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Investigation on Ginzburg-Landau equation via a tested approach to benchmark stochastic Davis-Skodje system

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Abstract We propose new numerical methods with adding a modified ordinary differential equation solver to the Milstein methods for solution of stiff stochastic systems. We study a general form of stochastic differential equations so that the Ginzburg-Landau equation and the Davis-Skodje model can be considered as special states of them. The efficiency of the method is experimented, in terms of the convergence rate and accuracy of approximate solution, employing some numerical examples, including stochastic Ginzburg-Landau equation and a paradigm of chemical reaction systems.

1. Introduction

One of the interesting topics in mathematical sciences is modeling the behavior of natural phenomena. These types of models usually include the integral equations, fractional differential equations [1–11], delay differential equations [12,13], partial differential equations [14–24] and random differential equations [25–29] or a combination of them. In particular, stochastic systems are very useful tools for modeling in various fields such as physics, chemistry, biology, mathematical finance and other sciences [30–38]. Due to the unavailability of the closed form solution for stochastic differential equations (SDEs), the approximate methods have been developed, for example see [30,38–42]. Modeling of a wide range of physical systems is performed using the Ginzburg–Landau equations [30,43–45]. The real and complex Ginzburg–Landau equations were first derived in the studies of long-wave amplitude phenomena in binary mixtures and reaction–diffusion systems, respectively. Then they became instances of the most comprehensive equations of modern physics. The (original or modified versions) of Ginzburg–Landau equations are powerful tools in

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studies of superconductivity and they are able to characterize abundant diversity of phenomena from nonlinear waves, superfluidity, pattern formation, liquid crystals and super symmetric conformal field theories.

More recently, some authors attempt to explore the different ordinary differential equation (ODE) solvers for combination with classic stochastic numerical methods as Milstein methods, for solution of stiff SDEs [46,47]. The present paper focuses on the improving the split-step forward methods for solutions of stiff SDEs of Itô type. These methods are based on the exponential Milstein scheme.

Davis and Skodje introduced a model includes of a two-dimensional system to compare various reduction methods [48,49]. The Davis-Skodje mechanism is a measurement criterion for model reduction methods because of the adjustable time scale separation. In Section 3 we apply our schemes to benchmark stochastic Davis-Skodje system and present numerical experiments. Also, we examine the strong convergence rate of the methods via the study of Ginzburg–Landau equations.

2. Method expression

We consider Itô SDEs [30,33,38,40,50]

$$\mathrm{d}X(t) = f(X(t))\mathrm{d}t + \sum_{j=1}^{m} g_j(X(t))\mathrm{d}W^j(t), \quad X(t_0) = X_0 \in \mathbb{R}^d, \quad t \in [t_0, T]. \tag{1}$$

Where $W^j(t)$ denotes the Wiener process, whose increment $\Delta W^j(t) = W^j(t+\Delta t) - W^j(t), j = 1, \ldots, m$ is a Gaussian random variable $\mathcal{N}(0, \Delta t)$, deterministic term $f : \mathbb{R}^d \to \mathbb{R}^d$ and $g = (g_1, \ldots, g_m) : \mathbb{R}^d \to \mathbb{R}^{d \times m}$ are the drift coefficient and diffusion matrix, respectively, and satisfy in the Lipschitz and linear growth conditions.

For solving (1), the following general split-step approaches has been considered [41,50],

$$\begin{cases} \bar{U}_k &= U_k + h\mathrm{Y}\left(\bar{U}_k, U_k\right) \\ U_{k+1} &= \bar{U}_k + \sum_{j=1}^{m} g_j\left(\bar{U}_k\right)\Delta W_k^j, \end{cases}$$

here $\mathrm{Y}\left(\bar{U}_k, U_k\right)$ is increment function of appropriate ODE solver. To provide our methods, we have been inspired by modified ODE solver as following form

$$\begin{aligned} \mathrm{Y}\left(\bar{U}_k, U_k\right) &= f(U_k) + \tfrac{h}{2} J_f(U_k) f(U_k) + \mathcal{O}(h^2) \\ &= f(U_k)\frac{\exp\left(hJ_f(U_k)\right) - 1}{hJ_f(U_k)} + \mathcal{O}(h^2), \end{aligned} \tag{2}$$

where J_f denotes a Jacobian Matrix. Now, we establish the following explicit schemes based on the Milstein approach, the first drifting split-step exponential modified Milstein (DRSSEMM) method

$$\begin{cases} \bar{U}_k &= U_k + hf(U_k)\frac{\exp\left(hJ_f(U_k)\right)-1}{hJ_f(U_k)} \\ U_{k+1} &= \bar{U}_k + \sum_{j=1}^{m} g_j(\bar{U}_k)\Delta W_k^j + \tfrac{1}{2}\sum_{j=1}^{m} L^1 g_j(\bar{U}_k)\left(\left(\Delta W_k^j\right)^2 - h\right), \end{cases} \tag{3}$$

and the second diffused split-step exponential modified Milstein (DISSEMM) method

$$\begin{cases} \bar{U}_k &= U_k + \sum_{j=1}^{m} g_j(U_k)\Delta W_k^j + \tfrac{1}{2}\sum_{j=1}^{m} L^1 g_j(U_k)\left(\left(\Delta W_k^j\right)^2 - h\right) \\ U_{k+1} &= \bar{U}_k + hf(\bar{U}_k)\frac{\exp\left(hJ_f(U_k)\right)-1}{hJ_f(U_k)}, \end{cases} \tag{4}$$

here U_k is the approximation to $X(t_k)$ for $t_k = kh$, $k = 0, 1, \ldots, N, N = 1, 2, \ldots$, h is defined as $h = t_k - t_{k-1}$, and each $\Delta W_k = W_{t_k} - W_{t_{k-1}}$ is an independent $\mathcal{N}(0, h)$.

3. Numerical experiments

We use error criteria

$$\varepsilon_{\mathrm{MA}} = \frac{1}{N}\sum_{i=1}^{N}\left|U_N^{(i)} - X_{t_N}^{(i)}\right|, \tag{5}$$

$$\varepsilon_{\mathrm{MS}} = \frac{1}{N}\sum_{i=1}^{N}\left(\left(U_N^{(i)} - X_{t_N}^{(i)}\right)^2\right)^{\frac{1}{2}}, \tag{6}$$

to compare the accuracy of numerical schemes. Where $U_N^{(i)}$ and $X_{t_N}^{(i)}$ denote as the numerical solutions and the exact solution at step point t_i in ith simulation of all N simulations, respectively.

Example 3.1. Consider the scalar stochastic Ginzburg–Landau equation in the Itô form [30]

$$\begin{aligned} \mathrm{d}X(t) = &\left(\left(\eta + \frac{1}{2}\sigma^2\right)X(t) - \lambda X^3(t)\right)\mathrm{d}t \\ &+ \sigma X(t)\mathrm{d}W(t), \quad X(0) \\ &= X_0 \in (0, \infty), \end{aligned} \tag{7}$$

where $\eta \geqslant 0, \sigma, \lambda > 0$. This equation is from the theory of superconductivity and derived by Ginzburg and Landau [44] to describe a phase transition in deterministic sense. Fig. 1 shows a log–log plot of the means of absolute errors, $\varepsilon_{\mathrm{MA}}$ (5), based on the 2000 sample paths with step sizes $2^{-13}, 2^{-12}, 2^{-11}$, 2^{-10} and 2^{-9} for DRSSEMM (3) and DISSEMM (4) methods at the terminal time $T = Nh = 1$ with

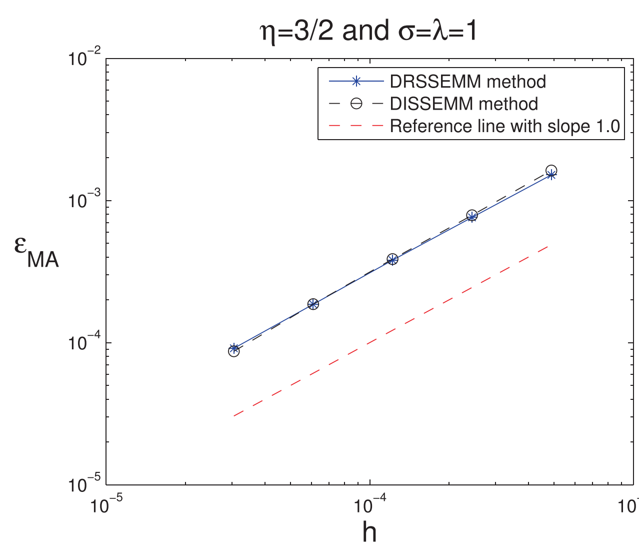

Fig. 1 The strong convergence rates of DRSSEMM and DISSEMM methods for nonlinear test Eq. (7) with $\eta = \frac{3}{2}$ and $\sigma = \lambda = 1$.

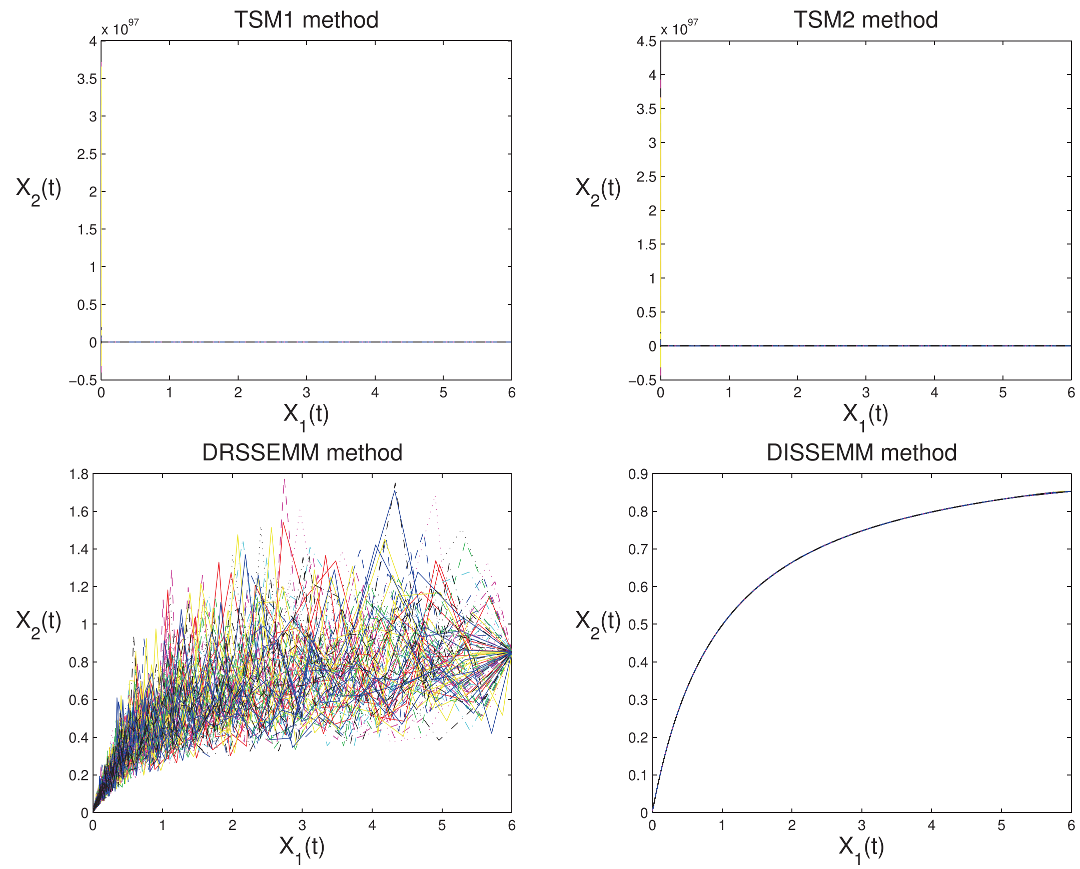

Fig. 3 Numerical simulations of nonlinear system (9) for Parameter I.

$\eta = \frac{3}{2}, \sigma = \lambda = 1$ and $X(0) = 2$. The simulations with step size 2^{-16} regarded as the exact solutions. A reference line of slope 1.0 is added as a dashed line type. The results are consistent with the strong errors close to order 1.0.

Example 3.2. Consider the following nonlinear SDE [30],

$$\begin{aligned} \mathrm{d}X(t) = &-\left(\alpha + \beta^2 X(t)\right)\left(1 - X^2(t)\right)\mathrm{d}t \\ &+ \beta\left(1 - X^2(t)\right)\mathrm{d}W(t), \quad X_0 \\ &= \frac{1}{2}, \quad t \in [0, 1]. \end{aligned} \tag{8}$$

The exact solution is given by

$$X(t) = \frac{(1 + X_0)\exp(-2\alpha t + 2\beta W(t)) + X_0 - 1}{(1 + X_0)\exp(-2\alpha t + 2\beta W(t)) - X_0 + 1}.$$

In Fig. 2 we display mean-square error, $\varepsilon_{\mathrm{MS}}$ (6), to against h for DRSSEMM (3), DISSEMM (4), TSM1 [42] and TSM2 [42] methods for ten different step sizes $h = 2^{-j}, j = 1, 2, \ldots, 10$ with fix parameters $\alpha = 3.0$, $\beta = 0.1$ and $\alpha = -5.0$, $\beta = 0.1$. The results of Fig. 2 specify the smaller mean-square errors of our suggested schemes than TSM1 and TSM2 methods to solve Eq. (8).

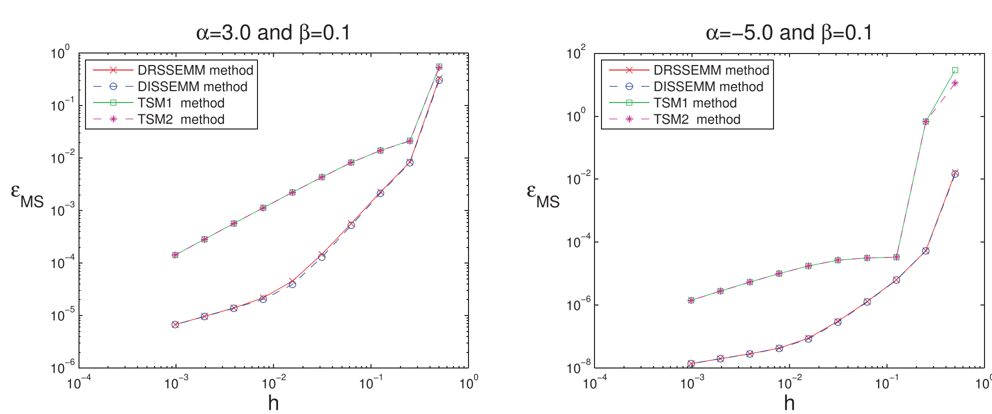

Fig. 2 Values of $\varepsilon_{\mathrm{MS}}$ (6), to against h for DRSSEMM (3), DISSEMM (4), TSM1 [42] and TSM2 [42] methods for nonlinear SDE (8).

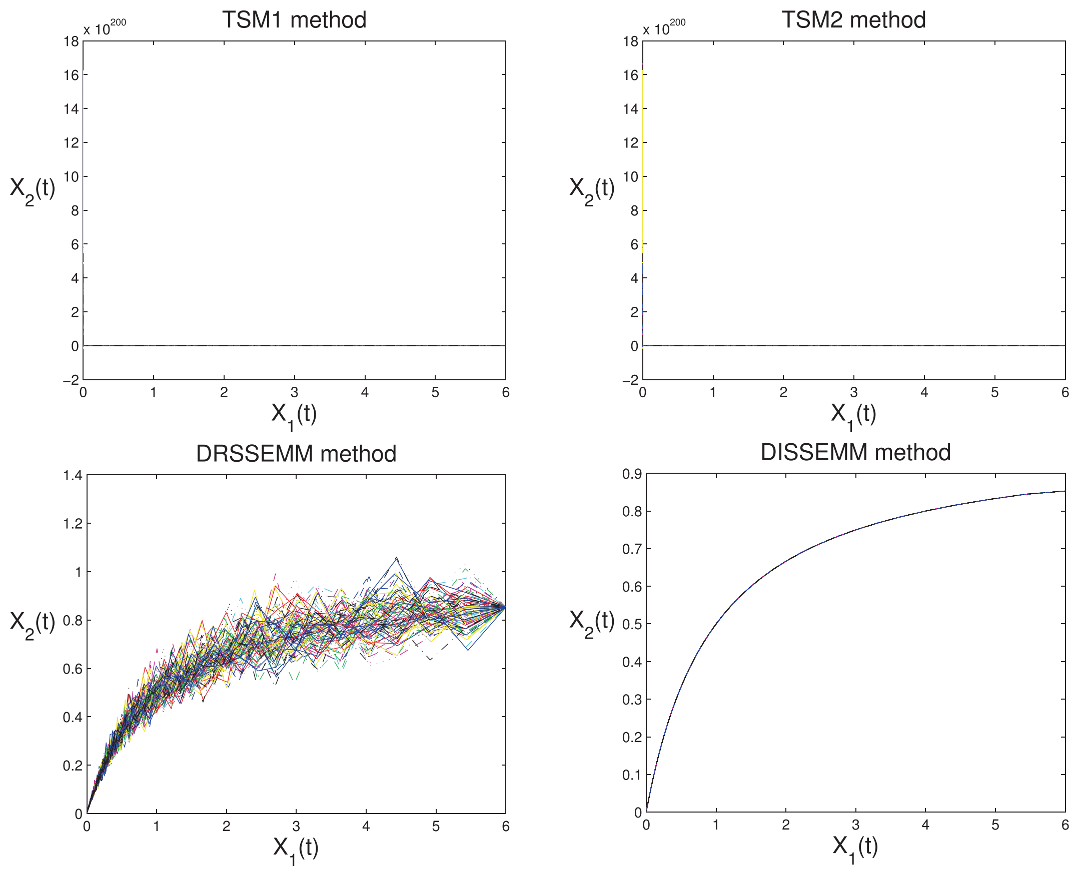

Fig. 4 Numerical simulations of nonlinear system (9) for Parameter II.

Example 3.3. The well-known Davis-Skodje mechanism is our third test case

$$\begin{aligned} dX_1(t) &= -X_1(t)dt + \mu X_1(t)dW_1(t) \\ dX_2(t) &= \left(-\gamma X_2(t) + \gamma \frac{X_1(t)}{1+X_1(t)} - \frac{X_1(t)}{(1+X_1(t))^2}\right)dt + \sigma\sqrt{\gamma}X_2(t)dW_2(t). \end{aligned} \tag{9}$$

This stiff chemical reaction system introduced in [48] and proved that stable equilibrium is point $(0,0)$. Also, $\gamma > 1$ gives a measure for the spectral gap or stiffness of the system. We simulated this system for three groups of parameters

- Case I: $\mu = \sigma = 0.1$ and $\gamma = 100$,
- Case II: $\mu = \sigma = 0.01$ and $\gamma = 1000$,
- Case III: $\mu = \sigma = 0.001$ and $\gamma = 10000$,

on the interval $[0, 10]$ with initial position at $(X_1(0), X_2(0)) = (6, 0.85)$. In Figs. 3–5, we simulated 100 sample paths of nonlinear system (9), using DRSSEMM (3), DISSEMM (4), TSM1 [42] and TSM2 [42] methods, with step size $h = 0.1$. We can see, our methods converge quickly towards the asymptotic solution $(0,0)$.

4. Conclusions and future research

In this work, we establish new stochastic methods to the strong approximation of stochastic differential equations in Itô sense. Two fully explicit methods, DRSSEMM (3), DISSEMM (4) methods are constructed based on modified ODE solver and Milstein methods. The ability of the proposed schemes have illustrated by several nonlinear stochastic differential equations. In particular, Fig. 1 shows that the order convergence rate of our methods equal to 1.0, for the stochastic Ginzburg–Landau Eq. (7). Moreover, the accuracy of our numerical schemes compared to the TSM1 and TSM2 methods is shown in Fig. 2. According to Fig. 2, the mean-square error (ε_{MS}), obtained by the proposed methods is even smaller than the obtained results by the TSM1 and TSM2 methods. Finally, in Figs. 3–5 we examine the behavior of the stochastic Davis-Skodje systems (9) by using the TSM1, TSM2, DRSSEMM and DISSEMM approaches. It can be seen that the our methods solutions are asymptotic stable and tend to the asymptotic solution $(0,0)$. The results reveal that the proposed numerical methods are very efficient, reliable and can be applied to stiff stochastic problems in applied sciences. In the future, we will construct new classes of stochastic methods based on the ODE solver (2) with higher convergence orders.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

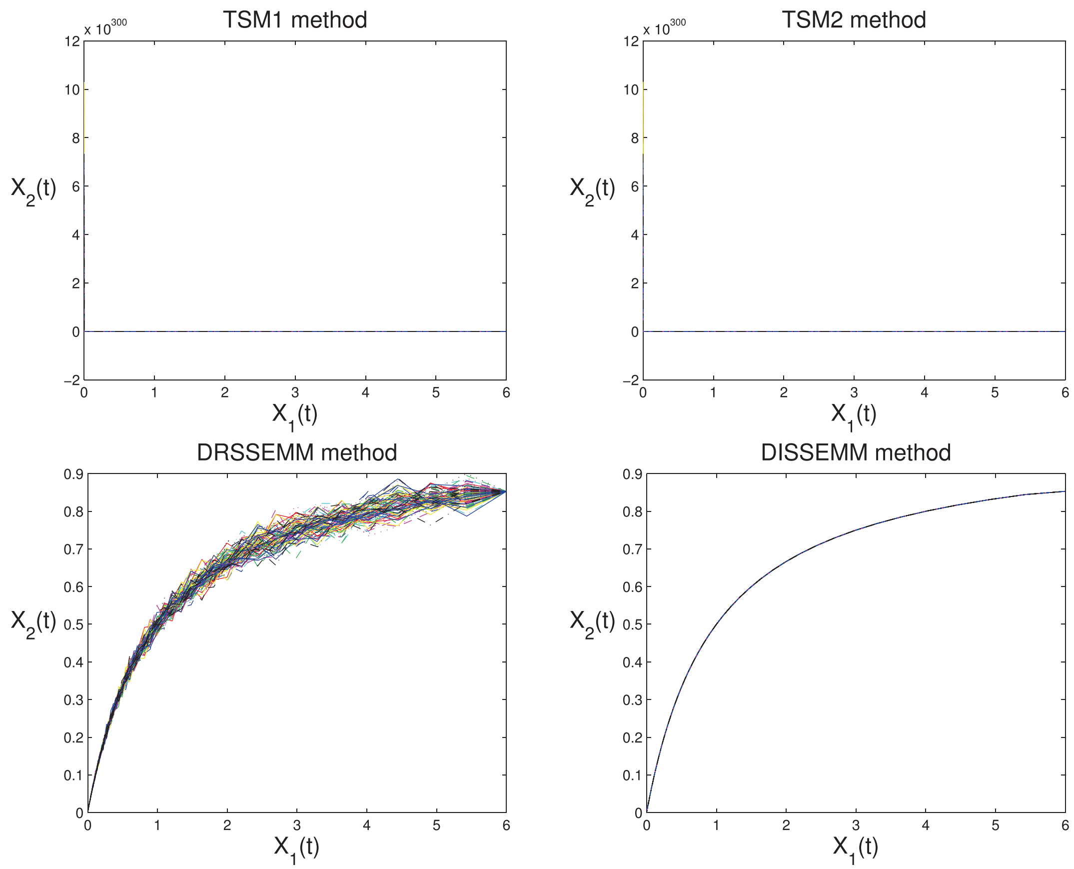

Fig. 5 Numerical simulations of nonlinear system (9) for Parameter III.

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