RESEARCH ARTICLE

Modeling the impact of temperature on fractional order dengue model with vertical transmission

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ARTICLE INFO	ABSTRACT
	A dengue epidemic model with fractional order derivative is formulated to analyze the effect of temperature on the spread of the vector-host transmitted dengue disease. The model is composed of a system of fractional order differential equations formulated within Caputo fractional operator. The stability of the equilibrium points of the considered dengue model is studied. The corresponding basic reproduction number R_0^α is derived and it is proved that if $R_0^\alpha < 1$, the disease-free equilibrium (DFE) is locally asymptotically stable. $L1$ method is applied to solve the dengue model numerically. Finally, numerical simulations are also presented to illustrate the analytical results showing the influence of the temperature on the dynamics of the vector-host interaction in dengue epidemics.

1. Introduction

A deeper understanding of mathematical models is essential to represent and to reliably control the transmission of the (epidemic/pandemic)diseases. The vector-borne diseases becomes an extensive threat with a significant affect on human and animal health. Distribution of vector-borne diseases is determined by complex demographic, environmental and social factors. Vector-borne diseases are responsible from more than 700 000 deaths annually as taking a part of 17% of all infectious diseases. Dengue fever, as a severe, flu-like illness influences infants, young children and adults, is mainly faced in urban and semi-urban areas of the countries in tropical and sub-tropical climates [1,2]. Dengue fever disease has no concrete treatment but early detection and efficient medical treatment reduces the death rates below 1%.

Dengue virus is carried from vector-host-vector mainly by the bites of infected female mosquitoes of the type Aedes aegypti. After virus incubation for 410 days, an infected mosquito is able to transmit the virus for the rest of its life [2].

All mosquito species go through four distinct phases during their life cycle: Egg, Larva (plural: larvae), Pupa (plural: pupae), Adult. The first three phases take place in water, but the adult is an active flying insect (see Figure 1). Only the female mosquitoes bite and they feed on the blood of humans or other animals.
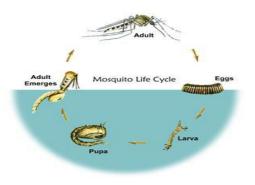

Figure 1. Mosquito Life Cycle [3].

In order to keep the dengue infection under control some adequate and powerful mathematical (compartmental) models and analysis have been

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suggested (see [4–15]). Abdelrazec et al. [15] developed a model for the dynamic study of transmission of dengue fever by means of a nonlinear rate of recovery to analyze the spread and control of the disease. A mathematical model for dengue is constructed in [8] to determine the chain of two epidemic diseases with different human populations. The reproduction number R_0^α as a threshold quantity of the epidemics is explained by means of the stability analysis. Their model shown that the ecological administration alone as a the vector control is not enough; it may postpone the spread of epidemics. The usage of a vaccine may control simultaneously against some serotypes. In Andraud et al. [16], deterministic models of dengue transmission are surveyed in terms of the assumptions for parameters, threshold values and control measures. The effect of seasonal variations in temperature and some other climate factors on the transmission dynamics of dengue diseases are epidemiologically discusses in several experimental research (e.g. see [17] the references therein) and mathematically analyzed in the recent studies [18–21]. The mathematical analysis in these references are based on the compartmental integer-order epidemic models that contain a system of ODE's. However, in general, integer-order systems are memoryless ([4–8, 10, 11, 15–20]).

Fractional calculus is the study of an extended form of integrals and derivatives in fractional orders. The most vital aspect of fractional derivatives is that, the models based on these operators hold memory which provides an important advantage for a well understanding of the behaviour of the entomological factors and so the dynamics of the epidemic diseases. Sardar et al. [12] investigated a compartmental dengue transmission model having the memory, in which the memory incorporated in the model exhibiting an arbitrary order differential operator. A threshold quantity R_0^α is derived, having the similarity with elementary reproduction number and determined that although the value of R_0^α is less than one, the disease-free equilibrium would not be constantly stable, and the system reveals a Hopf-type bifurcation. The SIR model of the fractional order differential equation of the dengue fever is investigated in [22], and a fractional order SEIR model with vertical transmission in a nonconstant population is studied by [23]. In [24] and [25], three definitions of fractional operators are carried out for MSEIR models of some other type of epidemics as varicella disease validated by an outbreak data. Moreover, in the studies of [26–29], dengue fever epidemics is modeled within real

data by the help of fractional operators of some types that varies by the definition of their kernel. Our goal in this study is to investigate the fractional order dynamical model of the dengue fever with temperature effect centered at the distribution of the human population into three categories (susceptible, infected, and resistant humans), whereas the population of the Aedes aegypti pre-adult female mosquitos (eggs and larvae) is distributed in two parts (susceptible and infected) and adult mosquitos population is divided in three parts (susceptible, infected but not infectious, and infectious) to understand the dynamics of dengue disease in a more accurate and realistic way.

The manuscript is organized as follows: after the related literature info in Introduction part in Section 1, the generalized non-integer order mathematical model of the temperature effect in the vector-host (vertical) transmission dynamics of dengue fever epidemics is introduced in Section 2 with a preliminary info about its parameter list and initial conditions. The disease-free equilibrium points of the system are newly obtained and the local asymptotic stability conditions are derived correspondingly which results the basic reproduction number of the system. Section 3 is dedicated to the numerical solution of the discretized version of generalized Caputo-based dengue model. Followingly, the numerical simulations are performed for the model (for different temperature values and fractional orders) in order to analyze the temperature effect on the dengue transmission dynamics by using the real data of the FongShan district, Kaohsiung, Taiwan. The discussions of the simulations are provided in Section 3 and the concluding remarks are given in Section 4.

2. Preliminaries and the proposed generalized dengue model

In this part, we present fractional operators where the Caputo fractional derivative is mainly considered. For a function $x(t)$ defined on a time interval $[0, T]$, the Caputo derivative and integral of $x(t)$ are denoted by $^CD^\alpha x(t)$ and $^CI^\alpha x(t)$, respectively, and defined as [30]

$$^CD^\alpha x(t) := \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\xi)^{-\alpha} \dot{x}(\xi) d\xi, \quad (1)$$

$$^CI^\alpha x(t) := \frac{1}{\Gamma(\alpha)} \int_0^t (t-\xi)^{\alpha-1} x(\xi) d\xi, \quad (2)$$

where $0 < \alpha < 1$ represents the order of the fractional operator.

We will examine a mathematical model of the dengue fever having arbitrary order, the parameter values used in this mathematical model are estimated based on the demographic data of Fong-Shan district, Kaohsiung in southern Taiwan ([20], [31]). Kaohshiung is the second largest cosmopolitan city of Taiwan. The data showed that Kaohsiung was at high risk for dengue fever from 1998 to 2010 and in total 2415 number of cases of dengue fever were reported. According to the WHO Dengue situation update by July 2019, approx. 29 300 cases reported in 2019 only in the Western Pasific Region ([1], [2]). In this study, a set of weekly meteorological data has been used which belongs to Kaohsiung district that was recorded by 11 supervising observatory locations of the Taiwan Environmental Protection Agency [3] in 2011, further weekly maximum, minimum and mean temperatures between the years of 2001 and 2010 are incorporated [20].

2.1. Fractional order vector-host dengue model

This study is based upon the model of vector-host transmission dynamics proposed in [32] to represent the transmission patterns of the dengue fever. The population has been separated into three main parts as host (human), vector (pre-adult female mosquito), and vector (adult female mosquito) population. There are two compartments of Aedes aegypti pre-adult female mosquito population (effective eggs and larvae) namely susceptible E_s and infected E_i. The three compartments of vector (adult) population are stated accordingly as: $M_s, M_e,$ and M_i, which are the values evaluated at time t of susceptible, infected (but not being infectious) and infectious female mosquitoes. Also, host (human population) is divided into three compartments as: $H_s, H_i,$ and H_r, these are the numbers at the time t of the susceptible, infected/infectious and recovered/immune human populations, respectively. This way dengue model defined by the following system [32] and correspondingly the model parameters are listed in Table 1:

$$
\begin{cases}
{}^C D^\alpha E_s = e_v^\alpha (1 - p(\frac{M_i}{M_s+M_e+M_i})) - \eta^\alpha E_s, \\
{}^C D^\alpha E_i = e_v^\alpha p(\frac{M_i}{M_s+M_e+M_i}) - \eta^\alpha E_i, \\
{}^C D^\alpha M_s = \eta^\alpha E_s - b^\alpha \frac{H_i}{N_h} M_s - \delta^\alpha M_s, \\
{}^C D^\alpha M_e = b^\alpha \frac{H_i}{N_h} M_s - \gamma^\alpha M_e - \delta^\alpha M_e, \\
{}^C D^\alpha M_i = \gamma^\alpha M_e + \eta^\alpha E_i - \delta^\alpha M_i, \\
{}^C D^\alpha H_s = R_{hb}^\alpha N_h - b^\alpha \frac{H_s}{N_h} M_i - R_{hd}^\alpha H_s, \\
{}^C D^\alpha H_i = b^\alpha \frac{H_s}{N_h} M_i - \xi^\alpha H_i - R_{hd}^\alpha H_i, \\
{}^C D^\alpha H_r = \xi^\alpha H_i - R_{hd}^\alpha H_r.
\end{cases}
\tag{3}
$$

The initial values are $H_s(0) = 341094$ (total human population), $H_i(0) = 26$ (number of confirmed cases), and $H_r(0) = 0$ according to the collected info in December, 2010 in FongShan district (Kaohsiung, Taiwan). The transmissible biting rate was taken as 0.33 on a daily basis (meaning that one bite occurs per three days for one female mosquito). At an initial time $t = 0$, the values for pre-adult and adult vector populations are set as $E_s(0) = 0$, $E_i(0) = 0$, $M_s(0) = 341120$, $M_e(0) = 0$, and $M_i(0) = 0$, respectively. The total adult mosquito population N_m was 341120 female mosquitoes that is same value with the human population size. The detailed assumptions of the considered (vertical) transmission dengue dynamics model can be seen in [20] and the references therein.

2.2. Equilibrium Points

The system has two types of disease free equilibrium points namely trivial disease-free equilibrium (DFE) E_0^α and biologically realistic disease-free equilibrium (BRDFE) $\widehat{E}_1^\alpha$. To find equilibrium points, we will solve the following system:
$${}^C D^\alpha E_s = 0, {}^C D^\alpha E_i = 0, {}^C D^\alpha M_s = 0, {}^C D^\alpha M_e = 0,$$
$${}^C D^\alpha M_i = 0, {}^C D^\alpha H_s = 0, {}^C D^\alpha H_i = 0, {}^C D^\alpha H_r = 0.$$
By solving above system, we obtain the following equilibrium points:
$$E_0^\alpha = (0, 0, 0, 0, 0, \overline{H}_s, 0, 0)$$
$$\widehat{E}_1^\alpha = (\widehat{E}_s, 0, \widehat{M}_s, 0, 0, \widehat{H}_s, 0, 0)$$
where
$$\overline{H}_s = \frac{R_{hb}^\alpha N_h}{R_{hd}^\alpha}, \widehat{E}_s = \frac{e_v^\alpha}{q^\alpha}, \widehat{M}_s = \frac{e_v^\alpha}{\delta^\alpha}, \widehat{H}_s = \frac{R_{hb}^\alpha N_h}{R_{hd}^\alpha}.$$
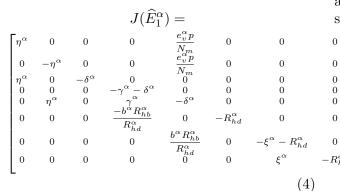
2.3. Stability analysis

Theorem 1. *The biologically realistic disease-free equilibrium (BRDFE) $\widehat{E}_1^\alpha$ of the system in Eq. (3) is locally asymptotically stable if*
$$R_1^\alpha = \frac{N_m + \eta^\alpha e_v^\alpha - \eta^\alpha \delta^\alpha N_m}{N_m} < 1.$$

Proof. The Jacobian matrix of the system in Eq. (3) evaluated biologically realistic disease-free equilibrium (BRDFE) $\widehat{E}_1^\alpha$ is given by

Table 1. Dengue model parameters (at temperature 25°C)

Symbol	Meaning and unit	Range of values	References
p	"proportion of eggs"	0.028	[33]
η	"pre-adult mosquito maturation rate (per day)"	0.099	Estimated
b	"the biting rate (per day)"	0.33	[32]
e_v	"Oviposition rate (per day)"	6.218	Estimated
γ	"Virus incubation rate in mosquito (per day)"	0.0607	Estimated
δ	"Adult mosquito death rate (per day)"	0.0331	Estimated
ξ	"Human recovery rate (per day)"	1/7	[32]
R_{hd}	"Human death rate (per day)"	0.000016	[31]
R_{hb}	"Human birth rate(per day)"	0.00002	[31]
N_m	"Total number of mosquitoes"	341120	Assumed
N_h	"Total size of human population"	341120	[31]

$$J(\widehat{E}_1^\alpha) =$$

$$\begin{bmatrix}
\eta^\alpha & 0 & 0 & 0 & \frac{e_v^\alpha p}{N_m} & 0 & 0 & 0 \\
0 & -\eta^\alpha & 0 & 0 & \frac{e_v^\alpha p}{N_m} & 0 & 0 & 0 \\
\eta^\alpha & 0 & -\delta^\alpha & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\gamma^\alpha - \delta^\alpha & 0 & 0 & 0 & 0 \\
0 & \eta^\alpha & 0 & \gamma^\alpha & -\delta^\alpha & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-b^\alpha R_{hb}^\alpha}{R_{hd}^\alpha} & 0 & -R_{hd}^\alpha & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{b^\alpha R_{hb}^\alpha}{R_{hd}^\alpha} & 0 & -\xi^\alpha - R_{hd}^\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \xi^\alpha & -R_{hd}^\alpha
\end{bmatrix}$$

$$(4)$$

The calculated eigenvalues are given by

$$\lambda_1 = -\eta^\alpha, \lambda_2 = -\delta^\alpha, \lambda_3 = -R_{hd}^\alpha,$$

$\lambda_4 = -R_{hd}^\alpha, \lambda_5 = -(\xi^\alpha + R_{hd}^\alpha), \lambda_6 = -(\gamma^\alpha + \delta^\alpha),$ remaining eigenvalues are the roots of the quadratic polynomial

$$\lambda^2 + (\eta^\alpha + \delta^\alpha)\lambda + (1 - R_1^\alpha) = 0,$$

by the Routh-Hurwitz stability criterion, $\widehat{E}_1^\alpha$ is locally asymptotically stable if and only if $R_1^\alpha < 1$.　　　□

2.4. The basic reproduction number (R_0^α)

The basic reproduction number R_0^α of the epidemic disease is known as the number of secondary infections caused by a unique infected individual. Hypothetically, when $R_0^\alpha < 1$ the transmissions chains are not beneficial and disease will eradicate from the population. Although if $R_0^\alpha > 1$, each generation will be raised by the number of infected humans and infection will be taken advantage of with the usage of next generation matrix approach. In this situation R_0^α is equal to the spectral radius of $K = FV^{-1}$, where F is a non-negative matrix of the infection items,

and V is the M-Matrix of the corresponding transition terms given as in below [13]

$$F = \begin{pmatrix}
0 & 0 & 0 & b^\alpha \\
0 & 0 & 0 & p\delta \\
b^\alpha \frac{N_m}{N_h} & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},$$

$$V^{-1} = \begin{pmatrix}
\frac{1}{\xi^\alpha + R_{hd}^\alpha} & 0 & 0 & 0 \\
0 & \frac{1}{\eta^\alpha} & 0 & 0 \\
0 & 0 & \frac{1}{\gamma^\alpha + \delta^\alpha} & 0 \\
0 & \frac{1}{\eta^\alpha} & \frac{1}{\gamma^\alpha} & \frac{1}{\delta^\alpha}
\end{pmatrix},$$

and so

$$FV^{-1} = \begin{pmatrix}
0 & \frac{b^\alpha}{\delta^\alpha} & \frac{b^\alpha}{\gamma^\alpha} & \frac{b^\alpha}{\delta^\alpha} \\
0 & p & \frac{p\delta^\alpha}{\gamma^\alpha} & p \\
\frac{b^\alpha N_m}{N_h}(\xi^\alpha + R_{hd}^\alpha) & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.$$

The eigenvalues of FV^{-1} is:

$$\frac{p}{2} \pm \frac{1}{2}\sqrt{p^2 + \frac{4b^\alpha N_m}{N_h \gamma^\alpha (\xi^\alpha + R_{hd}^\alpha)}}$$

Therefore, the dominant eigenvalue of FV^{-1} is

$$R_0^\alpha = \frac{p}{2} + \frac{1}{2}\sqrt{p^2 + \frac{4b^{2\alpha} N_m}{N_h \gamma^\alpha (\xi^\alpha + R_{hd}^\alpha)}}. \quad (5)$$

3. Numerical method and simulations

In this section, we will construct the discretization of the model given by Eq. (3) involving the Caputo fractional operator.

First, we rewrite system in Eq. (3) in a compact form so that the relations can be simplified

$$\begin{cases}
{}^C D^\alpha Y(t) = \mathcal{F}(Y(t)), & 0 < t < b < \infty, \\
Y(0) = Y^0,
\end{cases}$$

$$(6)$$

where $Y = (E_s, E_i, M_s, M_e, M_i, H_s, H_i, H_r) \in \mathbb{R}_+^8$, where $\mathcal{F}$, is a real-valued continuous vector

function having the Lipschitz condition

$$\|\mathcal{F}(Y_1(t)) - \mathcal{F}(Y_2(t))\| \leq L\|Y_1(t) - Y_2(t)\|, \quad L > 0, \tag{7}$$

and $Y^0 = (E_s^0, E_i^0, M_s^0, M_e^0, M_i^0, H_s^0, H_i^0, H_r^0)$ is the initial vector. By applying the fractional integral operator in Eq. (2) to Eq. (6) the following result is obtained

$$Y(t) = Y_0 + {}^C I^\alpha \mathcal{F}(Y(t)), \quad 0 < t < b < \infty, \tag{8}$$

where ${}^C I^\alpha$ represents the Riemann-Liouville integral operator. In order to propose a numerical scheme, the discretization of the time interval $[0, b]$ is done by equally spaced nodes so that time step size is $\tau = \frac{b}{N}$. Let Y_n denote the approximation of the true solution $Y(t_n)$ at point $t_n = n\tau$ and $0 = t_0 < t_1 < ... < t_N = b$, with $t_{n+1} - t_n = \tau$, $n = 0, 1, 2, ..., (N - 1)$. Then, we derive following numerical scheme for the Caputo operator using Euler method [34]

$$Y^{n+1} = Y^0 + \frac{\tau^q}{\Gamma(\alpha + 1)} \sum_{k=0}^n w_{k+1,j} \mathcal{F}(Y_k), \tag{9}$$

where (n=0,1,2,...,(N-1)) and

$$w_{k+1,j} = (k + 1 - j)^\alpha - (k - j)^\alpha$$

are the weights of the fractional Euler method. Thus, we obtain the following discretization of the model in Eq. (3)

$$
\begin{cases}
E_s^{n+1} = E_s^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(e_v^\alpha (1 - p(\frac{M_i^k}{N_m})) - \eta^\alpha E_s^k \right), \\
E_i^{n+1} = E_i^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(e_v^\alpha p(\frac{M_i^k}{N_m}) - \eta^\alpha E_i^k \right), \\
M_s^{n+1} = M_s^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(\eta^\alpha E_s^k - b^\alpha \frac{H_i^k}{N_h} M_s^k - \delta^\alpha M_s^k \right), \\
M_e^{n+1} = M_e^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(b^\alpha \frac{H_i^k}{N_h} M_s^k - \gamma^\alpha M_e^k - \delta^\alpha M_e^k \right), \\
M_i^{n+1} = M_i^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(\gamma^\alpha M_e^k + \eta^\alpha E_i^k - \delta^\alpha M_i^k \right), \\
H_s^{n+1} = H_s^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(R_{hb}^\alpha N_h - b^\alpha \frac{H_s^k}{N_h} M_i^k - R_{hd}^\alpha H_s^k \right), \\
H_i^{n+1} = H_i^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(b^\alpha \frac{H_s^k}{N_h} M_i^k - \xi^\alpha H_i^k - R_{hd}^\alpha H_i^k \right), \\
H_r^{n+1} = H_r^0 \\
\quad + \frac{\tau^q}{\Gamma(\alpha+1)} \sum_{k=0}^n w_{k+1,j} \left(\xi^\alpha H_i^k - R_{hd}^\alpha H_r^k \right),
\end{cases}
\tag{10}
$$

where $n = 0, 1, 2, ..., (N - 1)$, and $N_m = M_s + M_e + M_i$.

3.1. Simulation results and discussion

We solve the newly proposed fractional dengue model given by Eq. (3) numerically. The effect of temperature on the dynamics of dengue epidemics is presented by Fig 2 where the temperature values are considered as 18° C, 25° C and 28° C since the parameter values in Table 1 are known at these temperatures. It is stated that (see ref. [20] and the related references therein) e_v-oviposition rate, γ-virus incubation rate in mosquito, δ-adult mosquito death rate and η-pre-adult mosquito maturation rate are found as temperature dependent entomological parameters in the selected district of Taiwan. It is seen that the temperature is a significant environmental factor that affect the transmission in dengue epidemics. The obtained results in Fig 2 shows that the infected human (host) population and infected (pre-adult and adult) mosquito populations reached to their peak values with a higher amount at the temperature of 28° C approximately in the period of 50-100 days. Correspondingly, the susceptible human population showed a sharper decrease whereas the recovered human population size has a sharper increase (after 100 days approx.) at 28° C.

The impact of the fractional order is depicted in Fig 3. In this sense, the temperature effect in vector-host (vertical) transmission dynamics in dengue epidemics formulated by fractional order dengue model is investigated for different values of the fractional order such as $\alpha = 1, 0.8$ and $\alpha = 0.6$ at a fixed temperature of 25° C. The results for all considered values of α reach to steady state for each population and integer order case is recovered when $\alpha = 1$. The susceptible human and pre-adult mosquito populations, recovered human populations reach to different values in the asymptotic case as α differs for the fixed temperature value of 25° C. The fractional order α here enables us to include the memory into the evolution of the epidemic dynamics and also it gives us the opportunity to capture different behaviors of the system components inside the same model.

Moreover, Table 2 verify the Theorem 1 as the condition $R_1^\alpha < 1$ is satisfied for the considered 3-different temperature cases which means that the biologically realistic disease free equilibriums of the system are all asymptotically stable.

Table 2. Stability of $\widehat{E}_1^\alpha$

Equilibrium	R_1^1	$R_1^{0.8}$	$R_1^{0.6}$
$\widehat{E}_1^\alpha$ at 18° C	0.9761	0.9931	0.9761
$\widehat{E}_1^\alpha$ at 25° C	0.9677	0.9897	0.9677
$\widehat{E}_1^\alpha$ at 28° C	0.9633	0.9878	0.9633

(a) Pre-adult female mosquito-Susceptible

(b) Pre-adult female mosquito-Infected

(c) Adult female mosquito-Susceptible

(d) Adult female mosquito-Infected

(e) Adult female mosquito-Infectious

(f) Human population-Susceptible

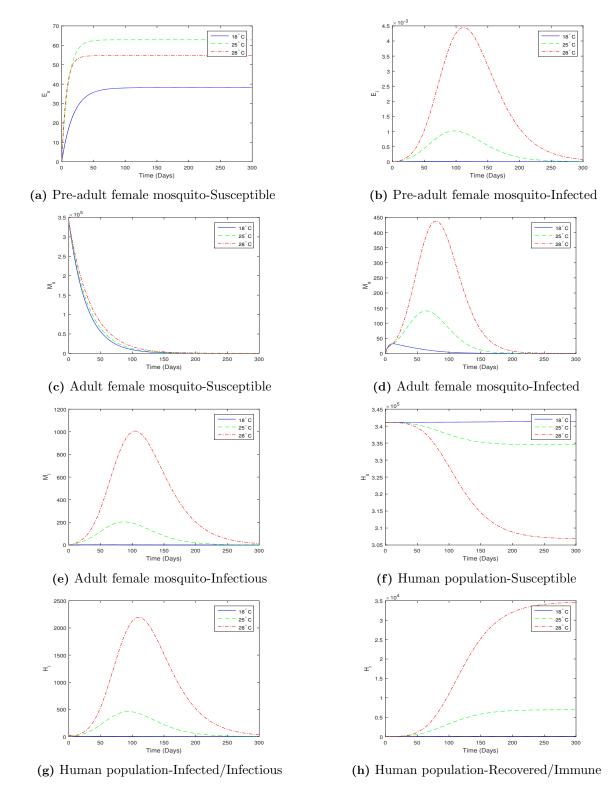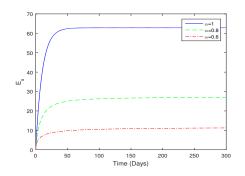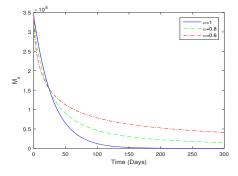**(g)** Human population-Infected/Infectious

(h) Human population-Recovered/Immune

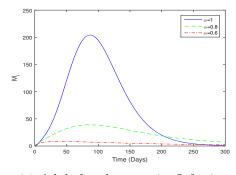**Figure 2.** Simulation results of the fractional order dengue model at temperature 18° C, 25° C and 28° C with $\alpha = 1$.

4. Conclusion

A fractional order dengue model is newly investigated that incorporate temperature dependent features in entomological parameters. In the modeling process four entomological parameters are used for this purpose, that includes the oviposition rate (e_v), pre-adult mosquito maturing rate η, adult mosquito death rate δ, and the virus incubation rate in mosquitoes γ. Weekly mean temperature data in Kaohsiung (2001-2010) [20] has been utilized to perform the simulations. We applied $L1$ method to solve the model numerically for various fractional order and temperatures. Further, the basic reproduction number is derived for the newly considered fractional order
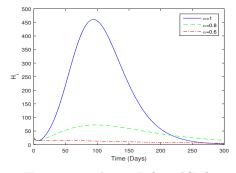
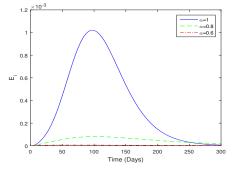**(a)** Pre-adult female mosquito-Susceptible
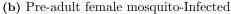
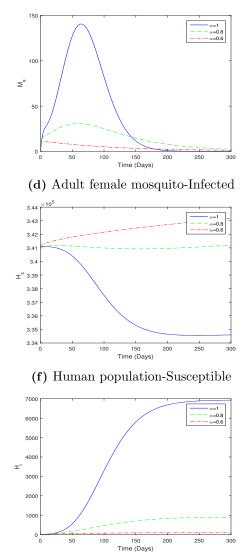**(b)** Pre-adult female mosquito-Infected

(c) Adult female mosquito-Susceptible

(d) Adult female mosquito-Infected

(e) Adult female mosquito-Infectious

(f) Human population-Susceptible

(g) Human population-Infected/Infectious

(h) Human population-Recovered/Immune

Figure 3. Simulation results of the fractional order dengue model at $\alpha = 1$, $\alpha = 0.8$, $\alpha = 0.6$ at temperature $25°$ C.

dengue model (that is formulated by using Caputo fractional operators). Correspondingly, stability analysis is performed and the local asymptotic stability of the disease-free equilibria is obtained. Simulation results refers that the highest danger of dengue transmission exists at temperature $28°$ C. The control of dengue transmission is based on the control of the aquatic and adult stages of the mosquito and so this study can contribute to adapt effective control strategies like the use of pesticide and other techniques.

Acknowledgments

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