AIMS Mathematics

AIMS Mathematics, 5(3): 2326–2345.
DOI: 10.3934/math.2020154
Received: 14 November 2019
Accepted: 23 February 2020
Published: 03 March 2020

http://www.aimspress.com/journal/Math

Research article

The deterministic and stochastic solutions of the Schrodinger equation with time conformable derivative in birefrigent fibers

Zeliha Korpinar[1], Mustafa Inc[2,3,*], Ali S. Alshomrani[4], Dumitru Baleanu[5,6]

[1] Mus Alparslan University, Faculty of Economic and Administrative Sciences, Department of Administration, 49250, Muş/Turkiye
[2] Fırat University, Science Faculty, Department of Mathematics, 23119 Elazığ/Turkiye
[3] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan
[4] King Abdulaziz University, Science Faculty, Department of Mathematics, 21589 Jeddah/ Saudi Arabia
[5] Cankaya University, Science Faculty, Department of Mathematics, 06530 Ankara /Turkiye
[6] Institute of Space Sciences, Magurele-Bucharest, Romania

*** Correspondence:** Email: minc@firat.edu.tr; Tel: +905468036564.

Abstract: In this manuscript, the deterministic and stochastic nonlinear Schrödinger equation with time conformable derivative is analysed in birefrigent fibers. Hermite transforms, white noise analysis and the modified fractional sub-equation method are used to obtain white noise functional solutions for this equation. These solutions consists of exact stochastic hyperbolic functions, trigonometric functions and wave solutions.

Keywords: the stochastic time fractional nonlinear Schrödinger equation; the modified fractional sub-equation method
Mathematics Subject Classification: 35Qxx, 35C08, 35L05

1. Introduction

In the recently, there have been many studies and developments in the fractional calculus field so far [1–9]. Random models that depend upon uncertainty in differential equations or partial differential equations have been analysed in many years ago in many fields of applied science such as sociology, physics, chemistry, sociology, economics, medicine and biology. Randomness effect in the input may arise because of several faults in the noted or measured data and empirical states [10–15]. Due to the truth that the stochastic types are more naturalistic than the deterministic types, we concentrate our work in this article on the wick-type stochastic the time nonlinear Schrödinger equation with conformable fractional derivatives. A lot of research on stochastic fractional differential equations has been done recently [16–18]. Ghany and Zakarya [16] studied exact traveling wave solutions for wick-type stochastic Schamel KdV equation, in [17] is obtained exact solutions for the stochastic time fractional Gardner equation, in [18] is studied white noise functional process for the fractional coupled KdV equations and is found some soliton solutions.

Birefringence in optical fibers is a natural phenomenal that occurs because of some components such as twists, bends and anisotropic stress of fibers. These cause to differential set delay and thus the splitting of pulses occur. An addition to detrimental positions of the random birefringence in optical fibers, a ruled and artificial birefringence presented in the fiber is sometimes studied to realise in-line fiber optical factors and tools. Artificially presented birefringence in optical fibers may be used to advance of high-birefringence (Hi-Bi) fibers. Such Hi-Bi fibers are realised by introducing a permanent high birefringence into the fiber during fabrication. The nonlinear Schrödinger's equation plays a vital role in various areas of biological, physical and engineering sciences. It appears in many applied fields, including fluid dynamics, nonlinear optics, plasma physics and proteinchemistry. In this article, we will study the nonlinear Schrödinger equation with the aid of time conformable derivative.

The presented equations for perturbation solitons in birefringent fibers given with Kerr law nonlinearity are expressed with the vector coupled nonlinear Schrödinger's equations (NLSE). The coupled NLSEs given with the spatio-temporal dispersion and Kerr law nonlinearity are as below [19,20].

$$iD_\tau q + a_1 D_{xx} q + b_1 D_{x\tau} q + (c_1 |r|^2 + d_1 |q|^2) q + i\{\alpha_1 D_x q + \lambda_1 D_x (|q|^2 q)$$
$$+ v_1 D_x (|q|^2) q + \theta_1 |q|^2 D_x q + \gamma_1 D_{xxx} q\} = 0, \tag{1.1}$$

$$iD_\tau r + a_2 D_{xx} r + b_2 D_{x\tau} r + (c_2 |q|^2 + d_2 |r|^2) r + i\{\alpha_2 D_x r + \lambda_2 D_x (|r|^2 r)$$
$$+ v_2 D_x (|r|^2) r + \theta_2 |r|^2 D_x r + \gamma_2 D_{xxx} r\} = 0. \tag{1.2}$$

$$(x,\tau) \in \mathrm{R} \times \mathrm{R}_+, \tau > 0.$$

Here $q = q(x,\tau)$ and $r = r(x,\tau)$ define wave profiles of two split pulses that are the function of spatial variable x and the temporal variable τ For $l = 1,2,$ a_l defines the coefficients of group-velocity dispersion, while b_l gives the coefficients of spatio-temporal dispersion, c_l defines the coefficients of cross-phase modulation, d_l defines the coefficients of self-phase modulations. Where the perturbation conditions α_l defines the inter-modal dispersion, λ_l is the self-steepening expression, v_l and θ_l are nonlinear dispersions and finally γ_l is the third order dispersion that must be taken into account in case the group-velocity dispersion is small.

We assume the following types without Spatio-temporal dispersion and Kerr Law nonlinearity without perturbation conditions, Hence $\alpha_l = 0, \lambda_l = 0, v_l = 0, \theta_l = 0, \gamma_l = 0, b_l = 0,$ Then the presented equation is modeled by the following dimensionless vector coupled nonlinear Schrödinger equations,

$$\begin{aligned} iD_\tau q + a_1 D_{xx} q + (c_1|r|^2 + d_1|q|^2)q &= 0, \\ iD_\tau r + a_2 D_{xx} r + (c_2|q|^2 + d_2|r|^2)r &= 0. \end{aligned}$$

The space-time coupled nonlinear Schrödinger equations with the aid of conformable derivative are given by [21–24]:

$$\begin{aligned} iD_\tau^\vartheta q + a_1 D_{xx}^{2\vartheta} q + (c_1|r|^2 + d_1|q|^2)q &= 0, \\ iD_\tau^\vartheta r + a_2 D_{xx}^{2\vartheta} r + (c_2|q|^2 + d_2|r|^2)r &= 0, \end{aligned} \tag{1.3}$$

where $\vartheta \in (0,1)$, c_1, c_2, d_1 and d_2 are nonzero constants, q and r are complex functions of x and τ that represent the amplitudes of circularly-polarized waves in a nonlinear optical fiber [21]. Eq (1.3) are developed by [21] and are named to playing great role in the pulse propagation through a two-mode optical fiber and the soliton wavelength division multiplexing. When $\vartheta = 1$, we have the original (1+1)-dimensional coupled nonlinear Schrödinger equations [21].

The nonlinear Schrödinger equation is an example of a universal nonlinear version that defines many physical nonlinear systems. The equation can be studied to nonlinear optics, hydrodynamics, quantum condensates, nonlinear acoustics, heat pulses in solids and various other nonlinear instability phenomenal. Such equations have been expressed with govern pulse propagation along orthogonal polarization axes in nonlinear optical fibers and in wavelength-division-multiplexed systems. These equations also type beam propagation inside crystals or photorefractive as well as water wave interactions. Solitary waves in these equations are often named vector solitons in the literature as they generally comprise two elements. In all the above physical cases, collision of vector solitons is an significant effect [25–31].

The stochastic coupled nonlinear Schrödinger equations with the aid of time conformable derivative are given by:

$$\begin{aligned} iD_\tau^\vartheta q(x,\tau) + a_1(\tau) D_{xx}^{2\vartheta} q(x,\tau) + (c_1(\tau)|r(x,\tau)|^2 + d_1(\tau)|q(x,\tau)|^2)q(x,\tau) &= 0, \\ iD_\tau^\vartheta r(x,\tau) + a_2(\tau) D_{xx}^{2\vartheta} r(x,\tau) + (c_2(\tau)|q(x,\tau)|^2 + d_2|r(x,\tau)|^2)r(x,\tau) &= 0, \end{aligned} \tag{1.4}$$

$$(x,\tau) \in \mathrm{R} \times \mathrm{R}_+, \tau > 0, \ 0 < \vartheta \le 1,$$

where $a_1(\tau)$, $a_2(\tau)$, $c_1(\tau)$ and $c_2(\tau)$ are limited mensurable or integrable functions on R_+. $D_\tau^\vartheta p(x,\tau)$ and $D_\tau^\vartheta r(x,\tau)$ are the time conformable derivative operator and d_1, d_2 are real valued constants.

The conformable derivative operator was exposed in [32]. This derivative operator can reform the failures of the other definitions. This important operator is the easiest, most ordinary and effectual description of the fractional derivative for order $\vartheta \in (0,1]$. We should note that the description can be generalised to need any ϑ. All the same, the order $\vartheta \in (0,1]$ is the most influential order.

The conformable derivative of order $\vartheta \in (0,1)$ described as the following statement [32]

$$_tD^\vartheta h(t) = \lim_{\eta \to 0} \frac{h(t + \eta t^{1-\vartheta}) - h(t)}{\eta}, \ h:(0,\infty) \to \mathrm{R}.$$

The description represents a ordinary formation of standard derivatives. Furthermore, the expression of the description represents that it is the most natural, and the most effectual definition.

The description for $0 \le \vartheta < 1$ gives with the standard expressions on polynomials (up to a constant).

Some characteristics of the conformable derivative are given by [32,33].

$$a)_t D^{\vartheta} t^{\alpha} = \alpha t^{\alpha-\vartheta}, \quad \forall \vartheta \in \mathrm{R},$$

$$b)_t D^{\vartheta}(hg) = h_t D^{\vartheta} g + g_t D^{\vartheta} h,$$

$$c)_t D^{\vartheta}(hog) = t^{1-\vartheta} g'(t) h'(g(t)),$$

$$d)_t D^{\vartheta}(\frac{h}{g}) = \frac{g_t D^{\vartheta} h - h_t D^{\vartheta} g}{g^2}.$$

This derivative is more advantageous than others because it's easy to apply. In the recently, there has been some researces on the conformable form of fractional computations [33–37].

The wick sense stochastic model of Eq (1.4) given with conformable derivatives expressed as below,

$$\begin{aligned} iD_{\tau}^{\vartheta} Q(x,\tau) + a_1(\tau) \blacklozenge D_{xx}^{2\vartheta} Q(x,\tau) + (c_1(\tau) \blacklozenge |R(x,\tau)|^2 + d_1 |Q(x,\tau)|^2) \blacklozenge Q(x,\tau) &= 0, \\ iD_{\tau}^{\vartheta} R(x,\tau) + a_2(\tau) \blacklozenge D_{xx}^{2\vartheta} R(x,\tau) + (c_2(\tau) \blacklozenge |Q(x,\tau)|^2 + d_2 |R(x,\tau)|^2) \blacklozenge R(x,\tau) &= 0, \end{aligned} \tag{1.5}$$

where " ♦ " give the Wick product on the Kondratiev distribution space $(S)_{-1}$, $a_1(\tau)$, $a_2(\tau)$, $c_1(\tau)$ and $c_2(\tau)$ are $(S)_{-1}$-valued functions.

We only consider it in a white noise environment, that is, we will discuss the Wick-type stochastic coupled nonlinear Schrödinger equations for time conformable derivative (1.5) to obtain the exact solutions of the stochastic coupled nonlinear Schrödinger equations with time conformable derivative.

Our focus in this study are to analyse new soliton and periodic wave solutions of coupled nonlinear Schrödinger equations by using time conformable derivative and to analyse new stochastic soliton and periodic wave solutions of the Wick-type stochastic coupled nonlinear Schrödinger equations by using time conformable derivatives. We use Hermite transform, white noise theory and the modified fractional sub-equation method [38,39]. In addition to, we use the inverse Hermite transform to find stochastic soliton and periodic wave solutions of the Wick-type stochastic coupled nonlinear Schrödinger equations by using time conformable derivative. Eventually, we give an application example to show how the stochastic solutions can be expression as Brownian motion functional solutions.

2. Deterministic case applications

In this section, we will obtain exact solutions of nonlinear Schrödinger equations. By using the Hermite transform of Eq (1.3), we define the deterministic equation as below

$$\begin{aligned} & iD_{\tau}^{\vartheta} q(x,\tau,z) + a_1(\tau,z) D_{xx}^{2\vartheta} q(x,\tau,z) + (c_1(\tau,z) |r(x,\tau,z)|^2 \\ & + d_1(\tau,z) |q(x,\tau,z)|^2) q(x,\tau,z) = 0, \\ & iD_{\tau}^{\vartheta} r(x,\tau,z) + a_2(\tau,z) D_{xx}^{2\vartheta} r(x,\tau,z) + (c_2(\tau,z) |q(x,\tau,z)|^2 \\ & + d_2 |r(x,\tau,z)|^2) r(x,\tau,z) = 0, \end{aligned} \tag{2.1}$$

where $z=(z_1,z_2,...)\in(\mathbb{C}^N)_c$ is a argument. To find travelling wave solutions to Eq (2.1), we give the transformations below

$$q(x,\tau,z)=q(\varsigma(x,\tau,z))e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\varpi\int_a^t\frac{\theta(\tau,z)}{\tau^{1-\vartheta}}d\tau),},$$
$$r(x,\tau,z)=r(\varsigma(x,\tau,z))e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\varpi\int_a^t\frac{\theta(\tau,z)}{\tau^{1-\vartheta}}d\tau),},$$

with,

$$\varsigma(x,\tau,z)=k(\frac{x^{\vartheta}}{\vartheta})+\varpi\int_a^t\frac{\theta(\tau,z)}{\tau^{1-\vartheta}}d\tau,$$

where $\theta\neq 0$, ϖ,k are arbitrary constants. So, Eq (2.1) can be reborn to NODEs. These equations are resolved it into real and imaginary sections as follows,

The real sections;

$$\begin{aligned}(-\varpi-k^2a_1(\tau,z))q(\varsigma)+(c_1(\tau,z)+d_1)q(\varsigma)r(\varsigma)^2+k^2a_1(\tau,z)q''(\varsigma) &= 0,\\ (-\varpi-k^2a_1(\tau,z))r(\varsigma)+c_2(\tau,z)q(\varsigma)^2r(\varsigma)+d_2r(\varsigma)^3+k^2a_2(\tau,z)r''(\varsigma) &= 0\end{aligned}\tag{2.2}$$

and by integrating with respect to ϕ once of imaginary sections and by integration constant neglect gives:

$$\begin{aligned}i\varpi\theta q(\varsigma)-2ik^2a_1(\tau,z)r(\varsigma) &= 0,\\ i\varpi\theta r(\varsigma)-2ik^2a_2(\tau,z)q(\varsigma) &= 0.\end{aligned}\tag{2.3}$$

From Eq (2.3) we obtain $\theta=\frac{2k^2a_1(\tau,z)}{\varpi}=\frac{2k^2a_2(\tau,z)}{\varpi}$ and $a_1(\tau,z)=a_2(\tau,z)$. We consider $a_1(\tau,z)=a_2(\tau,z)=a(\tau,z)$, we can write $\theta=\frac{2k^2a(\tau,z)}{\varpi}$.

- Assume the solutions of Eq (2.2) can express as a series expansion solution as below,

$$\begin{aligned}q(\varsigma) &= \sum_{i=0}^{N}\alpha_i(\tau,z)G^i(\varsigma)+\sum_{i=1}^{N}\beta_i(\tau,z)G^{-i}(\varsigma),\\ r(\varsigma) &= \sum_{i=0}^{N}\alpha_i(\tau,z)G^i(\varsigma)+\sum_{i=1}^{N}\beta_i(\tau,z)G^{-i}(\varsigma),\end{aligned}\tag{2.4}$$

where α_i $(i=0,1,...,n),\beta_i$ $(i=1,2,...,n)$ are functions to be determined later and $G(\varsigma)$ satisfies the fractional Riccati equation as below:

$$G'(\varsigma)=\ell+G^2(\varsigma),\tag{2.5}$$

where ρ is an arbitrary constants.

- N is control between the nonlinear terms and the highest order derivatives in Eq (2.2).

Some special solutions of Eq (2.5) are as below [40];

1) $\ell<0$,

$$\begin{aligned}G_1(\varsigma) &= -\sqrt{-\ell}\tanh_{\vartheta}(\sqrt{-\ell}\varsigma),\\ G_2(\varsigma) &= -\sqrt{-\ell}\coth_{\vartheta}(\sqrt{-\ell}\varsigma),\end{aligned}$$

2) $\ell > 0$,

$$\begin{aligned} G_3(\varsigma) &= \sqrt{\ell}\tan_{\vartheta}(\sqrt{\ell}\varsigma), \\ G_4(\varsigma) &= \sqrt{\ell}\cot_{\vartheta}(\sqrt{\ell}\varsigma), \end{aligned}$$

3) $\ell = 0, \quad \rho = const.$,

$$G_5(\varsigma) = -\frac{\Gamma(1+\vartheta)}{\varsigma^{\vartheta}+\rho},$$

Note. The generalized hyperbolic and trigonometric functions are as below[2];

$$\begin{aligned} \tan_{\vartheta}(\varsigma) &= \frac{E_{\vartheta}(i\varsigma^{\vartheta}) - E_{\vartheta}(-i\varsigma^{\vartheta})}{i(E_{\vartheta}(i\varsigma^{\vartheta}) + E_{\vartheta}(-i\varsigma^{\vartheta}))}, \cot_{\vartheta}(\varsigma) = \frac{i(E_{\vartheta}(i\varsigma^{\vartheta}) + E_{\vartheta}(-i\varsigma^{\vartheta}))}{E_{\vartheta}(i\varsigma^{\vartheta}) - E_{\vartheta}(-i\varsigma^{\vartheta})}, \\ \tanh_{\vartheta}(\varsigma) &= \frac{E_{\vartheta}(\varsigma^{\vartheta}) - E_{\vartheta}(-\varsigma^{\vartheta})}{E_{\vartheta}(\varsigma^{\vartheta}) + E_{\vartheta}(-\varsigma^{\vartheta})}, \coth_{\vartheta}(\varsigma) = \frac{E_{\vartheta}(\varsigma^{\vartheta}) + E_{\vartheta}(-\varsigma^{\vartheta}))}{E_{\vartheta}(\varsigma^{\vartheta}) - E_{\vartheta}(-\varsigma^{\vartheta})}, \end{aligned}$$

where $E_{\vartheta}(\varsigma) = \sum_{i=0}^{N} \frac{\varsigma^i}{\Gamma(1+i\vartheta)}$ is the Mittag-Leffler function.

From Eq (2.2), is found $N = 1$. Then we can choose the solution of Eq (2.2) are given by:

$$\begin{aligned} q(\varsigma) &= \alpha_0 + \alpha_1 G(\varsigma) + \alpha_2 G^{-1}(\varsigma), \\ r(\varsigma) &= \beta_0 + \beta_1 G(\varsigma) + \beta_2 G^{-1}(\varsigma), \end{aligned} \tag{2.6}$$

where $G(\varsigma)$ satisfied Eq (2.5).

Now, replacing (2.6) and (2.5) into (2.2), by equating the all coefficients of $G(\varsigma)$, we can solve equations. Then we obtain the following some groups of solutions:

One of the obtained these groups is given by;

$$\begin{aligned} \alpha_0 &= 0, \\ \alpha_1 &= -\frac{\begin{array}{c} i\sqrt{a(\tau,z)(c_1(\tau,z)+d_1-d_2)}a(\tau,z)(\varpi(c_1(\tau,z)+d_1)+2k^2(1-2\ell)a(\tau,z)d_2 \\ -((2\varpi+k^2(1-2\ell)a(\tau,z))c_1(\tau,z)+(2\varpi+k^2(1-2\ell)a(\tau,z))d_1-2\varpi d_2)) \end{array}}{3\sqrt{2a(\tau,z)}k\ell\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}, \\ \alpha_2 &= -\frac{ik\ell\sqrt{2a(\tau,z)(c_1(\tau,z)+d_1-d_2)}}{\sqrt{c_2(\tau,z)(c_1(\tau,z)+d_1)}}, \beta_0 = 0, \\ \beta_1 &= \frac{\begin{array}{c} -2\varpi a(\tau,z)(c_1(\tau,z)+d_1)+2k^2(1-2\ell)a(\tau,z)^2 d_2 + a(\tau,z)((\varpi+k^2(-1+2\ell)a(\tau,z))c_1(\tau,z) \\ +(\varpi+k^2(-1+2\ell)a(\tau,z))d_1+2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\sqrt{a(\tau,z)}\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}, \\ \beta_2 &= \frac{\sqrt{2}k\ell\sqrt{a(\tau,z)}}{\sqrt{-c_1(\tau,z)-d_1}}. \end{aligned} \tag{2.7}$$

The exact solutions of Eq (2.1) are given by;

1) Hyperbolic function solutions (when $\ell < 0$),

$$q_1(x,\tau,z) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)} \left(-\frac{\begin{array}{c} i\sqrt{a(\tau,z)(c_1(\tau,z)+d_1-d_2)}a(\tau,z)(\varpi(c_1(\tau,z)+d_1) \\ +2k^2(1-2\ell)a(\tau,z)d_2-((2\varpi+k^2(1-2\ell)a(\tau,z))c_1(\tau,z) \\ +(2\varpi+k^2(1-2\ell)a(\tau,z))d_1-2\varpi d_2)) \end{array}}{3\sqrt{2a(\tau,z)}k\ell\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}\right.$$

$$(-\sqrt{-\ell}\tanh_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)))$$

$$-\frac{ik\rho\sqrt{2a(\tau,z)(c_1(\tau,z)+d_1-d_2)}}{\sqrt{c_2(\tau,z)(c_1(\tau,z)+d_1)}\sqrt{-\ell}}\coth_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau))),$$

$$r_1(x,\tau,z) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)} \left(\frac{\begin{array}{c} -2\varpi a(\tau,z)(c_1(\tau,z)+d_1)+2k^2(1-2\ell)a(\tau,z)^2 d_2 \\ +a(\tau,z)((\varpi+k^2(-1+2\ell)a(\tau,z))c_1(\tau,z) \\ +(\varpi+k^2(-1+2\ell)a(\tau,z))d_1+2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\sqrt{a(\tau,z)}\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}\right.$$

$$(-\sqrt{-\ell}\tanh_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)))$$

$$-\frac{\sqrt{2}k\ell\sqrt{a(\tau,z)}}{\sqrt{-c_1(\tau,z)-d_1}\sqrt{-\rho}}\coth_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau))), \quad (2.8)$$

$$q_2(x,\tau,z) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)} \left(-\frac{\begin{array}{c} i\sqrt{2a(\tau,z)}\sqrt{c_1(\tau,z)-d_2}(\varpi c_1(\tau,z)+2k^2(1-2\ell)a(\tau,z)d_2 \\ -(2\varpi+k^2(1-2\ell)a(\tau,z))c_1(\tau,z)+2\varpi d_2) \end{array}}{6k\ell a(\tau,z)\sqrt{c_1(\tau,z)c_2(\tau,z)}(-c_1(\tau,z)+2d_2)}\right.$$

$$(-\sqrt{-\ell}\coth_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)))$$

$$-\frac{ik\ell\sqrt{2a(\tau,z)(c_1(\tau,z)+d_1-d_2)}}{\sqrt{c_2(\tau,z)(c_1(\tau,z)+d_1)}\sqrt{-\ell}}\tanh_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau))),$$

$$r_2(x,\tau,z) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)} \left(\frac{\begin{array}{c} -2\varpi a(\tau,z)(c_1(\tau,z)+d_1)+2k^2(1-2\ell)a(\tau,z)^2 d_2 \\ +a(\tau,z)((\varpi+k^2(-1+2\ell)a(\tau,z))c_1(\tau,z) \\ +(\varpi+k^2(-1+2\ell)a(\tau,z))d_1+2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\sqrt{a(\tau,z)}\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}\right.$$

$$(-\sqrt{-\ell}\coth_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)))$$

$$-\frac{\sqrt{2}k\ell\sqrt{a(\tau,z)}}{\sqrt{-c_1(\tau,z)-d_1}\sqrt{-\ell}}\tanh_{\vartheta}(\sqrt{-\ell}(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau))), \quad (2.9)$$

2) Trigonometric function solutions (when $\ell > 0$),

$$q_3(x,\tau,z) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}}d\tau)}\Big(-\frac{i\sqrt{a(\tau,z)(c_1(\tau,z)+d_1-d_2)}a(\tau,z)(\varpi(c_1(\tau,z)+d_1) + 2k^2(1-2\ell)a(\tau,z)d_2-((2\varpi+k^2(1-2\ell)a(\tau,z))c_1(\tau,z) + (2\varpi+k^2(1-2\ell)a(\tau,z))d_1-2\varpi d_2))}{3\sqrt{2a(\tau,z)}k\ell\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}$$

$$(\sqrt{\ell}\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau)))$$

$$-\frac{ik\ell\sqrt{2a(\tau,z)(c_1(\tau,z)+d_1-d_2)}}{\sqrt{c_2(\tau,z)(c_1(\tau,z)+d_1)}\sqrt{\ell}}\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau))),$$

$$r_3(x,\tau,z) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}}d\tau)}\Big(\frac{-2\varpi a(\tau,z)(c_1(\tau,z)+d_1)+2k^2(1-2\ell)a(\tau,z)^2d_2 + a(\tau,z)((\varpi+k^2(-1+2\ell)a(\tau,z))c_1(\tau,z) + (\varpi+k^2(-1+2\ell)a(\tau,z))d_1+2\varpi d_2)}{3\sqrt{2}k\ell\sqrt{a(\tau,z)}\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}$$

$$(\sqrt{\ell}\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau)))$$

$$+\frac{\sqrt{2}k\ell\sqrt{a(\tau,z)}}{\sqrt{-c_1(\tau,z)-d_1}\sqrt{\rho}}\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau))), \tag{2.10}$$

$$q_4(x,\tau,z) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}}d\tau)}\Big(-\frac{i\sqrt{a(\tau,z)(c_1(\tau,z)+d_1-d_2)}a(\tau,z)(\varpi(c_1(\tau,z)+d_1) + 2k^2(1-2\ell)a(\tau,z)d_2-((2\varpi+k^2(1-2\ell)a(\tau,z))c_1(\tau,z) + (2\varpi+k^2(1-2\ell)a(\tau,z))d_1-2\varpi d_2))}{3\sqrt{2a(\tau,z)}k\ell\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}$$

$$(\sqrt{\ell}\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau)))$$

$$-\frac{ik\ell\sqrt{2a(\tau,z)(c_1(\tau,z)+d_1-d_2)}}{\sqrt{c_2(\tau,z)(c_1(\tau,z)+d_1)}\sqrt{\ell}}\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau))),$$

$$r_4(x,\tau,z) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}}d\tau)}\Big(\frac{-2\varpi a(\tau,z)(c_1(\tau,z)+d_1)+2k^2(1-2\ell)a(\tau,z)^2d_2 + a(\tau,z)((\varpi+k^2(-1+2\ell)a(\tau,z))c_1(\tau,z) + (\varpi+k^2(-1+2\ell)a(\tau,z))d_1+2\varpi d_2)}{3\sqrt{2}k\ell\sqrt{a(\tau,z)}\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}$$

$$(\sqrt{\ell}\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau)))$$

$$+\frac{\sqrt{2}k\ell\sqrt{a(\tau,z)}}{\sqrt{-c_1(\tau,z)-d_1}\sqrt{\ell}}\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t\frac{2k^2a(\tau,z)}{\tau^{1-\vartheta}}d\tau))), \tag{2.11}$$

3) Wave solutions (when $\ell = 0$, $\rho = const.$),

$$q_5(x,\tau,z) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta}) + \int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)} \left(-\frac{\begin{array}{c} i\sqrt{a(\tau,z)(c_1(\tau,z)+d_1-d_2)}a(\tau,z)(\varpi(c_1(\tau,z)+d_1) \\ +2k^2(1-2\ell)a(\tau,z)d_2 - ((2\varpi + k^2(1-2\ell)a(\tau,z))c_1(\tau,z) \\ +(2\varpi + k^2(1-2\ell)a(\tau,z))d_1 - 2\varpi d_2)) \end{array}}{3\sqrt{2a(\tau,z)}k\ell\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}\right.$$

$$\left(-\frac{\Gamma(1+\vartheta)}{(k(\frac{x^{\vartheta}}{\vartheta}) + \int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)^{\vartheta} + \rho}\right)$$

$$\left. -\frac{ik\ell\sqrt{2a(\tau,z)(c_1(\tau,z)+d_1-d_2)}}{\sqrt{c_2(\tau,z)(c_1(\tau,z)+d_1)}} \frac{(k(\frac{x^{\vartheta}}{\vartheta}) + \int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)^{\vartheta} + \rho}{\Gamma(1+\vartheta)}\right),$$

$$r_5(x,\tau,z) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta}) + \int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)} \left(\frac{\begin{array}{c} -2\varpi a(\tau,z)(c_1(\tau,z)+d_1) + 2k^2(1-2\ell)a(\tau,z)^2 d_2 + a(\tau,z) \\ ((\varpi + k^2(-1+2\ell)a(\tau,z))c_1(\tau,z) \\ +(\varpi + k^2(-1+2\ell)a(\tau,z))d_1 + 2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\sqrt{a(\tau,z)}\sqrt{-c_1(\tau,z)-d_1}a(\tau,z)(c_1(\tau,z)+d_1-2d_2)}\right.$$

$$\left.\left(\left(-\frac{\Gamma(1+\vartheta)}{(k(\frac{x^{\vartheta}}{\vartheta}) + \int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)^{\vartheta} + \rho}\right)\right) + \frac{\sqrt{2}k\ell\sqrt{a(\tau,z)}}{\sqrt{-c_1(\tau,z)-d_1}\sqrt{\ell}} \frac{(k(\frac{x^{\vartheta}}{\vartheta}) + \int_a^t \frac{2k^2 a(\tau,z)}{\tau^{1-\vartheta}} d\tau)^{\vartheta} + \rho}{\Gamma(1+\vartheta)}\right). \tag{2.12}$$

3. Stochastic case applications

In this part, we use the inverse Hermite transform and Theorem 4.1.1 in [41] to analyse white noise functional solutions of Eq (1.2). The properties of generalized hyperbolic, trigonometric and exponential functions express that there is a limited open station $\Omega \subset \mathrm{R} \times \mathrm{R}_+$, $a < \infty$, $b > 0$ such that the solutions $q(x,\tau,z)$ and $r(x,\tau,z)$ of Eq (2.1) and whole its fractional derivatives which are contained in Eq (2.1) are uniformly limited for $(x,\tau,z) \in \Omega \times K_a(b)$, continuous with respect to $(x,\tau) \in \Omega$ for whole $z \in \Omega \times K_a(b)$ and analytic with respect to $z \in K_a(b)$, for whole $(x,\tau) \in \Omega$. From Theorem 4.1.1 in [42], there is $P(x,\tau) \in (S)_{-1}$ such that $q(x,\tau,z) = Q(x,\tau)(z)$ and $r(x,\tau,z) = R(x,\tau)(z)$ for whole $(x,\tau,z) \in \Omega \times K_a(b)$ and $Q(x,\tau)$, $R(x,\tau)$ solves Eq (1.5) in $(S)_{-1}$. Thus, we will investigate the white noise functional solutions of Eq (1.5) for $a(\tau) > 0$, $c_1(\tau) > 0$ and $c_2(\tau) > 0$ wit the aid of inverse Hermite transform for Eqs (2.8)–(2.12) as below.

1) Hyperbolic function random solutions (when $\ell < 0$),

$$Q_1(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)} (-\frac{i\sqrt{a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}\blacklozenge a(\tau)\blacklozenge(\varpi(c_1(\tau)+d_1) + 2k^2(1-2\ell)\blacklozenge a(\tau)d_2 - ((2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) + (2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))d_1 - 2\varpi d_2))}{3\sqrt{2a(\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge$$
$$(-\sqrt{-\ell}\tanh_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)))$$
$$-\frac{ik\ell\sqrt{2a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}}{\sqrt{c_2(\tau)\blacklozenge(c_1(\tau)+d_1)}\sqrt{-\ell}}\blacklozenge\coth_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau))),$$

$$R_1(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)} (\frac{-2\varpi a(\tau)\blacklozenge(c_1(\tau)+d_1)+2k^2(1-2\ell)\blacklozenge a(\tau)^2 d_2 + a(\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) + (\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))d_1 + 2\varpi d_2)}{3\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge$$
$$(-\sqrt{-\ell}\tanh_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)))$$
$$-\frac{\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}}{\sqrt{-c_1(\tau)-d_1}\sqrt{-\ell}}\blacklozenge\coth_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau))), \quad (3.1)$$

$$Q_2(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)} (-\frac{i\sqrt{a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}\blacklozenge a(\tau)\blacklozenge(\varpi(c_1(\tau)+d_1) + 2k^2(1-2\ell)\blacklozenge a(\tau)d_2 - ((2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) + (2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))d_1 - 2\varpi d_2))}{3\sqrt{2a(\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge$$
$$(-\sqrt{-\ell}\coth_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)))$$
$$-\frac{ik\ell\sqrt{2a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}}{\sqrt{c_2(\tau)\blacklozenge(c_1(\tau)+d_1)}\sqrt{-\ell}}\blacklozenge\tanh_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau))),$$

$$R_2(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)} (\frac{-2\varpi a(\tau)\blacklozenge(c_1(\tau)+d_1)+2k^2(1-2\ell)\blacklozenge a(\tau)^2 d_2 + a(\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) + (\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))d_1 + 2\varpi d_2)}{3\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge$$
$$(-\sqrt{-\ell}\coth_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau)))$$
$$-\frac{\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}}{\sqrt{-c_1(\tau)-d_1}\sqrt{-\ell}}\blacklozenge\tanh_\vartheta(\sqrt{-\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}}d\tau))), \quad (3.2)$$

2) Trigonometric function random solutions (when $\ell > 0$),

$$Q_3(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)} \left(-\frac{\begin{array}{c} i\sqrt{a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}\blacklozenge a(\tau)\blacklozenge(\varpi(c_1(\tau)+d_1) \\ +2k^2(1-2\ell)\blacklozenge a(\tau)d_2-((2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))d_1-2\varpi d_2)) \end{array}}{3\sqrt{2a(\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge\right.$$
$$(\sqrt{\ell}\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)))$$
$$+\frac{ik\ell\sqrt{2a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}}{\sqrt{c_2(\tau)\blacklozenge(c_1(\tau)+d_1)}\sqrt{\ell}}\blacklozenge\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau))),$$

$$R_3(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)} \left(\frac{\begin{array}{c} -2\varpi a(\tau)\blacklozenge(c_1(\tau)+d_1)+2k^2(1-2\ell)\blacklozenge a(\tau)^2 d_2+a(\tau) \\ \blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))d_1+2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge\right.$$
$$(\sqrt{\ell}\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)))$$
$$+\frac{\sqrt{2}k\rho\blacklozenge\sqrt{a(\tau)}}{\sqrt{-c_1(\tau)-d_1}\sqrt{\rho}}\blacklozenge\cot_\vartheta(\sqrt{\rho}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau))), \tag{3.3}$$

$$Q_4(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)} \left(-\frac{\begin{array}{c} i\sqrt{a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}\blacklozenge a(\tau)\blacklozenge(\varpi(c_1(\tau)+d_1) \\ +2k^2(1-2\ell)\blacklozenge a(\tau)d_2-((2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))d_1-2\varpi d_2)) \end{array}}{3\sqrt{2a(\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge\right.$$
$$(\sqrt{\ell}\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)))$$
$$+\frac{ik\ell\sqrt{2a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}}{\sqrt{c_2(\tau)\blacklozenge(c_1(\tau)+d_1)}\sqrt{\ell}}\blacklozenge\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau))),$$

$$R_4(x,\tau) = e^{i(-k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)} \left(\frac{\begin{array}{c} -2\varpi a(\tau)\blacklozenge(c_1(\tau)+d_1)+2k^2(1-2\ell)\blacklozenge a(\tau)^2 d_2+a(\tau) \\ \blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))d_1+2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge\right.$$
$$(\sqrt{\ell}\cot_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)))$$
$$+\frac{\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}}{\sqrt{-c_1(\tau)-d_1}\sqrt{\ell}}\blacklozenge\tan_\vartheta(\sqrt{\ell}(k(\frac{x^\vartheta}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau))), \tag{3.4}$$

3) Wave random solutions (when $\ell = 0$, $\rho = const.$),

$$Q_5(x,\tau) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)} \Big(-\frac{\begin{array}{c} i\sqrt{a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}\blacklozenge a(\tau)\blacklozenge(\varpi(c_1(\tau)+d_1) \\ +2k^2(1-2\ell)\blacklozenge a(\tau)d_2-((2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge a(\tau))d_1-2\varpi d_2)) \end{array}}{3\sqrt{2a(\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge$$
$$\Big(-\frac{\Gamma(1+\vartheta)}{(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)^{\vartheta}+\rho}\Big)$$
$$-\frac{ik\ell\sqrt{2a(\tau)\blacklozenge(c_1(\tau)+d_1-d_2)}}{\sqrt{c_2(\tau)\blacklozenge(c_1(\tau)+d_1)}}\blacklozenge\frac{(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)^{\vartheta}+\rho}{\Gamma(1+\vartheta)}\Big),$$

$$R_5(x,\tau) = e^{i(-k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)} \Big(\frac{\begin{array}{c} -2\varpi a(\tau)\blacklozenge(c_1(\tau)+d_1)+2k^2(1-2\ell)\blacklozenge a(\tau)^2 d_2+a(\tau) \\ \blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))\blacklozenge c_1(\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge a(\tau))d_1+2\varpi d_2) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}\blacklozenge\sqrt{-c_1(\tau)-d_1}\blacklozenge a(\tau)\blacklozenge(c_1(\tau)+d_1-2d_2)}\blacklozenge$$
$$\Big(-\frac{\Gamma(1+\vartheta)}{(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)^{\vartheta}+\rho}\Big)$$
$$+\frac{\sqrt{2}k\ell\blacklozenge\sqrt{a(\tau)}}{\sqrt{-c_1(\tau)-d_1}}\blacklozenge\frac{(k(\frac{x^{\vartheta}}{\vartheta})+\int_a^t \frac{2k^2 a(\tau)}{\tau^{1-\vartheta}} d\tau)^{\vartheta}+\rho}{\Gamma(1+\vartheta)}\Big), \tag{3.5}$$

4. Examples

In this part, we analyse special application example to show the accessibility of our consequences and to support the real help of these results. We interpret that the solutions of Eq (1.5) are powerfully count on the type of the given functions $c_1(\tau)$, $c_2(\tau)$ and $a(\tau)$. Thus, for different types of $c_1(\tau)$, $c_2(\tau)$ and $a(\tau)$, we can obtain different solutions of Eq (1.5) that come from Eqs (3.1)–(3.5). We demonstrate this by giving the following example.

When $\vartheta = 1$,

$$\tan_{\vartheta}(x) = \tan(x), \cot_{\vartheta}(x) = \cot(x), \tanh_{\vartheta}(x) = \tanh(x),$$
$$\coth_{\vartheta}(x) = \coth(x), E_{\vartheta}(x) = \exp(x).$$

Suppose $c_1(\tau) = \partial\, a(\tau)$, $c_2(\tau) = \rho\, a(\tau)$, $a(\tau) = h(\tau) + \lambda W_{\tau}$ and $d_1 = d_2 = 1$. Where ∂, ρ and λ are arbitrary constants, $h(\tau)$ is a limited mensurable function on $\mathbb{R}_+$ and W_{τ} is the Gaussian white noise that is the time derivative (in the strong sense in $(S)_{-1}$) of the Brownian motion B_{τ}.

The Hermite transform of W_{τ} is expressed by $\widetilde{W}_{\tau}(z) = \sum_{i=0}^{\infty} z_i \int_0^t \Psi_i(\tau)d\tau$ [42]. Using the describtion of $\widetilde{W}_{\tau}(z)$, Eqs (3.1)–(3.5) given the white noise functional solution of Eq (1.5) as below:

$$Q_1(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$

$$(-\frac{\begin{array}{c} i(h(\tau)+\lambda W_\tau)^2\sqrt{\partial}\blacklozenge(\varpi(\partial(h(\tau)+\lambda W_\tau)+1) \\ +2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau) \\ -((2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))-2\varpi)) \end{array}}{3\sqrt{2(h(\tau)+\lambda W_\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge$$

$$(-\sqrt{-\ell}\tanh(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$

$$-\frac{ik\ell(h(\tau)+\lambda W_\tau)\sqrt{2\partial}}{\sqrt{\ell(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)}\sqrt{-\ell}}\blacklozenge\coth(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$R_1(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$

$$(\frac{\begin{array}{c} -2\varpi(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau)^2 \\ +(h(\tau)+\lambda W_\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))+2\varpi) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{(h(\tau)+\lambda W_\tau)}\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge \tag{4.1}$$

$$(-\sqrt{-\ell}\tanh(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$

$$-\frac{\sqrt{2}k\ell\blacklozenge\sqrt{h(\tau)+\lambda W_\tau}}{\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\sqrt{-\ell}}\blacklozenge\coth(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$Q_2(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$

$$(-\frac{\begin{array}{c} i(h(\tau)+\lambda W_\tau)^2\sqrt{\partial}\blacklozenge(\varpi(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau) \\ -((2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))-2\varpi)) \end{array}}{3\sqrt{2(h(\tau)+\lambda W_\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge$$

$$(-\sqrt{-\ell}\coth(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$

$$-\frac{ik\ell(h(\tau)+\lambda W_\tau)\sqrt{2\partial}}{\sqrt{\ell(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)}\sqrt{-\ell}}\blacklozenge\tanh(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$R_2(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$

$$(\frac{\begin{array}{c} -2\varpi(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau)^2 \\ +(h(\tau)+\lambda W_\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))+2\varpi) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{(h(\tau)+\lambda W_\tau)}\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge \tag{4.2}$$

$$(-\sqrt{-\ell}\coth(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$

$$-\frac{\sqrt{2}k\ell\blacklozenge\sqrt{h(\tau)+\lambda W_\tau}}{\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\sqrt{-\ell}}\blacklozenge\tanh(\sqrt{-\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$Q_3(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$
$$(-\frac{\begin{array}{c} i(h(\tau)+\lambda W_\tau)^2\sqrt{\partial}\blacklozenge(\varpi(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau) \\ -((2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))-2\varpi)) \end{array}}{3\sqrt{2(h(\tau)+\lambda W_\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge$$
$$(\sqrt{\ell}\tan(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$
$$+\frac{ik\ell(h(\tau)+\lambda W_\tau)\sqrt{2\partial}}{\sqrt{\ell(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)}\sqrt{\ell}}\blacklozenge\cot(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$R_3(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$
$$(\frac{\begin{array}{c} -2\varpi(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau)^2 \\ +(h(\tau)+\lambda W_\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))+2\varpi) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{(h(\tau)+\lambda W_\tau)}\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge \tag{4.3}$$
$$(\sqrt{\ell}\tan(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$
$$+\frac{\sqrt{2}k\ell\blacklozenge\sqrt{h(\tau)+\lambda W_\tau}}{\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\sqrt{\ell}}\blacklozenge\cot(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$Q_4(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$
$$(-\frac{\begin{array}{c} i(h(\tau)+\lambda W_\tau)^2\sqrt{\partial}\blacklozenge(\varpi(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau) \\ -((2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))-2\varpi)) \end{array}}{3\sqrt{2(h(\tau)+\lambda W_\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge$$
$$(\sqrt{\ell}\cot(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$
$$+\frac{ik\ell(h(\tau)+\lambda W_\tau)\sqrt{2\partial}}{\sqrt{\ell(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)}\sqrt{\ell}}\blacklozenge\tan(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$R_4(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$
$$(\frac{\begin{array}{c} -2\varpi(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau)^2 \\ +(h(\tau)+\lambda W_\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))+2\varpi) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{(h(\tau)+\lambda W_\tau)}\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge \tag{4.4}$$
$$(\sqrt{\ell}\cot(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))$$
$$+\frac{\sqrt{2}k\ell\blacklozenge\sqrt{h(\tau)+\lambda W_\tau}}{\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\sqrt{\ell}}\blacklozenge\tan(\sqrt{\ell}(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c))),$$

$$Q_5(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$

$$(-\frac{\begin{array}{c} i(h(\tau)+\lambda W_\tau)^2\sqrt{\partial}\blacklozenge(\varpi(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau) \\ -((2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(2\varpi+k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))-2\varpi)) \end{array}}{3\sqrt{2(h(\tau)+\lambda W_\tau)}\blacklozenge k\ell\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge$$

$$(-\frac{\Gamma(1+\vartheta)}{(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)+\rho})$$

$$-\frac{ik\ell(h(\tau)+\lambda W_\tau)\sqrt{2\partial}}{\sqrt{\ell(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)}}\blacklozenge\frac{(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)+\rho}{\Gamma(1+\vartheta)}),$$

$$R_5(x,\tau) = e^{i(-kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)}$$

$$(\frac{\begin{array}{c} -2\varpi(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)+1)+2k^2(1-2\ell)\blacklozenge(h(\tau)+\lambda W_\tau)^2 \\ +(h(\tau)+\lambda W_\tau)\blacklozenge((\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))\blacklozenge\partial(h(\tau)+\lambda W_\tau) \\ +(\varpi+k^2(-1+2\ell)\blacklozenge(h(\tau)+\lambda W_\tau))+2\varpi) \end{array}}{3\sqrt{2}k\ell\blacklozenge\sqrt{(h(\tau)+\lambda W_\tau)}\blacklozenge\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}\blacklozenge(h(\tau)+\lambda W_\tau)\blacklozenge(\partial(h(\tau)+\lambda W_\tau)-1)}\blacklozenge \tag{4.5}$$

$$(-\frac{\Gamma(1+\vartheta)}{(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)+\rho})$$

$$+\frac{\sqrt{2}k\ell\blacklozenge\sqrt{h(\tau)+\lambda W_\tau}}{\sqrt{-\partial(h(\tau)+\lambda W_\tau)-1}}\blacklozenge\frac{(kx+2k^2\left\{\int_a^t h(\tau)d\tau+\lambda(B_\tau-\frac{\tau^2}{2})\right\}+c)+\rho}{\Gamma(1+\vartheta)}),$$

where we have already used the following relation [32]

$$\begin{aligned}
\tan^{\blacklozenge}(B_\tau) &= \tan(B_\tau-\frac{\tau^2}{2}), \\
\cot^{\blacklozenge}(B_\tau) &= \cot(B_\tau-\frac{\tau^2}{2}), \\
\tanh^{\blacklozenge}(B_\tau) &= \tanh(B_\tau-\frac{\tau^2}{2}), \\
\coth^{\blacklozenge}(B_\tau) &= \coth(B_\tau-\frac{\tau^2}{2}),
\end{aligned}$$

5. Physical reviews

In this section, we shown some figures to analyse the action of the obtained solutions of Eq (1.3).

In Figures 1 and 2, we show the evolutional effects of stochastic equation (1.5) for Brownian motion $B_\tau = random[0,1]\sinh 2\tau$. In Figures 3 and 4, we consider and we considered the effects of stochastic equation (1.5) without effect of stochastic term W_τ.

a) *b*)

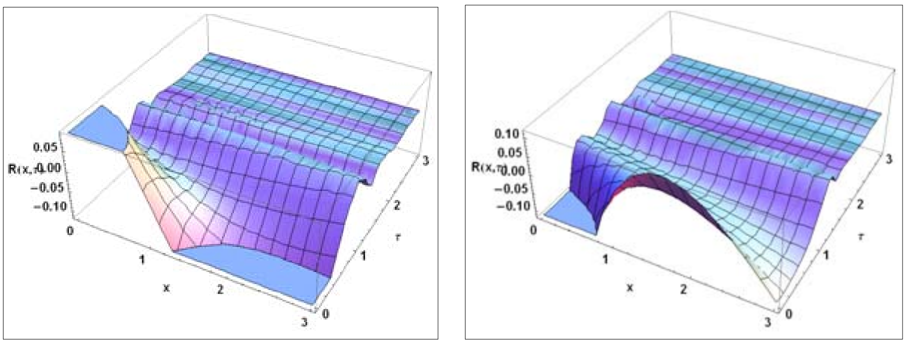

Figure 1. The 3D graphics of $Q_1(x,\tau)$ from the solutions (4.1) for Wick-type stochastic time fractional nonlinear Schrödinger equations (1.5) $(h(\tau)=\sinh 2\tau,\quad k=0.5, \rho=-1, \lambda=1, \rho=\frac{1}{2},\quad \partial=2,\quad c=0.3, \varpi=0.5, B_\tau=random[0,1]\sinh 2\tau)$ a) for real section, b) for imaginary section.

a) *b*)

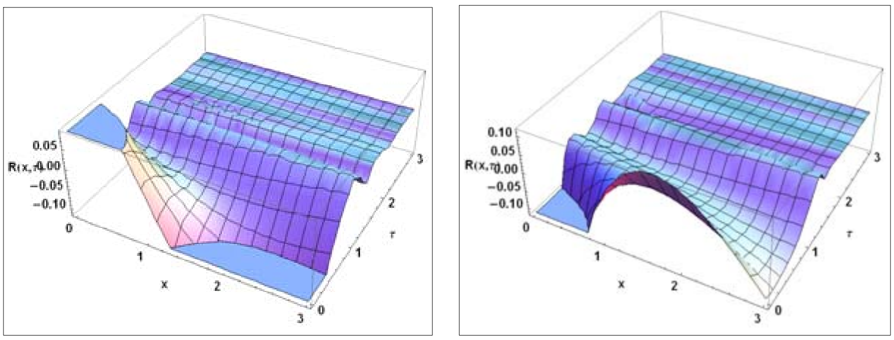

Figure 2. The 3D graphics of $R_1(x,\tau)$ from the solutions (4.1) for Wick-type stochastic time fractional nonlinear Schrödinger equations (1.5) $(h(\tau)=\sinh 2\tau,\quad k=0.5, \rho=-1, \lambda=1, \rho=\frac{1}{2},\quad \partial=2,\quad c=0.3, \varpi=0.5, B_\tau=random[0,1]\sinh 2\tau)$ a) for real section, b) for imaginary section.

a) *b*)

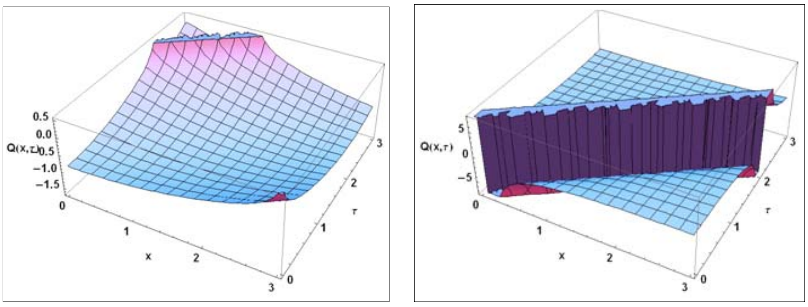

Figure 3. The 3D graphics of $Q_1(x,\tau)$ from the solutions (4.3) for Wick-type stochastic time fractional nonlinear Schrödinger equations (1.5) $(h(\tau)=\sinh 2\tau,$ $k=0.5, \rho=-1, \quad \lambda=1, \rho=\frac{1}{2}, \quad \partial=2, \quad c=0.3, \varpi=0.5, B_\tau=1,2)$ $a)$ for real section, $b)$ for imaginary section.

a) *b*)

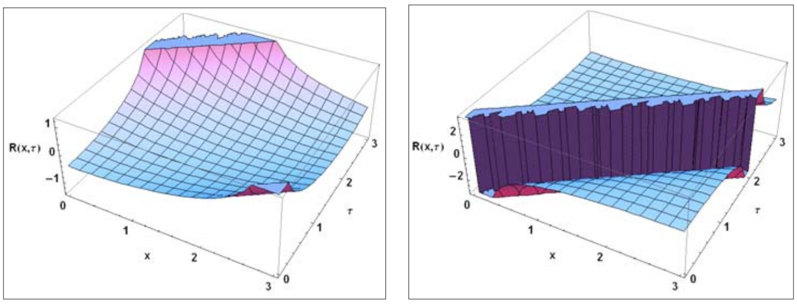

Figure 4. The 3D graphics of $R_1(x,\tau)$ from the solutions (4.3) for Wick-type stochastic time fractional nonlinear Schrödinger equations (1.5) $(h(\tau)=\sinh 2\tau,$ $k=0.5, \rho=-1, \quad \lambda=1, \rho=\frac{1}{2}, \quad \partial=2, \quad c=0.3, \varpi=0.5, B_\tau=1,2)$ $a)$ for real section, $b)$ for imaginary section.

6. conclusions

In this article, we investigated the coupled nonlinear Schrödinger equations with the aid of conformable derivative for deterministic and stochastic forms. In addition to, we studied Wick-model stochastic coupled nonlinear Schrödinger equations with time conformable derivatives. We analysed some exact solutions with the aid of the modified fractional sub-equation method, Hermite transform and White noise theory. We obtained stochastic hiperbolic and trigonometric wave solutions via inverse Hermite transform. Furthermore, we investigate an example, to show the stochastic solutions can be found as Brownian motion functional solutions. In addition to, if $\vartheta=1$, then the stochastic solutions (4.1)–(4.5) express a new set of stochastic solutions for the Wick-model stochastic coupled nonlinear Schrödinger equations by using integer derivatives.

This study emphasize that the modified fractional sub-equation method is adequate to solve the stochastic nonlinear equations in mathematical physics. The studied method in this paper is normal, direct and computerized method, which lets us to do confused and boring algebraic computation. It is expressed that the method can be also applied to other nonlinear stochastic differential equations in mathematical physics.

Acknowledgments

We would like to express our thanks to the anonymous referees who help us improved this paper.

Conflict of interest

All authors declare no conflict of interest in this paper.

References

1. A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, 2006.
2. I. Podlubny, *Fractional Differential Equation*, Academic Press, San Diego, 1999.
3. Z. Korpinar, M. Inc, M. Bayram, *Theory and application for the system of fractional Burger equations with Mittag leffler kernel*, Appl. Math. Comp., **367** (2020), 124781.
4. S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, Switzerland, 1993.
5. F. Tchier, M. Inc, Z. S. Korpinar, et al., *Solution of the time fractional reaction-diffusion equations with residual power series method*, Advances Mech. Eng., **8** (2016), 1–10.
6. M. S. Hashemi, *Invariant subspaces admitted by fractional differential equations with conformable derivatives*, Chaos, Solitons Fractals, **107** (2018), 161–169.
7. K. M. Owolabi, *Numerical analysis and pattern formation process for space-fractional superdiffusive systems*, Discrete Contin. Dyn. Syst.-S **12** (2019), 543–566.
8. M. Inc, Z. S. Korpinar, M. M. Al Qurashi, et al., *A new method for approximate solution of some nonlinear equations: Residual power series method*, Advances Mech. Eng., **8** (2016), 1–7.
9. Z. Korpinar, *On numerical solutions for the Caputo-Fabrizio fractional heat-like equation*, Therm. Sci., **22** (2018), 87–95.
10. A. M. El-Tawil, A. M. Sohaly, *Mean square convergent three points finite difference scheme for random partial differential equations*, J. Egypt. Math. Soc., **20** (2012), 188–204.
11. M. Sohaly, *Mean square Heun's method convergent for solving random differential initial value problems of first order*, Am. J. Comput. Math., **4** (2014), 474–481.
12. M. A. E. Abdelrahman, M. A. Sohaly, O. Moaaz, *The deterministic and stochastic solutions of the NLEEs in mathematical physics*, Int. J. Appl. Comput. Math, **5** (2019), 40.
13. T. Körpinar, R. C. Demirkol, Z. Körpinar, *Soliton propagation of electromagnetic field vectors of polarized light ray traveling in a coiled optical fiber in Minkowski space with Bishop equations*, Eur. Phys. J. D, **73** (2019), 203.
14. T. Körpinar, R. C. Demirkol, Z. Körpinar, *Soliton propagation of electromagnetic field vectors of polarized light ray traveling in a coiled optical fiber in the ordinary space*, Int. J. Geom. Methods Modern Phys., **16** (2019), 1950117.

15. T. Körpinar, R. C. Demirkol, Z. Körpinar, *Soliton propagation of electromagnetic field vectors of polarized light raytraveling along with coiled optical fiber on the unit 2-sphere S^2*, Revista Mexicana de Fisica **65** (2019), 626–633.
16. A. G. Hossam, H. Abd-Allah, *Exact traveling wave solutions for the wick-type stochastic Schamel KdV equation*, Phys. Res. Int., 2014 (2014), 937345.
17. Z. Korpinar, F. Tchier, M. Inc, et al., *On exact solutions for the stochastic time fractional Gardner equatio,* Physica Scripta, 2019, doi:10.1088/1402-4896/ab62d5.
18. A. G. Hossam, A. S. Okb El Bab, A. M. Zabel, et al., *The fractional coupled KdV equations: Exact solutions and white noise functional approach*, Chin. Phys. B, **22** (2013), 080501.
19. A. H. Bhrawy, A. A. Alshaery, E. M. Hilal, et al., *Optical solitons in birefringent fibers with spatio-temporal dispersion*, Optik, **125** (2014), 4935–4944.
20. M. M. Al Qurashi, Z. S. Korpinar, M. Inc, *Approximate solutions of bright and dark optical solitons in birefrigent fibers*, Optik, **140** (2017), 45–61.
21. R. W. Boyd, *Nonlinear Optics*, Academic, San Diego, 1992.
22. A. Esen, T. A. Sulaiman, H. Bulut, et al., *Optical solitons to the space-time fractional (1+1)-dimensional coupled nonlinear Schrödinger equation*, Optik, **167** (2018), 150–156.
23. S. Ali, M. Younis, M. O. Ahmad, et al., *Rogue wave solutions in nonlinear optics with coupled Schrödinger equations*, Optical Quantum Electron., **50** (2018), 266.
24. M. Eslami, *Exact traveling wave solutions to the fractional coupled nonlinear Schrodinger equations*, Appl. Math. Comp., **285** (2016), 141–148.
25. M. S. Osman, D. Lu, M. M. A. Khater, et al., *Complex wave structures for abundant solutions related to the complex Ginzburg-Landau model*, Optik-Int. J. Light Electron Optics, **192** (2019), 162927.
26. B. Younas, M. Younis, *Chirped solitons in optical monomode fibres modelled with Chen-Lee-Liu equation*, Pramana-J. Phys., **94** (2020), 3.
27. D. Lu, M. S. Osman, M. M. A. Khater, et al., *Analytical and numerical simulations for the kinetics of phase separation in iron (Fe-Cr-X (X=Mo, Cu)) based on ternary alloys*, Physica A, **537** (2020), 122634.
28. A. Arif, M. Younis, M. Imran, et al., *Solitons and lump wave solutions to the graphene thermophoretic motion system with a variable heat transmission*, Eur. Phys. J. Plus, **134** (2019), 303.
29. V. S. Kumar, H. Rezazadeh, M. Eslami, et al., *Jacobi elliptic function expansion method for solving KdV equation with conformable derivative and Dual-Power law nonlinearity*, Int. J. Appl. Comput. Math., **5** (2019), 127.
30. D. Lu, K. U. Tariq, M. S. Osman, et al., *New analytical wave structures for the (3+1)-dimensional Kadomtsev-Petviashvili and the generalized Boussinesq models and their applications*, Results Phys., **14** (2019), 102491.
31. M. S. Osman, A. M. Wazwaz, *A general bilinear form to generate different wave structures of solitons for a (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation*, Math Meth Appl Sci., **42** (2019), 6277–6283.
32. R. Khalil, M. Al Horani, A. Yousef, et al., *A new definition of fractional derivative*, J. Comput. Appl. Math., **264** (2014), 65–70.
33. A. Atangana, D. Baleanu, *New fractional derivative with nonlocal and non-singular kernel, theory and application to heat transfer model*, Therm. Sci., **20** (2016), 763–769.
34. M. Caputo, M. Fabrizio, *A new definition of fractional derivative without singular kernel*, Progr. Fract. Differ. Appl., **1** (2015), 73–85.

35. A. Atangana, B. T. Alkahtani, *Analysis of non-homogenous heat model with new trend of derivative with fractional order*, Chaos Solitons Fractals, **89** (2016), 566–571.
36. J. H. He, *Homotopy perturbation technique*, Comput. Methods Appl. Mech. Eng., **178** (1999), 257–262.
37. J. H. He, *Homotopy perturbation method: A new nonlinear analytical technique*, Appl. Math. Comput., **135** (2003), 73–79.
38. A. H. Ghany, A. S. Okb El Babb, A. M. Zabel, et al., *The fractional coupled KdV equations: Exact solutions and white noise functional approach*, Chin. Phys. B, **22** (2013), 080501.
39. A. H. Ghany, H. Abd-Allah, *Abundant solutions of Wick-type stochastic fractional 2D KdV equations*, Chin. Phys. B, **23** (2014), 060503.
40. S. Zang, *A generalized exp-function method for fractional Riccati differential equations*, Comm. Fractional Calculus, **1** (2010), 48–51.
41. H. Holden, B. Ø sendal, J. Ubøe, et al., *Stochastic Partial Differential Equations* Birhkäuser, Basel, 1996.
42. H. Holden, B. Ø sendal, J. Ubøe, et al., *Stochastic Partial Differential Equations*, Springer science-Business media, LLC, 2010.