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Research Article

A Novel Multicriteria Decision-Making Approach for Einstein Weighted Average Operator under Pythagorean Fuzzy Hypersoft Environment

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The experts used the Pythagorean fuzzy hypersoft set (PFHSS) in their research to discourse ambiguous and vague information in decision-making processes. The aggregation operator (AO) plays a prominent part in the sensitivity of the two forefront loops and eliminates anxiety from that perception. The PFHSS is the most influential and operative extension of the Pythagorean fuzzy soft set (PFSS), which handles the subparameterized values of alternatives. It is also a generalized form of Intuitionistic fuzzy hypersoft set (IFHSS) that provides better and more accurate assessments in the decision-making (DM) process. In this work, we present some operational laws for Pythagorean fuzzy hypersoft numbers (PFHSNs) and then formulate Pythagorean fuzzy hypersoft Einstein weighted average (PFHSEWA) operator based on developed operational laws. We discuss essential features such as idempotency, boundedness, and homogeneity for the proposed PFHSEWA operator. Furthermore, a DM approach has been developed based on the built-in operator to address multicriteria decision-making (MCDM) issues. A numerical case study of decision-making problems in real-life agricultural farming is considered to validate the settled technique's dominance and applicability. The consequences display that the planned model is more operative and consistent to handle inexact data based on PFHSS.

1. Introduction

In farming history, the subjugation of vegetations, wildlife, and the manufacturing and propagation methods used for high-yielding cultivation have been recorded. Farming started independently in numerous parts of the world, including a wide range of taxa. By 8000 BC, farming along the Nile was widely known. Around this time, farming developed autonomously in the Far East, most likely in China, and the main crop was rice instead of wheat. Modern agricultural practices result from an excessive water supply, extensive deforestation, and reduced soil fertility. Since there is lacking water to endure farming, it is compulsory to reexamine how to use essential water, land, and environmental resources to raise crop vintages. Highlighting the importance of the ecosystem, considering the balances among the atmosphere and livings, and balancing the privileges and benefits of a range of manipulators may explain. The discriminations rising from these steps need to be addressed, such as the redeployment of water resources from the poor to the rich and clearing land to make room for more profitable farmland. Scientific development supports farmers with apparatuses and facilities to help them become more affluent. Maintenance farming is agricultural expertise that evades land loss due to deforestation, decreases water contamination, and rallies carbon impounding. It is a sample of scientific invention. Agriculture has not been a simple task to meet the rising mandate for nutrition and now requires more analysis and expertise. Statisticians, hydrologists, and agriculturalists met in California to progress a plan to diminish crop water ingesting while still generating profits for farmers and meeting market demand. Scientific representations use information, including plant growing features and water supplies, to regulate which yields and zones should not be planted. Farmers are gratified with the sensible use of their tools, while mathematicians work with professional specialists.

MCDM is considered the most appropriate technique for finding the most acceptable alternative from all possible options, following criteria or attributes. In real-life circumstances, most decisions are taken when the objectives and limitations are usually indefinite or ambiguous. To overcome such ambiguities and anxieties, Zadeh offered the idea of the fuzzy set (FS) [1], a prevailing tool to handle the obscurities and uncertainties in DM. Such a set allocates to all objects a membership value ranging from 0 to 1. Experts mainly consider membership and a nonmembership value in the DM process that FS cannot handle. Atanassov [2] introduced the generalization of the FS, the idea of the intuitionistic fuzzy set (IFS) to overcome the limitation mentioned above. Wang and Liu [3] presented numerous operations on IFS, such as Einstein product and Einstein sum, and constructed two aggregation operators (AOs). They also discussed some essential properties of these operators and utilized their proposed AO to resolve multi-attribute decision making (MADM) for the IFS information. Atanassov [4] presented a generalized form of IFS in the light of ordinary interval values, called interval-valued intuitionistic fuzzy set (IVIFS). As a generalization of the IFS and IVIFS, Garg and Kaur [5] extended the concept of IFS and presented a novel idea of the cubic intuitionistic fuzzy set (CIFS). CIFS is a successful tool representing incomplete data by embedding IFS and IVIFS. They also discussed several desirable properties of CIFS.

The models mentioned above have been well-recognized by the specialists. Still, the existing IFS cannot handle the inappropriate and vague data because it is considered to envision the linear inequality between the membership and nonmembership grades. For example, if decision-makers choose membership and nonmembership values 0.7 and 0.6, respectively, then $0.7 + 0.6 \geq 1$. The IFS mentioned above theory cannot be applied to these data. To resolve the limitation described above, Yager [6] presented the idea of the Pythagorean fuzzy set (PFS) by amending the basic condition $a + b \leq 1$ to $a^2 + b^2 \leq 1$ and developed some results associated with score function and accuracy function. Ejegwa [7] extended the notion of PFS and presented a decision-making technique. Rahman et al. [8] formed the Einstein weighted geometric operator for PFS and presented a multiattribute group decision-making (MAGDM) methodology utilizing the proposed operator. Zhang and Xu [9] developed some basic operational laws and prolonged the technique for order preference by similarity to ideal solution (TOPSIS) method to resolve MCDM complications for PFS information. Pythagorean fuzzy power AOs along with essential characteristics were introduced by Wei and Lu [10]. They also recommended a DM technique to resolve MADM difficulties based on presented operators. Wang and Li [11] offered the interaction operational laws for PFNs and developed power Bonferroni mean operators under the PFS environment. They also discussed some definite cases of developed operators and discussed their basic characteristics. IIbahar et al. [12] offered the Pythagorean fuzzy proportional risk assessment technique to assess the professional health risk. Zhang [13] proposed a novel decision-making (DM) approach based on similarity measures to resolve multicriteria group decision-making (MCGDM) difficulties for the PFS.

Peng and Yang [14] introduced the division and subtraction operations for Pythagorean fuzzy numbers (PFNs), proved their basic properties, and presented a superiority and inferiority ranking approach under the PFS to overcome the MAGDM difficulties. Garg [15] introduced operational laws based on Einstein norms for PFNs, proposed generalized Pythagorean fuzzy Einstein average AOs, and then utilized these operators for DM. Garg [16] presented the generalized geometric AOs and established an MCDM approach based on developed operators. Garg [17] introduced logarithmic operational laws for the PFS and constructed various weighted operators based on presented logarithm operational laws. Gao et al. [18] developed numerous interaction aggregation operators under the PFS setting. Wang et al. [19] offered the interactive Hamacher operations for the PFS and settled a DM method to solve MCDM difficulties. Zulqarnain and Dayan [20] utilized the fuzzy TOPSIS to select the best alternative.

Peng and Yuan [21] explored some new inequalities of the Pythagorean fuzzy weighted average (PFWA) operator. They introduced some point operators under the PFS environment. They combined the Pythagorean fuzzy point operators with the generalized PFWA operator, developed a novel operator, and established a MADM methodology based on developed operators. Wang and Garg [22] presented the Archimedean-based interactive AOs for PFS and developed an algorithm to solve MADM problems. Rahman et al. [23] defined the interval-valued weighted AOs for interval-valued PFNs. They utilized the proposed operators to resolve the MADM issues under the interval-valued PFS. Wang and Li [24] used the interval-valued PFS, presented some novel PFS operators, and offered a DM approach to resolve the MCGDM complications. Arora and Garg [25] presented basic operational laws and suggested several selected AOs for linguistic IFS. To examine the ranking of normal IFS and IVIFS, Garg [26] gave novel algorithms for solving the MADM problems. Ma and Xu [27] modified the existing score function and accuracy function for PFNs and defined novel Pythagorean fuzzy weighted geometric and Pythagorean fuzzy weighted averaging operators.

All the methods mentioned above have too many applications in many fields. However, due to their inefficiency, these methods have many limitations in terms of parameterization tools. Presenting the solution of this sort of obscurity and ambiguity, Molodtsov [28] introduced the basic notions of soft sets (SSs) and debated some elementary operations with their possessions. Maji et al. [29] prolonged the idea of SS. They defined several basic operations, and binary operations for Maji et al. [30] further applied the SS theory to solve the DM problems using rough mathematics. Moreover, Maji et al. [31] combined two prevailing notions, such as FS and SS, and developed the idea of FSS, which is a more robust and reliable tool. They also presented basic operations and established and applied this concept in the study by Maji et al. [32] who demonstrated the intuitionistic fuzzy soft set (IFSS) theory and offered some basic operations with their essential properties. Deli and Çağman [33] developed the intuitionistic fuzzy parameterized soft sets and DM methodology properties. Later on, Garg and Arora [34] presented Maclaurin symmetric mean operator for dual hesitant fuzzy soft numbers. Arora and Garg [35] developed the correlation coefficients and introduced an MCDM technique based on the generated correlation coefficients to measure the affiliation of two IFSS. In 2018, Garg and Arora [36] proposed generalized Maclaurin symmetric mean AOs based on Archimedean t-norm under the IFS environment. Garg and Arora [37] developed the TOPSIS concept and presented correlation measures based on previously constructed correlations. Wang and Liu [38] introduced the Maclaurin symmetric mean AOs based on Schweizer–Sklar operations for IFS and established the MAGDM technique to solve DM issues. Liu and Wang [39] presented the Bonferroni mean AOs for q-rung orthopair fuzzy sets and settled a MADM approach to solving DM complications.

Nowadays, the conception and application consequences of soft sets and the earlier-mentioned several research developments are evolving speedily. Peng et al. [40] developed the concept of PFSS by merging two existing models, PFS and SS. They also discussed some fundamental operations with their basic properties. Athira et al. [41] established entropy measures for the PFSS. They also offered Euclidean distance and hamming distance for the PFSS and utilized their methods for DM [42]. Naeem et al. [43] developed the TOPSIS and VIKOR methods for PFSS and presented an approach for the stock exchange investment problem. Zulqarnain et al. [44] introduced the AOs under the PFSS environment and presented an application for the green supplier chain management. Zulqarnain et al. [45] developed the interaction AOs for PFSS and constructed a DM technique to resolve the MCDM problems. Zulqarnain et al. [46-47] formed the Einstein weighted average and geometric AOs for PFSS. They also proposed the MAGDM techniques using their developed operators for sustainable supplier selection and a business to finance money. Siddique et al. [48] proposed a novel DM technique for PFSS using a score matrix. Zulqarnain et al. [49] introduced the TOPSIS method for PFSS based on the correlation coefficient.

Samarandche [50] proposed the idea of the hypersoft set (HSS), which penetrates multiple subattributes in the parameter function f, which is a characteristic of the Cartesian product with the n attribute. Compared with SS and other existing concepts, Samarandche HSS is the most suitable theory which handles the multiple subattributes of the considered parameters. Several HSS extensions and their decision-making methods have been proposed. Several researchers developed different hybrid structures HSS and presented several AOs with their DM techniques [51-60]. Deli [61] introduced several hybrid structures for other extensions by merging neutrosophic sets and HSS. PFHSS is a hybrid intellectual structure of PFSS. The AOs stated formerly are based on the elementary algebraic product and algebra sum, which is not the only operation that can model the intersection and union of PFHSS. Similarly, Einstein operations contain Einstein product and Einstein sum, an excellent alternate to algebraic product and algebra sum. Moreover, there appears to be some study on aggregation techniques using Einstein operations on PFHSS. Wang and Liu [62] proposed the average AOs under the IFS setting and constructed the MADM approach under their considered environment. Liu and Wang [63] developed a MADM method based on interaction Einstein AOs under the IFS setting.

An enhanced sorting approach fascinates investigators to crack baffling and inadequate information. Rendering to the investigation outcomes, PFHSS plays a vital role in DM by collecting numerous sources into a single value. According to the most generally known knowledge, the emergence of PFSS and hypersoft set (HSS) hybridization has not been combined with the PFSS background. PFHSS is a hybrid intellectual structure of PFSS. So, to encourage the modern exploration of PFHSS, we will state AOs based on rough data. The main intentions of the current study are given as follows:

(i) The PFHSS efficiently deals with the complex apprehensions seeing the multi-sub-attributes of the DM method's considered factors. To reserve this value in attention, we prolong Einstein operational laws for PFHSS and establish the Einstein AOs for PFHSS.

(ii) The Einstein AOs for PFHSS are well-known attractive evaluation AOs. It has been detected that the prevailing AOs feature is insensitive to scratch the exact outcome over the DM method in some states. To overcome these particular obstacles, these AOs need to be reviewed. We determine inventive Einstein operational laws for Pythagorean fuzzy hypersoft numbers (PFHSNs).

(iii) Pythagorean fuzzy hypersoft Einstein weighted average and geometric operators have presented their essential properties expending advanced Einstein operational laws.

(iv) An innovative procedure was established on the intended operators to resolve the DM problem.

(v) Real-life agricultural farming is deliberated to endorse the developed method's supremacy and applicability. The significances show that the prearranged model is more operational and reliable to grip indefinite facts.

This study is systematized as follows. Basic knowledge of some important notions like SS, HSS, IFHSS, PFHSS, and Einstein norms are deliberated in section 2. Section 3 demarcated some basic operational laws for PFHSNs based on Einstein norms and established the PFHSEWA operator. Also, some dynamic properties of the planned operator have been debated in the same section. Section 4 also uses the agricultural example to explain several agricultural problems. The algorithm given in this section shows that it is realistic and appropriate. In Section 5, a comparison with some standing approaches is provided.

2. Preliminaries

This section remembers some fundamental notions such as soft set (SS), HSS, IFHSS, and PFHSS.

Definition 1 (see [28]). Let X and $\mathbb{N}$ be the universe of discourse and set of attributes, respectively. Let $\mathrm{P}(X)$ be the power set of X and $\mathscr{A}\subseteq\mathbb{N}$. A pair $(\Omega, \mathscr{A})$ is called a SS over X, and its mapping is expressed as follows:

$$\Omega : \mathscr{A} \longrightarrow \mathrm{P}(X). \tag{1}$$

Also, it can be defined as follows:

$$(\Omega, \boldsymbol{\mathscr{A}}) = \{\Omega(e) \in \boldsymbol{\mathscr{P}}(X) \colon e \in \mathbb{N}, \Omega(e) = \varnothing \text{if } e \notin \boldsymbol{\mathscr{A}}\}. \tag{2}$$

Definition 2 (see [50]). Let X be a universe of discourse and $\mathrm{P}(X)$ be a power set of X and $k = \{k_1, k_2, k_3, \ldots, k_n\}$, $(n \geq 1)$, and K_i represented the set of attributes and their corresponding subattributes such as $K_i \cap K_j = \varnothing$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3, \ldots, n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \dddot{\mathscr{A}} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of subattributes, where $1 \leq h \leq \alpha, 1 \leq k \leq \beta$, $1 \leq l \leq \gamma$, and $\alpha\beta\gamma \in N$, Then, the pair $(\Omega, K_1 \times K_2 \times K_3 \times \ldots \times K_n) = (\Omega\dddot{\mathscr{A}})$ is known as HSS defined as follows:

$$\Omega\colon K_1 \times K_2 \times K_3 \times \ldots \times K_n = \dddot{\mathscr{A}} \longrightarrow \mathrm{P}(X). \tag{3}$$

It is also defined as

$$(\Omega, \dddot{\mathscr{A}}) = \left\{\check{d}, \Omega_{\dddot{\mathscr{A}}}(\check{d})\colon \check{d} \in \dddot{\mathscr{A}}, \Omega_{\dddot{\mathscr{A}}}(\check{d}) \in \mathrm{P}(X)\right\}. \tag{4}$$

Definition 3 (see [50]). Let X be a universe of discourse and $\mathrm{P}(X)$ be a power set of X and $k = \{k_1, k_2, k_3, \ldots k_n\}$, $(n \geq 1)$, and K_i represented the set of attributes and their corresponding subattributes such as $K_i \cap K_j = \varnothing$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3, \ldots, n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \dddot{\mathscr{A}} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of subattributes, where $1 \leq h \leq \alpha, 1 \leq k \leq \beta$and $1 \leq l \leq \gamma$, and $\alpha\beta\gamma \in N$. And, IFS^X expresses the intuitionistic fuzzy power set over X. Then, the pair $(\Omega, K_1 \times K_2 \times K_3 \times \ldots \times K_n) = (\Omega\dddot{\mathscr{A}})$ is known as IFHSS defined as follows:

$$\Omega\colon K_1 \times K_2 \times K_3 \times \ldots \times K_n = \dddot{\mathscr{A}} \longrightarrow IFS^X. \tag{5}$$

It is also defined as

$$(\Omega, \dddot{\mathscr{A}}) = \left\{\left(\check{d}, \Omega_{\dddot{\mathscr{A}}}(\check{d})\right)\colon \check{d} \in \dddot{\mathscr{A}}, \Omega_{\dddot{\mathscr{A}}}(\check{d}) \in IFS^X\right\}, \tag{6}$$

where $\Omega_{\dddot{\mathscr{A}}}(\check{d}) = \left\{\left\langle \delta, a_{\Omega(\check{d})}(\delta), b_{\Omega(\check{d})}(\delta)\right\rangle\colon \delta \in X\right\}$, where $a_{\Omega(\check{d})}(\delta)$ and $b_{\Omega(\check{d})}(\delta)$ signify the membership value (Mem) and nonmembership value (NMem) of the subattributes: $a_{\Omega(\check{d})}(\delta), b_{\Omega(\check{d})}(\delta), \in [0, 1]$, and $0 \leq a_{\Omega(\check{d})}(\delta) + b_{\Omega(\check{d})}(\delta) \leq 1$.

Definition 4 (see [53]). Let X be a universe of discourse and $\mathrm{P}(X)$ be a power set of X and $k = \{k_1, k_2, k_3, \ldots k_n\}$, $(n \geq 1)$, and K_i represented the set of attributes and their corresponding subattributes such as $K_i \cap K_j = \varnothing$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3, \ldots, n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \dddot{\mathscr{A}} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of subattributes, where $1 \leq h \leq \alpha, 1 \leq k \leq \beta$, $1 \leq l \leq \gamma$ and $\alpha\beta\gamma \in N$. And, PFS^X expresses the Pythagorean fuzzy power set over X. Then, the pair $(\Omega, K_1 \times K_2 \times K_3 \times \ldots \times K_n) = (\Omega\dddot{\mathscr{A}})$ is known as PFHSS defined as follows:

$$\Omega\colon K_1 \times K_2 \times K_3 \times \ldots \times K_n = \dddot{\mathscr{A}} \longrightarrow PFS^X. \tag{7}$$

It is also defined as

$$(\Omega, \dddot{\mathscr{A}}) = \left\{\left(\check{d}, \Omega_{\dddot{\mathscr{A}}}(\check{d})\right)\colon \check{d} \in \dddot{\mathscr{A}}, \Omega_{\dddot{\mathscr{A}}}(\check{d}) \in PFS^X\right\}, \tag{8}$$

where $\Omega_{\dddot{\mathscr{A}}}(\check{d}) = \left\{\delta, a_{\Omega(\check{d})}(\delta), b_{\Omega(\check{d})}(\delta)\colon \delta \in \mathscr{U}\right\}$, where $a_{\Omega(\check{d})}(\delta)$ and $b_{\Omega(\check{d})}(\delta)$ signify the Mem and NMem values of the attributes: $a_{\Omega(\check{d})}(\delta), b_{\Omega(\check{d})}(\delta), \in [0, 1]$, and $0 \leq (a_{\Omega(\check{d})}(\delta))^2 + (b_{\Omega(\check{d})}(\delta))^2 \leq 1$.

A Pythagorean fuzzy hypersoft number (PFHSN) can be stated as $\Omega = \left\{(a_{\Omega(\check{d})}(\delta), b_{\Omega(\check{d})}(\delta))\right\}$, where $0 \leq (a_{\Omega(\check{d})}(\delta))^2 + (b_{\Omega(\check{d})}(\delta))^2 \leq 1$.

Remark 1. If $(a_{\Omega(\check{d})}(\delta))^2 + (b_{\Omega(\check{d})}(\delta))^2$ and $a_{\Omega(\check{d})}(\delta) + b_{\Omega(\check{d})}(\delta) \leq 1$ both are holds, then PFHSS was reduced to IFHSS [58].

For readers' suitability, the PFHSN $\Omega_{\delta_i}(\check{d}_j) = \left\{(a_{\Omega(\check{d}_j)}(\delta_i), \mathrm{b}_{\Omega(\check{d}_j)}(\delta_i))|\delta_i \in \mathscr{U}\right\}$ can be written as $\mathfrak{F}_{\check{d}_{ij}} = a_{\Omega(\check{d}_{ij})}, b_{\Omega(\check{d}_{ij})}$. The score function [65] for $\mathfrak{F}_{\check{d}_{ij}}$ is expressed as follows:

$$\mathbb{S}\left(\mathfrak{F}_{\check{d}_{ij}}\right) = a^2_{\Omega\left(\check{d}_{ij}\right)} - b^2_{\Omega\left(\check{d}_{ij}\right)}\mathbb{S}\left(\mathfrak{F}_{\check{d}_{ij}}\right) \in [-1, 1], \tag{9}$$

However, in some cases, the above-defined score function cannot handle the scenario. For example, if we consider two PFHSNs, such as $\mathfrak{J}_{\check{d}_{11}} = .4, .7$ and $\mathfrak{J}_{\check{d}_{12}} = .5, .8$. The score function cannot deliver relevant results to subtract the PFHSNs. So, in such situations, it is tough to achieve the most suitable alternative $\mathbb{S}(\mathfrak{J}_{\check{d}_{11}}) = .3 = \mathbb{S}(\mathfrak{J}_{\check{d}_{12}})$. To intimidated such problems, the accuracy function [65] had been developed.

$$H\left(\mathfrak{J}_{\check{d}_{ij}}\right)a^2_{\Omega(\check{d}_{ij})} + b^2_{\Omega(\check{d}_{ij})}H\left(\mathfrak{J}_{\check{d}_{ij}}\right) \in [0,1]. \tag{10}$$

The following comparison laws have been projected to compute two PFHSNs $\mathfrak{J}_{\check{d}_{ij}}$ and $\mathfrak{T}_{\check{d}_{ij}}$:

(1) If $\mathbb{S}(\mathfrak{J}_{\check{d}_{ij}}) > \mathbb{S}(\mathfrak{T}_{\check{d}_{ij}})$, then $\mathfrak{J}_{\check{d}_{ij}} > \mathfrak{T}_{\check{d}_{ij}}$

(2) If $\mathbb{S}(\mathfrak{J}_{\check{d}_{ij}}) = \mathbb{S}(\mathfrak{T}_{\check{d}_{ij}})$, then

(3) If $H(\mathfrak{J}_{\check{d}_{ij}}) > H(\mathfrak{T}_{\check{d}_{ij}})$, then $\mathfrak{J}_{\check{d}_{ij}} > \mathfrak{T}_{\check{d}_{ij}}$

(4) If $H(\mathfrak{J}_{\check{d}_{ij}}) = H(\mathfrak{T}_{\check{d}_{ij}})$, then $\mathfrak{J}_{\check{d}_{ij}} = \mathfrak{T}_{\check{d}_{ij}}$

Definition 5. Einstein sum $\oplus_\varepsilon$ and Einstein product $\oplus_\varepsilon$ are good alternatives of algebraic t-norm and t-conorm, respectively, given as follows:

$$a \oplus_\varepsilon b = \frac{a+b}{1+(a.b)} \text{ and } a \otimes_\varepsilon b = \frac{a.b}{1+(1-a).(1-b)}, \tag{11}$$

$\forall (a,b) \in [0,1]^2$.

Under the Pythagorean fuzzy environment, Einstein sum $\oplus_\varepsilon$ and Einstein product $\oplus_\varepsilon$ are defined as follows:

$$a \oplus_\varepsilon b = \sqrt{\frac{a^2+b^2}{1+\left(a^2.b^2\right)}},$$

$$a \otimes_\varepsilon b = \frac{a.b}{\sqrt{1+\left(1-a^2\right).\left(1-b^2\right)}}, \tag{12}$$

$\forall (a,b) \in [0,1]^2$,

where $a \oplus_\varepsilon b$ and $a \oplus_\varepsilon b$ are known as t-norm and t-conorm, respectively, satisfying the bounded, monotonicity, commutativity, and associativity properties.

3. Einstein Weighted Aggregation Operators for Pythagorean Fuzzy Hypersoft Set

This section will introduce a novel Einstein weighted AO such as the PFHSEWA operator for PFHSNs with essential properties.

3.1. Operational Laws for PFHSNs

Definition 6. Let $\mathfrak{J}_{\check{d}_k} = (a_{\check{d}_k}, b_{\check{d}_k})$, $\mathfrak{J}_{\check{d}_{11}} = (a_{\check{d}_{11}}, b_{\check{d}_{11}})$, and $\mathfrak{J}_{\check{d}_{12}} = (a_{\check{d}_{12}}, b_{\check{d}_{12}})$ represent the PFHSNs and ∂ is a positive real number. Then, operational laws for PFHSNs based on Einstein norms can be expressed as follows:

1 $\mathfrak{J}_{\check{d}_{11}} \oplus_\varepsilon \mathfrak{J}_{\check{d}_{12}} = \left(\sqrt{(1+a^2_{\check{d}_{12}}) - (1-a^2_{\check{d}_{12}})} / \sqrt{(1+a^2_{\check{d}_{12}}) + (1-a^2_{\check{d}_{12}})}\right), \left(\sqrt{2b^2_{\check{d}_{12}}} / \sqrt{(2-b^2_{\check{d}_{12}}) + b^2_{\check{d}_{12}}}\right)$

$$\mathfrak{J}_{\check{d}_{11}} \otimes_\varepsilon \mathfrak{J}_{\check{d}_{12}} = \left(\frac{\sqrt{2\alpha^2_{\check{d}_{12}}}}{\sqrt{\left(2-\alpha^2_{\check{d}_{12}}\right)+\alpha^2_{\check{d}_{12}}}}, \frac{\sqrt{\left(1+b^2_{\check{d}_{12}}\right)-\left(1-b^2_{\check{d}_{12}}\right)}}{\sqrt{\left(1+b^2_{\check{d}_{12}}\right)+\left(1-b^2_{\check{d}_{12}}\right)}}\right) \tag{13}$$

2 $\partial\mathfrak{J}_{\check{d}_k} = \left(\sqrt{(1+a^2_{\check{d}_k})^\partial - (1-a^2_{\check{d}_k})^\partial} / \sqrt{(1+a^2_{\check{d}_k})^\partial + (1-a^2_{\check{d}_k})^\partial}\right), \left(\sqrt{2(b^2_{\check{d}_k})^\partial} / \sqrt{(2-b^2_{\check{d}_k})^\partial + (b^2_{\check{d}_k})^\partial}\right)$

3 $\mathfrak{J}^\partial_{\check{d}_k} = \sqrt{2(a)^\partial} / \sqrt{(2-a^2_{\check{d}_k})^\partial + (a^2_{\check{d}_k})^\partial}$, $\left(\sqrt{(1+b^2_{\check{d}_k})^\partial - (1-b^2_{\check{d}_k})^\partial} / \sqrt{(1+b^2_{\check{d}_k})^\partial + (1-b^2_{\check{d}_k})^\partial}\right)$

Definition 7. Let $\mathfrak{J}_{\check{d}_{ij}} = (a_{\check{d}_{ij}}, b_{\check{d}_{ij}})$ be a collection of PFHSNs, then the PFHSEWA operator is defined as follows:

$$PFHSEWA = \left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) \oplus^m_{\varepsilon j=1} \lambda_j \left(\oplus^n_{\varepsilon i=1} \theta_i \mathfrak{J}_{\check{d}_{ij}}\right), \tag{14}$$

where $(i = 1, 2, \ldots, n)$, $(j = 1, 2, \ldots, m)$, and θ_i and λ_j represent the weighted vectors such that $\theta_i > 0$, $\sum^n_{i=1} \theta_i = 1$, and $\lambda_j > 0$ and $\sum^n_{j=1} \lambda_j = 1$.

Theorem 1. *Let $\mathfrak{J}_{\check{d}_{ij}} = (a_{\check{d}_{ij}}, b_{\check{d}_{ij}})$ be a collection of PFHSNs, then the aggregated value attained by equation (3) given as*

$$PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \oplus^m_{j=1} \lambda_j \left(\oplus^n_{i=1} \theta_i \mathfrak{J}_{\check{d}_{ij}}\right),$$

$$= \frac{\sqrt{\prod^m_{j=1}\left(\prod^n_{i=1}\left(1+\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} - \prod^m_{j=1}\left(\prod^n_{i=1}\left(1-\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod^m_{j=1}\left(\prod^n_{i=1}\left(1+\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} + \prod^m_{j=1}\left(\prod^n_{i=1}\left(1-\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod^m_{j=1}\left(\prod^n_{i=1}\left(b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod^m_{j=1}\left(\prod^n_{i=1}\left(2-b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} + \prod^m_{j=1}\left(\prod^n_{i=1}\left(b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}, \tag{15}$$

where $(i = 1, 2, \ldots, n)$, $(j = 1, 2, \ldots, m)$ *and* θ_i *and* λ_j *represent the weight vectors such that* $\theta_i > 0$, $\sum_{i=1}^{n} \theta_i = 1$, *and* $\lambda_j > 0$, $\sum_{j=1}^{n} \lambda_j = 1$.

Proof. We will prove it by using mathematical induction. For $n = 1$, we get $\theta_i = 1$

$$PFHSEWA = \left(\mathfrak{F}_{\breve{d}_{11}}, \mathfrak{F}_{\breve{d}_{12}}, \ldots, \mathfrak{F}_{\breve{d}_{nm}}\right) = \oplus_{j=1}^{m} \lambda_j \mathfrak{F}_{\breve{d}_{1j}}$$
$$= \left\langle \frac{\sqrt{\prod_{j=1}^{m}\left(1 + a_{\breve{d}_{1j}}^2\right)^{\lambda_j} - \prod_{j=1}^{m}\left(1 - a_{\breve{d}_{1j}}^2\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(1 + a_{\breve{d}_{1j}}^2\right)^{\lambda_j} + \prod_{j=1}^{m}\left(1 - a_{\breve{d}_{1j}}^2\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{m}\left(b_{\breve{d}_{1j}}^2\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(2 - b_{\breve{d}_{1j}}^2\right)^{\lambda_j} + \prod_{j=1}^{m}\left(b_{\breve{d}_{1j}}^2\right)^{\lambda_j}}} \right\rangle,$$
$$= \left\langle \frac{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - a_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - a_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(2 - b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \right\rangle. \tag{16}$$

For $m = 1$, we get $\lambda_j = 1$.

$$PFHSEWA\left(\mathfrak{F}_{\breve{d}_{11}}, \mathfrak{F}_{\breve{d}_{12}}, \ldots, \mathfrak{F}_{\breve{d}_{nm}}\right) = \oplus_{i=1}^{n} \theta_i \mathfrak{F}_{\breve{d}_{i1}}$$
$$= \left\langle \frac{\sqrt{\prod_{i=1}^{n}\left(1 + a_{\breve{d}_{i1}}^2\right)^{\theta_i} - \prod_{i=1}^{n}\left(1 - a_{\breve{d}_{i1}}^2\right)^{\theta_i}}}{\sqrt{\prod_{i=1}^{n}\left(1 + a_{\breve{d}_{i1}}^2\right)^{\theta_i} + \prod_{i=1}^{n}\left(1 - a_{\breve{d}_{i1}}^2\right)^{\theta_i}}}, \frac{\sqrt{2\prod_{i=1}^{n}\left(b_{\breve{d}_{i1}}^2\right)^{\theta_i}}}{\sqrt{\prod_{i=1}^{n}\left(2 - b_{\breve{d}_{i1}}^2\right)^{\theta_i} + \prod_{i=1}^{n}\left(b_{\breve{d}_{i1}}^2\right)^{\theta_i}}} \right\rangle$$
$$= \left\langle \frac{\sqrt{\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 + \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 + \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(2 - b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \right\rangle. \tag{17}$$

So, equation (4) is true for $n = 1$ and $m = 1$.

Suppose that equation holds for $n = \delta_2$, $m = \delta_1 + 1$ and for $n = \delta_2 + 1$, $m = \delta_1$,

$$\oplus_{j=1}^{\delta_1+1} \lambda_j\left(\oplus_{i=1}^{\delta_2} \theta_i \mathfrak{F}_{\breve{d}_{ij}}\right) = \left\langle \frac{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1 + \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1 - \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1 + \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1 - \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(b_{\breve{d}_{a_{ij}}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(2 - b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \right\rangle,$$
$$\oplus_{j=1}^{\delta_1} \lambda_j\left(\oplus_{i=1}^{\delta_2+1} \theta_i \mathfrak{F}_{\breve{d}_{ij}}\right) = \left\langle \frac{\sqrt{\prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(1 + \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(1 - \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(1 + \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(1 - \alpha_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(2 - b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1}\left(\prod_{i=1}^{\delta_2+1}\left(b_{\breve{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \right\rangle. \tag{18}$$

Now, we prove the equation for $m = \delta_1 + 1$ and $=$ $n = \delta_2 + 1$:

$$\oplus_{j=1}^{\delta_1+1}\lambda_j\left(\oplus_{i=1}^{\delta_2+1}\theta_i\mathfrak{J}_{\check{d}_{ij}}\right) = \oplus_{j=1}^{\delta_1+1}\lambda_j\left(\oplus_{i=1}^{\delta_2}\theta_i\mathfrak{J}_{\check{d}_{ij}}\oplus\theta_{i+1}\mathfrak{J}_{\check{d}_{(\delta_2+1)j}}\right)$$
$$= \left(\oplus_{j=1}^{\delta_1+1}\oplus_{i=1}^{\delta_2}\theta_i\lambda_j\,\mathfrak{J}_{\check{d}_{ij}}\right)\left(\oplus_{j=1}^{\delta_1+1}\lambda_j\theta_{i+1}\mathfrak{J}_{\check{d}_{(\delta_2+1)j}}\right)$$
$$= \left\langle \frac{\sqrt{\prod_{j=2}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}} \oplus \frac{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\left(1+\alpha_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\left(1+\alpha_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\left(1+\alpha_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1+1}\left(\left(1+\alpha_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j}}}\right.$$
$$\left.\frac{\sqrt{2\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(2+b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \oplus \frac{\sqrt{2\prod_{j=1}^{\delta_1+1}\left(\left(b_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{\delta_1+1}\left(\left(2-b_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\left(b_{\check{d}_{(\delta_2+1)j}}^2\right)^{\theta\delta_2+1}\right)^{\lambda_j}}}\right\rangle$$
$$= \left\langle \frac{\sqrt[r]{\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(1-\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\left(\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(1+\alpha_{\check{d}_{ij}}^2\right)\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(1-\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\left(\prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(2-b_{\check{d}_{ij}}^2\right)\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{\delta_1+1}\left(\prod_{i=1}^{\delta_2+1}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}\right\rangle$$
$$= \oplus_{j=1}^{\delta_1+1}\lambda_j\left(\oplus_{i=1}^{\delta_2+1}\theta_i\mathfrak{J}_{\check{d}_{ij}}\right). \tag{19}$$

So, it is true for $m = \delta_1 + 1$ and $n = \delta_2 + 1$. □

3.2. Example. Let $\mathscr{R} = \{\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4\}$ be a set of experts with the given weight vector $\theta_i = (0.1, 0.3, 0.3, 0.3)^T$. The team of experts is going to describe the attractiveness of a house under-considered set of attributes $A = \{d_1 =$ lawn, $d_2 =$ security system$\}$ with their corresponding subattributes Lawn$d_1 = \{d_{11} =$ with grass, $d_{12} =$ without grass$\}$ and security system$d_2 = \{d_{21} =$ guards, $d_{22} =$ cameras$\}$. Let $\dddot{\mathscr{A}} = d_1 \times d_2$ be a set of subattributes $\dddot{\mathscr{A}} = d_1 \times d_2 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} = \{(d_{11}, d_{21}), (d_{11}, d_{22}), (d_{12}, d_{21}), (d_{12}, d_{22})\}$.

$\dddot{\mathscr{A}} = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ represents the set subattributes with weights with weight vector $\lambda_j = (0.2, 0.2, 0.2, 0.4)^T$. The supposed rating values for all attributes in the form of PFSNs $(\mathscr{H}, \dddot{\mathscr{A}}) = (a_{ij}, b_{ij})_{4\times4}$ are given as follows:

$$(\mathscr{H}, \dddot{\mathscr{A}}) = \begin{matrix} (0.5, 0.8) & (0.7, 0.5) & (0.4, 0.6) & (0.7, 0.4) \\ (0.5, 0.6) & (0.9, 0.1) & (0.3, 0.7) & (0.4, 0.5) \\ (0.4, 0.8) & (0.7, 0.5) & (0.4, 0.6) & (0.3, 0.5) \\ (0.3, 0.7) & (0.6, 0.5) & (0.5, 0.4) & (0.5, 0.7) \end{matrix} \tag{20}$$

As we know that

$$PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \left\langle \frac{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}\right\rangle, \tag{21}$$

$$PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{44}}\right) = \left\langle \begin{matrix} \dfrac{\sqrt{\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{4}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^4\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1+\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\alpha_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}, \\ \dfrac{\sqrt{2\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(2-b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \end{matrix} \right\rangle$$

$$
= \left\langle \frac{\sqrt{\begin{array}{c}
\left\{(1.25)^{0.1}(1.25)^{0.3}(1.16)^{0.3}(1.09)^{0.3}\right\}^{0.2}\left\{(1.49)^{0.1}(1.81)^{0.3}(1.49)^{0.3}(1.36)^{0.3}\right\}^{0.2}\\
\left\{(1.16)^{0.1}(1.09)^{0.3}(1.16)^{0.3}(1.25)^{0.3}\right\}^{0.2}\left\{(1.49)^{0.1}(1.16)^{0.3}(1.09)^{0.3}(1.25)^{0.3}\right\}^{0.4}\\
-\\
\left[\left\{(0.75)^{0.1}(0.75)^{0.3}(0.84)^{0.3}(0.91)^{0.3}\right\}^{0.2}\left\{(0.51)^{0.1}(0.19)^{0.3}(0.51)^{0.3}(0.64)^{0.3}\right\}^{0.2}\right.\\
\left.\left\{(0.84)^{0.1}(0.91)^{0.3}(0.84)^{0.3}(0.75)^{0.3}\right\}^{0.2}\left\{(0.51)^{0.1}(0.84)^{0.3}(0.91)^{0.3}(0.75)^{0.3}\right\}^{0.4}\right.
\end{array}}}{\sqrt{\begin{array}{c}
\left\{(1.25)^{0.1}(1.25)^{0.3}(1.16)^{0.3}(1.09)^{0.3}\right\}^{0.2}\left\{(1.49)^{0.1}(1.81)^{0.3}(1.49)^{0.3}(1.36)^{0.3}\right\}^{0.2}\\
\left\{(1.16)^{0.1}(1.09)^{0.3}(1.16)^{0.3}(1.25)^{0.3}\right\}^{0.2}\left\{(1.49)^{0.1}(1.16)^{0.3}(1.09)^{0.3}(1.25)^{0.3}\right\}^{0.4}\\
+\\
\left[\left\{(0.75)^{0.1}(0.75)^{0.3}(0.84)^{0.3}(0.91)^{0.3}\right\}^{0.2}\left\{(0.51)^{0.1}(0.19)^{0.3}(0.51)^{0.3}(0.64)^{0.3}\right\}^{0.2}\right.\\
\left.\left\{(0.84)^{0.1}(0.91)^{0.3}(0.84)^{0.3}(0.75)^{0.3}\right\}^{0.2}\left\{(0.51)^{0.1}(0.84)^{0.3}(0.91)^{0.3}(0.75)^{0.3}\right\}^{0.4}\right.
\end{array}}}, \frac{\sqrt{2\left[\begin{array}{c}
\left(\left\{(0.64)^{0.1}(0.36)^{0.3}(0.64)^{0.3}(0.49)^{0.3}\right\}^{0.2}\left\{(0.25)^{0.1}(0.01)^{0.3}(0.25)^{0.3}(0.25)^{0.3}\right\}^{0.2}\right.\\
\left.\left\{(0.36)^{0.1}(0.49)^{0.3}(0.36)^{0.3}(0.16)^{0.3}\right\}^{0.2}\left\{(0.16)^{0.1}(0.25)^{0.3}(0.25)^{0.3}(0.49)^{0.3}\right\}^{0.4}\right)
\end{array}\right]}}{\sqrt{\begin{array}{c}
\left\{(1.36)^{0.1}(1.64)^{0.3}(1.36)^{0.3}(1.51)^{0.3}\right\}^{0.2}\left\{(1.75)^{0.1}(1.99)^{0.3}(1.75)^{0.3}(1.75)^{0.3}\right\}^{0.2}\\
\left\{(1.64)^{0.1}(1.51)^{0.3}(1.64)^{0.3}(1.84)^{0.3}\right\}^{0.2}\left\{(1.84)^{0.1}(1.75)^{0.3}(1.75)^{0.3}(1.51)^{0.3}\right\}^{0.4}\\
+\\
\left\{(0.64)^{0.1}(0.36)^{0.3}(0.64)^{0.3}(0.49)^{0.3}\right\}^{0.2}\left\{(0.25)^{0.1}(0.01)^{0.3}(0.25)^{0.3}(0.25)^{0.3}\right\}^{0.2}\\
\left\{(0.36)^{0.1}(0.49)^{0.3}(0.36)^{0.3}(0.16)^{0.3}\right\}^{0.2}\left\{(0.16)^{0.1}(0.25)^{0.3}(0.25)^{0.3}(0.49)^{0.3}\right\}^{0.4}
\end{array}}} \right\rangle
$$

$$
= \left\langle \frac{\sqrt{\begin{array}{c}(1.0324)(1.0897)(1.0309)(1.0734)-\\ \left[(0.9616)(0.8350)(0.9638)(0.9105)\right]\end{array}}}{\sqrt{\begin{array}{c}(1.0324)(1.0897)(1.0309)(1.0734)+\\ \left[(0.9616)(0.8350)(0.9638)(0.9105)\right]\end{array}}}, \frac{\sqrt{2\left[(0.8695)(0.6247)(0.7909)(0.6116)\right]}}{\sqrt{\begin{array}{c}(1.0822)(1.1270)(1.1061)(1.2313)+\\ (0.8695)(0.6247)(0.7909)(0.6116)\end{array}}} \right\rangle
$$

$$
= 0.5263, 0.5225. \tag{22}
$$

Lemma 1. *Let* $\mathfrak{J}_{\check{d}_{ij}} = a_{\check{d}_{ij}}, b_{\check{d}_{ij}}$, *where* $\theta_i > 0$, $\sum_{i=1}^{n} \theta_i = 1$, *and* $\lambda_j > 0$, $\sum_{j=1}^{n} \lambda_j = 1$, *then*

$$
\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\mathfrak{J}_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} = \sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\mathfrak{J}_{\check{d}_{ij}}. \tag{23}
$$

Theorem 2. *Let* $\mathfrak{J}_{\check{d}_{ij}} = a_{\check{d}_{ij}}, b_{\check{d}_{ij}}$ *be a collection of PFHSNs, then*

$$
PFHSWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) \geq PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right), \tag{24}
$$

where $(i = 1, 2, \ldots, n)$, $(j = 1, 2, \ldots, m)$, *and* θ_i *and* λ_j *represent the weight vectors such as* $\theta_i > 0$, $\sum_{i=1}^{n} \theta_i = 1$, *and* $\lambda_j > 0$, $\sum_{j=1}^{n} \lambda_j = 1$.

Proof. As we know that

$$\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}} \leq \sqrt{\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(1+a_{\breve{d}_{ij}}^{2}\right)+\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(1-a_{\breve{d}_{ij}}^{2}\right)}$$

$$\sqrt{\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(1+a_{\breve{d}_{ij}}^{2}\right)+\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(1-a_{\breve{d}_{ij}}^{2}\right)} = \sqrt{2}$$

$$\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+a_{\breve{d}_{ij}}^{2}\right)^{\theta_i)}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-a_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}} \leq \sqrt{2} \tag{25}$$

$$\frac{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+a_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-a_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+a_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-a_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}} \leq \sqrt{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-a_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}},$$

again

$$\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}} \leq \sqrt{\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(2-b_{\breve{d}_{ij}}^{2}\right)+\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(b_{\breve{d}_{ij}}^{2}\right)}$$

$$\sqrt{\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(2-b_{\breve{d}_{ij}}^{2}\right)+\sum_{j=1}^{m}\lambda_j\sum_{i=1}^{n}\theta_i\left(b_{\breve{d}_{ij}}^{2}\right)} \leq \sqrt{2}$$

$$\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}} \leq \sqrt{2} \tag{26}$$

$$\frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}^{2}\right)^{\theta_i}\right)\Big)^{\lambda_j}}} \geq \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\breve{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}.$$

Let $PFHSWA(\mathfrak{J}_{\breve{d}_{11}},\mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}}) = \mathfrak{J}_{\breve{d}_k} = (a_{\mathfrak{J}_{\breve{d}_k}}, b_{\mathfrak{J}_{\breve{d}_k}})$ and PFHSEWA $(\mathfrak{J}_{\breve{d}_{11}},\mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}}) = \mathfrak{J}_{\breve{d}_k}^{\varepsilon} = (a_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}, b_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}})$. Then, inequalities (A) and (B) can be transformed into the forms $a_{\mathfrak{J}_{\breve{d}_k}} \geq a_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}$ and $b_{\mathfrak{J}_{\breve{d}_k}} \leq b_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}$ respectively. So, $S(\mathfrak{J}_{\breve{d}_k}) = a_{\mathfrak{J}_{\breve{d}_k}}^2 - b_{\mathfrak{J}_{\breve{d}_k}}^2 \geq a_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}^2 - b_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}^2 = S(\mathscr{H}^{\varepsilon})$. Hence, $S(\mathfrak{J}_{\breve{d}_k}) \geq S(\mathfrak{J}_{\breve{d}_k}^{\varepsilon})$.

If $SS(\mathfrak{J}_{\breve{d}_k}) > S(\mathfrak{J}_{\breve{d}_k}^{\varepsilon})$, then $PFHSWA(\mathfrak{J}_{\breve{d}_{11}}, \mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}}) > PFHSEWA(\mathfrak{J}_{\breve{d}_{11}},\mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}})$ (C).

If $S(\mathfrak{J}_{\breve{d}_k}) = S(\mathfrak{J}_{\breve{d}_k}^{\varepsilon})$, then $S(\mathfrak{J}_{\breve{d}_k}) = a_{\mathfrak{J}_{\breve{d}_k}}^2 - b_{\mathfrak{J}_{\breve{d}_k}}^2 = a_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}^2 - b_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}^2 = S(\mathfrak{J}_{\breve{d}_k}^{\varepsilon})$.

So, $a_{\mathfrak{J}_{\breve{d}_k}} = a_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}$ and $b_{\mathfrak{J}_{\breve{d}_k}} = b_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}$; then, by accuracy function, $A(\mathfrak{J}_{\breve{d}_k}) = a_{\mathfrak{J}_{\breve{d}_k}}^2 + b_{\mathfrak{J}_{\breve{d}_k}}^2 = a_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}^2 + b_{\mathfrak{J}_{\breve{d}_k}^{\varepsilon}}^2 = A(\mathfrak{J}_{\breve{d}_k}^{\varepsilon})$. Thus, $PFHSWA(\mathfrak{J}_{\breve{d}_{11}}, \mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}}) > PFHSEWA(\mathfrak{J}_{\breve{d}_{11}},\mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}})$(D).

From (C) and (D), we get

$PFHSWA(\mathfrak{J}_{\breve{d}_{11}},\mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}}) > PFHSEWA(\mathfrak{J}_{\breve{d}_{11}},\mathfrak{J}_{\breve{d}_{12}},\ldots,\mathfrak{J}_{\breve{d}_{nm}})$. □

3.3. Example. Using the data given in example 3.1,

$$PFHSWA\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{44}}\right)=\left\langle\sqrt{1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}},\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(b_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right\rangle$$

$$\begin{aligned}PFHSWA\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{44}}\right)&=\left\langle\sqrt{1-\begin{bmatrix}\left[\left\{(0.75)^{0.1}(0.75)^{0.3}(0.84)^{0.3}(0.91)^{0.3}\right\}^{0.2}\left\{(0.51)^{0.1}(0.19)^{0.3}(0.51)^{0.3}(0.64)^{0.3}\right\}^{0.2}\right.\\ \left.\left\{(0.84)^{0.1}(0.91)^{0.3}(0.84)^{0.3}(0.75)^{0.3}\right\}^{0.2}\left\{(0.51)^{0.1}(0.84)^{0.3}(0.91)^{0.3}(0.75)^{0.3}\right\}^{0.4}\right]\end{bmatrix}},\right.\\ &\left.\quad\begin{pmatrix}\left\{(0.8)^{0.1}(0.6)^{0.3}(0.8)^{0.3}(0.7)^{0.3}\right\}^{0.2}\left\{(0.5)^{0.1}(0.1)^{0.3}(0.5)^{0.3}(0.5)^{0.3}\right\}^{0.2}\\ \left\{(0.6)^{0.1}(0.7)^{0.3}(0.6)^{0.3}(0.4)^{0.3}\right\}^{0.2}\left\{(0.4)^{0.1}(0.5)^{0.3}(0.5)^{0.3}(0.7)^{0.3}\right\}^{0.4}\end{pmatrix}\right\rangle\\ &=\left\langle\sqrt{1-[(0.9616)(0.8350)(0.9638)(0.9105)]},((0.9324)(0.7904)(0.8893)(0.7820))\right\rangle\\ &=\langle 0.5404,0.5125\rangle.\end{aligned}\tag{27}$$

Hence, from examples 3.1 and 3.2, it is proved that $PFHSWA(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}})>PFHSEWA(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}})$.

3.4. Properties of PFHSEWA Operator

3.4.1. Idempotency. If $\mathfrak{J}_{\check{d}_{ij}}=\mathfrak{J}_{\check{d}_k}=(a_{\check{d}_{ij}},b_{\check{d}_{ij}})$ $\forall i,j$, then PFHSEWA $(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}})=\mathfrak{J}_{\check{d}_k}$.

Proof. We know that

$$\begin{aligned}&PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)\\ &=\left\langle\frac{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}},\frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}\right\rangle\\ &=\left\langle\frac{\sqrt{\left(\left(1+\alpha_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}-\left(\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}}}{\sqrt{\left(\left(1+\alpha_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}+\left(\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}}},\frac{\sqrt{2\left(\left(b_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}}}{\sqrt{\left(\left(2-b_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}+\left(\left(b_{\check{d}_{ij}}^{2}\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}}}\right\rangle\\ &=\frac{\sqrt{\left(1+\alpha_{\check{d}_{ij}}^{2}\right)-\left(1-\alpha_{\check{d}_{ij}}^{2}\right)}}{\sqrt{\left(1+\alpha_{\check{d}_{ij}}^{2}\right)+\left(1-\alpha_{\check{d}_{ij}}^{2}\right)}},\frac{\sqrt{2b_{\check{d}_{ij}}^{2}}}{\sqrt{\left(2-b_{\check{d}_{ij}}^{2}\right)+\left(b_{\check{d}_{ij}}^{2}\right)}}\\ &=a_{\check{d}_{ij}},b_{\check{d}_{ij}}\\ &=\mathfrak{J}_{\check{d}_k}.\end{aligned}\tag{28}$$

□

3.4.2. Boundedness. Let $\mathfrak{J}_{\check{d}_{ij}}=(a_{\check{d}_{ij}},b_{\check{d}_{ij}})$ be a Collection PFHSNs and $\mathfrak{J}_{\min}=\min(\mathfrak{J}_{\check{d}_{ij}})\mathfrak{J}_{\max}=\max(\mathfrak{J}_{\check{d}_{ij}})$. Then, $\mathfrak{J}_{\min}\leq PFHSEWA\leq(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}})\leq\mathfrak{J}_{\max}$.

Proof. Let $g(y)=\sqrt{1-y^2/1+y^2}, y\in 0,1]]$, then $(d/dy)(g(y))=(-2y/(1+y^2)^2)\sqrt{1+y^2/1-y^2}<0$ which shows that $g(y)$ is decreasing function on $0,1]]$. So,

$a_{\min} \le a_{ij} \le a_{\max}, \forall i, j$. Hence, $g(a_{\max}) \le g(a_{\breve{d}_{ij}}) \le g(a_{\min})$, $\forall i, j$

$\Rightarrow \sqrt{1 - a_{\max}^2 /} \quad 1 + a_{\max}^2 \le \sqrt{1 - a_{\breve{d}_{ij}}^2 / 1 + a_{\breve{d}_{ij}}^2} \le \sqrt{1 - a_{\min}^2 / 1 + a_{\min}^2}$, $(i = 1, 2, \ldots, n)$ and $(j = 1, 2, \ldots, m)$.

Let θ_i and λ_j represent the weight vectors such as $\theta_i > 0$, $\sum_{i=1}^n \theta_i = 1$, and $\lambda_j > 0$, $\sum_{j=1}^n \lambda_j = 1$. We have

$$
\begin{aligned}
&\Leftrightarrow \sqrt{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\max}^2/1 + a_{\max}^2\right)^{\theta_i} \right)^{\lambda_j}} \le \sqrt{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} \le \sqrt{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\min}^2/1 + a_{\min}^2\right)^{\theta_i} \right)^{\lambda_j}} \\
&\Leftrightarrow \sqrt{\left(\left(1 - a_{\max}^2/1 + a_{\max}^2\right)^{\sum_{i=1}^n \theta_i} \right)^{\sum_{j=1}^m \lambda_j}} \le \sqrt{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} \le \sqrt{\left(\left(1 - a_{\min}^2/1 + a_{\min}^2\right)^{\sum_{i=1}^n \theta_i} \right)^{\sum_{j=1}^m \lambda_j}} \\
&\Leftrightarrow \sqrt{1 + \left(\frac{1 - a_{\max}^2}{1 + a_{\max}^2}\right)} \le \sqrt{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} \le \sqrt{1 + \left(\frac{1 - a_{\min}^2}{1 + a_{\min}^2}\right)} \\
&\Leftrightarrow \sqrt{\frac{2}{1 + a_{\max}^2}} \le \sqrt{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} \le \sqrt{\frac{2}{1 + a_{\min}^2}} \\
&\Leftrightarrow \sqrt{\frac{1 + a_{\min}^2}{2}} \le \left(\frac{1}{\sqrt{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}}} \right) \le \sqrt{\frac{1 + a_{\max}^2}{2}} \\
&\Leftrightarrow \sqrt{1 + a_{\min}^2} \le \sqrt{\frac{2}{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}}} \le \sqrt{1 + a_{\max}^2} \\
&\Leftrightarrow \sqrt{1 + a_{\min}^2 - 1} \le \sqrt{\frac{2}{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} - 1} \le \sqrt{1 + a_{\max}^2 - 1} \\
&\Leftrightarrow \sqrt{a_{\min}^2} \le \sqrt{\frac{2}{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} - 1} \le \sqrt{a_{\max}^2} \\
&\Leftrightarrow a_{\min} \le \sqrt{\frac{2}{1 + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2/1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}} - 1} \le a_{\max} \\
&a_{\min} \le \frac{\sqrt{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 + a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j} + \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - a_{\breve{d}_{ij}}^2\right)^{\theta_i} \right)^{\lambda_j}}} \le a_{\max}.
\end{aligned}
\tag{29}
$$

Let $f(x) = \sqrt{2 - x^2/x^2}$, $x \in 0,1]]$, then $(d/dx)(f(x)) = (-2/x^3)\sqrt{x^2/2 - x^2} < 0$. So, $f(x)$ is decreasing function on $0,1]]$. Since $b_{\min} \le b_{ij} \le b_{\max}$, $\forall i, j$, then $f(b_{\max}) \le f(b_{ij}) \le f(b_{\min})$. So, $\sqrt{(2 - b_{\max}^2/b_{\max}^2)} \le \sqrt{2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2} \le \sqrt{(2 - b_{\min}^2/b_{\min}^2)}$, $(i = 1, 2, \ldots, n)$ and $(j = 1, 2, \ldots, m)$. Let θ_i and λ_j represent the weight vectors such as $\theta_i > 0$, $\sum_{i=1}^{n}\theta_i = 1$, and $\lambda_j > 0$, $\sum_{j=1}^{n}\lambda_j = 1$. We have

$$\begin{aligned}
&\Leftrightarrow \sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\max}^2/b_{\max}^2\right)^{\theta_i}\right)^{\lambda_j}} \le \sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}} \le \sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\min}^2/b_{\min}^2\right)^{\theta_i}\right)^{\lambda_j}} \\
&\Leftrightarrow \sqrt{\left(\left(2 - b_{\max}^2/b_{\max}^2\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}} \le \sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}} \le \sqrt{\left(\left(2 - b_{min}^2/b_{min}^2\right)^{\sum_{i=1}^{n}\theta_i}\right)^{\sum_{j=1}^{m}\lambda_j}} \\
&\Leftrightarrow \sqrt{1 + \frac{2 - b_{\max}^2}{b_{\max}^2}} \le \sqrt{1 + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}} \le \sqrt{1 + \frac{2 - b_{\min}^2}{b_{\min}^2}} \\
&\Leftrightarrow \sqrt{\frac{2}{b_{\max}^2}} \le \sqrt{1 + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}} \le \sqrt{\frac{2}{b_{\min}^2}} \\
&\Leftrightarrow \sqrt{\frac{b_{\min}^2}{2}} \le \frac{1}{\sqrt{1 + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \le \sqrt{b_{\max}^2/2} \\
&\Leftrightarrow b_{\min} \le \sqrt{\frac{2}{1 + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2/b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \le b_{\max} \\
&b_{\min} \le \frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2 - b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^2\right)^{\theta_i}\right)^{\lambda_j}}} \le b_{\max}.
\end{aligned} \tag{30}$$

Let $PFHSEWA(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}) = \mathfrak{J}_{\check{d}_k}$. Then, inequalities (E) and (F) can be written as $a_{\min} \le a \le a_{\max}$ and $b_{\min} \le b \le b_{\max}$. Thus, S $(\mathfrak{J}_{\check{d}_k}) = a^2 - b^2 \le a_{\max}^2 - b_{\min}^2 = S(\mathfrak{J}_{\check{d}_k^{\max}})$ and S $(S(\mathfrak{J}_{\check{d}_k}) = a^2 - b^2 \ge a_{\min}^2 - b_{\max}^2 = S(\mathfrak{J}_{\check{d}_k^{\min}})$.

If $S(\mathfrak{J}_{\check{d}_k}) < S(\mathfrak{J}_{\check{d}_k^{\max}})$ and $S(\mathfrak{J}_{\check{d}_k}) > S(\mathfrak{J}_{\check{d}_k^{\min}})$, then we have

$$\mathfrak{J}_{\check{d}_k^{\min}} < PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) < \mathfrak{J}_{\check{d}_k^{\min}}, \tag{31}$$

If $S(\mathfrak{J}_{\check{d}_k}) = S(\mathfrak{J}_{\check{d}_k^{\max}})$, then we have $a^2 = a_{\max}^2$ and $b^2 = b_{\max}^2$. Thus, $S(\mathfrak{J}_{\check{d}_k}) = a^2 - b^2 == a_{\max}^2 - b_{\max}^2 = S(\mathfrak{J}_{\check{d}_k^{\max}})$. Therefore,

$$PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \mathfrak{J}_{\check{d}_k^{\max}}. \tag{32}$$

If S $(\mathfrak{J}_{\check{d}_k}) = S(\mathfrak{J}_{\check{d}_k\min})$, then we have $a^2 - b^2 = a_{\min}^2 - b_{\min}^2 \Rightarrow a^2 = a_{\min}^2$ and $b^2 = b_{\min}^2$.

Thus, $A(\mathfrak{J}_{\check{d}_k}) = a^2 + b^2 = a_{\min}^2 + b_{\min}^2 = A(\mathfrak{J}_{\check{d}_k^{\min}})$. So,

$$PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \mathfrak{J}_{\check{d}_k^{\min}}. \tag{33}$$

$$\mathfrak{J}_{\check{d}_k^{\min}} \le PFHSEWA(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}) \le \mathfrak{J}_{\check{d}_k^{\max}}.$$ □

3.4.3. Homogeneity. Prove that $PFHSEWA(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}) = \partial P\ FHSEWA(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}})$ for $\partial > 0$.

Proof. Let $\mathfrak{J}_{\check{d}_{ij}}$ be a PFHSN and ∂ is a positive number, then by $\partial\mathfrak{J}_{\check{d}_{ij}} = (\sqrt{(1+a^2_{\check{d}_{ij}})^{\partial} - (1-a^2_{\check{d}_{ij}})^{\partial}} / \sqrt{(1+a^2_{\check{d}_{ij}})^{\partial} + (1-a^2_{\check{d}_{ij}})^{\partial}}), \sqrt{2(b^2_{\check{d}_{ij}})^{\partial}} / \sqrt{(2-b^2_{\check{d}_{ij}})^{\partial} + (b^2_{\check{d}_{ij}})^{\partial}}$.
So,

$$
\begin{aligned}
&PFHSEWA\left(\partial\mathfrak{J}_{\check{d}_{11}}, \partial\mathfrak{J}_{\check{d}_{12}} \partial\mathfrak{J}_{\check{d}_{nm}}\right) \\
&= \left\langle \frac{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j} + \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}}} \right\rangle \\
&= \left\langle \frac{\sqrt{\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial} - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial}}}{\sqrt{\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial} + \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial}}}, \frac{\sqrt{\left(2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial}}}{\sqrt{\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial} + \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b^2_{\check{d}_{ij}}\right)^{\theta_i}\right)^{\lambda_j}\right)^{\partial}}} \right\rangle \\
&= PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}} \mathfrak{J}_{\check{d}_{nm}}\right).
\end{aligned} \tag{34}
$$

□

4. Novel Multicriteria Decision-Making Approach

This section has developed a DM approach for solving MCDM problems based on the proposed PFHSEWA operator and numerical examples.

4.1. Proposed Approach. Consider $\mathfrak{H} = \{\mathfrak{H}^1, \mathfrak{H}^2, \mathfrak{H}^3, \ldots, \mathfrak{H}^s\}$ be a set of alternatives and $O = \{O_1, O_2, O_3, \ldots, O_r\}$ be a set of experts. The weights of experts are given as $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_n)^T$ such that $\theta_i > 0$, $\sum_{i=1}^{n} \theta_i = 1$. Let L= $\{d_1, d_2, \ldots, d_m\}$ express the set of attributes with their corresponding multi-sub-attributes such as $\mathfrak{L}' = \{(d_{1\rho} \times d_{2\rho} \times \ldots \times d_{m\rho})$ for all $\rho \in \{1, 2, \ldots, t\}\}$ with weights $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_n)^T$ such that $\theta_i > 0$, $\sum_{i=1}^{n} \theta_i = 1$, and can be stated as $\mathfrak{L}' = \{\check{d}_{\partial}\colon \partial \in \{1, 2, \ldots, m\}\}$. The group of experts $\{\kappa^i, i = 1, 2, \ldots, n\}$ assess the alternatives $\{\mathfrak{H}^{(z)}, z = 1, 2, \ldots S\}$ under the chosen subattributes $\{\check{d}_{\partial}, \partial = 1, 2, \ldots, k\}$ in the form of PFHSNs such as $(\mathfrak{H}^{(z)}_{\check{d}_{\partial}})_{n\times m} = (\alpha_{\check{d}_{ij}}, b_{\check{d}_{ij}})_{n*m}$, where $0 \leq \alpha_{\check{d}_{ij}}, b_{\check{d}_{ij}} \leq 1$ and $0 \leq (\alpha_{\check{d}_{ij}})^2 + (b_{\check{d}_{ij}})^2 \leq 1$ for all i, k. The experts provide their opinion in the form of PFHSNs $\mathscr{L}_k$ for each alternative and present the step-wise algorithm to obtain the most suitable alternative.

Step 1. Obtain decision matrices $F = (\mathfrak{J}_{\check{d}_{ij}})_{n*m}$ in the form of PFHSNs for alternatives relative to attributes.

$$
\left(\mathfrak{H}^{(z)}_{\check{d}_{ik}}, \mathfrak{L}'\right)_{n\times\partial} = \begin{matrix} O_1 \\ O_2 \\ \vdots \\ O_n \end{matrix} \begin{pmatrix} \left(\alpha^{(z)}_{\check{d}_{11}}, b^{(z)}_{\check{d}_{11}}\right) & \left(\alpha^{(z)}_{\check{d}_{12}}, b^{(z)}_{\check{d}_{12}}\right) & \cdots & \left(\alpha^{(z)}_{\check{d}_{1\partial}}, b^{(z)}_{\check{d}_{1\partial}}\right) \\ \left(\alpha^{(z)}_{\check{d}_{21}}, b^{(z)}_{\check{d}_{21}}\right) & \left(\alpha^{(z)}_{\check{d}_{22}}, b^{(z)}_{\check{d}_{22}}\right) & \cdots & \left(\alpha^{(z)}_{\check{d}_{2\partial}}, b^{(z)}_{\check{d}2\partial}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\alpha^{(z)}_{\check{d}_{n1}}, b^{(z)}_{\check{d}_{n1}}\right) & \left(\alpha^{(z)}_{\check{d}_{n2}}, b^{(z)}_{\check{d}_{n2}}\right) & \cdots & \left(\alpha^{(z)}_{\check{d}_{n\partial}}, b^{(z)}_{\check{d}_{n\partial}}\right) \end{pmatrix}. \tag{35}
$$

Step 2. Use the normalization formula to normalize the decision matrix and convert the rating value of the cost type parameter to the benefit type parameter.

$$
M_{\check{d}_{ij}} = \begin{cases} \mathfrak{J}^c_{\check{d}_{ij}} = \left(b_{\check{d}_{ij}}, \alpha_{\check{d}_{ij}}\right) \text{costtypeparameter}, \\ \mathfrak{J}_{\check{d}_{ij}} = \left(\alpha_{\check{d}_{ij}}, b_{\check{d}_{ij}}\right) \text{benefittypeparameter}. \end{cases} \tag{36}
$$

Step 3. Use the settled PFHSEWA operator to collect the PFHSNs $\mathfrak{J}_{\check{d}_{ij}}$ for each alternative $\mathfrak{H} = \{\mathfrak{H}^1, \mathfrak{H}^2, \mathfrak{H}^3, \ldots, \mathfrak{H}^s\}$ into the decision matrix $\mathscr{L}_k$.

Step 4. Use equation (1) to calculate the scores for all alternatives.

Step 5. Choose the alternative with the highest score.

Step 6. Rank the alternatives.

The graphical demonstration of the planned model is given in Figure 1.

4.2. Numerical Example. In this section, a practical MCDM problem comprises decisive adequate agricultural models in numerous kinds of farming to confirm that the conventional approach is pertinent and reasonable.

4.2.1. Case Study. Green agriculture claims sustainable growth ideas to farming, such as confirming food production and fiber while preserving financial and societal constrictions to ensure the long-term viability of production. For example, sustainable agriculture diminishes the practice of pesticides that are harmful to the health of agriculturalists and customers. Accuracy farming and intellectual farming

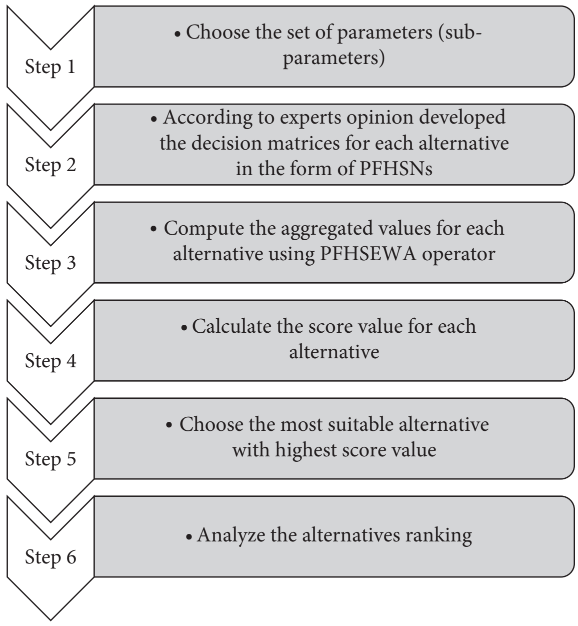

FIGURE 1: Flowchart of the proposed model.

are the core mechanisms of sustainable farming. Increasing crops and raising livestock are agricultural professions or businesses. Farming comprises raising animals and growing crops, which deliver nutrition and raw ingredients. Agriculture was initiated approximately 5,000 years earlier, but the particular time and source are indefinite. Agriculture is a technique of lifecycle, not just a profession. We are all farmers, and we like farming no matter whether we are at home or in the fields. This love of gardening must be a lifelong practice, regardless of age. Due to this land devastation, food prices will skyrocket, and we will have to pay more for daily food needs. Farmers must focus on increasing production through agricultural robots to get out of this situation. The use of robots in farming is an illustration of inspiration beyond origination. Agriculture, as an industry, will grow into a high-tech sector in the new era. Agribots or agri-robots are other terms for agricultural robots [64]. Five key alternatives are interrelated to sustainable agriculture such as good crop production ($\mathfrak{H}^1$), environmental protection ($\mathfrak{H}^2$), natural resources availability ($\mathfrak{H}^3$), food security and productivity ($\mathfrak{H}^4$), and availability of machines ($\mathfrak{H}^5$). In addition, the abovementioned five alternatives are evaluated using four parameters. The attribute of robotic agriculture is given as follows: $\mathfrak{L}\{d_1 = \text{Qualityproduction}, d_2 = \text{Completionofatime–consumingproject}, d_3 = \text{Consistentroleincompletingaproject}, d_4 = \text{Limitingtheneedformanuallabor}\}$. The corresponding subattributes of the considered parameters are $\text{Qualityproduction} = d_1 = \{d_{11} = \text{High} - \text{qualityproduction}, d_{12} = \text{Low} - \text{qualityproduction}\}\{d_{11} = \text{High–qualityproduction}, d_{12} = \text{Low} - \text{qualityproduction}\}$, $\text{Completionofatime} - \text{consumingproject} = d_2 = \{d_{21} = \text{Shortterm}, d_{22} = \text{Longterm}\} = \{d_{21} = \text{Shortterm}, d_{22} = \text{Long term}\}$, $\text{Consistentroleincompletingaproject} = d_3 = \{d_{31} = \text{Projectbudgetingandforecasting}, d_{32} = \text{Developingariskmanagementplan}\} = \{d_{31} = \text{Projectbudgetingandforecasting}, d_{32} = \text{Developingariskmanagementplan}\}$, $\text{Limiting the need for manual labor} = d_4 = \{d_{41} = \text{Limiting the need for manual labor}\} = \{d_{41} = \text{Limiting the need for manual labor}\}$. Let $\mathfrak{L}' = d_1 \times d_2 \times d_3 \times d_4$ be a set of subattributes.

$\mathfrak{L}' = d_1 \times d_2 \times d_3 \times d_4 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} \times \{d_{31}, d_{32}\} \times \{d_{41}\} = \{ (d_{11}, d_{21}, d_{31}, d_{41}), (d_{11}, d_{21}, d_{32}, d_{41}), (d_{11}, d_{22}, d_{31}, d_{41}), (d_{11}, d_{22}, d_{32}, d_{41}), (d_{12}, d_{21}, d_{31}, d_{41}), (d_{12}, d_{21}, d_{32}, d_{41}), (d_{12}, d_{22}, d_{31}, d_{41}), (d_{12}, d_{22}, d_{32}, d_{41})\}$, $\mathfrak{L}' = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4, \check{d}_5, \check{d}_6, \check{d}_7, \check{d}_8\}$ be a set of all sub-attributes with weights $(0.12, 0.18, 0.1, 0.15, 0.05, 0.22, 0.08, 0.1)^T$. Let $\{O_1, O_2, O_3\}$ be a set of three experts with weights $(0.143, 0.514, 0.343)^T$. To judge the optimum alternative, specialists provide their preferences in the form of PFHSNs.

4.2.2. PFHSEWA Operator

Step 1. According to the expert's opinion, Pythagorean fuzzy hypersoft decision matrices for all alternatives are given in Tables 1–5.

Step 2. There is no need to normalize because all parameters are the same type.

Step 3. Apply the proposed PFHSEWA operator to the obtained data (Tables 1–5), and then we get the opinions of decision-makers on alternatives in the form of PFHSN, for example,

Table 1: PFHS decision matrix for $\mathfrak{H}^1$.

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$	$\check{d}_5$	$\check{d}_6$	$\check{d}_7$	$\check{d}_8$
O_1	(0.3,0.8)	(0.7 0.3)	(0.6,0.7)	(0.5,0.4)	(0.2,0.4)	(0.4,0.6)	(0.5,0.8)	(0.9,0.3)
O_2	(0.7,0.6)	(0.3,0.4)	(0.6,0.5)	(0.3,0.9)	(0.5,0.4)	(0.4,0.6)	(0.7,0.5)	(0.4,0.8)
O_3	(0.5,0.7)	(0.8,0.5)	(0.7,0.4)	(0.4,0.3)	(0.4,0.9)	(0.2,0.4)	(0.8,0.4)	(0.7,0.5)

Table 2: PFHS decision matrix for $\mathfrak{H}^2$.

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$	$\check{d}_5$	$\check{d}_6$	$\check{d}_7$	$\check{d}_8$
O_1	(0.6,0.7)	(0.4,0.6)	(0.3,0.4)	(0.9,0.2)	(0.3,0.8)	(0.2,0.4)	(0.7,0.5)	(0.4,0.5)
O_2	(0.8,0.5)	(0.7,0.4)	(0.9,0.2)	(0.7,0.4)	(0.4,0.5)	(0.9,0.3)	(0.2,0.7)	(0.3,0.8)
O_3	(0.8,0.5)	(0.7,0.4)	(0.8,0.5)	(0.5,0.2)	(0.5,0.7)	(0.7,0.5)	(0.7,0.6)	(0.6,0.4)

Table 3: PFHS decision matrix for $\mathfrak{H}^3$.

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$	$\check{d}_5$	$\check{d}_6$	$\check{d}_7$	$\check{d}_8$
O_1	(0.7,0.3)	(0.2,0.5)	(0.1,0.6)	(0.3,0.4)	(0.4 0.6)	(0.8,0.4)	(0.6,0.7)	(0.2,0.5)
O_2	(0.3,0.7)	(0.4,0.5)	(0.4,0.8)	(0.3,0.4)	(0.6,0.7)	(0.3,0.4)	(0.9,0.2)	(0.7,0.2)
O_3	(0.6,0.8)	(0.4,0.5)	(0.6,0.5)	(0.6,0.4)	(0.7,0.5)	(0.8,0.4)	(0.5,0.8)	(0.4 0.5)

Table 4: PFHS decision matrix for $\mathfrak{H}^4$.

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$	$\check{d}_5$	$\check{d}_6$	$\check{d}_7$	$\check{d}_8$
O_1	(0.8,0.4)	(0.2,0.9)	(0.2,0.4)	(0.4,0.6)	(0.6,0.5)	(0.5,0.6)	(0.4,0.5)	(0.8,0.3)
O_2	(0.5,0.4)	(0.7,0.6)	(0.9,0.3)	(0.8,0.5)	(0.9,0.2)	(0.2,0.4)	(0.4,0.6)	(0.6,0.5)
O_3	(0.5,0.7)	(0.9,0.3)	(0.3,0.5)	(0.5,0.7)	(0.3,0.5)	(0.8,0.5)	(0.7,0.5)	(0.2,0.5)

Table 5: PFHS decision matrix for $\mathfrak{H}^5$.

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$	$\check{d}_5$	$\check{d}_6$	$\check{d}_7$	$\check{d}_8$
O_1	(0.5,0.7)	(0.8,0.5)	(0.7,0.4)	(0.4,0.3)	(0.4,0.9)	(0.2,0.4)	(0.8,0.4)	(0.7,0.5)
O_2	(0.8,0.5)	(0.7,0.4)	(0.8,0.5)	(0.5,0.2)	(0.5,0.7)	(0.7,0.5)	(0.7,0.6)	(0.6,0.4)
O_3	(0.5,0.4)	(0.4,0.8)	(0.5,0.6)	(0.3,0.4)	(0.7,0.6)	(0.7,0.5)	(0.4,0.9)	(0.5,0.2)

$$
\begin{aligned}
& PFHSEWA\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}}\mathfrak{J}_{\check{d}_{nm}}\right) \\
& =\left\langle \frac{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1+\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\alpha_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}, \frac{\sqrt{2\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}}{\sqrt{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(2-b_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}+\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(b_{\check{d}_{ij}}^{2}\right)^{\theta_i}\right)^{\lambda_j}}} \right\rangle
\end{aligned}
\tag{37}
$$

$$
\mathscr{L}_2=\left\langle
\frac{\sqrt{\begin{array}{c}
\left\{(1.36)^{0.12}(1.16)^{0.18}(1.09)^{0.1}(1.81)^{0.15}(1.09)^{0.05}(1.04)^{0.22}(1.49)^{0.08}(1.16)0.1\right\}^{0.143}\\
\left\{(1.64)^{0.12}(1.49)^{0.18}(1.81)^{0.1}(1.49)^{0.15}(1.16)^{0.05}(1.81)^{0.22}(1.04)^{0.08}(1.09)0.1\right\}^{0.514}\\
\left\{(1.64)^{0.12}(1.49)^{0.18}(1.64)^{0.1}(1.25)^{0.15}(1.25)^{0.05}(1.49)^{0.22}(1.49)^{0.08}(1.36)0.1\right\}^{0.343}\\
-\\
\left\{(0.64)^{0.12}(0.84)^{0.18}(0.91)^{0.1}(0.19)^{0.15}(0.91)^{0.05}(0.96)^{0.22}(0.51)^{0.08}(0.84)^{0.1}\right\}^{0.143}\\
\left\{(0.36)^{0.12}(0.51)^{0.18}(0.19)^{0.1}(0.51)^{0.15}(0.84)^{0.05}(0.19)^{0.22}(0.96)^{0.08}(0.91)^{0.1}\right\}^{0.514}\\
\left\{(0.36)^{0.12}(0.51)^{0.18}(0.36)^{0.1}(0.75)^{0.15}(0.75)^{0.05}(0.51)^{0.22}(0.51)^{0.08}(0.64)^{0.1}\right\}^{0.343}
\end{array}}}
{\sqrt{\begin{array}{c}
\left\{(1.36)^{0.12}(1.16)^{0.18}(1.09)^{0.1}(1.81)^{0.15}(1.09)^{0.05}(1.04)^{0.22}(1.49)^{0.08}(1.16)^{0.1}\right\}^{0.143}\\
\left\{(1.64)^{0.12}(1.49)^{0.18}(1.81)^{0.1}(1.49)^{0.15}(1.16)^{0.05}(1.81)^{0.22}(1.04)^{0.08}(1.09)^{0.1}\right\}^{0.514}\\
\left\{(1.64)^{0.12}(1.49)^{0.18}(1.64)^{0.1}(1.25)^{0.15}(1.25)^{0.05}(1.49)^{0.22}(1.49)^{0.08}(1.36)0.1\right\}^{0.343}\\
+\\
\left\{(0.64)^{0.12}(0.84)^{0.18}(0.91)^{0.1}(0.19)^{0.15}(0.91)^{0.05}(0.96)^{0.22}(0.51)^{0.08}(0.84)^{0.1}\right\}^{0.143}\\
\left\{(0.36)^{0.12}(0.51)^{0.18}(0.91)^{0.1}(0.51)^{0.15}(0.84)^{0.05}(0.91)^{0.22}(0.96)^{0.08}(0.91)^{0.1}\right\}^{0.514}\\
\left\{(0.36)^{0.12}(0.51)^{0.18}(0.36)^{0.1}(0.75)^{0.15}(0.75)^{0.05}(0.51)^{0.22}(0.51)^{0.08}(0.64)^{0.1}\right\}^{0.343}
\end{array}}},
\frac{\sqrt{2\left[\begin{array}{c}
\left\{(0.49)^{0.12}(0.36)^{0.18}(0.16)^{0.1}(0.04)^{0.15}(0.64)^{0.05}(0.16)^{0.22}(0.25)^{0.08}(0.25)^{0.1}\right\}^{0.143}\\
\left\{(0.25)^{0.12}(0.16)^{0.18}(0.04)^{0.1}(0.16)^{0.15}(0.25)^{0.05}(0.09)^{0.22}(0.49)^{0.08}(0.64)^{0.1}\right\}^{0.514}\\
\left\{(0.25)^{0.12}(0.16)^{0.18}(0.25)^{0.1}(0.04)^{0.15}(0.49)^{0.05}(0.25)^{0.22}(0.36)^{0.08}(0.16)^{0.1}\right\}^{0.343}
\end{array}\right]}}
{\sqrt{\begin{array}{c}
\left\{(1.51)^{0.12}(1.64)^{0.18}(1.84)^{0.1}(1.96)^{0.15}(1.36)^{0.05}(1.84)^{0.22}(1.75)^{0.08}(1.75)^{0.1}\right\}^{0.143}\\
\left\{(1.75)^{0.12}(1.84)^{0.18}(1.96)^{0.1}(1.84)^{0.15}(1.75)^{0.05}(1.91)^{0.22}(1.51)^{0.08}(1.36)^{0.1}\right\}^{0.514}\\
\left\{(1.75)^{0.12}(1.84)^{0.18}(1.75)^{0.1}(1.96)^{0.15}(1.51)^{0.05}(1.75)^{0.22}(1.64)^{0.08}(1.84)^{0.1}\right\}^{0.343}\\
+\\
\left\{(0.49)^{0.12}(0.36)^{0.18}(0.16)^{0.1}(0.04)^{0.15}(0.64)^{0.05}(0.16)^{0.22}(0.25)^{0.08}(0.25)^{0.1}\right\}^{0.143}\\
\left\{(0.25)^{0.12}(0.16)^{0.18}(0.04)^{0.1}(0.16)^{0.15}(0.25)^{0.05}(0.09)^{0.22}(0.49)^{0.08}(0.64)^{0.1}\right\}^{0.514}\\
\left\{(0.25)^{0.12}(0.16)^{0.18}(0.25)^{0.1}(0.04)^{0.15}(0.49)^{0.05}(0.25)^{0.22}(0.36)^{0.08}(0.16)^{0.1}\right\}^{0.343}
\end{array}}}
\right\rangle
\tag{38}
$$

$$\langle 0.7105, 0.4250\rangle$$

$$\mathscr{L}_3=\left\langle \frac{\sqrt{\begin{array}{c}\left\{(1.49)^{0.12}(1.04)^{0.18}(1.01)^{0.1}(1.09)^{0.15}(1.16)0.05(1.64)0.22(1.36)0.08(1.04)0.1\right\}0.143\\ \left\{(1.09)^{0.12}(1.16)^{0.18}(1.16)^{0.1}(1.09)^{0.15}(1.36)0.05(1.09)0.22(1.81)0.08(1.49)0.1\right\}0.514\\ \left\{(1.36)^{0.12}(1.16)^{0.18}(1.36)^{0.1}(1.36)^{0.15}(1.49)0.05(1.64)0.22(1.25)0.08(1.16)0.1\right\}0.343\\ -\\ \left\{(0.51)^{0.12}(0.96)^{0.18}(0.01)^{0.1}(0.91)^{0.15}(0.84)^{0.05}(0.36)^{0.22}(0.64)^{0.08}(0.96)^{0.1}\right\}^{0.143}\\ \left\{(0.91)^{0.12}(0.84)^{0.18}(0.84)^{0.1}(0.91)^{0.15}(0.64)^{0.05}(0.91)^{0.22}(0.19)^{0.08}(0.51)^{0.1}\right\}^{0.514}\\ \left\{(0.64)^{0.12}(0.84)^{0.18}(0.64)^{0.1}(0.64)^{0.15}(0.51)^{0.05}(0.36)^{0.22}(0.75)^{0.08}(0.84)^{0.1}\right\}^{0.343}\end{array}}}{\sqrt{\begin{array}{c}\left\{(1.49)^{0.12}(1.04)^{0.18}(1.01)^{0.1}(1.09)^{0.15}(1.16)^{0.05}(1.64)^{0.22}(1.36)^{0.08}(1.04)^{0.1}\right\}^{0.143}\\ \left\{(1.09)^{0.12}(1.16)^{0.18}(1.16)^{0.1}(1.09)^{0.15}(1.36)^{0.05}(1.09)^{0.22}(1.81)^{0.08}(1.49)^{0.1}\right\}^{0.514}\\ \left\{(1.36)^{0.12}(1.16)^{0.18}(1.36)^{0.1}(1.36)^{0.15}(1.49)^{0.05}(1.64)^{0.22}(1.25)^{0.08}(1.16)^{0.1}\right\}^{0.343}\\ +\\ \left\{(0.51)^{0.12}(0.96)^{0.18}(0.01)^{0.1}(0.91)^{0.15}(0.84)^{0.05}(0.36)^{0.22}(0.64)^{0.08}(0.96)^{0.1}\right\}^{0.143}\\ \left\{(0.91)^{0.12}(0.84)^{0.18}(0.84)^{0.1}(0.91)^{0.15}(0.64)^{0.05}(0.91)^{0.22}(0.19)^{0.08}(0.51)^{0.1}\right\}^{0.514}\\ \left\{(0.64)^{0.12}(0.84)^{0.18}(0.64)^{0.1}(0.64)^{0.15}(0.51)^{0.05}(0.36)^{0.22}(0.75)^{0.08}(0.84)^{0.1}\right\}^{0.343}\end{array}}}, \frac{\sqrt{2\left[\begin{array}{c}\left\{(0.09)^{0.12}(0.25)^{0.18}(0.36)^{0.1}(0.16)0.15(0.36)^{0.05}(0.16)^{0.22}(0.49)^{0.08}(0.25)^{0.1}\right\}^{0.143}\\ \left\{\left\{(0.49)^{0.12}(0.25)^{0.18}(0.64)^{0.1}(0.16)0.15(0.49)^{0.05}(0.16)^{0.22}(0.04)^{0.08}(0.04)^{0.1}\right\}^{0.514}\right\}\\ \left\{(0.64)^{0.12}(0.25)^{0.18}(0.25)^{0.1}(0.16)0.15(0.25)^{0.05}(0.16)^{0.22}(0.64)^{0.08}(0.25)^{0.1}\right\}^{0.343}\end{array}\right]}}{\sqrt{\begin{array}{c}\left\{(1.91)^{0.12}(1.75)^{0.18}(1.64)^{0.1}(1.84)0.15(1.64)^{0.05}(1.84)^{0.22}(1.51)^{0.08}(1.75)^{0.1}\right\}^{0.143}\\ \left\{(1.51)^{0.12}(1.75)^{0.18}(1.36)^{0.1}(1.84)0.15(1.51)^{0.05}(1.84)^{0.22}(1.96)^{0.08}(1.96)^{0.1}\right\}^{0.514}\\ \left\{(1.36)^{0.12}(1.75)^{0.18}(1.75)^{0.1}(1.84)0.15(1.75)^{0.05}(1.84)^{0.22}(1.36)^{0.08}(1.75)^{0.1}\right\}^{0.343}\\ +\\ \left\{(0.09)^{0.12}(0.25)^{0.18}(0.36)^{0.1}(0.16)^{0.15}(0.36)^{0.05}(0.16)^{0.22}(0.49)^{0.08}(0.25)^{0.1}\right\}^{0.143}\\ \left\{(0.49)^{0.12}(0.25)^{0.18}(0.64)^{0.1}(0.16)^{0.15}(0.49)^{0.05}(0.16)^{0.22}(0.04)^{0.08}(0.04)^{0.1}\right\}^{0.514}\\ \left\{(0.64)^{0.12}(0.25)^{0.18}(0.25)^{0.1}(0.16)^{0.15}(0.25)^{0.05}(0.16)^{0.22}(0.64)^{0.08}(0.25)^{0.1}\right\}0.343\end{array}}} \right\rangle \quad (39)$$

$$\langle 0.5834, 0.4680\rangle$$

$$\mathscr{L}_4=\left\langle\frac{\sqrt{\begin{array}{c}\left\{(1.64)^{0.12}(1.04)^{0.18}(1.04)^{0.1}(1.16)^{0.15}(1.36)^{0.05}(1.25)^{0.22}(1.16)^{0.08}(1.64)^{0.1}\right\}^{0.143}\\ \left\{(1.25)^{0.12}(1.49)^{0.18}(1.81)^{0.1}(1.64)^{0.15}(1.81)^{0.05}(1.04)^{0.22}(1.16)^{0.08}(1.36)^{0.1}\right\}^{0.514}\\ \left\{(1.25)^{0.12}(1.81)^{0.18}(1.09)^{0.1}(1.25)^{0.15}(1.09)^{0.05}(1.64)^{0.22}(1.49)^{0.08}(1.04)^{0.1}\right\}^{0.343}\\ -\\ \left\{(0.36)^{0.12}(0.96)^{0.18}(0.96)^{0.1}(0.84)^{0.15}(0.64)0.05(0.75)^{0.22}(0.84)^{0.08}(0.36)^{0.1}\right\}^{0.143}\\ \left\{(0.75)^{0.12}(0.51)^{0.18}(0.19)^{0.1}(0.36)^{0.15}(0.19)0.05(0.96)^{0.22}(0.84)^{0.08}(0.64)^{0.1}\right\}^{0.514}\\ \left\{(0.75)^{0.12}(0.19)^{0.18}(0.91)^{0.1}(0.75)^{0.15}(0.91)0.05(0.36)^{0.22}(0.51)^{0.08}(0.96)^{0.1}\right\}^{0.343}\end{array}}}{\sqrt{\begin{array}{c}\left\{(1.64)^{0.12}(1.04)^{0.18}(1.04)^{0.1}(1.16)^{0.15}(1.36)^{0.05}(1.25)^{0.22}(1.16)^{0.08}(1.64)^{0.1}\right\}^{0.143}\\ \left\{(1.25)^{0.12}(1.49)^{0.18}(1.81)^{0.1}(1.64)^{0.15}(1.81)^{0.05}(1.04)^{0.22}(1.16)^{0.08}(1.36)^{0.1}\right\}^{0.514}\\ \left\{(1.25)^{0.12}(1.81)^{0.18}(1.09)^{0.1}(1.25)^{0.15}(1.09)^{0.05}(1.64)^{0.22}(1.49)^{0.08}(1.04)^{0.1}\right\}^{0.343}\\ +\\ \left\{(0.36)^{0.12}(0.96)^{0.18}(0.96)^{0.1}(0.84)^{0.15}(0.64)^{0.05}(0.75)^{0.22}(0.84)^{0.08}(0.36)^{0.1}\right\}^{0.143}\\ \left\{(0.75)^{0.12}(0.51)^{0.18}(0.19)^{0.1}(0.36)^{0.15}(0.19)^{0.05}(0.96)^{0.22}(0.84)^{0.08}(0.64)^{0.1}\right\}^{0.514}\\ \left\{(0.75)^{0.12}(0.19)^{0.18}(0.91)^{0.1}(0.75)^{0.15}(0.91)^{0.05}(0.36)^{0.22}(0.51)^{0.08}(0.96)^{0.1}\right\}^{0.343}\end{array}}},\frac{\sqrt{2\left[\begin{array}{c}\left\{(0.16)^{0.12}(0.81)^{0.18}(0.16)^{0.1}(0.36)^{0.15}(0.25)^{0.05}(0.36)^{0.22}(0.25)^{0.08}(0.09)^{0.1}\right\}^{0.143}\\ \left\{(0.16)^{0.12}(0.36)^{0.18}(0.09)^{0.1}(0.25)^{0.15}(0.04)^{0.05}(0.16)^{0.22}(0.36)^{0.08}(0.25)^{0.1}\right\}^{0.514}\\ \left\{(0.49)^{0.12}(0.09)^{0.18}(0.25)^{0.1}(0.49)^{0.15}(0.25)^{0.05}(0.25)^{0.22}(0.25)^{0.08}(0.25)^{0.1}\right\}^{0.343}\end{array}\right]}}{\sqrt{\begin{array}{c}(1.84)^{0.12}(1.19)^{0.18}(1.84)^{0.1}(1.64)^{0.15}(1.75)^{0.05}(1.64)^{0.22}(1.75)^{0.08}(1.91)^{0.1 0.143\}}\\ \left\{(1.84)^{0.12}(1.64)^{0.18}(1.91)^{0.1}(1.75)^{0.15}(1.96)^{0.05}(1.84)^{0.22}(1.64)^{0.08}(1.75)^{0.1}\right\}^{0.514}\\ \left\{(1.51)^{0.12}(1.91)^{0.18}(1.75)^{0.1}(1.51)^{0.15}(1.75)^{0.05}(1.75)^{0.22}(1.75)^{0.08}(1.75)^{0.1}\right\}^{0.343}\\ +\\ \left\{(0.16)^{0.12}(0.81)^{0.18}(0.16)^{0.1}(0.36)^{0.15}(0.25)^{0.05}(0.36)^{0.22}(0.25)^{0.08}(0.09)^{0.1}\right\}^{0.143}\\ \left\{(0.16)^{0.12}(0.36)^{0.18}(0.09)^{0.1}(0.25)^{0.15}(0.04)^{0.05}(0.16)^{0.22}(0.36)^{0.08}(0.25)^{0.1}\right\}^{0.514}\\ \left\{(0.49)^{0.12}(0.09)^{0.18}(0.25)^{0.1}(0.49)^{0.15}(0.25)^{0.05}(0.25)^{0.22}(0.25)^{0.08}(0.25)^{0.1}\right\}^{0.343}\end{array}}}\right\rangle \tag{40}$$

$$\langle 0.6521, 0.4253\rangle$$

$$\mathscr{L}_5 = \left\langle \frac{\sqrt{\begin{array}{c} \left\{(1.25)^{0.12}(1.64)0.18(1.49)^{0.1}(1.16)^{0.15}(1.16)^{0.05}(1.04)^{0.22}(1.64)^{0.08}(1.49)^{0.1}\right\}^{0.143} \\ \left\{(1.64)^{0.12}(1.49)0.18(1.64)^{0.1}(1.25)^{0.15}(1.25)^{0.05}(1.49)^{0.22}(1.49)^{0.08}(1.36)^{0.1}\right\}^{0.514} \\ \left\{(1.25)^{0.12}(1.16)0.18(1.25)^{0.1}(1.09)^{0.15}(1.49)^{0.05}(1.49)^{0.22}(1.16)^{0.08}(1.25)^{0.1}\right\}^{0.343} \\ - \\ \left\{(1.25)^{0.12}(1.16)^{0.18}(1.25)^{0.1}(1.09)^{0.15}(1.49)^{0.05}(1.49)^{0.22}(1.16)^{0.08}(1.25)^{0.1}\right\}^{0.343} \\ \left\{(0.36)^{0.12}(0.51)^{0.18}(0.36)^{0.1}(0.75)^{0.15}(0.75)^{0.05}(0.51)^{0.22}(0.51)^{0.08}(0.64)^{0.1}\right\}^{0.514} \\ \left\{(0.75)^{0.12}(0.84)^{0.18}(0.75)0.1(0.91)^{0.15}(0.51)^{0.05}(0.51)^{0.22}(0.84)^{0.08}(0.75)^{0.1}\right\}^{0.343} \end{array}}}{\sqrt{\begin{array}{c} \left\{(1.25)^{0.12}(1.64)^{0.18}(1.49)^{0.1}(1.16)^{0.15}(1.16)^{0.05}(1.04)^{0.22}(1.64)^{0.08}(1.49)^{0.1}\right\}^{0.143} \\ \left\{(1.64)^{0.12}(1.49)^{0.18}(1.64)^{0.1}(1.25)^{0.15}(1.25)^{0.05}(1.49)^{0.22}(1.49)^{0.08}(1.36)^{0.1}\right\}^{0.514} \\ \left\{(1.25)^{0.12}(1.16)^{0.18}(1.25)^{0.1}(1.09)^{0.15}(1.49)^{0.05}(1.49)^{0.22}(1.16)^{0.08}(1.25)^{0.1}\right\}^{0.343} \\ + \\ \left\{(0.75)^{0.12}(0.36)^{0.18}(0.51)^{0.1}(0.84)^{0.15}(0.84)^{0.05}(0.96)^{0.22}(0.36)^{0.08}(0.51)^{0.1}\right\}^{0.143} \\ \left\{(0.36)^{0.12}(0.51)^{0.18}(0.36)^{0.1}(0.75)^{0.15}(0.75)^{0.05}(0.51)^{0.22}(0.51)^{0.08}(0.64)^{0.1}\right\}^{0.514} \\ \left\{(0.75)^{0.12}(0.84)^{0.18}(0.75)^{0.1}(0.91)^{0.15}(0.51)^{0.05}(0.51)^{0.22}(0.84)^{0.08}(0.75)^{0.1}\right\}^{0.343} \end{array}}}, \frac{\sqrt{2\left[\begin{array}{c} \left\{(0.49)^{0.12}(0.25)^{0.18}(0.16)^{0.1}(0.09)^{0.15}(0.81)^{0.05}(0.16)^{0.22}(0.16)^{0.08}(0.25)^{0.1}\right\}^{0.143} \\ \left\{(0.25)^{0.12}(0.16)^{0.18}(0.25)^{0.1}(0.04)^{0.15}(0.49)^{0.05}(0.25)^{0.22}(0.36)^{0.08}(0.16)^{0.1}\right\}^{0.514} \\ \left\{(0.16)^{0.12}(0.64)^{0.18}(0.36)^{0.1}(0.16)^{0.15}(0.36)^{0.05}(0.25)^{0.22}(0.81)^{0.08}(0.04)^{0.1}\right\}^{0.343} \end{array}\right]}}{\sqrt{\begin{array}{c} \left\{(1.51)^{0.12}(1.75)^{0.18}(1.84)^{0.1}(1.91)^{0.15}(1.19)^{0.05}(1.84)^{0.22}(1.84)^{0.08}(1.75)^{0.1}\right\}^{0.143} \\ \left\{(1.75)^{0.12}(1.84)^{0.18}(1.75)^{0.1}(1.96)^{0.15}(1.51)^{0.05}(1.75)^{0.22}(1.64)^{0.08}(1.84)^{0.1}\right\}^{0.514} \\ \left\{(1.84)^{0.12}(1.36)^{0.18}(1.64)^{0.1}(1.84)^{0.15}(1.64)^{0.05}(1.75)^{0.22}(1.19)^{0.08}(1.96)^{0.1}\right\}^{0.343} \\ + \\ \left\{(0.49)^{0.12}(0.25)^{0.18}(0.16)^{0.1}(0.09)^{0.15}(0.81)^{0.05}(0.16)^{0.22}(0.16)^{0.08}(0.25)^{0.1}\right\}^{0.143} \\ \left\{(0.25)^{0.12}(0.16)^{0.18}(0.25)^{0.1}(0.04)^{0.15}(0.49)^{0.05}(0.25)^{0.22}(0.36)^{0.08}(0.16)^{0.1}\right\}^{0.514} \\ \left\{(0.16)^{0.12}(0.64)^{0.18}(0.36)^{0.1}(0.16)^{0.15}(0.36)^{0.05}(0.25)^{0.22}(0.81)^{0.08}(0.04)^{0.1}\right\}^{0.343} \end{array}}} \right\rangle$$

$$\langle 0.6260, 0.4583 \rangle$$

$$\mathscr{L}_1 = \left\langle \frac{\sqrt{\begin{array}{c} \left\{(1.09)^{0.12}(1.49)^{0.18}(1.36)^{0.1}(1.25)^{0.15}(1.04)^{0.05}(1.16)^{0.22}(1.25)^{0.08}(1.81)^{0.1}\right\}^{0.143} \\ \left\{(1.49)^{0.12}(1.09)^{0.18}(1.36)^{0.1}(1.09)^{0.15)}(1.25)^{0.05}(1.16)^{0.22}(1.49)^{0.08}(1.09)^{0.1}\right\}^{0.514} \\ \left\{(1.25)^{0.12}(1.64)^{0.18}(1.49)^{0.1}(1.16)^{0.15}(1.16)^{0.05}(1.04)^{0.22}(1.64)^{0.08}(1.49)^{0.1}\right\}^{0.343} \\ - \\ \left\{(0.91)^{0.12}(0.51)^{0.18}(0.64)^{0.1}(0.75)^{0.15}(0.96)^{0.05}(0.84)^{0.22}(0.75)^{0.08}(0.19)^{0.1}\right\}^{0.143} \\ \left\{(0.51)^{0.12}(0.91)^{0.18}(0.64)^{0.1}(0.91)^{0.15}(0.75)^{0.05}(0.84)^{0.22}(0.51)^{0.08}(0.91)^{0.1}\right\}^{0.514} \\ \left\{(0.75)^{0.12}(0.36)^{0.18}(0.51)^{0.1}(0.84)^{0.15}(0.84)^{0.05}(0.96)^{0.22}(0.36)^{0.08}(0.51)^{0.1}\right\}^{0.343} \end{array}}}{\sqrt{\begin{array}{c} \left\{(1.09)^{0.12}(1.49)^{0.18}(1.36)^{0.1}(1.25)^{0.15}(1.04)^{0.05}(1.16)^{0.22}(1.25)^{0.08}(1.81)^{0.1}\right\}^{0.143} \\ \left\{(1.49)^{0.12}(1.09)^{0.18}(1.36)^{0.1}(1.09)^{0.15}(1.25)^{0.05}(1.16)^{0.22}(1.49)^{0.08}(1.09)^{0.1}\right\}^{0.514} \\ \left\{(1.25)^{0.12}(1.64)^{0.18}(1.49)^{0.1}(1.16)^{0.15}(1.16)^{0.05}(1.04)^{0.22}(1.64)^{0.08}(1.49)^{0.1}\right\}^{0.343} \\ + \\ \left\{(0.91)^{0.12}(0.51)^{0.18}(0.64)^{0.1}(0.75)^{0.15}(0.96)^{0.05}(0.84)^{0.22}(0.75)^{0.08}(0.19)^{0.1}\right\}^{0.143} \\ \left\{(0.51)^{0.12}(0.91)^{0.18}(0.64)^{0.1}(0.91)^{0.15}(0.75)^{0.05}(0.84)^{0.22}(0.51)^{0.08}(0.91)^{0.1}\right\}^{0.514} \\ \left\{(0.75)^{0.12}(0.36)^{0.18}(0.51)^{0.1}(0.84)^{0.15}(0.84)^{0.05}(0.96)^{0.22}(0.36)^{0.08}(0.51)^{0.1}\right\}^{0.343} \end{array}}}, \frac{\sqrt{2\left[\begin{array}{c} \left\{(0.64)^{0.12}(0.09)^{0.18}(0.49)^{0.1}(0.16)^{0.15}(0.16)^{0.05}(0.36)^{0.22}(0.64)^{0.08}(0.09)^{0.1}\right\}^{0.143} \\ \left\{(0.36)^{0.12}(0.16)^{0.18}(0.25)^{0.1}(0.81)^{0.15}(0.16)^{0.05}(0.36)^{0.22}(0.25)^{0.08}(0.64)^{0.1}\right\}^{0.514} \\ \left\{(0.49)^{0.12}(0.25)^{0.18}(0.16)^{0.1}(0.09)^{0.15}(0.81)^{0.05}(0.16)^{0.22}(0.16)^{0.08}(0.25)^{0.1}\right\}^{0.343} \end{array}\right]}}{\sqrt{\begin{array}{c} \left\{(1.36)^{0.12}(1.91)^{0.18}(1.51)^{0.1}(1.84)^{0.15}(1.84)^{0.05}(1.64)^{0.22}(1.36)^{0.08}(1.91)^{0.1}\right\}^{0.143} \\ \left\{(1.64)^{0.12}(1.84)^{0.18}(1.75)^{0.1}(1.19)^{0.15}(1.84)^{0.05}(1.64)^{0.22}(1.75)^{0.08}(1.36)^{0.1}\right\}^{0.514} \\ \left\{(1.51)^{0.12}(1.75)^{0.18}(1.84)^{0.1}(1.91)^{0.15}(1.19)^{0.05}(1.84)^{0.22}(1.84)^{0.08}(1.75)^{0.1}\right\}^{0.343} \\ + \\ \left\{(0.64)^{0.12}(0.09)^{0.18}(0.49)^{0.1}(0.16)^{0.15}(0.16)^{0.05}(0.36)^{0.22}(0.64)^{0.08}(0.09)^{0.1}\right\}^{0.143} \\ \left\{(0.36)^{0.12}(0.16)^{0.18}(0.25)^{0.1}(0.81)^{0.15}(0.16)^{0.05}(0.36)^{0.22}(0.25)^{0.08}(0.64)^{0.1}\right\}^{0.514} \\ \left\{(0.49)^{0.12}(0.25)^{0.18}(0.16)^{0.1}(0.09)^{0.15}(0.81)^{0.05}(0.16)^{0.22}(0.16)^{0.08}(0.25)^{0.1}\right\}^{0.343} \end{array}}} \right\rangle \tag{41}$$

$$\langle 0.5387, 0.5299\rangle$$

Step 4. Use equation (1) $\mathbb{S} = a^2_{\mathscr{F}(\breve{d}_{ij})} - b^2_{\mathscr{F}(\breve{d}_{ij})}$ to compute the score values for all alternatives.

$\mathbb{S}(\mathscr{H}_1) = 0.0088$, $\mathbb{S}(\mathscr{H}_2) = 0.2855$, $\mathbb{S}(\mathscr{H}_3) = 0.1154$, $\mathbb{S}(\mathscr{H}_4) = 0.2268$, and $\mathbb{S}(\mathscr{H}_5) = 0.1677$.

Step 5. After calculation, we get the ranking of alternatives $\mathbb{S}(\mathscr{H}_2) > \mathbb{S}(\mathscr{H}_4) > \mathbb{S}(\mathscr{H}_5) > \mathbb{S}(\mathscr{H}_3) > \mathbb{S}(\mathscr{H}_1)$. So, $\mathfrak{H}^2 > \mathfrak{H}^4 > \mathfrak{H}^5 > \mathfrak{H}^3 > \mathfrak{H}^1$.

Hence, the best alternative is $\mathfrak{H}^2$.

Table 6: Feature analysis of different models with a proposed model.

	Fuzzy information	Aggregated parameters information	Einstein aggregated parameters information	Multi-sub-attributes information of each attribute
IFWA [66]	✓	×	×	×
IFEWA [62]	✓	✓	✓	×
IFSWA [35]	✓	✓	×	×
IFHSWA [58]	✓	✓	×	✓
PFSWA [44]	✓	✓	×	×
PFEWA [27]	✓	✓	✓	×
PFSEOWA [46]	✓	✓	✓	×
PFHSWA [65]	✓	✓	×	✓
Proposed operator	✓	✓	✓	✓

Table 7: Comparison of proposed operators with some existing operators.

Approach	H^1	H^2	H^3	H^4	H^5	Alternatives ranking
PFWA operator	0.0039	0.0644	0.0433	−0.0179	−0.0376	$\mathfrak{H}^2 > \mathfrak{H}^3 > \mathfrak{H}^1 > \mathfrak{H}^4 > \mathfrak{H}^5$
PFEWA operator	−0.3306	0.5957	0 .1383	−0.1661	0.1092	$\mathfrak{H}^2 > \mathfrak{H}^3 > \mathfrak{H}^5 > \mathfrak{H}^4 > \mathfrak{H}^1$
PFSWA operator	0.0293	0.0938	0.0783	0.0694	0.0369	$\mathfrak{H}^2 > \mathfrak{H}^3 > \mathfrak{H}^4 > \mathfrak{H}^5 > \mathfrak{H}^1$
PFHSWA operator	0.1975	0.3513	0.2632	0.2297	0.1204	$\mathfrak{H}^2 > \mathfrak{H}^3 > \mathfrak{H}^4 > \mathfrak{H}^1 > \mathfrak{H}^5$
PFHSEWA operator	0.0088	0.2855	0.1154	0.2268	0.1677	$\mathfrak{H}^2 > \mathfrak{H}^4 > \mathfrak{H}^5 > \mathfrak{H}^3 > \mathfrak{H}^1$

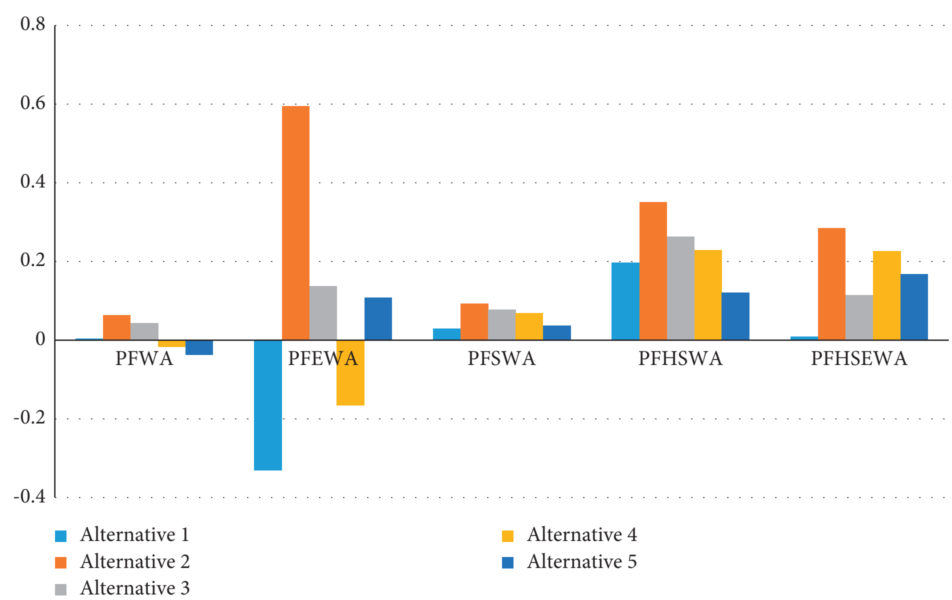

Figure 2: Graphical results of comparative studies.

5. Comparative Studies and Supremacy of the Proposed Model

To demonstrate the efficiency of the anticipated approach, some existing techniques under the PFS, PFSS, and proposed PFHSS model were compared.

5.1. Supremacy of the Proposed Method. The deliberate approach is proficient and convincing; we have constructed a pioneering MCDM model under the PFHSS setting over the PFHSEWA operator. Our advanced model is extra brilliant than prevailing techniques and can convey the utmost subtle connotations in MCDM obstacles. The collective model is flexible and conversant, adjusting to potential instability, commitment, and production. Different models have exhaustive ranking processes, so there is an instantaneous variance among the positions of the offered method to be realistic conferring to their conventions. This systematic exploration and estimation determine that outcomes gained from present techniques are volatile equated to hybrid structures. It is informal to syndicate inadequate and indeterminate facts in DM methods. Hence, our deliberated methodology will be more capable, authoritative, superior, and better than various hybrid-structured FS. Table 6 presents the supremacy analysis of the anticipated technique and some standing models.

5.2. Comparative Studies. To validate the usefulness of the projected technique, we compare the obtained results with some existing techniques under the environment of PFS and PFSS. A summary of all numerical and graphical outcomes is given in Table 7 and Figure 2. Firstly, we present a comparison with methods proposed by Siddique et al. [65] and Zulqarnain et al. [44]. Their proposed AOs are based on algebraic norms, while the proposed operators in this work are based on Einstein norms. Secondly, we compare the PFEWA operator proposed by Garg [15]. He developed the DM technique for PFNs by utilizing Einstein norms that cannot accommodate the parametrized values of the alternatives. On the other hand, our established approach competently deals with parametrized values of the alternatives and delivers better information than existing techniques. This work recommends innovative Einstein AO, such as PFHSEWA, to integrate the evaluation materials and then use the score function to calculate the substitute score. Therefore, it is inevitable that, based on the above facts, the plan operator in this work is more influential, consistent, and effective. The graphical ranking order of the alternatives of our proposed model with existing models is given in Figure 2.

6. Conclusion

Mathematical validation in agri-farming developments feats all resources while integrating objectives under economic, superior, and protection boundaries. Studies must be delimited for the most acceptable decision, accessing judgment requirements. In genuine DM, the assessment of alternative details carried by the expert is regularly incorrect, rough, and impetuous, so PFHSNs can be used to comport this indeterminate information. The core goal of this research is to use Einstein's norms to develop some operational laws for PFHSS. Then, a new operator, such as PFHSEWA, was developed according to the designed operational laws. In addition, some basic properties are proposed, such as the idempotence, homogeneity, and boundedness of the developed PFHSEWA operator. Furthermore, a DM approach has been designed to address MCDM problems based on endorsed operators. To certify the robustness of the settled approach, we provide an inclusive mathematical illustration for selecting the best agricultural robots in agri-farming. A comparative analysis with some current methods is presented. Finally, based on the outcomes attained, it is determined that the technique projected in this research is the most practical and effective way to solve the problem of MCDM. Future research focuses on developing more decision-making methods in the PFHSS environment using other operators, such as Einstein's hybrid geometric and Einstein's hybrid average operator.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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