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Fuzzy clustering to classify several time series models with fractional Brownian motion errors

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Abstract In real world problems, scientists aim to classify and cluster several time series processes that can be used for a dataset. In this research, for the first time, based on fuzzy clustering method, an approach is applied to classify and cluster several time series models with fractional Brownian motion errors as candidates to fit on a dataset. The ability of the introduced technique is studied using simulation and real world example.

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1. Introduction

Recently, statistical methods including regression and time series models [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24], mathematical methods including numerical analysis and optimization [25–42],[43], and artificial intelligence methods including machine learning and deep learning [44–50], frequently applied to model the natural phenomena. In data modeling problems such as time series modeling in agricultural, biological, climatological, economic, financial, hydrological, management and signal processing researches, scientists aim to cluster and classify the stochastic mechanism of several time series that can be fitted on a specific dataset. They usually use the values of different selection criterions likes R-Squared (R^2), mean absolute or square error (MAE or MSE), Akaike information criterion (AIC) and Bayesian information criterion (BIC), to compare models and then the model with the minimum MSE (or AIC or BIC) and maximum R^2 will be selected as the best. For example, in hydrological researches, Zarei and Mahmoudi [14], studied several models about the changes in a drought index. Also Zarei et al. [51,52] studied the variability in drought's trend and pattern using different time series models.

The problem of comparison and classification of two or several processes have been considered in many statistical researches [6–8],[20,21],[53–85]. Recently, Mahmoudi et al. [6], developed a novel approach to compare two stationary processes. Their proposed test was more powerful than other methods.

Most of previous studies focused on two following cases: (1) comparison of two or several processes; (2) clustering of several independent processes. Therefore the clustering of several time series fitted on a specific dataset (specially, non-stationary processes or time series with fractional Brownian motion noises) was lacked. In this research, for the first time, an approach is applied to classify and cluster several stationary and non-stationary time series models fitted on a dataset, specially when the noises follow fractional Brownian motion processes. At first, all of time series models are fitted on observed dataset. Then, for observed dataset, the predicted values of all models are estimated based on fitted models formulas. Finally, the fuzzy clustering technique is used to cluster the models based on the similarities between predicted values. The ability of the proposed technique is studied using simulation and real world example. The proposed technique has many advantages. First, the power of this method is acceptable. Second, the introduced approach is not too computational and time consuming. More ever, this technique can be applied to cluster different dependent stationary and non-stationary time series processes with dependent or independent noises.

2. Clustering several time series models

Assume we have k time series models as candidates to fit on X_t. Let write the equations of these k time series models by

$$X_t = f_i(X_{t-1}, X_{t-2}, \cdots) + B_H^{(i)}(t), \quad i = 1, \cdots, k, \tag{2.1}$$

where $B_H^{(i)}(t), i = 1, \cdots, k$, are independent fractional Brownian motion errors, and $f_i, i = 1, \cdots, k$, are known parametric functions with unknown parameters. In other words, $f_i, i = 1, \cdots, k$, are known time series models.

From (2.1), we have

$$Projection(X_t | X_{t-1}, X_{t-2}, \cdots) = f_i(X_{t-1}, X_{t-2}, \cdots),$$

which is a projection for X_t given $X_{t-1}, X_{t-2}, \cdots$.

Some researches focused on testing $H_0 : f_1 = \cdots = f_k$. In other words, they considered the comparison of k processes. Some others studied the clustering of several independent processes. But the clustering of several time series (specially, non-stationary processes or time series with fractional Brownian motion noises) fitted on a specific dataset is lacked. In this study, we are interested to know the rates of similarities of these k time series models and to cluster these models based on their similarities.

Consider n observations $X_1, \cdots, X_n$, as a sample dataset of size n from X_t. We are interested in clustering $f_1, \cdots, f_k$. The following steps should be run:

Step (1): At first, all of time series models are fitted on dataset by

$$\widehat{X}_t^{(i)} = \widehat{f}_i(X_{t-1}, X_{t-2}, \cdots), i = 1, \cdots, k. \tag{2.2}$$

In other words, we estimate the unknown parameters of the known functions $f_1, \cdots, f_k$.

Step (2): The predicted values of X_t is estimated based on all k models by

$$\widehat{X}_1^{(i)}, \cdots, \widehat{X}_n^{(i)}, i = 1, \cdots, k.$$

In other words, $\widehat{X}_j^{(i)}, j = 1, \cdots, n$, is the predicted value of dataset $X_j, j = 1, \cdots, n$, based on i^{th} fitted time series model.

Now, we have k predicted values $\left(\widehat{X}_1^{(1)}, \cdots, \widehat{X}_n^{(1)}\right)$, ..., $\left(\widehat{X}_1^{(k)}, \cdots, \widehat{X}_n^{(k)}\right)$, for observed dataset $X_1, \cdots, X_n$. Note that $\left(\widehat{X}_1^{(1)}, \cdots, \widehat{X}_n^{(1)}\right)$, ..., $\left(\widehat{X}_1^{(k)}, \cdots, \widehat{X}_n^{(k)}\right)$, can be considered as a sample of size n from $f_1, \cdots, f_k$, respectively.

Step (3): Now, using the couples $\left(\widehat{X}_1^{(1)}, \cdots, \widehat{X}_n^{(1)}\right)$, ..., $\left(\widehat{X}_1^{(k)}, \cdots, \widehat{X}_n^{(k)}\right)$, the fuzzy clustering methods is applied to know the rates of similarities of these k time series models and also to cluster these models based on their computed similarities.

As can be seen, this technique is not too computational and time consuming and can be also applied to cluster different dependent stationary or non-stationary processes. In next section the power of the introduced technique is studied.

3. Simulation study

This section deals with studying the ability of the proposed technique in simulated datasets. The simulation study is setup as following:

Table 1 Empirical power of the introduced technique for Example 1.

H	ϕ_1			Actual Number of Clusters	n			
	First	Second	Third		50	75	100	500
0.25	0.1	0.1	0.1	1	0.9593	0.9624	0.9798	0.9957
0.25	0.1	0.1	0.5	2	0.9512	0.9617	0.9785	0.9927
0.25	0.1	0.1	0.9	2	0.9521	0.9680	0.9758	0.9954
0.25	0.1	0.5	0.1	2	0.9524	0.9660	0.9876	0.9915
0.25	0.1	0.5	0.5	2	0.9600	0.9691	0.9809	0.9949
0.25	0.1	0.5	0.9	3	0.9505	0.9602	0.9802	0.9955
0.75	0.1	0.1	0.1	1	0.9543	0.9627	0.9790	0.9978
0.75	0.1	0.1	0.5	2	0.9543	0.9671	0.9849	0.9983
0.75	0.1	0.1	0.9	2	0.9540	0.9673	0.9734	0.9969
0.75	0.1	0.5	0.1	2	0.9532	0.9670	0.9898	1.0000
0.75	0.1	0.5	0.5	2	0.9571	0.9639	0.9763	0.9950
0.75	0.1	0.5	0.9	3	0.9529	0.9623	0.9728	0.9944

Table 2 Empirical power of the introduced technique for Example 2.

H	θ_1			Actual Number of Clusters	n			
	First	Second	Third		50	75	100	500
0.25	0.1	0.1	0.1	1	0.9503	0.9632	0.9785	0.9939
0.25	0.1	0.1	0.5	2	0.9503	0.9613	0.9826	0.9966
0.25	0.1	0.1	0.9	2	0.9596	0.9670	0.9799	0.9956
0.25	0.1	0.5	0.1	2	0.9534	0.9658	0.9748	0.9913
0.25	0.1	0.5	0.5	2	0.9567	0.9685	0.9804	0.9965
0.25	0.1	0.5	0.9	3	0.9587	0.9672	0.9753	0.9948
0.75	0.1	0.1	0.1	1	0.9565	0.9621	0.9701	0.9940
0.75	0.1	0.1	0.5	2	0.9531	0.9634	0.9837	0.9922
0.75	0.1	0.1	0.9	2	0.9568	0.9616	0.9713	0.9906
0.75	0.1	0.5	0.1	2	0.9532	0.9696	0.9861	0.9925
0.75	0.1	0.5	0.5	2	0.9536	0.9664	0.9869	0.9918
0.75	0.1	0.5	0.9	3	0.9547	0.9681	0.9894	0.9994

Step 1: For each time series model, three samples of size n are separately generated.
Step 2: The proposed approach was applied to cluster the three samples.
Step 3: Step1 and 2 were repeated 10,000 times.
Step 4: The empirical power $(\widehat{\pi})$ was computed by

$$\widehat{\pi} = \frac{T}{10000},$$

such that T is equal to the number of runs that the value of the estimated number of clusters after using the proposed approach is similar to actual number of clusters.

Remark 1:. If the parameters of three populations are the same, the true value of number of clusters is equal to 1. If only the parameters of two populations are the same and the parameters of other population differs, the true value is 2, and if the parameter settings of three populations are different, the true value is 3.

Example 1:. Assume the AR(1) process,

$$X_t = \phi_1 X_{t-1} + B_H(t).$$

For the first, the second and the third models, we assumed $\phi_1 = 0.1$, $\phi_1 \in \{0.1, 0.5\}$, and $\phi_1 \in \{0.1, 0.5, 0.9\}$ respectively. Also, we consider the Hurst parameter $H \in \{0.25, 0.75\}$.

Example 2:. Consider the MA(1) model,

$$X_t = B_H(t) + \theta_1 B_H(t-1).$$

For the first, the second and the third models, we assumed $\theta_1 = 0.1$, $\theta_1 \in \{0.1, 0.5\}$, and $\theta_1 \in \{0.1, 0.5, 0.9\}$ respectively. Also, we consider the Hurst parameter $H \in \{0.25, 0.75\}$.

Example 3:. Assume the ARMA(1,1) model,

$$X_t = \phi_1 X_{t-1} + B_H(t) + \theta_1 B_H(t-1).$$

We assume $\phi_1 = 0.2$, for all models and for the first to the third models, we let $\theta_1 = 0.1$, $\theta_1 \in \{0.1, 0.5\}$, and $\theta_1 \in \{0.1, 0.5, 0.9\}$ respectively. Also, we consider the Hurst parameter $H \in \{0.25, 0.75\}$.

Example 4:. Consider the PAR(1) process

$$X_t = \phi_t X_{t-1} + B_H(t),$$

such that

Table 3 Empirical power of the introduced technique for Example 3.

H	θ_1			Actual Number of Clusters	n			
	First	Second	Third		50	75	100	500
0.25	0.1	0.1	0.1	1	0.9599	0.9656	0.9770	0.9985
0.25	0.1	0.1	0.5	2	0.9521	0.9628	0.9701	0.9985
0.25	0.1	0.1	0.9	2	0.9514	0.9656	0.9774	0.9979
0.25	0.1	0.5	0.1	2	0.9589	0.9685	0.9758	0.9940
0.25	0.1	0.5	0.5	2	0.9566	0.9606	0.9813	0.9912
0.25	0.1	0.5	0.9	3	0.9521	0.9649	0.9846	0.9906
0.75	0.1	0.1	0.1	1	0.9585	0.9636	0.9842	0.9985
0.75	0.1	0.1	0.5	2	0.9544	0.9667	0.9743	0.9935
0.75	0.1	0.1	0.9	2	0.9552	0.9600	0.9877	0.9919
0.75	0.1	0.5	0.1	2	0.9590	0.9606	0.9735	0.9951
0.75	0.1	0.5	0.5	2	0.9533	0.9622	0.9708	0.9919
0.75	0.1	0.5	0.9	3	0.9548	0.9671	0.9726	0.9973

Table 4 Empirical power of the introduced technique for Example 4.

H	ϕ_1			Actual Number of Clusters	n			
	First	Second	Third		50	75	100	500
0.25	0.1	0.1	0.1	1	0.9569	0.9657	0.9800	0.9985
0.25	0.1	0.1	0.5	2	0.9536	0.9652	0.9809	0.9903
0.25	0.1	0.1	0.9	2	0.9509	0.9603	0.9801	0.9902
0.25	0.1	0.5	0.1	2	0.9502	0.9648	0.9871	0.9938
0.25	0.1	0.5	0.5	2	0.9589	0.9645	0.9735	0.9939
0.25	0.1	0.5	0.9	3	0.9588	0.9615	0.9762	0.9987
0.75	0.1	0.1	0.1	1	0.9553	0.9649	0.9765	0.9985
0.75	0.1	0.1	0.5	2	0.9515	0.9632	0.9801	0.9950
0.75	0.1	0.1	0.9	2	0.9554	0.9679	0.9720	0.9927
0.75	0.1	0.5	0.1	2	0.9586	0.9657	0.9743	0.9952
0.75	0.1	0.5	0.5	2	0.9526	0.9669	0.9821	0.9993
0.75	0.1	0.5	0.9	3	0.9594	0.9644	0.9888	0.9940

Table 5 Empirical power of the introduced technique for Example 5.

H	θ_1			Actual Number of Clusters	n			
	First	Second	Third		50	75	100	500
0.25	0.1	0.1	0.1	1	0.9593	0.9672	0.9841	0.9938
0.25	0.1	0.1	0.5	2	0.9546	0.9640	0.9804	0.9974
0.25	0.1	0.1	0.9	2	0.9550	0.9676	0.9717	0.9916
0.25	0.1	0.5	0.1	2	0.9556	0.9642	0.9758	0.9916
0.25	0.1	0.5	0.5	2	0.9520	0.9640	0.9771	0.9933
0.25	0.1	0.5	0.9	3	0.9503	0.9600	0.9854	0.9900
0.75	0.1	0.1	0.1	1	0.9597	0.9683	0.9827	0.9969
0.75	0.1	0.1	0.5	2	0.9594	0.9620	0.9884	0.9924
0.75	0.1	0.1	0.9	2	0.9588	0.9624	0.9704	0.9977
0.75	0.1	0.5	0.1	2	0.9548	0.9608	0.9720	0.9924
0.75	0.1	0.5	0.5	2	0.9576	0.9620	0.9704	0.9961
0.75	0.1	0.5	0.9	3	0.9595	0.9668	0.9837	0.9979

$$\phi_t = 0.6 + \phi_1 \cos(2\pi t/T)$$

For the first, the second and the third models, we assumed $\phi_1 = 0.1$, $\phi_1 \in \{0.1, 0.5\}$, and $\phi_1 \in \{0.1, 0.5, 0.9\}$ respectively. Also, we consider the Hurst parameter $H \in \{0.25, 0.75\}$.

Example 5:. Consider the PMA(1) process

$$X_t = B_H(t) + \theta_t B_H(t-1),$$

such that

$$\theta_t = 1 + \theta_1 \cos(2\pi t/T)$$

For the first, the second and the third models, we assumed $\theta_1 = 0.1$, $\theta_1 \in \{0.1, 0.5\}$, and $\theta_1 \in \{0.1, 0.5, 0.9\}$ respectively. Also, we consider the Hurst parameter $H \in \{0.25, 0.75\}$.

Table 6 Empirical power of the introduced technique for Example 6.

H	θ_1			Actual Number of Clusters	n			
	First	Second	Third		50	75	100	500
0.25	0.1	0.1	0.1	1	0.9550	0.9633	0.9772	0.9966
0.25	0.1	0.1	0.5	2	0.9535	0.9670	0.9750	0.9965
0.25	0.1	0.1	0.9	2	0.9592	0.9613	0.9768	0.9960
0.25	0.1	0.5	0.1	2	0.9543	0.9672	0.9777	0.9957
0.25	0.1	0.5	0.5	2	0.9534	0.9685	0.9874	0.9988
0.25	0.1	0.5	0.9	3	0.9583	0.9670	0.9754	0.9991
0.75	0.1	0.1	0.1	1	0.9515	0.9612	0.9770	0.9928
0.75	0.1	0.1	0.5	2	0.9504	0.9682	0.9739	0.9984
0.75	0.1	0.1	0.9	2	0.9517	0.9663	0.9873	0.9997
0.75	0.1	0.5	0.1	2	0.9587	0.9685	0.9800	0.9945
0.75	0.1	0.5	0.5	2	0.9592	0.9696	0.9752	0.9970
0.75	0.1	0.5	0.9	3	0.9524	0.9693	0.9870	0.9981

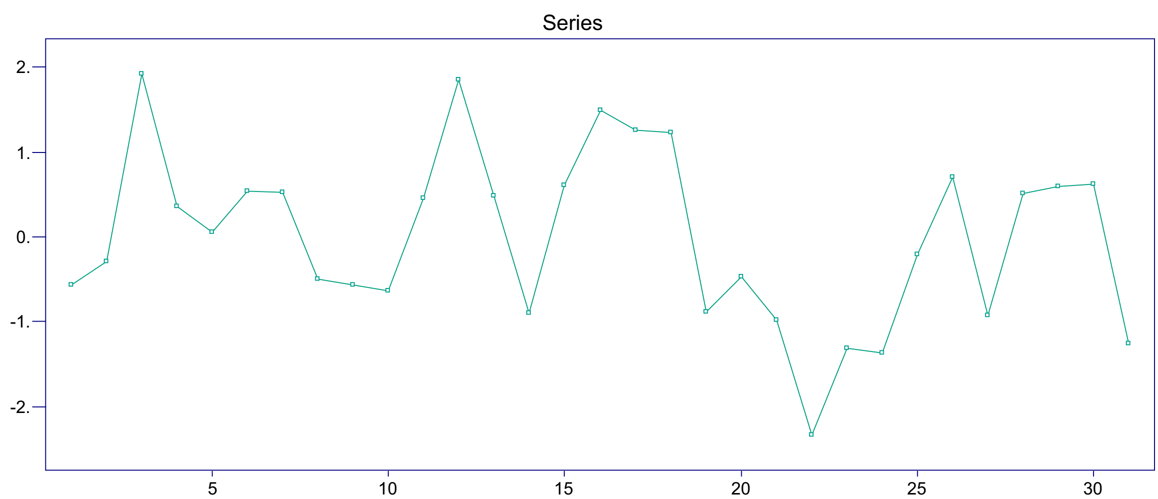

Fig. 1 Annual RDI time series of Zahedan synoptic station (1980–2010).

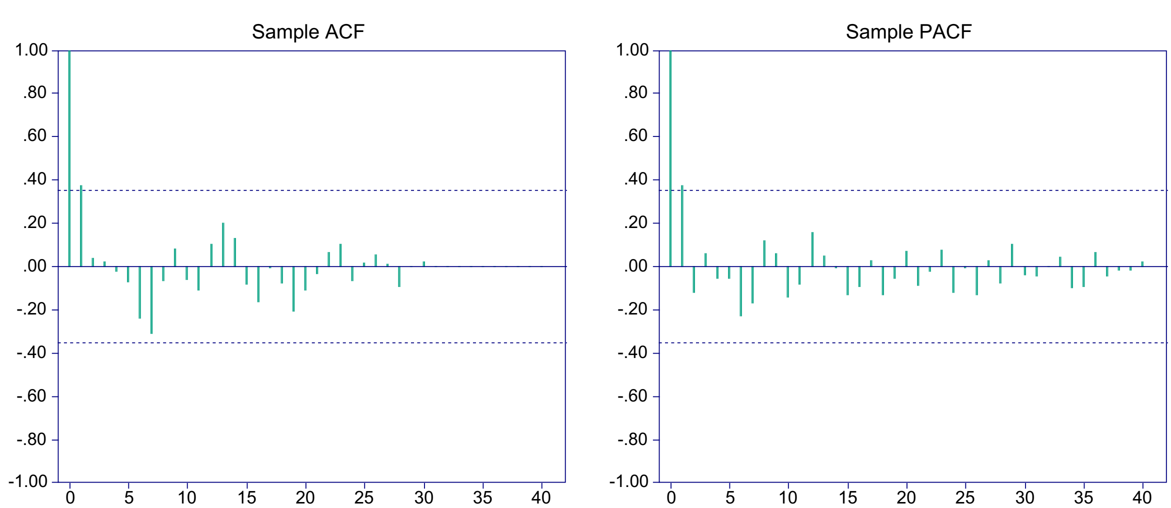

Fig. 2 Sample ACF/PACF for annual RDI time series of Zahedan synoptic station (1980–2010).

Example 6:. Consider the PARMA(1,1) process

$$X_t - \phi_t X_{t-1} = B_H(t) + \theta_t B_H(t-1),$$

such that

$$\phi_t = 0.6 + \phi_1 \cos(2\pi t/T) \text{ and } \theta_t = 1 + \theta_1 \cos(2\pi t/T).$$

We assume $\phi_1 = 0.2$, for all models and for the first to the third models, we let $\theta_1 = 0.1$, $\theta_1 \in \{0.1, 0.5\}$, and $\theta_1 \in \{0.1, 0.5, 0.9\}$ respectively. Also, we consider the Hurst parameter $H \in \{0.25, 0.75\}$.

The values of the estimated power $(\widehat{\pi})$ for Examples 1–6, are reported in Tables 1–6, respectively. As it can be seen, the values of $\widehat{\pi}$ are very close to the one, specially when the value of n grows. In other words, the introduced approach can nicely discriminate and cluster different fitted models. It can be concluded that the introduced approach has many advantages. First, the power of this method is acceptable. Second, the introduced approach is not too computational and time consuming. More ever, this technique can be applied to cluster different stationary and non-stationary time series processes.

4. Real data

Now, the behavior of the introduced procedure is studied in real world applications. The values of the annual reconnaissance drought index (RDI) from an Iranian synoptic station (Zahedan) from 1980 to 2010 have been considered. Figs. 1 and 2 indicate the dataset, sample auto-correlation (ACF) and partial auto-correlation functions (PACF) for RDI time series of Zahedan synoptic station. As can be seen in ACF/PACF plot, AR(1), MA(1) and ARMA(1,1) models can be candidate models to fit on this dataset.

Table 7 shows the results for the fitted models. As this table indicates, based on AICC and variances, the MA(1) model is better than other models.

We are interested in clustering these three models. To do this aim, the proposed technique is applied. By using Kaiser Index (the number of eigen-values of correlation matrix that are more than 1), the number of clusters was considered to be 2. Table 8 and Figs. 3 and 4 provide the results of the fuzzy clustering technique in. As it can be seen, these three models can be divided in two distinct clusters; cluster 1: MA(1), and cluster 2: AR(1) and ARMA(1,1). In other words, MA(1) is best model to describe the annual RDI time series of Zahedan synoptic station (1980–2010). Also, there is no significant difference between AR(1) and ARMA(1,1) models.

Table 7 Best models for annual RDI time series of Zahedan synoptic station (1980–2010).

Model	Model's Formula	AICC
AR(1)	$X_t = 0.3869X_{t-1} + B_H(t)$	87.5374
MA(1)	$X_t = B_H(t) + 0.5033B_H(t-1)$	86.7242
ARMA(1,1)	$X_t = 0.6538X_{t-1} + B_H(t) - 0.2635B_H(t-1)$	91.4619

Table 8 The probabilities of membership in two clusters using fuzzy clustering.

Model	Cluster 1	Cluster 2
AR(1)	0.38232564	0.617674356
MA(1)	0.99402695	0.005973045
ARMA(1,1)	0.02219271	0.977807293

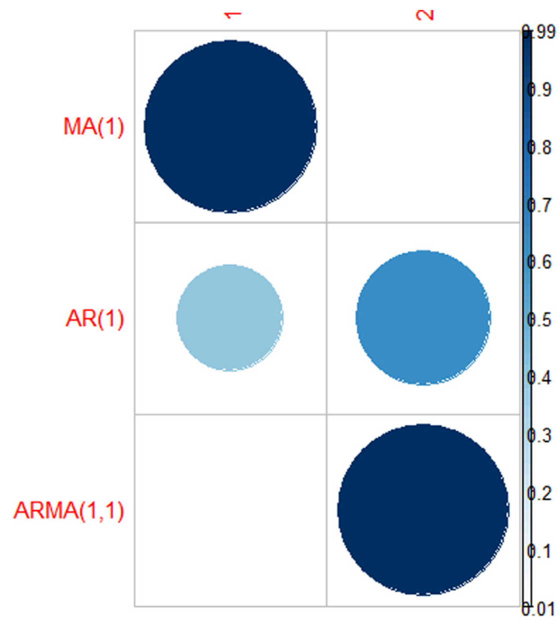

Fig. 3 Fuzzy clustering method to classify the time series models fitted on the RDI.

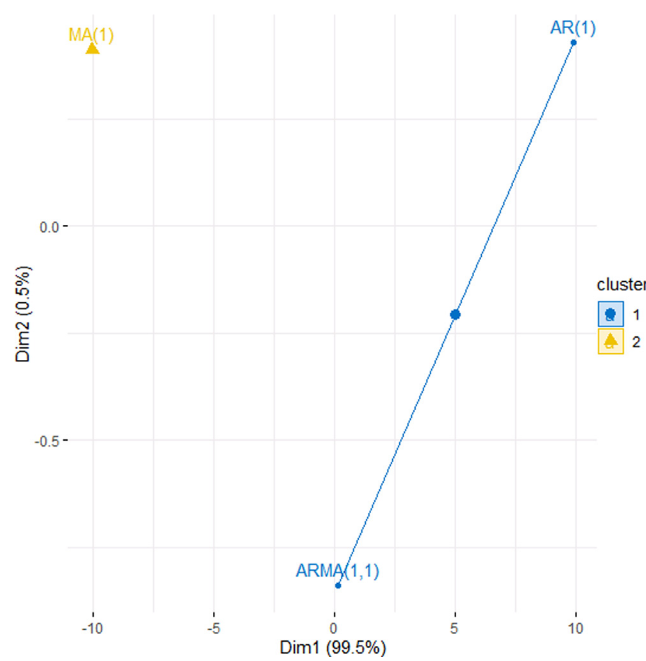

Fig. 4 Fuzzy clustering plot to classify the time series models fitted on the RDI.

5. Conclusion

Classifying and clustering several time series models fitted on a dataset is an important problem in data analysis. The clustering of several dependent time series fitted on a specific dataset (specially, non-stationary processes or time series with fractional Brownian motion noises) was lacked. In this research, we tried to solve this important problem. At first, all of time series models were fitted on dataset. Then, for observed dataset, the predicted values of all models were estimated based on fitted models formulas. Finally, the fuzzy clustering technique was applied to cluster the models based on the similarities between predicted values. The ability of the proposed technique was studied using simulation. The results indicated that for small sample size ($n = 50$), the power of the proposed method to classify different models was more than 95%. The power was increased to more than 99% as the sample size grew to ($n = 50$). Consequently the introduced approach could nicely discriminate and cluster different fitted models. The introduced approach had many advantages. The power of this method was acceptable. Moreover, the introduced approach was not too computational and time consuming. Furthermore, this technique could be applied to cluster different dependent stationary and non-stationary time series processes. The applicability of the introduced procedure in real world applications was also investigated by studying RDI values from an Iranian synoptic station. Three candidate models AR(1), MA(1) and ARMA(1,1) have been fitted to dataset. Then the introduced approach was used two cluster the fitted models. Based on the results, these models were classified into 2 clusters; cluster 1: MA(1), and cluster 2: AR(1), and ARMA(1,1).

CRediT authorship contribution statement

Mohammad Reza Mahmoudi: Data curation, Validation. **Dumitru Baleanu:** Conceptualization, Methodology, Software, Supervision. **Sultan Noman Qasem:** Validation, Visualization. **Amirhosein Mosavi:** Validation, Visualization. **S Band:** Validation, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Further reading

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