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Mathematical modeling and analysis of the novel Coronavirus using Atangana–Baleanu derivative

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ABSTRACT

The novel Coronavirus infection disease is becoming more complex for the humans society by giving death and infected cases throughout the world. Due to this infection, many countries of the world suffers from great economic loss. The researchers around the world are very active to make a plan and policy for its early eradication. The government officials have taken full action for the eradication of this virus using different possible control strategies. It is the first priority of the researchers to develop safe vaccine against this deadly disease to minimize the infection. Different approaches have been made in this regards for its elimination. In this study, we formulate a mathematical epidemic model to analyze the dynamical behavior and transmission patterns of this new pandemic. We consider the environmental viral concentration in the model to better study the disease incidence in a community. Initially, the model is constructed with the derivative of integer-order. The classical epidemic model is then reconstructed with the fractional order operator in the form of Atangana–Baleanu derivative with the nonsingular and nonlocal kernel in order to analyze the dynamics of Coronavirus infection in a better way. A well-known estimation approach is used to estimate model parameters from the COVID-19 cases reported in Saudi Arabia from March 1 till August 20, 2020. After the procedure of parameters estimation, we explore some basic mathematical analysis of the fractional model. The stability results are provided for the disease free case using fractional stability concepts. Further, the uniqueness and existence results will be shown using the Picard–Lendelof approach. Moreover, an efficient numerical scheme has been proposed to obtain the solution of the model numerically. Finally, using the real fitted parameters, we depict many simulation results in order to demonstrate the importance of various model parameters and the memory index on disease dynamics and possible eradication.

Introduction

Infectious diseases are disorders caused by different organisms such as viruses, bacteria, fungi or parasites. A number of outbreaks of different infectious diseases have been reported across the globe taking the lives of millions of humans each year. These outbreaks are not only a serious issue for public health, but also having a significant impact on the whole societies, economic interest and political systems and is felt in both developing and developed countries across the world. The world is facing different types of deadly infectious diseases. Although some of these infections have been eradicated from the world or almost wiped out still many diseases such as HIV/AIDS and TB persist with little or even no hope of getting them under control. In addition, the emergence of new infectious diseases is another serious threat to the world. The novel Coronavirus infectious disease or COVID-19 is one of the newly emerged deadly infectious disease. It was initiated in the mainland China during the end of 2019 and spread to the rest of the word within no time. Many facts are still to explore about the novel COVID-19 pandemic. Initially, the transmission of the COVID-19 infection was observed from animals to humans. The first virus transmission among humans was observed in Guangdong a province of China on January 20 [1,2]. New research confirmed that this virus can also be transferred to humans through environment. As on 28 August 2020, the Who reports show the cumulative cases confirmed with this infection are higher than 24,021,218 with the total death cases were 821,462 and the recovered cases are more than 15,907,858 [3]. The Kingdom of Saudi Arabia (KSA) is among those Arabian countries seriously affected from the COVID-19 pandemic and has the highest graph of the infected cases. The first case confirmed with COVID-19 is reported in Saudi Arabia on 2nd of March 2020. Later on after few days on 14th of March, the second case was detected when a companion came from

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Iran. Currently, Saudi Arabia is placed at fourteen position in the list of high confirmed infected cases. As of 28 August, the Kingdom has reported 311,855 confirmed COVID-19 cases, with 286,255 recoveries and 3785 deaths.

Recently, many approaches are used for the analysis and investigations of the Coronavirus infection and its early eliminations. In this regards, the modeling tools are gaining much attention from researchers point of view. The mathematical modeling is effectively and confidently being used to provide to help the public health decision-making in order to set effective control measure for diseases. These models are helpful to study the dynamical behavior and spatiotemporal patterns of infectious disease and consequently, can be used to set effective control measures to eradicate the disease. Mathematical models are mostly based on differential and integral operators with integer or non-integer orders. A rich literature regarding deterministic compartmental epidemic models have been formulated on the dynamics of effective controlling strategies of the ongoing novel COVID-19 pandemic in different countries and territories in the world. The importance of some control measures on the dynamics and future prediction of COVID-19 outbreak in Pakistan has been presented in [4]. The authors in [4] also have developed the optimal control problem using optimal control concepts. A deterministic mathematical model incorporating the incidence of infection through environmental route in Ghana is suggested in [5]. The authors carried out the cost effective analysis and used the confirmed COVID-19 cases in Ghana to estimate the model parameters. A new transmission model on COVID-19 describing the dynamical behavior of COVID-19 has been studied in [6]. Additionally, the authors [6] simulated the proposed model for real parameters estimated from infected cases in Nigeria. The effective mitigating measure for the COVID-19 in the population of Canada have been presented in [7].

Mathematical modeling of real-world physical problems with fractional derivatives gains much attention from the researchers and scientists in recent days. It is due to some serious limitations of classical integer order derivative, which can be overcome through fractional order operators. The crossover behavior and fading memory effects found in many physical and biological processes can be captured only using fractional order derivatives. The model with fractional operators provides a better understanding of disease dynamics. In the recent era, a class of fractional operators based on either singular or nonsingular kernels were studied in literature. But, among these, the operators which gain more interest are known as Caputo derivative developed in [8], the Caputo–Fabrizio (CF) recently described in [9] and the Atangana–Baleanu (AB) operator developed in the sense of Caputo shown in [10]. These operators were effectively utilized to express many problems in science and other field as well [11–13]. The Atangana–Baleanu derivative has a nonsingular as well as nonlocal kernel and is the most recent and generalized fractional operator. The application of ABC derivative can be found in almost every field of science such as [14–17] and references therein. Recently, a number of the epidemic transmission models describing the COVID-19 dynamics with fractional order have been developed and studied in the literature. The transmission of COVID-19 pandemic using a mathematical model in ABC sense are explored in [18]. Moreover, in the study [18], the authors have taken the Wuhan infected cases for parameter estimation in order to make a realistic study. Baleanu et al. [19] analyzed transmission procedures of COVID-19 with the help of a compartmental epidemic model in CF sense. The application of the Caputo operator to describe the dynamical behavior of COVID-19 through a new model is recently studied in [20]. A realistic and novel approach based on fractional–fractal operator is used to construct a new compartmental model based on the dynamics of the current COVID-19 pandemic in [21,22]. The application of well-known classical Caputo-type operator coupled with real data to studying the transmission dynamics of novel COVID-19 can be found in [23,24].

In the current study, a fractional epidemic model with environmental transmission of the virus that contributes the further spread of infection among humans population is considered. The model in integer case was studied recently in [25]. The model is developed by taking into consideration the environmental contributions of the latent, infected and asymptomatic infected population. The model under consideration is taken in the from of fractional order differential operators instead of integer order. The ABC derivative is used to derive the proposed epidemic compartmental model and obtain their results. In the first stage, the model construction is briefly described using the integer order differential equations. The classical COVID-19 model is then simulated to provide a better fit to COVID-19 cases in KSA and ultimately to estimate the updated parameters. The present paper has been arranged in various sections as follows: The related concepts of fractional operator and its integral has been suggested in Section "Some basics of fractional calculus". The novel COVID-19 model for the real data analysis of Saudi Arabia in integer case is presented in Section "Integer order model". The model in fractional case is developed in "The COVID-19 model in fractional case" Section, while its necessary mathematical aspects are studied in Section "Theoretical investigations of the fractional model". The Section "Numerical solution of the ABC model" presents an iterative scheme of the considered model and the simulation results along with discussion, which are performed in Section "Simulations and discussion". Finally, the conclusion is given in Section "Conclusion".

Some basics of fractional calculus

The related concepts regarding the fractional operators used in the given paper are shown in the following:

Definition 1. Let $y(t) \in C^n$, then the fractional derivative of Caputo type with order ρ in $(n-1, n]$ such that $n \in \mathbb{N}$ defined in [8] is given by:

$$ {}^{C}D_t^{\rho}(y(t)) = \frac{1}{\Gamma(n-\rho)} \int_0^t \frac{y^n(\zeta)}{(t-\zeta)^{\rho+1-n}} d\zeta. \tag{1} $$

Clearly, ${}^{C}D_t^{\rho}(y(t))$ tends to $y'(t)$ when $\rho \to 1$.

Definition 2. We define the Atangana–Baleanu derivative defined in [10] and shown by the following:

$$ {}_{a}^{\mathrm{ABC}}D_t^{\rho} y(t) = \frac{AB(\rho)}{1-\rho} \int_a^t y'(x) E_{\rho}\left[\frac{-\rho(t-x)^{\rho}}{1-\rho}\right] dx, \tag{2} $$

where, $0 \le \rho \le 1$ and $y(t) \in C[a, b]$. The expression $AB(\rho)$ in the above integral equation defines the normalized function.

Definition 3. The Atangana–Baleanu derivative defined above has the following integral [10]:

$$ {}_{0}^{\mathrm{ABC}}I_t^{\rho} y(t) = \frac{1-\rho}{AB(\rho)} y(t) + \left[\frac{\rho}{AB(\rho)\Gamma(\rho)} \int_0^t y(x)(t-x)^{\rho-1}\right] dx. \tag{3} $$

Let us express the model proposed in the fractional derivative of ABC case by the following way [16]:

$$ ({}_{t_0}^{\mathrm{ABC}}D_t^{\rho} x)(t) = Y(t, x), \tag{4} $$

where $Y : \mathbb{D} \subset \mathbb{R}^n \times \mathbb{R}_+ \longrightarrow \mathbb{R}^n$ and $\rho \in (0, 1]$. So, to obtain the global asymptotical stability of the model in the Atangana–Baleanu sense, the results for (4) can be used.

Theorem 2.1 (*[16]*). *The model (4) in Atangana–Baleanu derivative must possess: (1): Let $Y(t, x(t))$ describes a positive definite function, (2): $Y(t) = Y(t, x(t))$ must be continuously differentiable, (3): Further, ${}_{t_0}^{\mathrm{ABC}}D_t^{\rho} Y(t, x(t))$ must be negative definite for every $\rho \in (0, 1]$. Then, under the condition that a function shown by G having class K exists with $Y(x) = G(x)$ and further, if Y increases then G increases, so, then the equilibrium point shown by $x = 0$ is asymptotically stable at the initial time $t = 0$.*

Integer order model

The present section explores the modeling of the novel Coronavirus. We begin the modeling process be denoting the host population by $N(t)$ dividing into five mutually exclusive epidemiological classes based on dynamics of COVID-19 infection. These classes consist of susceptible S, exposed E, infectious showing symptoms of disease I, infected with no disease symptoms I_a and the individuals recovered are denoted R respectively. So, that

$$N = S + E + I + I_a + R.$$

The symptomatically-infected individuals are infectious and have clinical symptoms of COVID-19 whereas the asymptomatically-infected people are capable to transmit the infection and show mild or even not yet shown any disease symptoms. We define the class $B(t)$ that defines concentration of the virus present in the contaminated surfaces in environment. The assumption taken in consideration into the model formulations are:

(i) The individuals in the symptomatically-infected class have the ability to infect other health individuals fully and spread the disease further.
(ii) The individuals in the asymptomatically-infected class that do not show the symptoms but may transmit the infections and infect other individuals in the population.
(iii) Individuals in the exposed class within the incubations which may or may not yet show any disease signs are capable to transmit the disease further.
(iv) The COVID-19 related death is only considered in symptomatically-infected class.
(v) The exposed, symptomatic, asymptomatic COVID-19 population has contributed the virus to the environment and is one of the transmission routes of the infection.

Taking the above discussion and assumptions in account, we formulate the following dynamical model for the COVID-19:

$$\begin{cases} \frac{d}{dt}S &= \Pi - (\eta_1 E + \eta_2 I + \eta_3 I_a + \eta_4 B)\frac{S}{N} - \mu S, \\ \frac{d}{dt}E &= (\eta_1 E + \eta_2 I + \eta_3 I_a + \eta_4 B)\frac{S}{N} - (\kappa + \mu)E, \\ \frac{d}{dt}I &= \kappa(1-\tau)E - (\mu + \zeta + \delta_1)I, \\ \frac{d}{dt}I_a &= \tau\kappa E - (\mu + \delta_2)I_a, \\ \frac{d}{dt}R &= \delta_1 I + \delta_2 I_a - \mu R, \\ \frac{d}{dt}B &= \varpi_1 E + \varpi_3 I_a + \varpi_2 I - \phi B, \end{cases} \tag{5}$$

where the additional non-negative initial conditions

$$S(0) = S_0 \geq 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, I_a(0) = I_{a0} \geq 0, \\ R(0) = R_0 \geq 0, B(0) = B_0 \geq 0. \tag{6}$$

In the model described in (5), the individuals in the susceptible class is generated by the birth rate Π, while the death naturally in each class can be shown by μ. The death rate due to COVID-19 infection is denoted by ζ, which only appears in symptomatically-infected class. The parameters η_1, η_2 and η_3 are the effective transmission rates of infection due to exposed, symptomatically-infected and asymptomatically-infected individuals, respectively. The parameter η_4 denotes the generation of infection due to environment. We denote the incubation period of the individuals by κ. The exposed individuals at the end of incubation period, a proportion τ, where $(0 < \tau < 1)$, remain in the I which develops symptoms compartment while the rest join I_a class which have no or mild disease symptoms. The parameters δ_1 and δ_2 account for the recovery rates of symptomatically and asymptomatically infected people. The contribution of the virus to the environment due to exposed, and the population of both infected comparers (i.e. I and I_a) are shown respectively by ϖ_1, ϖ_2 and ϖ_3. The virus can be removed from the environment using the parameter ϕ. We define the following expressions to make the model easier for onward analysis:

$$\lambda(t) = \frac{(\eta_1 E + \eta_2 I + \eta_3 I_a + \eta_4 B)}{N}, \ \mathrm{k}_1 = (\kappa + \mu), \\ \mathrm{k}_2 = (\mu + \zeta + \delta_1), \text{ and } \mathrm{k}_3 = (\mu + \delta_2).$$

Thus, the model (5) can be reformulated as follows:

$$\begin{cases} \frac{d}{dt}S &= \Pi - \lambda(t)S - \mu S, \\ \frac{d}{dt}E &= \lambda(t)S - \mathrm{k}_1 E, \\ \frac{d}{dt}I &= \kappa(1-\tau)E - \mathrm{k}_2 I, \\ \frac{d}{dt}I_a &= \tau\kappa E - \mathrm{k}_3 I_a, \\ \frac{d}{dt}R &= \delta_1 I + \delta_2 I_a - \mu R, \\ \frac{d}{dt}B &= \varpi_1 E + \varpi_2 I + \varpi_3 I_a - \phi B. \end{cases} \tag{7}$$

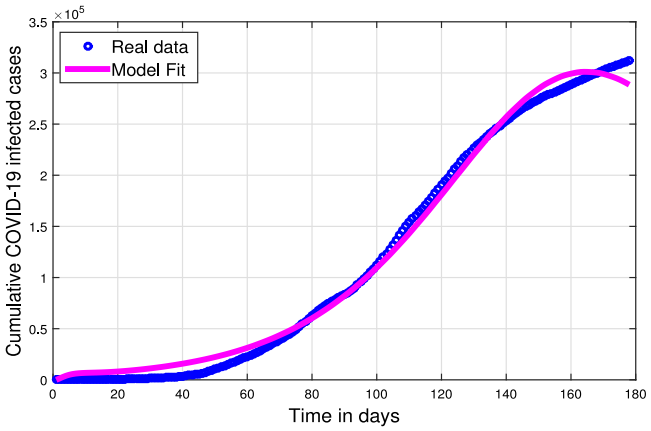

Fig. 1. Data since 1 March to 20 August 2020 versus model fitting.

Estimation of parameters

The estimation of parameters is one of the essential parts for the mathematical models in epidemiology. Various approaches are used for the estimation of parameters of a biological model having real data. Among these techniques, the nonlinear least square curve fitting is one of them. We used the nonlinear least square curve fitting method in order to determine the best values of the parameters involved in the model under consideration. We use the method of nonlinear least square curve fitting method by considering the infected cases of KSA for a selected period of time as shown in Fig. 1. In parameters estimation, we get the value of the parameter Π and μ using the literature. The remaining parameters have been gotten through the fitting. The procedure used in this paper for estimation is obtained from the work in [4], while the readers can see the details procedure of this technique in [4]. After using the technique, the desired result for the data fitting is shown in Fig. 1 while, Table 1 contains the respective estimated values. We have the $\mathcal{R}_0 \approx 1.2915$ for the given data of the proposed period. During the estimations, the initial values of various population classes are set out as: $S(0) = 34\,811\,870$, $E(0) = 2000$, $I(0) = 1$, $I_a(0) = 0$, $R(0) = 0$ and $B(0) = 30\,000$.

Table 1
Biological description of parameters with respective numerical values.

Notation	Biological meaning	Value/day	Source
Π	Recruitment rate	$\mu * N(0)$	Estimated
μ	Natural death rate	$1/(74.87 * 365)$	[26]
ξ	COVID-19 induced mortality rate in I compartment	0.0103	Fitted
η_1	Transmissibility rate relative to exposed class	0.2259	Fitted
η_2	Transmissibility rate due to symptomatic class	0.01298	Fitted
η_3	Transmissibility rate due to asymptomatic class	0.4579	Fitted
η_4	The transmission rate at which the environment contributes	0.0969	Fitted
κ	Incubation period	0.5625	Fitted
τ	Fraction of the asymptomatically-infected people	0.1142	Fitted
δ_1	Recovery rate of I class	0.3346	Fitted
ϕ_2	Recovery or removal rate of I_a class	0.0867	Fitted
ϖ_1	Contribution due to E	0.2616	Fitted
ϖ_2	Contribution due to I	0.0100	Fitted
ϖ_3	Contribution due to I_a	0.1815	Fitted
ϕ	Virus decay	0.2786	Fitted

The COVID-19 model in fractional case

The classical integer order epidemic models developed for an infectious having its own importance in epidemiology and its importance cannot be ignored. However, there are certain number of limitations of the models developed via classical differential equations. Such as the non existing of the memory or nonlocal effects, not able to capture the crossover behavior of a physical or a biological process. In the result, the mathematical models developed via the integer order derivatives are not suitable in some cases due to the properties mentioned above. To deal with limitations, the fractional operator specifically the ABC operator comprises the memory effects and the crossover behavior of the model. Therefore, to explore the COVID-19 dynamics more realistically, the proposed model (7) described in Section "Some basics of fractional calculus" is reformulated with the replacement of classical derivative by the one having fractional order in ABC sense. Thus, the fractional epidemic model for COVID-19 with the nonlocal kernel is formulated through the following system:

$$\begin{cases} {}^{ABC}D_t^{\rho} S &= \Pi - \lambda(t)S - \mu S, \\ {}^{ABC}D_t^{\rho} E &= \lambda(t)S - \mathrm{k}_1 E, \\ {}^{ABC}D_t^{\rho} I &= \kappa(1-\tau)E - \mathrm{k}_2 I, \\ {}^{ABC}D_t^{\rho} I_a &= \tau\kappa E - \mathrm{k}_3 I_a, \\ {}^{ABC}D_t^{\rho} R &= \delta_1 I + \delta_2 I_a - \mu R, \\ {}^{ABC}D_t^{\rho} B &= \varpi_1 E + \varpi_2 I + \varpi_3 I_a - \phi B. \end{cases} \tag{8}$$

The notation ${}^{ABC}D_t^{\rho}$ represents the time fractional derivative in ABC sense and the parameter $\rho \in (0,1]$ denotes the arbitrary order of the ABC operator.

Theoretical investigations of the fractional model

The current section presents the basic mathematical features of the ABC Coronavirus transmission model as formulated in (8). Initially, we investigate the biologically-feasible region for the ABC model. After this, we evaluate the model equilibria, the basic reproductive quantity and then explore the asymptotic stability of the model at the disease free equilibrium. Finally, the existence and uniqueness are provided. We proceed as follows:

Biologically-feasible region

Now, summing the first five equations of the system (8), we are lead to the following equation

$$^{ABC}D_t^{\rho} N(t) = \Pi - \mu N(t) - \zeta I \tag{9}$$

$$\leq \Pi - \mu N(t).$$

Solving Eq. (9) via Laplace transform, we deduce that

$$\lim_{t\to\infty} N(t) \leq \Pi/\mu.$$

As a result, we derived the biologically feasible region as shown in the following set:

$$\Phi = \left\{ (S,E,I,A,R) \in \mathbb{R}_+^5 : N \leq \frac{\Pi}{\mu},\ B \in \mathbb{R}_+ : B(t) \leq \frac{\Pi}{\mu}\frac{\varpi_1+\varpi_2+\varpi_3}{\phi} \right\}.$$

Model equilibria

This section explores the equilibrium points of the fractional ABC case described in (8). Also, we evaluate the basic reproductive number $\mathcal{R}_0$ and then prove the asymptotical stability of the disease free equilibrium. Moreover, we will explore impartment features of fractional models known as the existence and uniqueness (EU). The fractional model describing the COVID-19 dynamics (8) possess usually two types of steady states. The one is known as the disease free equilibrium (DFE) and the second one is named as the endemic equilibrium (EE). The DFE is obtained by solving the system by equating the right sides of (8) to zero at the infection free state. Thus, the following expression is derived for DFE of the model (8):

$$\mathrm{K}_0 = \left(\frac{\Pi}{\mu}, 0, 0, 0, 0, 0\right).$$

In order to derive $\mathcal{R}_0$ of the model, we follow the next generation matrix approach [27]. Using the approached mentioned earlier, we have the desired results for our system (8) given by:

$$F = \begin{pmatrix} \eta_1 & \eta_2 & \eta_3 & \eta_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\quad V = \begin{pmatrix} \mathrm{k}_1 & 0 & 0 & 0 \\ -(1-\tau)\kappa & \mathrm{k}_2 & 0 & 0 \\ -\tau\kappa & 0 & \mathrm{k}_3 & 0 \\ -\varpi_1 & -\varpi_2 & -\varpi_3 & \phi \end{pmatrix}.$$

Thus, using the well-known criteria developed in [27], we finally, evaluate the desired expression as follows:

$$\mathcal{R}_0 = \underbrace{\frac{\kappa(1-\tau)(\eta_4\varpi_2+\eta_2\phi)}{\mathrm{k}_1\mathrm{k}_2\phi}}_{\mathcal{R}_1} + \underbrace{\frac{\kappa\tau(\eta_4\varpi_3+\eta_3\phi)}{\mathrm{k}_1\mathrm{k}_3\phi}}_{\mathcal{R}_2} + \underbrace{\frac{(\eta_4\varpi_1+\eta_1\phi)}{\mathrm{k}_1\phi}}_{\mathcal{R}_3}.$$

Existence of EE

We focus on the existence of EE for the epidemic model (8) in ABC sense in this section. The EE is denoted by

$$\mathrm{K}_1^{**} = (S^{**}, E^{**}, I^{**}, I_a^{**}, R^{**}, B^{**}),$$

where, the expressions involved are obtain by solving the equations in (8) at steady state.

$$S^{**} = \frac{\Pi}{\lambda+\mu},\ E^{**} = \frac{\lambda S^{**}}{\mathrm{k}_1},\ I^{**} = \frac{(1-\tau)\kappa E^{**}}{\mathrm{k}_2},$$
$$I_a^{**} = \frac{\tau\kappa E_h^{**}}{\mathrm{k}_3},\ R^{**} = \frac{\delta_1 I^{**} + \delta_2 I_a^{**}}{\mu},$$
$$B^{**} = \frac{\varpi_3 I_a^{**} + \varpi_2 I^{**} + \varpi_1 E^{**}}{\phi}, \tag{10}$$

where,

$$\lambda^{**} = \frac{(\eta_1 E^{**} + \eta_2 I^{**} + \eta_3 A^{**} + \eta_4 B^{**})}{N^{**}}. \tag{11}$$

After substituting the values involved (10) into (12), we obtain the following equation in λ^{**}:

$$A_1\lambda^{**} + A_2 = 0, \tag{12}$$

where;

$$A_1 = \phi\Big(\kappa\mathrm{k}_3(\mu+\delta_1)(1-\tau) + \mathrm{k}_2(\kappa\tau(\delta_2+\mu) + \mu\mathrm{k}_3)\Big),$$
$$A_2 = \mathrm{k}_1\mathrm{k}_2\mathrm{k}_3\mu\phi(1-\mathcal{R}_0). \tag{13}$$

Thus, the following result regarding the existence of EE for the COVID-19 (8) is parented.

Theorem 5.1. *A unique EE of the model (8) describing the COVID-19 dynamics exists if and only if $\mathcal{R}_0 > 1$.*

Stability results of DFE

For the proof of the local asymptotical stability (LAS) of the model around the DFE, the necessary condition to be shown is that for the entire eigenvalues of the matrix (14) lie outside the closed angular sector $|arg(\Pi)| \le 1$ [28]. Although the Matignon's result was applied to the Caputo case, but it can be extended to systems having ABC fractional order (FO) derivative. To proceed, let us evaluate the corresponding Jacobian matrix of (8) as:

$$J_{\mathcal{K}_0} = \begin{pmatrix} -\mu & -\eta_1 & -\eta_2 & -\eta_3 & 0 & -\eta_4 \\ 0 & -\mathrm{k}_1+\eta_1 & \eta_2 & \eta_3 & 0 & \eta_4 \\ 0 & \kappa(1-\tau) & -\mathrm{k}_2 & 0 & 0 & 0 \\ 0 & \kappa\tau & 0 & -\mathrm{k}_3 & 0 & 0 \\ 0 & 0 & \delta_1 & \delta_2 & -\mu & 0 \\ 0 & \varpi_1 & \varpi_2 & \varpi_3 & 0 & -\phi \end{pmatrix}. \tag{14}$$

Next, we introduce the following result for the desired stability of the model.

Theorem 5.2. *The DFE $\mathcal{K}_0$ of the COVID-19 model in ABC case is LAS if $\forall$ eigenvalues λ_i of J_{K_0} satisfy the necessary condition:*

$$|arg(\lambda_i)| > \frac{\rho\pi}{2} \quad for\ \ i = 0, 1, \dots, 6. \tag{15}$$

Proof. It is clearly observed from J_{K_0} that the two eigenvalues *i.e.*, $-\mu$ and $-\mu$ are negative and clearly satisfy the necessary condition (15) for all $\rho \in (0,1]$. Moreover, after some manipulations, it is easy to show that the remaining four eigenvalues will have negative real parts whenever, $\mathcal{R}_0 < 1$ and thus, following [28], the arguments of the remaining eigenvalues fulfill the desired condition as described (15). Hence, K_0 is LAS whenever, $\mathcal{R}_0 < 1$. □

The GAS of the COVID-19 model (8) at the DFE is discussed in the following result. The result for this purpose is stated as follows:

Theorem 5.3. *For any $\rho \in (0,1]$, the DFE $\mathcal{K}_0$ of (8) is GAS in the region Φ if $\mathcal{R}_0$ <1.*

Proof. We introduce the following suitable Lyapunov function in order to proceed

$$\mathcal{F}(t) = C_1E(t) + C_2I(t) + C_3I_a(t) + C_4B(t),$$

where, C_j, for j= 1, … , 4, describe unknown positive constants to be chosen later. The time fractional ABC derivative of $\mathcal{F}(t)$ leads to the following

$${}^{ABC}D_t^{\rho}\mathcal{F}(t) = C_1\,{}^{ABC}D_t^{\rho}E + C_2\,{}^{ABC}D_t^{\rho}I + C_3\,{}^{ABC}D_t^{\rho}A + C_4\,{}^{ABC}D_t^{\rho}M.$$

Using the system defined in (8), we get

$$\begin{aligned}
{}^{ABC}D_t^{\rho}\mathcal{F}(t) &= C_1\Big\{(\eta_1E + \eta_2I + \eta_3I_a + \eta_4B)\frac{S}{N} - \mathrm{k}_1E\Big\} \\
&\quad + C_2\Big\{\kappa(1-\tau)E - \mathrm{k}_2I\Big\} \\
&\quad + C_3\Big\{\tau\kappa E - \mathrm{k}_3I_a\Big\} + C_4\Big\{\varpi_1E + \varpi_2I + \varpi_3I_a - \phi B\Big\} \\
&\le C_1\Big\{(\eta_1E + \eta_2I + \eta_3A + \eta_4B) - \mathrm{k}_1E\Big\} \\
&\quad + C_2\Big\{\kappa(1-\tau)E - \mathrm{k}_2I\Big\} \\
&\quad + C_3\Big\{\tau\kappa E - \mathrm{k}_3I_a\Big\} + C_4\Big\{\varpi_1E + \varpi_2I + \varpi_3I_a - \phi B\Big\} \\
&\quad as\ \ \frac{S}{N} \le 1 \\
&= \Big\{C_1\eta_1 + C_2\kappa(1-\tau) - C_3\kappa\tau + C_4\varpi_1 - C_1\mathrm{k}_1\Big\}E \\
&\quad + \Big\{C_1\eta_2 + C_4\varpi_2 - C_2\mathrm{k}_2\Big\}I \\
&\quad + \Big\{C_1\eta_3 + C_4\varpi_3 - C_3\mathrm{k}_3\Big\}I_a + \Big\{C_1\eta_4 - C_4\phi\Big\}B, \\
&= C_1\mathrm{k}_1\Big\{\frac{C_1\eta_1 + C_2\kappa(1-\tau) - C_3\kappa\tau + C_4\varpi_1}{C_1\mathrm{k}_1} - 1\Big\}E \\
&\quad + \Big\{C_1\eta_2 + C_4\varpi_2 - C_2\mathrm{k}_2\Big\}I \\
&\quad + \Big\{C_1\eta_3 + C_4\varpi_3 - C_3\mathrm{k}_3\Big\}I_a + \Big\{C_1\eta_4 - C_4\phi\Big\}B.
\end{aligned}$$

Let us choose the constants values as

$$C_1 = \phi,\ C_2 = \frac{\phi\eta_2 + \varpi_2\eta_4}{\mathrm{k}_2},\ \ C_3 = \frac{\phi\eta_3 + \varpi_3\eta_4}{\mathrm{k}_3},\ \text{ and }\ C_4 = \eta_4,$$

and then after some simplifications, we have

$${}^{ABC}D_t^{\rho}\mathcal{F}(t) \le \phi\mathrm{k}_1\Big\{\mathcal{R}_0 - 1\Big\}E.$$

It is clear that when $\mathcal{R}_0 < 1$ then ${}^{ABC}D_t^{\rho}\mathcal{F}(t)$ is negative, due to the non-negativity of all parameters. Thus,with the help of Theorem 2.1, that the DFE $\mathcal{K}_0$ is GAS in the region Φ. □

Existence and uniqueness of the solution

This subsection is focused on proving the EU of fractional compartmental epidemic model (8). The well-known fixed point theory is utilized for the said purpose. Firstly, the COVID-19 model (8) is shown through a general initial value problem as follows:

$$\begin{cases} {}^{ABC}D_t^{\rho}\mathrm{v}(t) = F(t, \mathrm{v}(t)), \\ \mathrm{v}(0) = \mathrm{v}_0,\ \ 0 < t < \mathcal{T} < \infty. \end{cases} \tag{16}$$

In the problem described in (16), the vector $\mathrm{v} = (S, E, I, I_a, R, B)$ denotes state variables while $\mathrm{v}_0 = (S(0), E(0), I(0), I_a(0), R(0), B(0))$ denotes the corresponding initial condition. Moreover, F defines a continuous vector function as follows:

$$\mathrm{F} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{pmatrix} = \begin{pmatrix} \Pi - (\lambda(t)+\mu)S \\ \lambda(t)S - F_1E \\ (1-\tau)\kappa E - F_2I \\ \tau\kappa E - F_3I_a \\ \delta_1I + \delta_2I_a - \mu R \\ \varpi_1E + \varpi_2I + \varpi_3I_a - \phi B \end{pmatrix}.$$

Additionally, the function F satisfies the Lipschitz condition expressed as below:

$$\|F(t, \mathrm{v}_1(t)) - F(t, \mathrm{v}_2(t))\| \le \mathrm{D}\|\mathrm{v}_1(t) - \mathrm{v}_2(t)\|,\ \ \mathrm{D} > 0. \tag{17}$$

Next, we introduce the following theorem regarding the EU of the model solution.

Theorem 5.4. *There exists a unique solution of the epidemic model (16), if the condition shown in (18) holds:*

$$\frac{(1-\rho)}{ABC(\rho)}\mathtt{D}+\frac{\rho}{ABC(\rho)\Gamma(\rho)}\mathcal{T}^{\rho}_{max}\mathtt{D}<1. \tag{18}$$

Proof. Taking the ABC integral operator over the problem (16), which gives a nonlinear Volterra integral equation in the following form:

$$\mathtt{v}=\upsilon_0+\frac{1-\rho}{ABC(\rho)}F(t,\mathtt{v})+\frac{\rho}{ABC(\rho)\Gamma(\rho)}\int_0^t(t-\varsigma)^{\rho-1}F(x,\upsilon(x))dx. \tag{19}$$

Let $\mathcal{J}=(0,\mathcal{T})$, and the operator $\mathtt{F}:C(\mathcal{J},\mathbb{R}^6)\to C(\mathcal{J},\mathbb{R}^6)$ be defined as

$$\mathtt{F}[\mathtt{v}]=\upsilon_0+\frac{1-\rho}{ABC(\rho)}F(t,\mathtt{v})+\frac{\rho}{ABC(\rho)\Gamma(\rho)}\int_0^t(t-x)^{\rho-1}F(x,\upsilon(x))dx. \tag{20}$$

Thus, the expression in (19) takes the form:

$$\mathtt{v}=\mathtt{F}[\mathtt{v}]. \tag{21}$$

Further, let $\|.\|_{\mathcal{J}}$ be the supremum norm over $\mathcal{J}$ and expressed as:

$$\|\mathtt{v}\|_{\mathcal{J}}=\sup_{t\in\mathcal{J}}\|\mathtt{v}\|,\quad \mathtt{v}\in C. \tag{22}$$

It is clear that $C(\mathcal{J},\mathbb{R}^6)$ with the respective norm $\|.\|_{\mathcal{J}}$ constructs a Banach space. Also, the inequality given in (23) can be easily demonstrated as:

$$\left\|\int_0^t F(t,x)\mathtt{v}(x)dx\right\|\le\mathcal{T}\|F(t,x)\|_{\mathcal{J}}\|\mathtt{v}\|_{\mathcal{J}}, \tag{23}$$

with $\mathtt{v}\in C(\mathcal{J},\mathbb{R}^6)$, $F(t,x)\in C(\mathcal{J}^2,\mathbb{R})$ such that

$$\|F(t,x)\|_{\mathcal{J}}=\sup_{t,x\in\mathcal{J}}|F(t,x)|. \tag{24}$$

Using Eq. (21), we obtain

$$\begin{aligned}&\|\mathtt{F}[\mathtt{v}_1(t)]-\mathtt{F}[\mathtt{v}_2(t)]\|_{\mathcal{J}}\\&\le\left\|\frac{(1-\rho)}{ABC(\rho)}(F(t,\mathtt{v}_1(t))-F(t,\mathtt{v}_2(t)))+\frac{\rho}{ABC(\rho)\Gamma(\rho)}\times\right.\\&\left.\int_0^t(t-x)^{\rho-1}(F(x,\mathtt{v}_1(x))-F(x,\mathtt{v}_2(x)))dx\right\|_{\mathcal{J}}.\end{aligned} \tag{25}$$

Further, utilizing Eqs. (17), (23) along with triangular inequality, Eq. (25) leads to the following form:

$$\|\mathtt{F}[\mathtt{v}_1(t)]-\mathtt{F}[\mathtt{v}_2(t)]\|_{\mathcal{J}}\le\left(\frac{(1-\rho)\mathtt{D}}{B(\rho)}+\frac{\rho\mathtt{D}}{B(\rho)\Gamma(\rho)}\mathcal{T}^{\rho}_{max}\right)\|\mathtt{v}_1(t)-\mathtt{v}_2(t)\|_{\mathcal{J}}. \tag{26}$$

Finally, we get the following equation:

$$\|\mathtt{F}[\mathtt{v}_1(t)]-\mathtt{F}[\mathtt{v}_2(t)]\|_{\mathcal{J}}\le\mathtt{M}\|\mathtt{v}_1(t)-\mathtt{v}_2(t)\|_{\mathcal{J}}, \tag{27}$$

where,

$$\mathtt{M}=\frac{(1-\rho)\mathtt{D}}{ABC(\rho)}+\frac{\rho\mathtt{D}}{ABC(\rho)\Gamma(\rho)}\mathcal{T}^{\rho}_{max}.$$

If the condition given in (18) holds then the operator $\mathtt{F}$ will be a contraction. Thus, the desired model shown by (16) will possess unique solution. □

Numerical solution of the ABC model

It is obvious that the biological models are often complicated due to their nonlinearity. The nonlinearity together with fractional derivative the model becomes more complex and difficult to solve analytically. In this regard, the numerical techniques are often considered for its solution graphically. Different numerical methods are considered for the solution of fractional order numerically. Among these methods, the method developed in [29] based on Lagrange interpolation polynomial of two-step is appropriate for the given problem. We will present the details of the scheme in the present section and then write the scheme for the proposed system. To get the approximate solution of the problem, we use the method in [29]. Considering the Atangana–Baleanu integral described in (3) and applying upon the general problem described in (16), then we have the following:

$$\mathtt{v}(t)-\mathtt{v}(0)=\frac{(1-\rho)}{AB(\rho)}F(t,\mathtt{v})+\frac{\rho}{AB(\rho)\times\Gamma(\rho)}\int_0^t F(\varsigma,\upsilon(\varsigma))(t-\varsigma)^{\rho-1}d\varsigma. \tag{28}$$

Further, at $t=t_{\mathtt{m}+1}=(\mathtt{m}+1)h$, we have

$$\begin{aligned}\mathtt{v}(t_{\mathtt{m}+1})-\mathtt{v}(0)&=\frac{1-\rho}{AB(\rho)}F(t_{\mathtt{m}},\mathtt{v}(t_{\mathtt{m}}))+\\&\quad\frac{\rho}{AB(\rho)\times\Gamma(\rho)}\int_0^{t_{\mathtt{m}+1}}F(\varsigma,\upsilon(\varsigma))(t_{m+1}-\varsigma)^{\rho-1}d\varsigma,\\&=\frac{1-\rho}{AB(\rho)}F(t_{\mathtt{m}},\mathtt{v}(t_{\mathtt{m}}))+\\&\quad\frac{\rho}{AB(\rho)\times\Gamma(\rho)}\sum_{j=0}^{\mathtt{m}}\int_{t_j}^{t_{j+1}}F(\varsigma,\mathtt{v}(\varsigma))(t_{\mathtt{m}+1}-\varsigma)^{\rho-1}d\varsigma.\end{aligned} \tag{29}$$

Now, utilizing the polynomial interpolation concept, we approximate the function $F(\varsigma,\mathtt{v}(\varsigma))$ by two-step Lagrange polynomial over the closed interval $[t_j,t_{j+1}]$ as follows:

$$\mathcal{G}(\varsigma,F(\varsigma))\cong\mathcal{P}_k(\varsigma)=\frac{F(t_j,\mathtt{v}(t_j))}{h}(\varsigma-t_{j-1})-\frac{F(t_{j-1},\mathtt{v}(t_{j-1}))}{h}(\varsigma-t_j). \tag{30}$$

Thus, Eq. (29) becomes:

$$\begin{aligned}\mathtt{v}(t_{\mathtt{m}+1})&=\mathtt{v}(0)+\frac{1-\rho}{AB(\rho)}F(t_{\mathtt{m}},\mathtt{v}(t_{\mathtt{m}}))+\\&\quad\frac{\rho}{AB(\rho)\times\Gamma(\rho)}\sum_{j=0}^{\mathtt{m}}\left(\frac{F(t_j,\mathtt{v}(t_j))}{h}\int_{t_j}^{t_{j+1}}(\varsigma-t_{j-1})(t_{\mathtt{m}+1}-\varsigma)^{\rho-1}d\varsigma\right.\\&\quad\left.-\frac{F(t_{j-1},\mathtt{v}(t_{j-1}))}{h}\int_{t_j}^{t_{j+1}}(\varsigma-t_j)(t_{\mathtt{m}+1}-\varsigma)^{\rho-1}d\varsigma\right).\end{aligned} \tag{31}$$

By simplifying the above Eq. (31), the final approximate form can be shown by the below expression:

$$\begin{aligned}\mathtt{v}(t_{\mathtt{m}+1})&=\mathtt{v}(t_0)+\frac{1-\rho}{AB(\rho)}F(t_{\mathtt{m}},\mathtt{v}(t_{\mathtt{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathtt{m}}\\&\quad\left(\frac{h^{\rho}F(t_j,\mathtt{v}(t_j))}{\Gamma(\rho+2)}\left\{(\mathtt{m}-j+1)^{\rho}(\mathtt{m}-j+2+\rho)\right.\right.\\&\quad\left.-(\mathtt{m}-j)^{\rho}(\mathtt{m}-j+2+2\rho)\right\}\\&\quad-\frac{h^{\rho}F(t_{j-1},\mathtt{v}(t_{j-1}))}{\Gamma(\rho+2)}\left\{(\mathtt{m}-j+1)^{\rho+1}\right.\\&\quad\left.\left.-(\mathtt{m}-j)^{\rho}(\mathtt{m}-j+1+\rho)\right\}\right).\end{aligned} \tag{32}$$

Using the above scheme (32), we can write the system (8) below that presents the numerical scheme:

$$\begin{aligned}S(t_{\mathtt{m}+1})&=S(t_0)+\frac{1-\rho}{AB(\rho)}F_1(t_{\mathtt{m}},\mathtt{v}(t_{\mathtt{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathtt{m}}\\&\quad\left(\frac{h^{\rho}F_1(t_j,\mathtt{v}(t_j))}{\Gamma(\rho+2)}\left\{(\mathtt{m}+1-j)^{\rho}(\mathtt{m}+2-j+\rho)\right.\right.\\&\quad\left.-(\mathtt{m}-j)^{\rho}(\mathtt{m}+2-j+2\rho)\right\}\\&\quad-\frac{h^{\rho}F_1(t_{j-1},\mathtt{v}(t_{j-1}))}{\Gamma(\rho+2)}\\&\quad\left.\times\left\{(\mathtt{m}-j+1)^{\rho+1}-(\mathtt{m}-j)^{\rho}(\mathtt{m}-j+1+\rho)\right\}\right),\\E(t_{\mathtt{m}+1})&=E(t_0)+\frac{1-\rho}{AB(\rho)}F_2(t_{\mathtt{m}},\mathtt{v}(t_{\mathtt{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathtt{m}}\\&\quad\left(\frac{h^{\rho}F_2(t_j,\mathtt{v}(t_j))}{\Gamma(\rho+2)}\left\{(\mathtt{m}+1-j)^{\rho}(\mathtt{m}+2-j+\rho)\right.\right.\\&\quad\left.\left.-(\mathtt{m}-j)^{\rho}(\mathtt{m}+2-j+2\rho)\right\}\right.\end{aligned}$$

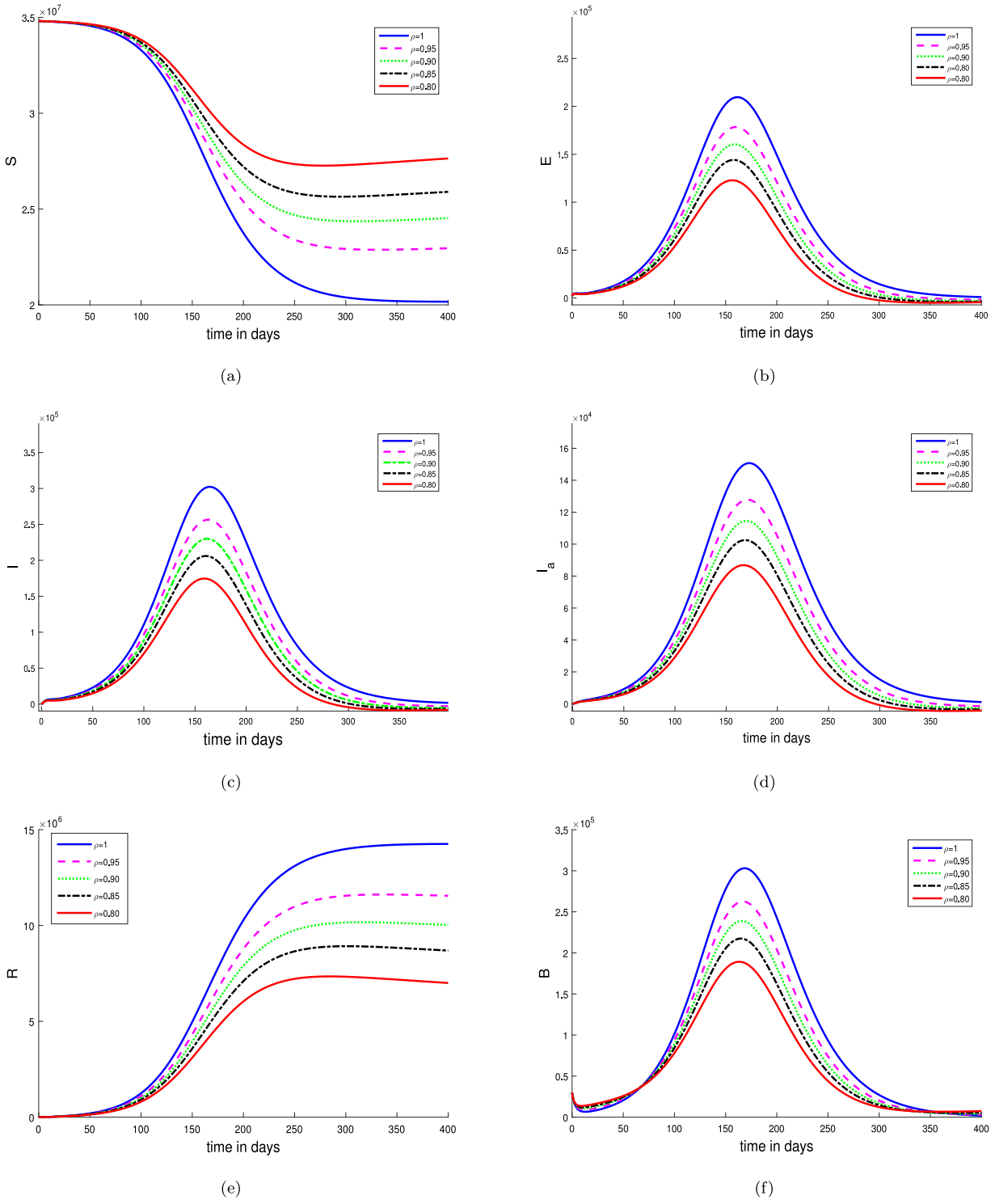

Fig. 2. Numerical solution of (8) with different ν.

$$-\frac{h^{\rho}F_2(t_{j-1},\nu(t_{j-1}))}{\Gamma(\rho+2)}$$
$$\times\Big\{(\mathtt{m}+1-j)^{\rho+1}-(\mathtt{m}-j)^{\rho}(\mathtt{m}-j+1+\rho)\Big\}\Big),$$

$$I(t_{\mathtt{m}+1}) = I(t_0)+\frac{1-\rho}{B(\rho)}F_3(t_{\mathtt{m}},\nu(t_{\mathtt{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathtt{m}}$$
$$\Big(\frac{h^{\rho}F_3(t_j,\nu(t_j))}{\Gamma(\rho+2)}\Big\{(\mathtt{m}+1-j)^{\rho}(\mathtt{m}+2-j+\rho)$$

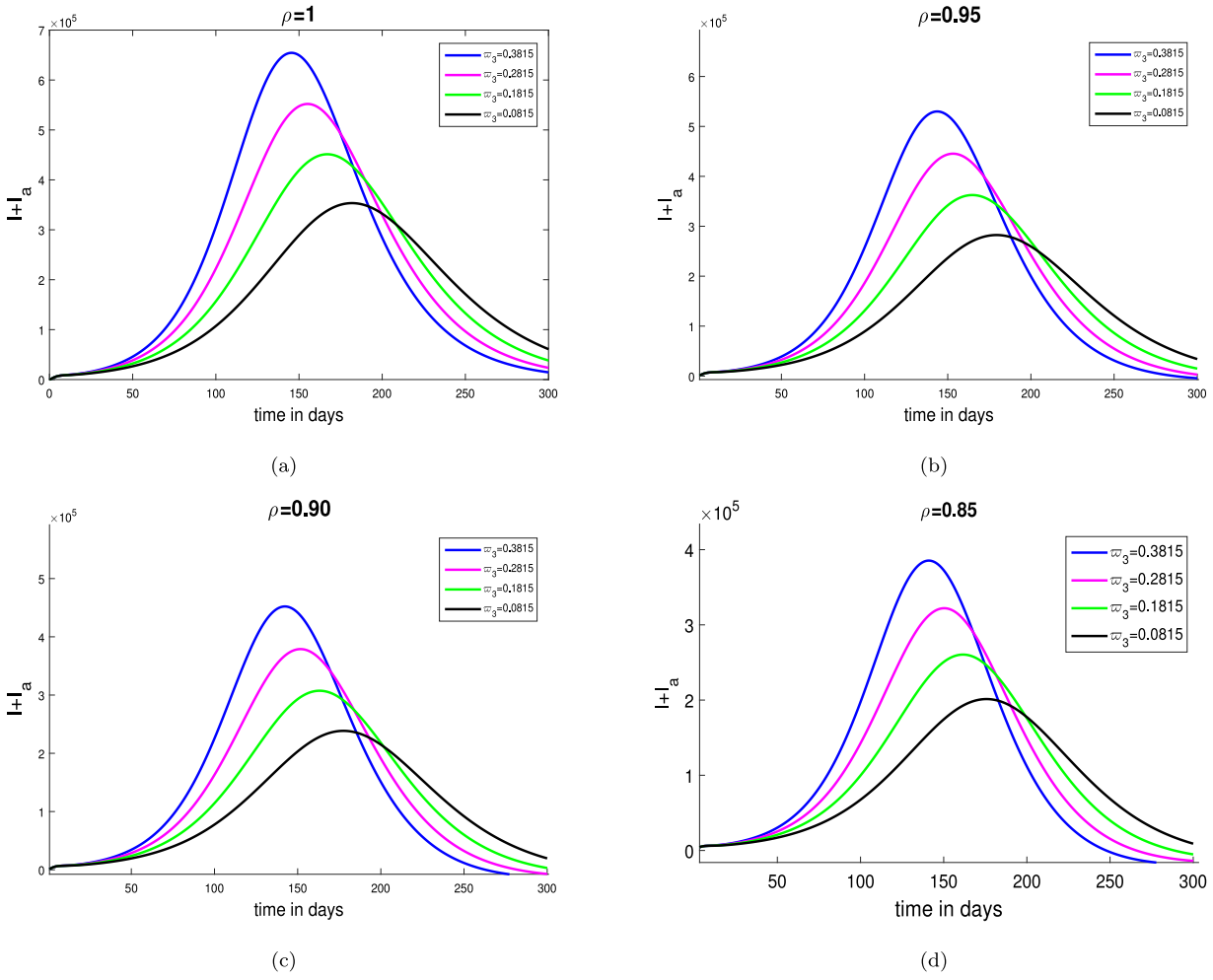

Fig. 3. Numerical solution of (8) with many ρ and ϖ_3.

$$
\begin{aligned}
&-(\mathrm{m}-j)^{\rho}(\mathrm{m}+2-j+2\rho)\Big\}\\
&-\frac{h^{\rho}F_3(t_{j-1},\mathrm{v}(t_{j-1}))}{\Gamma(\rho+2)}\\
&\times\Big\{(\mathrm{m}+1-j)^{\rho+1}-(\mathrm{m}-j)^{\rho}(\mathrm{m}-j+1+\rho)\Big\}\Big),\\
I_a(t_{\mathrm{m}+1}) \;=\; & I_a(t_0)+\frac{1-\rho}{AB(\rho)}F_4(t_{\mathrm{m}},\mathrm{v}(t_{\mathrm{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathrm{m}}\\
&\Big(\frac{h^{\rho}F_4(t_j,\mathrm{v}(t_j))}{\Gamma(\rho+2)}\Big\{(\mathrm{m}+1-j)^{\rho}(\mathrm{m}+2-j+\rho)\\
&-(\mathrm{m}-j)^{\rho}(\mathrm{m}+2-j+2\rho)\Big\}\\
&-\frac{h^{\rho}F_4(t_{j-1},\mathrm{v}(t_{j-1}))}{\Gamma(\rho+2)}\\
&\times\Big\{(\mathrm{m}+1-j)^{\rho+1}-(\mathrm{m}-j)^{\rho}(\mathrm{m}-j+1+\rho)\Big\}\Big),\\
R(t_{\mathrm{m}+1}) \;=\; & R(t_0)+\frac{1-\rho}{AB(\rho)}F_5(t_{\mathrm{m}},\mathrm{v}(t_{\mathrm{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathrm{m}}\\
&\Big(\frac{h^{\rho}F_5(t_j,\mathrm{v}(t_j))}{\Gamma(\rho+2)}\Big\{(\mathrm{m}+1-j)^{\rho}(\mathrm{m}+2-j+\rho)\\
&-(\mathrm{m}-j)^{\rho}(\mathrm{m}+2-j+2\rho)\Big\}\\
&-\frac{h^{\rho}F_5(t_{j-1},\mathrm{v}(t_{j-1}))}{\Gamma(\rho+2)}\\
&\times\Big\{(\mathrm{m}+1-j)^{\rho+1}-(\mathrm{m}-j)^{\rho}(\mathrm{m}-j+1+\rho)\Big\}\Big),
\end{aligned}
$$

$$
\begin{aligned}
B(t_{\mathrm{m}+1}) \;=\; & B(t_0)+\frac{1-\rho}{AB(\rho)}F_6(t_{\mathrm{m}},\mathrm{v}(t_{\mathrm{m}}))+\frac{\rho}{AB(\rho)}\sum_{j=0}^{\mathrm{m}}\\
&\Big(\frac{h^{\rho}F_6(t_j,\mathrm{v}(t_j))}{\Gamma(\rho+2)}\Big\{(\mathrm{m}+1-j)^{\rho}(\mathrm{m}+2-j+\rho)\\
&-(\mathrm{m}-j)^{\rho}(\mathrm{m}+2-j+2\rho)\Big\}\\
&-\frac{h^{\rho}F_6(t_{j-1},\mathrm{v}(t_{j-1}))}{\Gamma(\rho+2)}\Big\{(\mathrm{m}+1-j)^{\rho+1}\\
&-(\mathrm{m}-j)^{\rho}(\mathrm{m}-j+1+\rho)\Big\}\Big).
\end{aligned}
\tag{33}
$$

Simulations and discussion

We derive the numerical scheme using the two-step Lagrange interpolation polynomial for the approximate solution of the model involves fractional derivative. Then, we particularize the scheme for our model by giving the scheme shown in (33). As we taken the daily reported cases, so the time unit considered in the numerical solution in days. The respective values of the biological parameters involved in the model obtained through the real cases of KSA data shown in Table 1 are used for the simulation purpose. Firstly, we depict the influence of variation in the memory index (or fractional order ρ) upon the dynamics of all population classes of the proposed model. After that, we analyzed the influence of various model parameters such as ϖ_3 and ϕ on the COVID-19 incidence in the population of Saudi Arabia. The dynamics of the susceptible population for various values of fractional order ρ of the ABC derivative are shown in Fig. 2(a). It is observed that susceptible

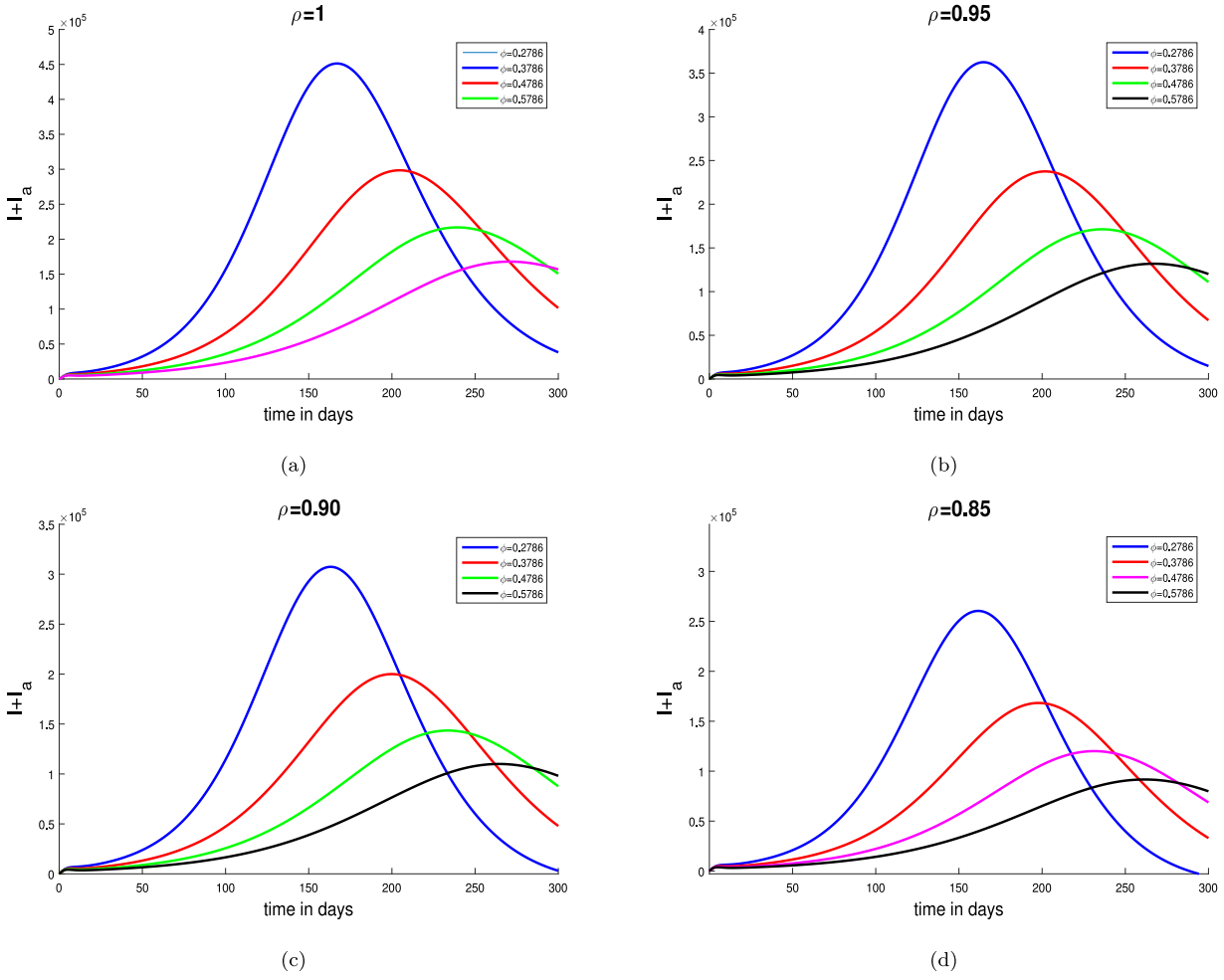

Fig. 4. Numerical solution of (8) with many ρ and ϕ.

individuals are increasing with the decrease in ρ. The impact of memory index ρ over the exposed, symptomatically and asymptomatically COVID-19 infective individuals is shown in sub-plots (b), (c) and (d) Fig. 2, respectively. It is observed that peaks of population in the exposed, symptomatically and asymptomatically infected classes are decreasing significantly for the smaller values of ρ. The dynamical behavior of the recovered people is depicted in Fig. 2(e). The recovered people shows a decreasing effect with a decrease in the fractional order ρ. The impact of memory index ρ on the dynamics of virus concentration in the environment B is analyzed in Fig. 2(f). Further, Fig. 3(a-d) depicts the influence of the variation in the viral contribution due asymptomatically-infected individuals ϖ_3 on the COVID-19 incidence. We have analyzed this impact for various values of fractional order ρ. In all cases, it is observed that the peaks cumulative symptomatically and asymptomatically infected curves decreased well with the decrease in the parameter ϖ_3. Moreover, it is observed that the decrease in pandemic peaks is comparatively faster for smaller values of ρ as shown in Fig. 3(b-d). Finally, the impact of parameter ϕ for various values of ρ is shown in Fig. 4(a-d). It is observed that infected curves become flatten with the increase in parameter ϕ by different rates to their baseline values as shown in Fig. 4(a). Furthermore, the decrease in the peaks of symptomatically and asymptomatically infected curves is slightly faster for smaller values of fractional order ρ showing the significance of the memory index on disease dynamics.

Conclusion

In the last few decades, mathematical models have been used as a useful tool to understand the complex dynamics and to determine the future trend of an infectious disease. Although, the epidemic models developed via classical integer order differential systems have its own significance to explore a disease dynamics. But the mathematical model designed through the fractional operators are more useful than the ordinary case, due to data fitting, the memory effects and the crossover behavior etc. We studied a new fractional model for understanding the complex dynamics of Coronavirus in the Kingdom of Saudi Arabia reported cases using non-singular kernel operator. We studied the model and provided its mathematical results in details. We found that the given model is locally asymptotically stable at the infection free state when its threshold below 1. We used the nonlinear least square method for the obtaining of parameters estimations and then provide the good fit to the proposed data. We also proved the existence and uniqueness criteria of the fractional COVID-19 model. The proposed parameters obtained through estimation are considered further to obtain its numerical results. We studied graphically the model equations solution with memory index and found that the solution converges for the arbitrary order. We then varied the important sensitive parameters for different fractional orders. Reducing the fractional order, the number of infective compartment decreases well. The results obtained in this paper through this fractional model can be useful further for the scientists and researchers working on the COVID-19 infection in KSA. The results can be useful to have the policy for the future spread and control of Coronavirus in the country.

CRediT authorship contribution statement

Ebraheem Alzahrani: Investigation, Conceptualization, Methodology, Formal analysis, Validation, Visualization, Software. **M.M. El-Dessoky:** Software, Data curation, Investigation, Methodology, Visualization, Writing - original draft, Conceptualization, Validation, Writing - review & editing, Formal analysis. **Dumitru Baleanu:** Visualization, Software, Validation, Formal analysis, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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