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New Aspects of ZZ Transform to Fractional Operators With Mittag-Leffler Kernel

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In this paper, we discuss the relationship between the Zain Ul Abadin Zafar (ZZ) transform with Laplace and Aboodh transforms. Further, the ZZ transform is applied to the fractional derivative with the Mittag-Leffler kernel defined in both the Caputo and Riemann-Liouville sense. In order to illustrate the validity and applicability of the transform, we solve some illustrative examples.

Keywords: ZZ transform, fractional calculus, aboodh transform, non-singular kernel, mittag-leffler kernel

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1. OUTLINE AND MOTIVATION

In recent years, fractional calculus (FC) has gained considerable achievements in various fields of science and engineering. Many physical problems [1–7] are modeled by using fractional differential equations (FDE) more accurately than classical differential equations [8–11]. Earlier, various real-life problems were modeled by using the Caputo and Riemann-Liouville (R-L) fractional derivatives. However, Caputo and Fabrizio proposed a new idea that reflects the exponential kernel [12] to address a new way of modeling phenomena with non-local effects. Further, in [13], a new fractional operator (AB) with a Mittag-Leffler kernel was developed. So, in this regard, many researchers [14–16] have given their interest in this definition to solve various problems/models. In fact, in modeling real phenomena, we need a variety of fractional operators to thoroughly describe the complexity of the problem studied. Some other studies regarding fractional calculus and special functions can be found in the literature [17–26].

In the present study, we establish the relationship between the ZZ transform (ZZT) with the Aboodh transform (AT), and the Laplace transform (LT) having their various applications given in [27–31]. Next, the ZZT has been applied to AB fractional operators defined in the Caputo and R-L sense, which are described in terms of theorems. Later, we have solved some test examples defined in the AB sense using this ZZT. The contribution of the present authors to this manuscript are (i) firstly establishing the relationship among ZZT, LT, and AT, (ii) secondly applying ZZT to fractional differential equations defined in the AB derivative to get the solution of the problems. The ZZ transform is the generalization of some famous transforms and we can relate this transformation to other well-known transforms. If we divide the ZZ transform by the transformed variable, then we get the Natural transform. Similarly, relations with other integral transforms in terms of theorems have been included in this paper. The main benefit of this transformation is that it may converge to the Sumudu transform and is advantageous in solving FDEs with variable coefficients.

The organization of the paper is as follows: In section Preliminaries and Basic Definitions, we establish the connection between the Aboodh and ZZ transform; we prove some significant results and create the relationships between AB derivatives with ZZT. In section Applications, some FDEs are solved using ZZT. Finally, a conclusion section is included in section Conclusion.

2. PRELIMINARIES AND BASIC DEFINITIONS

Definition 2.1

The Aboodh transform is obtained on the set of functions

$$B = \left\{ f(t) : \exists\, M, m_1, m_2 > 0, \left| f(t) \right| < Me^{-st} \right\}$$

and is defined as [27, 28]

$$A\left\{ f(t) \right\} = \frac{1}{s} \int_0^\infty f(t)\, e^{-st} dt,\, t > 0 \text{ and } m_1 \leq s \leq m_2$$

Theorem 2.1

Let us consider G and Fas the Aboodh and Laplace transforms of $f(t) \in B$ then [32]

$$G(s) = \frac{F(s)}{s}. \tag{2.1}$$

The ZZT was introduced by Zain Ul Abadin Zafar [29, 30]. It generalizes the Aboodh and Laplace integral transforms. In the following definition, we discuss the definition of ZZT.

Definition 2.2 (ZZ Transform)

Suppose $f(t)\ \forall\, t \geq 0$ is a function then the ZZT $Z(v, s)$ of $f(t)$ is defined as [29, 30]

$$ZZ\left(f(t) \right) = Z(v, s) = s \int_0^\infty f(vt)\ e^{-st} dt.$$

Similar to the Aboodh and Laplace transforms, the ZZT is also linear. The MLF is an extension of exponential function which is defined as.

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(1 + n\alpha)}, \quad \text{Re}(\alpha) > 0.$$

Definition 2.3

Let us consider a function $\xi(x, t) \in H^1\left(a, b\right)$, then for $\alpha \in (0, 1)$, the Atangana-Baleanu Caputo (ABC) derivative is written as [13].

$${}^{ABC}{}_aD_t^\alpha \xi(x, t) = \frac{\psi(\alpha)}{1 - \alpha} \int_a^t \xi'(x, \tau)\, E_\alpha \left(\frac{-\alpha(t - \tau)^\alpha}{1 - \alpha} \right) d\tau.$$

Definition 2.4

Let $\xi(x, t) \in H^1\left(a, b\right)$, then for $\alpha \in (0, 1)$, the Atangana-Baleanu Riemann-Liouville (ABR) derivative is given as [13]

$${}_a^{ABR}D_t^\alpha \xi(x, t) = \frac{\psi(\alpha)}{1 - \alpha} \frac{d}{dt} \int_a^t \xi(x, \tau)\, E_\alpha \left(\frac{-\alpha(t - \tau)^\alpha}{1 - \alpha} \right) d\tau,$$

where $\psi(\alpha)$is a function with the conditions $\psi(0) = \psi(1) = 1$and $b > a$.

Theorem 2.2

The LT of ABC and ABR derivative are, respectively, given as [13]

$$\mathrm{L}\left\{ {}_a^{ABC}D_t^\alpha \xi(x, t) \right\}(s) = \frac{\psi(\alpha)}{1 - \alpha} \frac{s^\alpha L\{\xi(x, t)\} - s^{\alpha - 1}\xi(x, 0)}{s^\alpha + \frac{\alpha}{1 - \alpha}} \tag{2.2}$$

and

$$\mathrm{L}\left\{ {}_a^{ABR}D_t^\alpha \xi(x, t) \right\}(s) = \frac{\psi(\alpha)}{1 - \alpha} \frac{s^\alpha L\{\xi(x, t)\}}{s^\alpha + \frac{\alpha}{1 - \alpha}}. \tag{2.3}$$

The following theorems have been proposed where it is assumed that $f(t) \in H^1\left(a, b\right)$, $b > a$and $\alpha \in (0, 1)$.

Theorem 2.3

The AT of ABC derivative is given as.

$$\begin{aligned} G(s) &= A\left\{ {}_a^{ABC}D_t^\alpha \xi(x, t) \right\}(s) \\ &= \frac{1}{s} \left[\frac{\psi(\alpha)}{1 - \alpha} \frac{s^\alpha L\{\xi(x, t)\} - s^{\alpha - 1}\xi(x, 0)}{s^\alpha + \frac{\alpha}{1 - \alpha}} \right]. \end{aligned} \tag{2.4}$$

Proof: Using Theorem 2.1 and Equation. (2.2), we may get the desired result.

Theorem 2.4

The Aboodh transform of ABR derivative is written as.

$$G(s) = A\left\{ {}_a^{ABR}D_t^\alpha \xi(x, t) \right\}(s) = \frac{1}{s} \left[\frac{\psi(\alpha)}{1 - \alpha} \frac{s^\alpha L\{\xi(x, t)\}}{s^\alpha + \frac{\alpha}{1 - \alpha}} \right]. \tag{2.5}$$

Proof

Applying the Theorem 2.1 and Equation (2.3), we obtain the required result.

The connection between the transforms of Aboodh and ZZ is given in the theorem below.

Theorem 2.5

If $G(s)$ and$Z(v, s)$ are the Aboodh and ZZ transforms of$f(t) \in B$. Then, we obtain

$$Z(v, s) = \frac{s^2}{v^2} G\left(\frac{s}{v} \right).$$

Proof. From the definition of ZZ transform we have

$$Z(v, s) = s \int_0^\infty f(vt)\ e^{-st} dt. \tag{2.6}$$

Substituting $vt = \tau$ in Equation (2.6) we get

$$Z(v,s) = \frac{s}{v}\int_0^{\infty} f(\tau)\, e^{-\frac{s\tau}{v}}\, d\tau. \tag{2.7}$$

The right-hand side of the above Equation (2.7) may be written as.

$$Z(v,s) = \frac{s}{v} F\left(\frac{s}{v}\right), \tag{2.8}$$

where $F(.)$ denotes the Laplace transform of $f(t)$.

Applying the Theorem 2.1, Equation (2.8) can be expressed as

$$Z(v,s) = \frac{s}{v}\frac{F\left(\frac{s}{v}\right)}{\left(\frac{s}{v}\right)} \times \left(\frac{s}{v}\right) = \left(\frac{s}{v}\right)^2 G\left(\frac{s}{v}\right), \tag{2.9}$$

where $G(.)$ denotes the Aboodh transform of $f(t)$.

Theorem 2.6

ZZ transform of $f(t) = t^{\alpha-1}$ is given as

$$Z(v,s) = \Gamma(\alpha)\left(\frac{v}{s}\right)^{\alpha-1}. \tag{2.10}$$

Proof. The Aboodh transform of $f(t) = t^{\alpha}, \alpha \geq 0$ is

$$G(s) = \frac{\Gamma(\alpha)}{s^{\alpha+1}},$$

$$\text{Now,} \quad G\left(\frac{s}{v}\right) = \frac{\Gamma(\alpha)\, v^{\alpha+1}}{s^{\alpha+1}}.$$

Using Equation (2.9), we obtain.

$$Z(v,s) = \frac{s^2}{v^2} G\left(\frac{s}{v}\right) = \frac{s^2}{v^2}\frac{\Gamma(\alpha)\, v^{\alpha+1}}{s^{\alpha+1}} = \Gamma(\alpha)\left(\frac{v}{s}\right)^{\alpha-1}.$$

Theorem 2.7

Let $\alpha, \omega \in C$ and $\mathrm{Re}(\alpha) > 0$, then the ZZ transform of $E_{\alpha}(\omega t^{\alpha})$ is given as

$$ZZ\left\{\left(E_{\alpha}\left(\omega t^{\alpha}\right)\right)\right\} = Z(v,s) = \left(1 - \omega\left(\frac{v}{s}\right)^{\alpha}\right)^{-1} \tag{2.11}$$

Proof. We know that Aboodh transform of $E_{\alpha}(\omega t^{\alpha})$ is written as.

$$G(s) = \frac{F(s)}{s} = \frac{s^{\alpha-1}}{s\,(s^{\alpha} - \omega)}, \tag{2.12}$$

$$\text{So,} \quad G\left(\frac{s}{v}\right) = \frac{\left(\frac{s}{v}\right)^{\alpha-1}}{\left(\frac{s}{v}\right)\left(\left(\frac{s}{v}\right)^{\alpha} - \omega\right)}, \tag{2.13}$$

Using the Theorem 2.9, we obtain.

$$\begin{aligned} Z(v,s) &= \left(\frac{s}{v}\right)^2 G\left(\frac{s}{v}\right) = \left(\frac{s}{v}\right)^2 \frac{\left(\frac{s}{v}\right)^{\alpha-1}}{\left(\frac{s}{v}\right)\left(\left(\frac{s}{v}\right)^{\alpha} - \omega\right)} \\ &= \frac{\left(\frac{s}{v}\right)^{\alpha}}{\left(\frac{s}{v}\right)^{\alpha} - \omega} = \left(1 - \omega\left(\frac{v}{s}\right)^{\alpha}\right)^{-1}. \end{aligned}$$

Theorem 2.8

If $G(s)$ and $Z(v,s)$ are the Aboodh and ZZ transforms of $f(t)$. Then the ZZT of ABC derivative is written as.

$$ZZ\left\{{}_0^{ABC}D_t^{\alpha} f(t)\right\} = \left[\frac{\psi(\alpha)}{1-\alpha}\frac{\frac{s^{\alpha+2}}{v^{\alpha+2}} G\left(\frac{s}{v}\right) - \frac{s^{\alpha}}{v^{\alpha}} f(0)}{\frac{s^{\alpha}}{v^{\alpha}} + \frac{\alpha}{1-\alpha}}\right]. \tag{2.14}$$

Proof. Using the Equations (2.1) and (2.4), we have

$$G\left(\frac{s}{v}\right) == \frac{v}{s}\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha+1} G\left(\frac{s}{v}\right) - \left(\frac{s}{v}\right)^{\alpha-1} f(0)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right]. \tag{2.15}$$

So, the ZZ transform of ABC is given as.

$$\begin{aligned} Z(v,s) &= \left(\frac{s}{v}\right)^2 G\left(\frac{s}{v}\right) \\ &= \left(\frac{s}{v}\right)^2 \frac{v}{s}\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha+1} G\left(\frac{s}{v}\right) - \left(\frac{s}{v}\right)^{\alpha-1} f(0)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right] \\ &= \left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha+2} G\left(\frac{s}{v}\right) - \left(\frac{s}{v}\right)^{\alpha} f(0)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right] \end{aligned}$$

Theorem 2.9

Let us assume that$G(s)$ and $Z(v,s)$ are the Aboodh and ZZ transform of $f(t)$. Then the ZZ transform of ABR derivative is given as

$$ZZ\left\{{}_0^{ABR}D_t^{\alpha} f(t)\right\} = \left[\frac{\psi(\alpha)}{1-\alpha}\frac{\frac{s^{\alpha+2}}{v^{\alpha+2}} G\left(\frac{s}{v}\right)}{\frac{s^{\alpha}}{v^{\alpha}} + \frac{\alpha}{1-\alpha}}\right]. \tag{2.16}$$

Proof. Using the Equations (2.1) and (2.5), we get

$$G\left(\frac{s}{v}\right) = \frac{v}{s}\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha+1} G\left(\frac{s}{v}\right)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right]. \tag{2.17}$$

From the Equation (2.9), the ZZ transform of ABR is written as.

$$\begin{aligned} Z(v,s) = \left(\frac{s}{v}\right)^2 G\left(\frac{s}{v}\right) &= \left(\frac{s}{v}\right)^2 \left(\frac{v}{s}\right)\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha+1} G\left(\frac{s}{v}\right)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right] \\ &= \left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha+2} G\left(\frac{s}{v}\right)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right]. \end{aligned}$$

3. APPLICATIONS

Let us consider the following initial value problem (IVP) defined in ABC sense [15]

$$\begin{cases} {}_{0}^{ABC}D_{t}^{\alpha} y(t) = f\left(t, y(t)\right), \ t > 0, \\ y(0) = k, \quad k \in \Re. \end{cases} \tag{3.1}$$

Suppose $Z(v,s)$ and $T(v,s)$ are the ZZ transforms of $y(t)$and f, respectively. Then by taking the ZZT on both sides of Equation (3.1) and using Equations (2.9) and (2.14), we may get

$$\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha} Z(v,s) - \left(\frac{s}{v}\right)^{\alpha} y(0)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right] = T(v,s).$$

$$\left[\psi(\alpha)\frac{Z(v,s)-k}{\left(1-\alpha+\alpha\left(\frac{v}{s}\right)^{\alpha}\right)}\right] = T(v,s)$$

$$\text{Thus, } Z(v,s) = \frac{1-\alpha+\alpha\left(\frac{v}{s}\right)^{\alpha}}{\psi(\alpha)} T(v,s) + k. \tag{3.2}$$

Then, by applying the inverse ZZT on both sides of Equation (3.2), we obtain the exact solution.

Similarly, we may solve Equation (3.1) defined in ABR derivative.

Example 3.1

Let us take the following fractional IVP [15]

$$\begin{cases} {}_{0}^{ABC}D_{t}^{\alpha} y(t) = y(t), \ t > 0, \\ y(0) = 1. \end{cases} \tag{3.3}$$

Firstly, we apply the ZZT on both sides of Equation (3.3) which gives

$$\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha} Z(v,s) - \left(\frac{s}{v}\right)^{\alpha} y(0)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right] = Z(v,s). \tag{3.4}$$

Simplifying Equation (3.4) and using the initial condition, we have

$$\left[\psi(\alpha)\frac{Z(v,s)-1}{\left(1-\alpha+\alpha\left(\frac{v}{s}\right)^{\alpha}\right)}\right] = Z(v,s). \tag{3.5}$$

The simplification of Equation (3.5) gives us the following:

$$Z(v,s) = \frac{1}{1-\frac{1-\alpha+\alpha\left(\frac{v}{s}\right)^{\alpha}}{\psi(\alpha)}} = \frac{\psi(\alpha)}{\psi(\alpha)-1+\alpha-\alpha\left(\frac{v}{s}\right)^{\alpha}}. \tag{3.6}$$

Equation (3.6) may be rewritten as.

$$Z(v,s) = \frac{\psi(\alpha)}{(\psi(\alpha)-1+\alpha)}\left(1-\frac{\alpha}{\psi(\alpha)-1+\alpha}\left(\frac{v}{s}\right)^{\alpha}\right)^{-1} \tag{3.7}$$

Applying the inverse of the ZZT on Equation (3.7) and using Equation (2.11), Equation (3.7) is reduced to

$$y(t) = \frac{\psi(\alpha)}{(\psi(\alpha)-1+\alpha)} E_{\alpha}\left(\frac{\alpha}{\psi(\alpha)-1+\alpha} t^{\alpha}\right), \tag{3.8}$$

where $E_{\alpha}(t)$ is the MLF.

Substituting $\alpha = 1$ in Equation (3.8), we obtain

$$y(t) = E_1(t) = e^t, \tag{3.9}$$

which is the exact solution of Equation (3.3) when $\alpha = 1$.

Example 3.2

Considering the following fractional IVP [15]

$$\begin{cases} {}_{0}^{ABC}D_{t}^{\alpha} y(t) = \eta\, t, \ t > 0, \\ y(0) = 0. \end{cases} \tag{3.10}$$

Taking the ZZT on both sides of Equation (3.10) and plugging the initial condition, we get

$$\left[\frac{\psi(\alpha)}{1-\alpha}\frac{\left(\frac{s}{v}\right)^{\alpha} Z(v,s) - \left(\frac{s}{v}\right)^{\alpha} y(0)}{\left(\frac{s}{v}\right)^{\alpha} + \frac{\alpha}{1-\alpha}}\right] = \eta\left(\frac{v}{s}\right),$$

$$\left[\psi(\alpha)\frac{Z(v,s)}{\left(1-\alpha+\alpha\left(\frac{v}{s}\right)^{\alpha}\right)}\right] = \eta\left(\frac{v}{s}\right),$$

$$\begin{aligned} Z(v,s) &= \eta\left(\frac{v}{s}\right)\frac{\left(1-\alpha+\alpha\left(\frac{v}{s}\right)^{\alpha}\right)}{\psi(\alpha)} \\ &= \frac{\eta}{\psi(\alpha)}\left[(1-\alpha)\left(\frac{v}{s}\right)+\alpha\left(\frac{v}{s}\right)^{\alpha+1}\right]. \end{aligned} \tag{3.11}$$

Applying inverse ZZT on both sides of Equation (3.11), we obtain

$$y(t) = \frac{\eta}{\psi(\alpha)}\left[(1-\alpha)\,t + \frac{\alpha}{\Gamma(\alpha+2)} t^{\alpha+1}\right]. \tag{3.12}$$

It is noticed that if we put $\alpha = 0$, then Equation (3.12) reduces to $y(t) = \eta\, t$ and substituting $\alpha = 1$ in Equation (3.12), we obtain $y(t) = \eta\,\frac{t^2}{2}$. Plugging $\alpha = 0.5$, we get $y(t) = \frac{\eta}{\psi(0.5)}\left[\frac{t}{2} + \frac{2}{3\sqrt{\pi}} t^{\frac{3}{2}}\right]$.

4. CONCLUSION

In this manuscript, the ZZT is debated and the associated properties of ZZT are established. Some theorems related to the connection between the ZZ, Aboodh, and Laplace transforms are successfully proven. ZZT was applied to FDEs within the AB derivatives. Besides, some fractional initial value problems are solved in order to illustrate the validity and performance of this transformation.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary materials, further inquiries can be directed to the corresponding author/s.

AUTHOR CONTRIBUTIONS

RJ: conceptualization, writing–original draft, methodology, software, and validation. SC and DB: project administration and supervision. MA and DB: funding acquisition. SC, DB, and MA: contributed to the analysis, discussion of the results, and help in revision. All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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