result.

Theorem 1. For strictly positive data that has been discussed, the rational quartic spline interpolant defined on the interval $[x_1, x_n]$ is positive in each subinterval $[x_1, x_n], i = 1,2, \dots n-1$ the following sufficient conditions are satisfied:

$$\gamma_i > Max\left\{0, \left[\frac{-2\alpha_i f_i - \alpha_i h_i d_i}{f_i}\right], \left[\frac{-2\beta_i f_{i+1} + \beta h_i d_{i+1}}{f_{i+1}}\right]\right\} \tag{8}$$

Proof: The sufficient conditions of the positivity are derived from the condition in Eq (7) as follows:

By using Eq (7), it can be observed that $\alpha_i f_i, \beta_i f_{i+1}$ and B_i are positive, so the $P_i(\theta)$ will be positive value if

$$A_i > 0, C_i > 0 \tag{9}$$

$A_i > 0$ If

$$\gamma_i > \frac{-2\alpha_i f_i - \alpha_i h_i d_i}{f_i} \tag{10}$$

$C_i > 0$ If

$$\gamma_i > \frac{-2\beta_i f_{i+1} + \beta_i h_i d_{i+1}}{f_{i+1}} \tag{11}$$

The above constraints can be combined and rewritten as:

$$\gamma_i > Max\left\{0, \left[\frac{-2\alpha_i f_i - \alpha_i h_i d_i}{f_i}\right], \left[\frac{-2\beta_i f_{i+1} + \beta h_i d_{i+1}}{f_{i+1}}\right]\right\} \tag{12}$$

For the computer implementation, condition in Eq (12) can be rewritten as:

$$\gamma_i = \lambda_i + Max\left\{0, \left[\frac{-2\alpha_i f_i - \alpha_i h_i d_i}{f_i}\right], \left[\frac{-2\beta_i f_{i+1} + \beta h_i d_{i+1}}{f_{i+1}}\right]\right\} \tag{13}$$

where $\lambda_i > 0$.

Algorithm for computer implementation as follow:

Step 1: Input the positive data for $\{(x_i, f_i), i = 1,1,2, \dots, n\}$.

Step 2: Calculate the first derivative, d_i using Arithmetic Mean Method (AMM).

Step 3: Stimulate the value free parameters when $\alpha_i > 0, \beta_i > 0$ and values of λ_i.

Step 4: Calculate the value of $\gamma_i$by using the sufficient condition in Equation (13).

Step 5: Construct the positive interpolating curve with C^1 continuity.

Figure 1 shows flowchart for algorithm to obtain the positivity preserving interpolation curve.

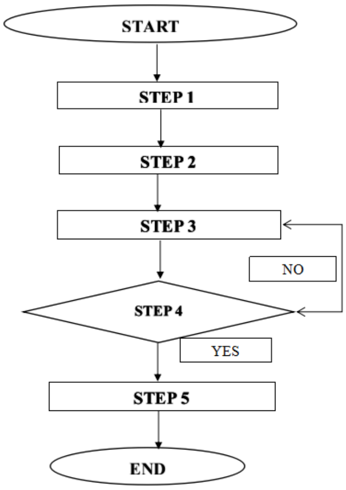


Figure 1. Flowchart for the proposed scheme algorithm.

4. Numerical results

This section is devoted to the numerical results for the positivity preserving interpolation including the comparison with the existing schemes. We test the scheme to four different positive data sets.

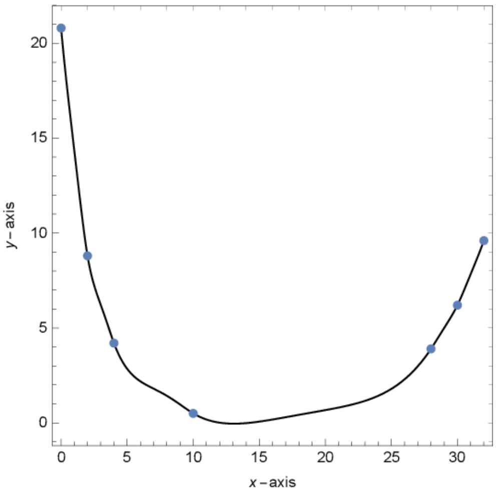
4.1. Example 1

Table 1 shown positive data from demonstrates the level of oxygen from the experiment conducted. Data used Butt and Brodlie [11].

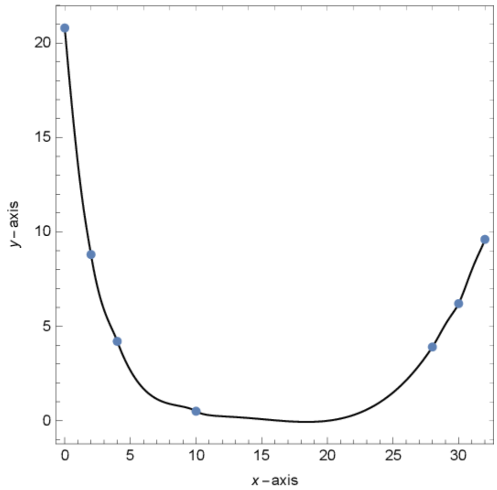
Table 1. Data from Butt and Brodlie [11].

i	1	2	3	4	5	6	7
x_i	0	2	4	10	28	30	32
f_i	20.8	8.8	4.2	0.5	3.9	6.3	9.6
d_i	−7.85	−4.15	−1.8792	−0.4153	1.0539	1.425	1.975

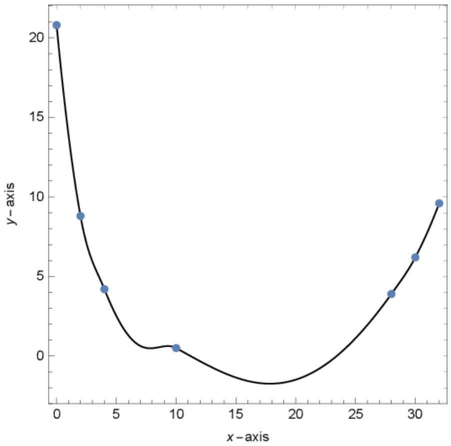
Figure 2 shows the positivity preserving interpolating curve for Table 1 using the existing scheme and the proposed scheme with a value of $\lambda_i = 0.1$. Figure 2(a) shows the interpolating curve by the quartic polynomial when $\alpha_i = 1, \beta_i = 1, \gamma_i = 2$ and the curve gives the negative result as shown in Figure 3(a). Figure 2(b) and Figure 2(c) show the scheme of Hussain et al. [12] and Sarfraz et al. [17] and both schemes show the negative result on the interval [10,28] as shown in Figure 3(b) and Figure 3(c) respectively. Figure 2(d) until Figure 2(f) show the Karim and Pang [12], Qin et al. [21] and the proposed scheme using the same value of $\alpha_i = 1, \beta_i = 1$ respectively. For Figure 2(e) not visual pleasing, because on interval [10,28] straight line compared with the others scheme. Figure 2(g) and Figure 2(h) show the interpolating the positivity curve by the proposed schemes with different values shown in Table 2. The values for γ_i obtained from Theorem 1.



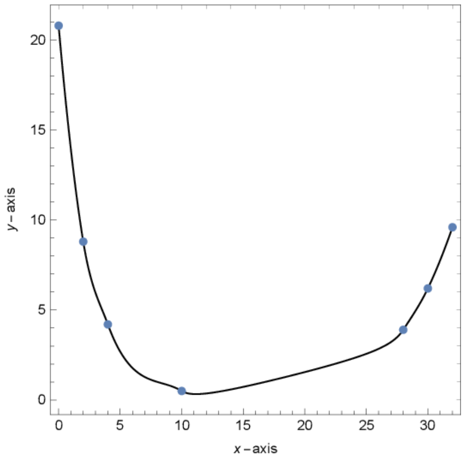
(a) Quartic Polynomial



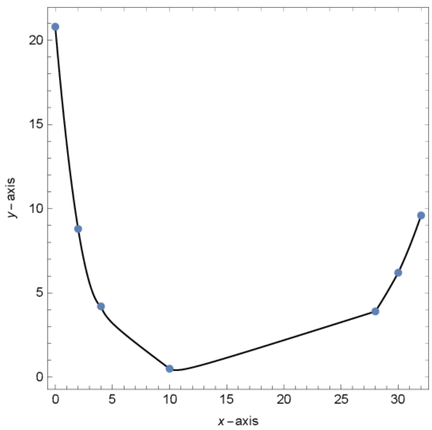
(b) Hussain et al. [15]



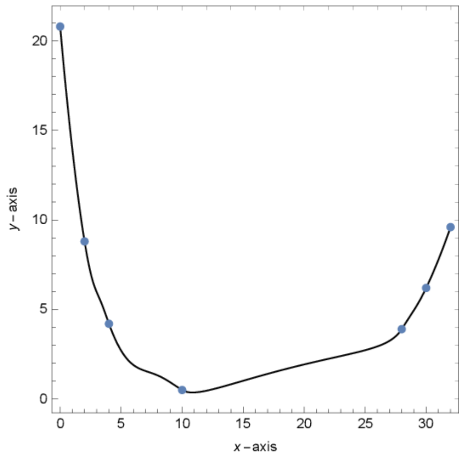
(c) Sarfraz et al. [17]



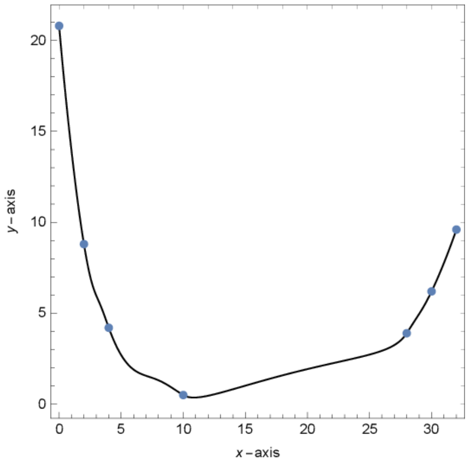
(d) Karim and Pang [12]



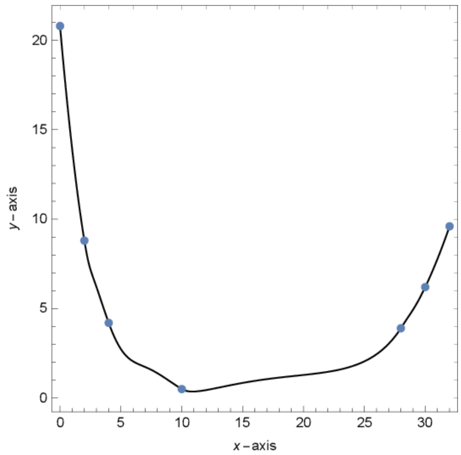
(e) Qin et al. [21]



(f) The Proposed Scheme

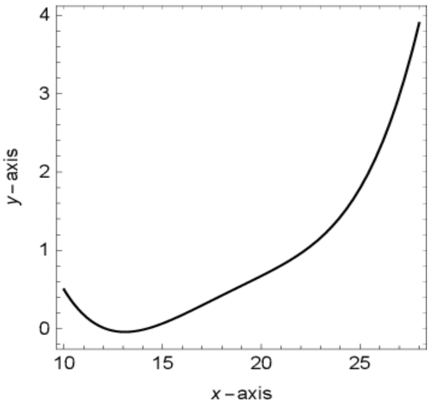


(g) The Proposed Scheme

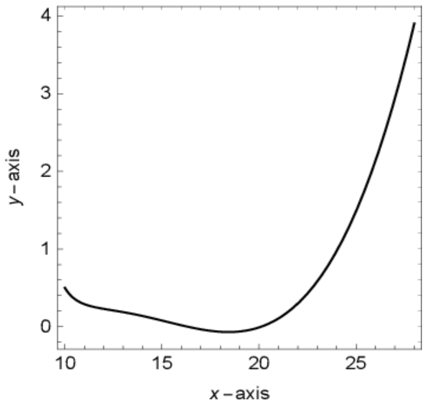


(h) The Proposed Scheme

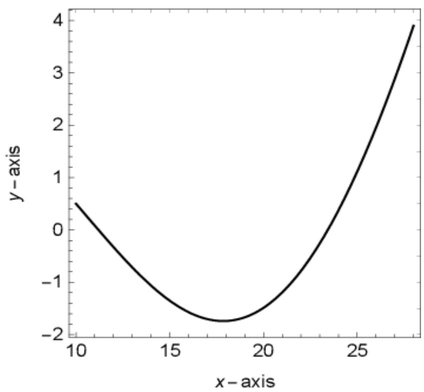
Figure 2. The Interpolating Curve.



(a) Quartic Polynomial



(b) Hussain et al. [15]



(c) Sarfraz et al. [17]

Figure 3. Zooming negative curve on interval [10,28].

Table 2. Value of parameters for the proposed scheme.

	i	1	2	3	4	5	6	7
	d_i	−7.85	−4.15	−1.879	−0.415	1.054	1.425	1.975
Figure 2(f)	α_i	1.0	1.0	1.0	1.0	1.0	1.0	-
	β_i	1.0	1.0	1.0	1.0	1.0	1.0	-
	γ_i	0.1	0.1	0.7845	13.0472	0.1	0.1	-
Figure 2(g)	α_i	1	1	1	1	1	1	-
	β_i	0.1	0.1	0.1	0.1	0.1	0.1	-
	γ_i	0.1	0.1	0.7845	13.0074	0.1	0.1	-
Figure 2(h)	α_i	1	1	1	1	2.5	1	-
	β_i	3	6	2.5	9.5	2.5	1.5	-
	γ_i	0.1	0.1	0.7845	32.4680	0.1	0.1	-

Figure 4 shows the interpolating curve for positivity preserving by the various value of $\lambda_i = 0.1$, $\lambda_i = 0.5$ (dashed) and $\lambda_i = 0.25$ (dotted). From this figure, we can conclude that value of $\lambda_i = 0.5$ more suitable to be chosen for this positive data.

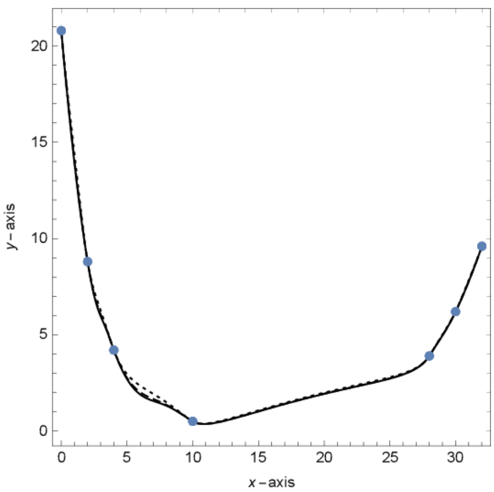


Figure 4. Interpolating curve by varying λ_i .

4.2. Example 2

Table 3 shows a positive data are taken from Hussain and Ali [22].

Figure 5 shows the positivity preserving interpolation for positive data shown in Table 3. Figure 5(a) shows the interpolation curve for default quartic polynomial. Figure 5(b) and Figure 5(c) show curve for positivity used Hussain et al. [15] and Sarfraz et al. [17] schemes respectively. Figure 6 shows the zooming negative curve on the interval [3,8] for Figure 5(c) which is Sarfraz et al. [23] scheme. Figure 5(d) show the interpolating curve for positivity for Karim and Pang [12] meanwhile Figure 5(e) shows the positivity preserving interpolation for Qin et al. [21]. Figure 5(f) until Figure 5(h) show the interpolating curve for the proposed scheme by using values of different parameters shown in Table 4. The value of the parameter γ_i is calculated from Theorem 1 have been discussed in Eq (13). Figure 5(f) gives a loose curve compared to the other proposed schemes.

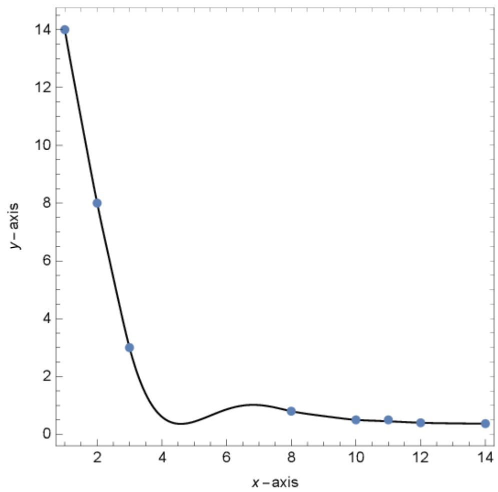
Figure 6 shows that Sarfraz et al. [17] fails to preserve the positivity of the positive data sets. Figure 7 shows the positivity interpolating curve by varying values of λ_i. Figure 7 shows the values of $\lambda_i = 1$, $\lambda_i = 0.5$ (dashed) and $\lambda_i = 2.5$ (dotted). The figure above used the same value of parameters α_i and β_i but the different values of γ_i due to the calculation in Theorem 1. Curve with $\lambda_i = 2.5$ gives a slightly different curve compared with $\lambda_i = 0.5$ and $\lambda_i = 1$.

Table 3. Data from [11].

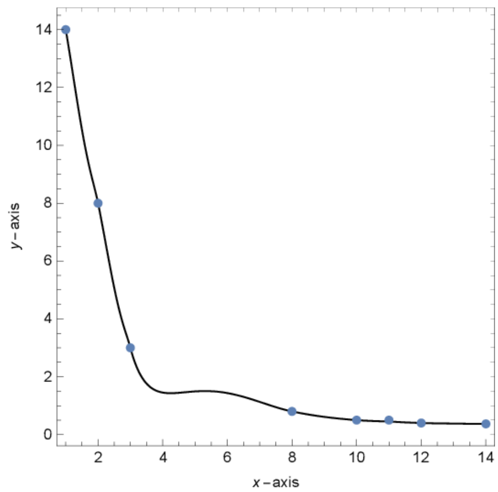
i	1	2	3	4	5	6	7	8
x_i	1	2	3	8	10	11	12	14
f_i	14	8	3	0.8	0.5	0.45	0.4	0.37
d_i	−6.5	−5.5	−4.24	−0.232	−0.083	−0.05	−0.038	0.0083

Table 4. Value of parameters for the proposed scheme.

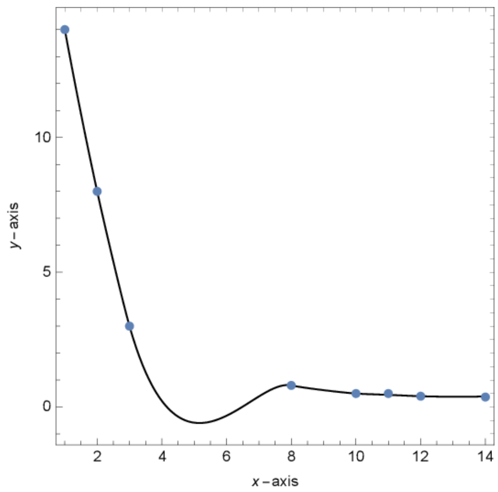
	i d_i	1 −6.5	2 −5.5	3 −4.24	4 −0.232	5 −0.083	6 −0.05	7 −0.038	8 0.0083
Figure 5(f)	α_i	1	1	1	1	1	1	1	-
	β_i	2.5	2.5	2.5	2.5	2.5	2.5	2.5	-
	γ_i	0.1	0.1	5.17	0.1	0.1	0.1	0.1	-
Figure 5(g)	α_i	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-
	β_i	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-
	γ_i	0.1	0.1	1.37	0.1	0.1	0.1	0.1	-
Figure 5(h)	α_i	2.5	2.5	10	2.5	2.5	2.5	2.5	-
	β_i	2.5	2.5	0.01	2.5	2.5	2.5	2.5	-
	γ_i	0.1	0.1	50.77	0.1	0.1	0.1	0.1	-



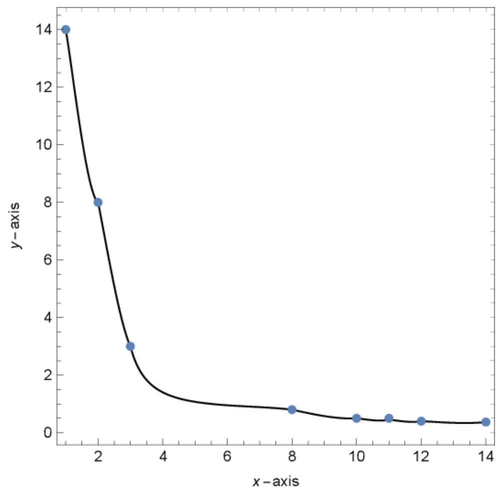
a) The Quartic Polynomial



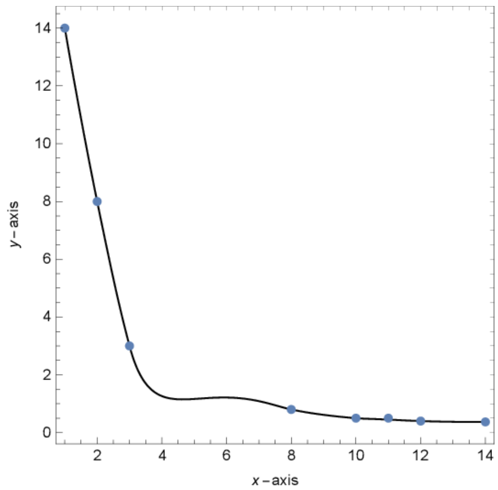
b) Hussain et al. [15]



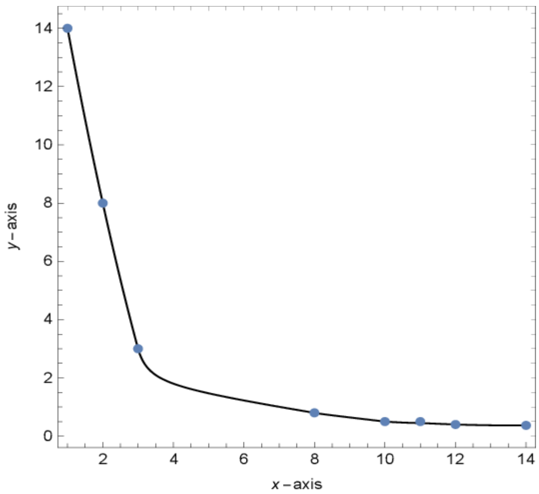
c) Sarfraz et al. [17]



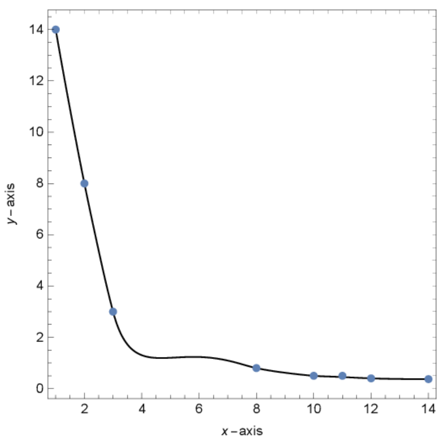
d) Karim and Pang [12]



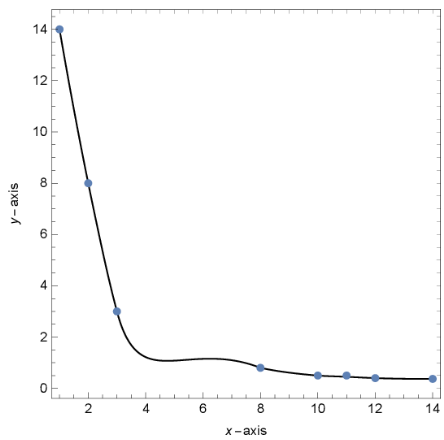
e) Qin et al. [21]



f) The Proposed Scheme



g) The Proposed Scheme



h) The Proposed Scheme

Figure 5. The Interpolating Curve.

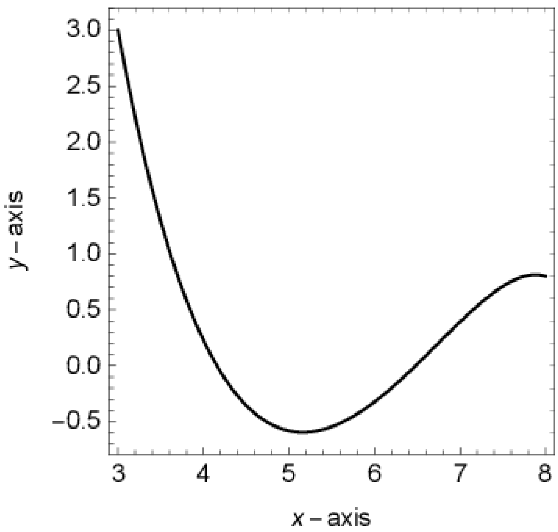


Figure 6. Zooming curve for Figure 5(c).

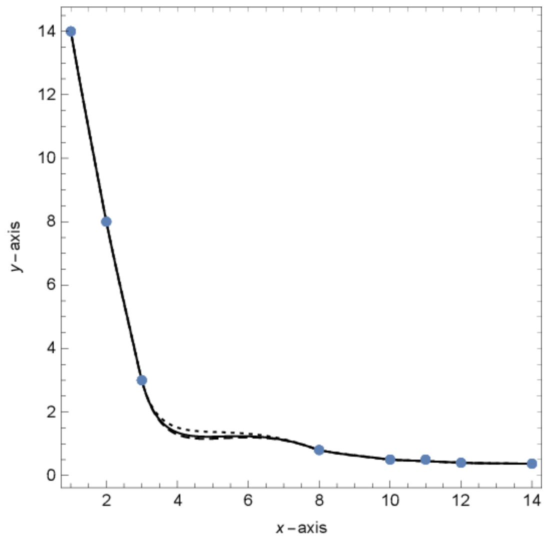


Figure 7. Interpolating curve by varying λ_i.

4.3. Example 3

Table 5 shows a positive data from Wu et al. [23].

Figure 8 shows the shape-preserving for positivity interpolation curve for Table 5. Figure 8(a) shows the quartic polynomial interpolating curve with value $\alpha_i = 1, \beta_i = 1, \gamma_i = 2$. Figure 9 shows the zooming for Figure 8(a) that show the negative result for positivity interpolation. Figure 8(b) until Figure 8(e) show the positivity curve for Hussain et al. [15], Sarfraz et al. [17], Karim and Pang [12] and Qin et al. [21] respectively. For these figures, the curve nearly shows the negative result for the interval [3,7]. Figure 8(e) not visual pleasing curve interpolation on interval [1.2, 1.8] compared with the others scheme. Curve interpolation for the proposed scheme shown in Figure 8(f) until Figure 8(h) by using the values of different α_i and β_i shown in Table 6.

Figure 10 shows the positivity interpolating curve by varying values of λ_i for all $i = 1,2, \ldots n - 1$.

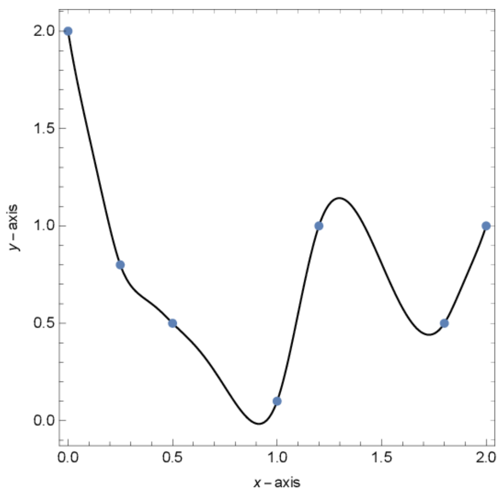
Figure 10 shows the values of $\lambda_i = 0.1$, $\lambda_i = 0.5$ (dashed) and $\lambda_i = 2.5$ (dotted). The Figure above used the same value of parameters α_i and β_i but different values of γ_i due to the calculation in Theorem 1. Interval [0.25, 0.5] with $\lambda_i = 2.5$ gives smoother curve compared with others value. Curve with $\lambda_i = 0.1$ is smoother or pleasing visually on the interval [1.2, 1.8]. For the other intervals, it gives the same curve for all three values of λ_i.

Table 5. Data from [23].

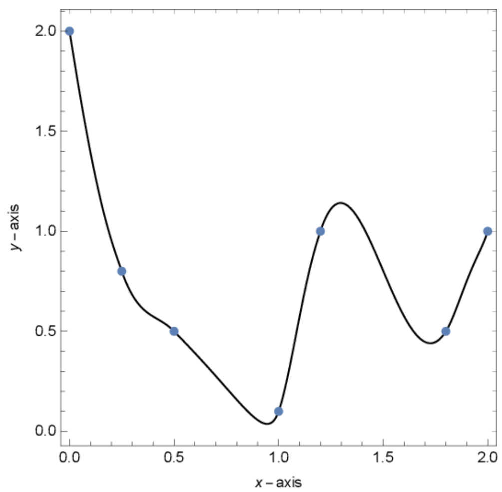
i	1	2	3	4	5	6	7
x_i	0	0.25	0.5	1	1.2	1.8	2
f_i	2	0.8	0.5	0.1	1	0.5	1
d_i	−6.6	−3	−1.067	2.986	3.167	1.667	3.333

Table 6. Value of parameters for the proposed scheme.

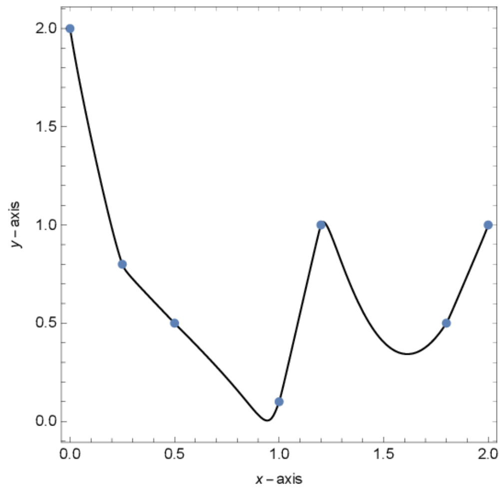
	i	1	2	3	4	5	6	7
	d_i	−6.6	−3	−1.067	2.986	3.167	1.667	3.333
Figure	α_i	1.0	1.0	1.0	1.0	1.0	1.0	-
8(f)	β_i	1.0	1.0	1.0	1.0	1.0	1.0	-
	γ_i	0.1	0.1	13.01	0.1	0.1	0.1	-
Figure	α_i	0.5	0.3	2.5	2.5	5.3	0.1	-
8(g)	β_i	2.5	10.5	1.0	1.0	5.3	5.5	-
	γ_i	0.1	0.1	13.03	0.1	0.1	0.1	-
Figure	α_i	2.5	0.1	2.5	2.5	0.5	0.5	-
8(h)	β_i	0.15	10.5	0.1	0.1	4.5	0.5	-
	γ_i	0.1	0.1	1.393	0.1	0.1	0.1	-



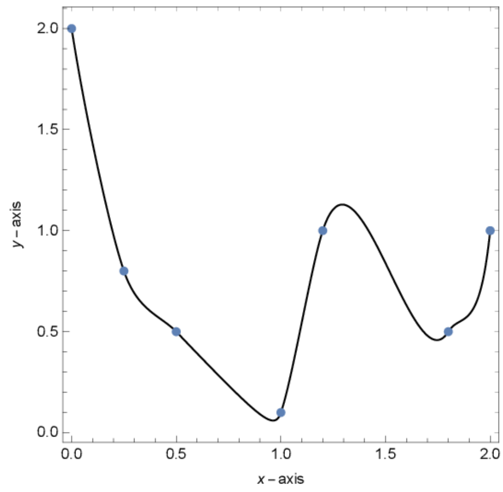
a) The Quartic Polynomial



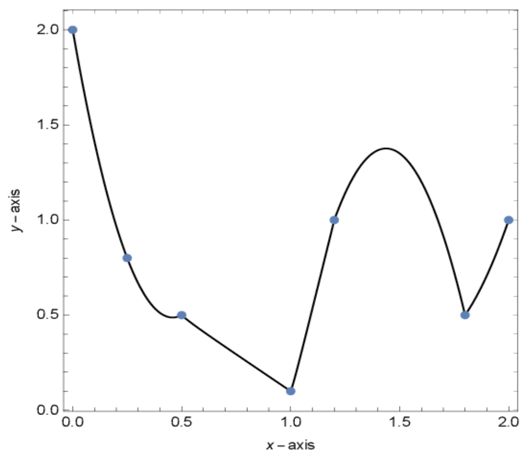
b) Hussain et al. [15]



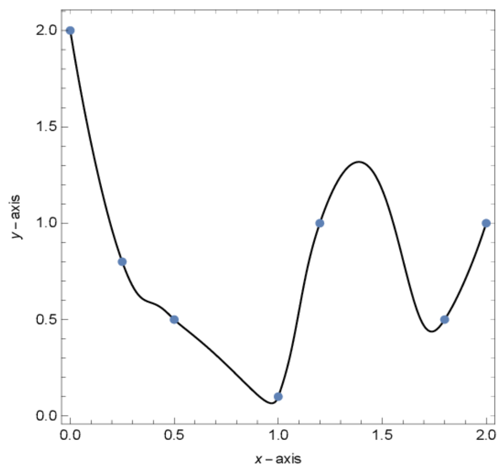
c) Sarfraz et al. [17]



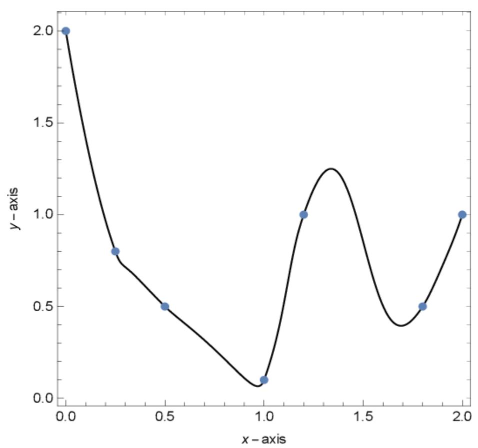
d) Karim and Pang [12]



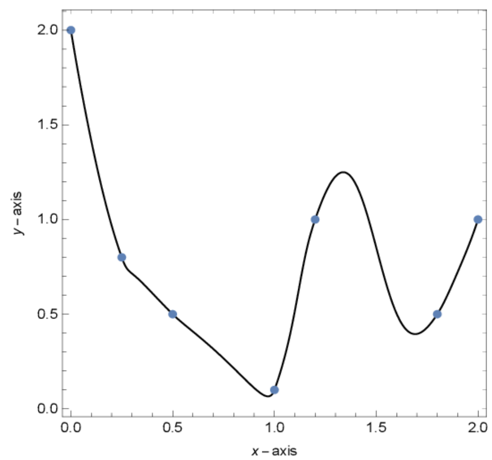
e) Qin et al. [21]



f) The Proposed Scheme



g) The Proposed Scheme



h) The Proposed Scheme

Figure 8. The Interpolating curve.

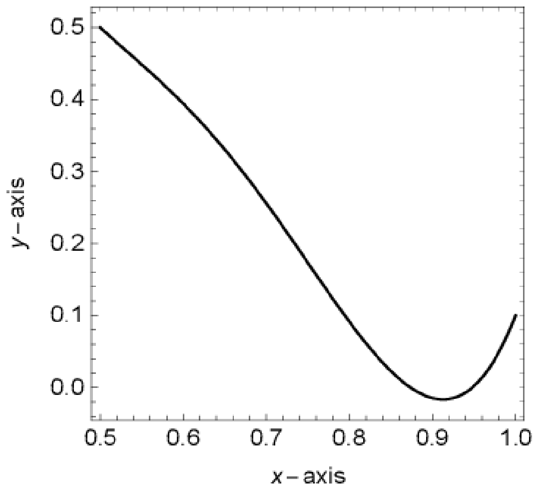


Figure 9. Zooming negative result for Figure 8(a).

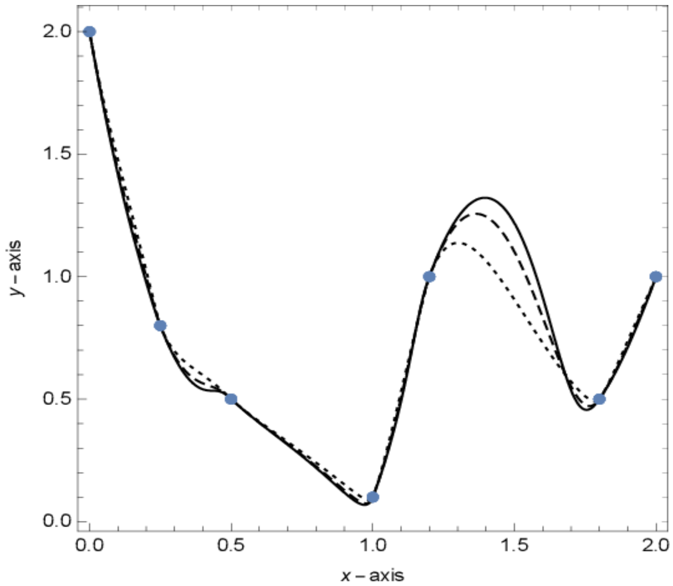


Figure 10. Interpolating curve by varying λ_i.

4.4. Example 4

Table 7 shows a positive data from Hussain et al. [24].

Figure 11 shows the interpolating curve for positivity preserving interpolation curve by comparing the existing schemes and the proposed scheme of a rational quartic spline. Figure 11(a) is default quartic polynomial interpolating curve when $\alpha_i = 1$, $\beta_i = 1$, $\gamma_i = 2$ and that figure shows the negative result on the interval [0,1] as shown in Figure 12(a). Figure 11(b) and Figure 11(c) show the scheme of Hussain et al. [15] and Sarfraz et al. [17], these schemes show the negative result on the interval [0,1] as shown in Figure 12(b) and Figure 12(c) respectively. Figure 11(d) shows the scheme of Karim and Pang [12] meanwhile Figure 11(e) shows the interpolating curve from Qin et al. [21] scheme. The figure, not visual pleasing interpolating curve, because on interval [1.0, 1.7] have straight line compared with the others scheme. Figure 11(f) until Figure 11(h) show the curve interpolation for positivity preserving by the proposed schemes with the values of parameters as shown in Table 8. Figure 11(f) gives a more pleasing visually curve compared with other proposed schemes.

Figure 13 shows the interpolating curve for positivity preserving by the various value of $\lambda_i = 0.1, \lambda_i = 0.5$ (dashed) and $\lambda_i = 2.5$ (dotted). Value of $\lambda_i = 2.5$ more converges to straight -line curve interpolation. For $\lambda_i = 0.1$, give more pleasing curve interpolation.

Table 9 summarize of all presented schemes in this study for positivity preserving interpolation. From all four examples, Hussain et al. [15] fails to preserve the positivity for Examples 1 and 4. Meanwhile Sarfraz et al. [17] fails to preserve the positivity for Examples 1, 2 and 4. Finally, the proposed scheme preserves the positivity of the data for all examples.

Table 7. Positive data from [24].

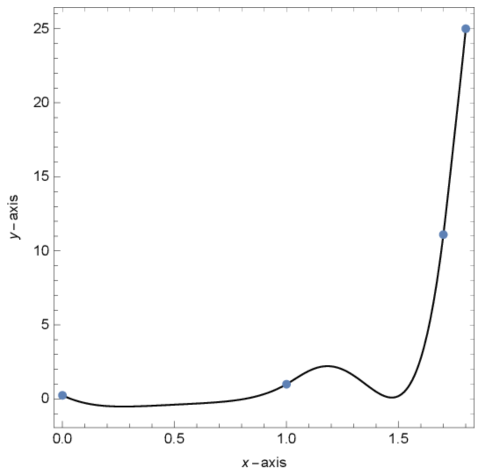
i	1	2	3	4
x_i	0	1	1.7	1.8
f_i	0.25	1	11.11	25
d_i	−7.2962	8.7962	123.4286	154.5714

Table 8. Value of parameters for the proposed scheme.

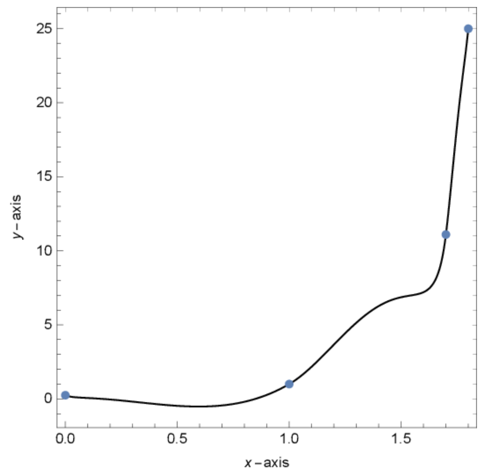
	i	1	2	3	4
	d_i	−7.2962	8.7962	123.4286	154.5714
Figure 11(f)	α_i	1.5	1.5	1.5	-
	β_i	0.1	0.1	1.0	-
	γ_i	40.8772	0.5783	0.1	-
Figure 11(g)	α_i	2.5	2.5	0.5	-
	β_i	0.5	2.5	1.0	-
	γ_i	68.062	14.5592	0.1	-
Figure 11(h)	α_i	0.5	0.01	2.5	-
	β_i	2.5	0.01	2.0	-
	γ_i	17.0905	0.1578	0.1	-

Table 9. Comparison schemes for positivity preserving interpolation.

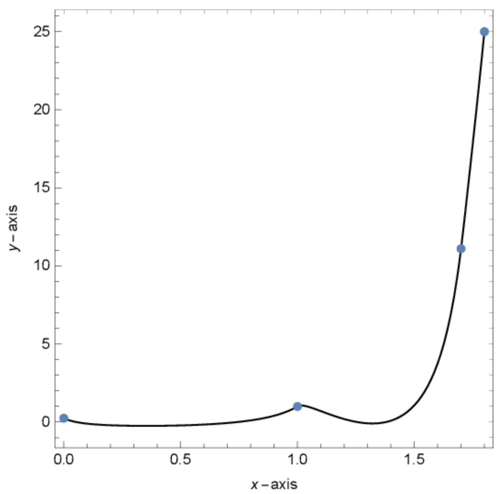
Scheme	Example 1	Example 2	Example 3	Example 4
Hussain et al. [15]	No	Yes	Yes	No
Sarfraz et al. [17]	No	No	Yes	No
The proposed scheme	Yes	Yes	Yes	Yes



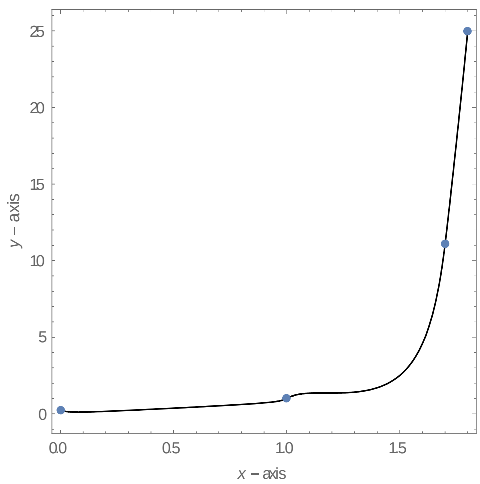
a) The Quartic Polynomial



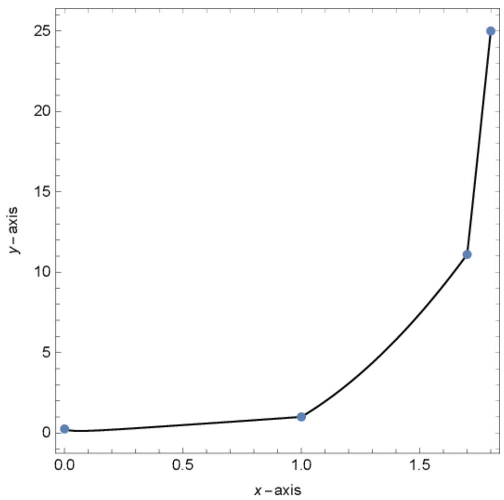
b) Hussain et al. [15]



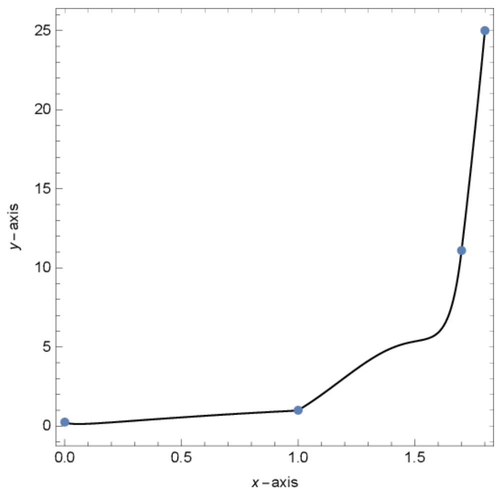
c) Sarfraz et al. [17]



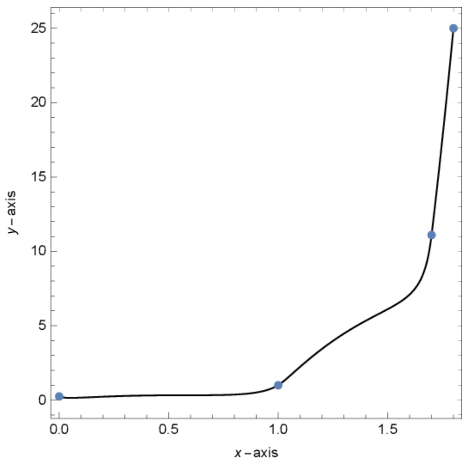
d) Karim and Pang [12]



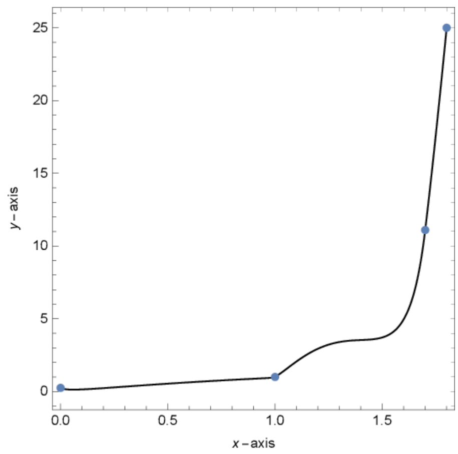
e) Qin et al. [21]



f) The Proposed Scheme

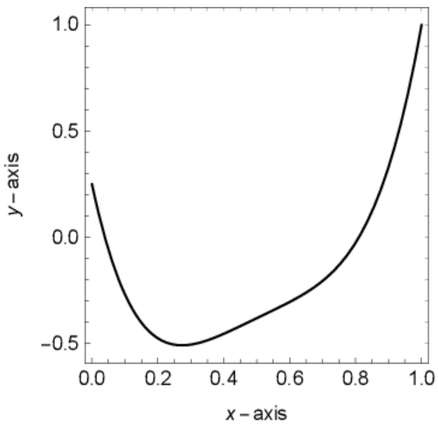


g) The Proposed Scheme

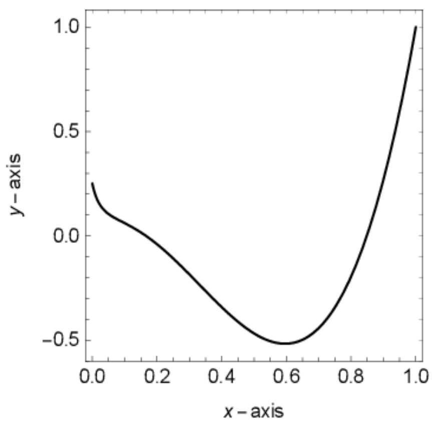


h) The Proposed Scheme

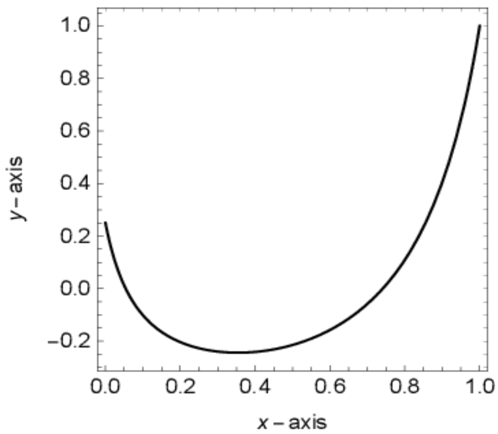
Figure 11. Interpolating curve by varying λ_i.



(a)Quartic Polynomial



(b)Hussain et al. [15]



(c) Sarfraz et al. [17]

Figure 12. Zooming negative curve on interval [0, 1].

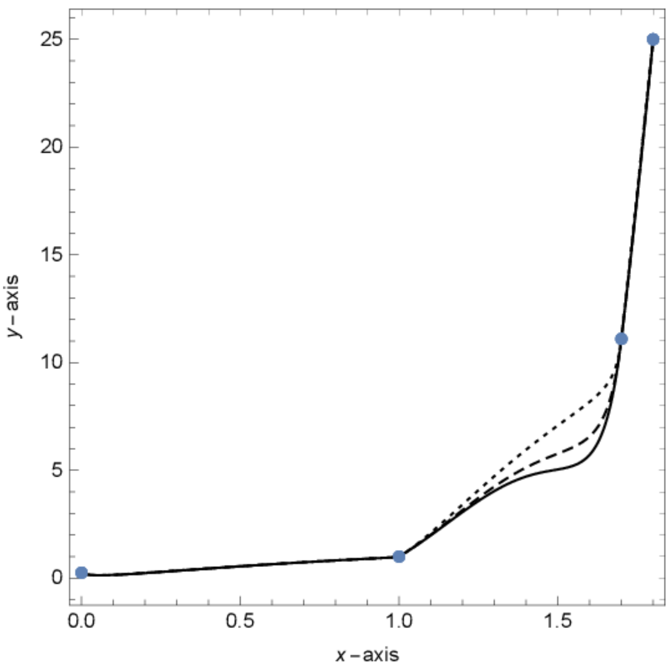


Figure 13. Interpolating curve by varying λ_i.

Final Remark: As seen in Figures 2 and 11, Qin et al. [21] shows interpolation curves that approaching to straight line. Even though the curves have no oscillation, however in terms of visually pleasing, their scheme is not. If we compare with other existing schemes, for instance Karim and Pang [12], the interpolating curves using [12] also show effect 'small oscillation' which not destroying the characteristic of the data. From Figures 4 and 13, we can see that, the proposed interpolating curves is approaching linear segments when $\lambda_i = 2.5$ which is quite similar with Qin et al. [21] scheme. Furthermore, based on Figure 8(e), Qin et al. [21] is not visually pleasing since the interpolating curves are oscillated. Compared with our proposed schemes, all interpolating is visually pleasing. Without doubt, the proposed positivity-preserving using rational quartic spline provide good alternative to the existing schemes.

5. Conclusions

To achieve a smooth for positive data, a new C^1 rational quartic spline (quartic/quadratic) with three parameters is proposed. The two free parameters are manipulated in Eq (13) to preserve the shape of the positivity preserving interpolation. This paper is an extension to the works of Hussain et al. [15] and Sarfraz et al. [17] that did not preserve the positivity results everywhere for positive data. From numerical and graphical results, we found that the proposed scheme preserves the positivity preserving interpolation everywhere than some established schemes such as Hussain et al. [15] and Sarfraz et al. [17]. By manipulating the value of $\lambda_i, i = 1,2,\ldots,n-1$, we found that the smallest value of λ_i, resulted in a smooth curve interpolation for positivity preserving. One possible extension to the present study is constructing positivity preserving interpolation by using the proposed rational quartic spline with C^2 continuity. We may adopt the main idea in Zhu [25,26].

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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