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Research article

Numerical method for pricing discretely monitored double barrier option by orthogonal projection method

Kazem Nouri[1,*], **Milad Fahimi**[1], **Leila Torkzadeh**[1] **and Dumitru Baleanu**[2,3]

[1] Department of Mathematics, Faculty of Mathematics, Statistics and Computer Sciences, Semnan University, P. O. Box 35195-363, Semnan, Iran
[2] Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, Ankara, Turkey
[3] Institute of Space Sciences, Magurele–Bucharest, Romania

* **Correspondence:** E-mail: knouri@semnan.ac.ir, knouri.h@gmail.com.

Abstract: In this paper, we consider discretely monitored double barrier option based on the Black-Scholes partial differential equation. In this scenario, the option price can be computed recursively upon the heat equation solution. Thus we propose a numerical solution by projection method. We implement this method by considering the Chebyshev polynomials of the second kind. Finally, numerical examples are carried out to show accuracy of the presented method and demonstrate acceptable accordance of our method with other benchmark methods.

Keywords: double barrier option; orthogonal projection; Chebyshev polynomial; black-Scholes model
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1. Introduction

Barrier options are one of the oldest types of exotic derivatives. Since 1967, barrier options have been traded in over-the-counter(OTC) markets, especially in the US markets. Barrier options are considered as a type of exotic option because they are more complex than basic ordinary options. The most important advantage of barrier options is that flexibility and possibility of matching with risk hedging which make them more attractive to hedgers and traders in financial markets. Also, their payoff depend on whether or not the underlying asset has reached or exceeded a predetermined price.

Nowadays, we have impressive growth in the variety of barrier options, and the most frequently used standard barrier options are knock-in and knock-out options. A barrier option is called knock-out if it is deactivated when the price of the underlying asset hits the barrier. Similarly, a barrier option is called knock-in if it is activated when the price of underlying asset hits the barrier. From another side, an important matter is how frequently the barrier and the price of underlying asset. Although most of researchers assume barrier options in continuous form of monitoring [1–3], but most of real financial contracts are based on discrete monitoring for example daily, weekly or monthly. Therefore discrete monitoring provides an additional degree of realism and also adapts the process of option pricing with real contracts in financial markets. The Black-Scholes model [4] is a mathematical model for pricing an option contract. In addition to the Black–Scholes option valuation theory, we can benefit from works previously developed by researchers, such as Fusai et al. [5] and Awasthi and Tk [6]. The options contracts can be priced using mathematical models such as the Black-Scholes model. There are many models and methods to propose approximations based on a variety of different numerical approaches, such as recent research studies [7–9].

Barrier options were proposed and analyzed by researchers over the last three decades. In the academic literature, the pricing of barrier option arises in the research of Merton [10] who presented a closed-form solution for single barrier options based on European call option. Fusai et al. [5] presented the analytical solution for discrete barrier options. In discrete monitoring case, the trinomial method has been presented by Kamrad and Ritchken [11], and also Kowk used the binomial and trinomial for pricing path-dependent options [12]. The quadrature method QUAD-K20 and QUAD-K30 are applied for pricing discrete barrier option in [13]. In [14–17] the process of pricing single and double barrier option has been considered. In the work of [16], finite element method is applied for pricing double barrier option.

The projection methods have been used in various fields, especially in financial mathematics. These methods give a general flexible way to solve functional equation. In this paper, we introduce orthogonal projection operators based on the Chebyshev polynomials and we implement new algorithm for pricing discretely monitored double barrier option. We present numerical results which confirm acceptable accordance with other benchmark values.

The main advantages of the proposed numerical method is that Chebyshev polynomials are more robust than their alternatives for projection, and they form a special class of orthogonal polynomials especially suited for approximating with higher accuracy. In our method, the move between the coefficients of a Chebyshev expansion of a function and the values of the function quickly performed by orthogonal projection operators involved with integration. Also, it is important to note that an implication of our work is that our numerical method is simple to apply and to extend and provides a reliable framework which can be used either for pricing more complex derivative instruments. This paper is organized as follows. In Section 2, preliminaries and notations are presented. In Section 3, the Chebyshev polynomials and their properties are introduced. Moreover, the shifted Chebyshev polynomial of the second kind is prepared for applying in approximation. In Section 4, we implement orthogonal projection method based on the Chebyshev polynomial of the second kind for pricing discretely monitored double barrier option and finally, Section 5 is devoted to present numerical results and illustrate our highly accurate results.

2. Preliminaries

In this paper we focus on the standard Black-Scholes framework. We assume the following notation: S is the price of underlying asset, σ is its volatility, r is risk-free interest rate, t denotes the current time and V is the value of barrier option defined as a function of underlying asset price at time

$t \in (t_{m-1}, t_m)$ with monitoring dates $0 = t_0 < t_1 < t_2 < ... < t_M = T$ which are determined in equal discrete time intervals, i.e., $t_m = m\tau$ where $\tau = \frac{T}{M}$ and T denotes the time of expiration and E is the strike or exercise price.

In this paper we assume that the price of barrier option $P(S, t)$ satisfies the Black-Scholes equation

$$-\frac{\partial P}{\partial t} + rS\frac{\partial P}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P}{\partial S^2} - rP = 0. \tag{2.1}$$

Also the process S_t is described by the following geometric Brownian motion (GBM) [18],

$$dS_t = rS_t dt + \sigma S_t dB_t, \tag{2.2}$$

where B_t is a standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}_t, \mathbb{P})$, in which Ω is a nonempty set called the sample space and $\mathcal{F}_t$ is filtration satisfying the usual conditions, and $\mathbb{P}$ is a probability function. First, we consider the classification of knock-out single barrier option for call position. According to [19, 20], this classification can be expressed as follows:

(1) Down-and-out is an option that terminates when the price of the underlying asset declines to a predetermined level and its payoff is

$$DOC = (S_t - E)^+ I_{\{min_{0\le t\le T} S_t > L\}} = (S_t - E) I_{\{min_{0\le t\le T} S_t > L,\ S_t > E\}}. \tag{2.3}$$

(2) Up-and-out is an option that terminates when the price of the underlying asset increases to a predetermined level and its payoff is

$$UOC = (S_t - E)^+ I_{\{max_{0\le t\le T} S_t < U\}} = (S_t - E) I_{\{max_{0\le t\le T} S_t < U,\ S_t > E\}}. \tag{2.4}$$

Where L is lower barrier, U is upper barrier and I is the indicator function. Therefore, the payoff of knock-out double barrier option can be obtained by relations (2.3) and (2.4),

$$\begin{aligned} OC_{Double} &= (S_t - E)^+ I_{\{max_{0\le t\le T}(E,L) < S_t < U\}} \\ &= (S_t - E) I_{\{max_{0\le t\le T}(E,L) < S_t < U,\ S_t > E\}}. \end{aligned} \tag{2.5}$$

In our specific case, the initial and boundary conditions can be written as

$$\begin{cases} P(S, t_0, 0) = (S - E) I_{(\max(E,L) < S_t < U)}, \\ P(S, t_m, 0) = P(S, t_m, m-1) I_{(L < S_t < U)}, \ m = 1, 2, ..., M-1, \end{cases}$$

and

$$P(S, t_m, m-1) = \lim_{t \to t_m} P(S, t, m-1).$$

Now, we apply the following transformation

$$\begin{cases} \theta = \ln(\frac{U}{L}), \quad \mu = r - \frac{\sigma^2}{2}, \quad \delta = \max\{E^*, 0\}, \quad E^* = \ln(\frac{E}{L}), \\ Z = \ln(\frac{S}{L}), \quad C(Z, t, m) = P(S, t, m). \end{cases}$$

So, we obtain

$$\begin{cases} Z = \ln(\frac{S}{L}) \Rightarrow \frac{\partial Z}{\partial S} = \frac{1}{S}, \\ \frac{\partial P}{\partial t} = \frac{\partial C}{\partial t} = C_t, \\ \frac{\partial P}{\partial S} = \frac{\partial P}{\partial Z} \times \frac{\partial Z}{\partial S} = \frac{\partial C}{\partial Z} \times \frac{\partial Z}{\partial S} = C_Z \times \frac{1}{S}, \\ \frac{\partial^2 P}{\partial S^2} = \frac{\partial}{\partial S}(\frac{\partial P}{\partial S}) = \frac{\partial}{\partial S}(\frac{1}{S}C_Z) = -\frac{1}{S^2}C_Z + \frac{1}{S^2}C_{ZZ}. \end{cases}$$

According to (2.1), we obtain

$$\begin{aligned} &-C_t + rS(\frac{1}{S}C_Z) + \frac{1}{2}\sigma^2 S^2(-\frac{1}{S^2}C_Z + \frac{1}{S^2}C_{ZZ}) = rC \\ &\Rightarrow -C_t + (r - \frac{1}{2}\sigma^2)C_Z + \frac{1}{2}\sigma^2 C_{ZZ} = rC \\ &\Rightarrow -C_t + \mu C_Z + \frac{1}{2}\sigma^2 C_{ZZ} = rC. \end{aligned}$$

Since,

$$\begin{cases} Z = \ln(\frac{S}{L}) \Rightarrow S = Le^Z, \\ E^* = \ln(\frac{E}{L}) \Rightarrow E = Le^{E^*}, \end{cases}$$

the initial conditions can be rewritten as

$$\begin{cases} P(S, t_0, 0) = C(Z, t_0, 0) = S - E = L(e^Z - e^{E^*}), \quad \delta < Z < \theta, \\ \text{while } \max(E, L) < S < U, \\ P(S, t_m, 0) = C(Z, t_m, m-1), \; 0 < Z < \theta. \end{cases}$$

Now apply further transformation

$$C(Z, t_m, m) = e^{\alpha Z + \beta t} h(Z, t, m), \tag{2.6}$$

where

$$\alpha = -\frac{\mu}{\sigma^2}, \; \beta = \alpha\mu + \alpha^2\frac{\sigma^2}{2} - r, \; \lambda^2 = \frac{\sigma^2}{2}.$$

Therefore,

$$C_t = (\beta h + h_t)e^{\alpha Z + \beta t}, \tag{2.7}$$

$$C_Z = (\alpha h + h_Z)e^{\alpha Z + \beta t}, \tag{2.8}$$

$$C_{ZZ} = \frac{\partial}{\partial Z}(\frac{\partial C}{\partial Z}) = \frac{\partial}{\partial Z}(\alpha h + h_Z)e^{\alpha Z + \beta t} = (\alpha^2 h + 2\alpha h_Z + h_{ZZ})e^{\alpha Z + \beta t}. \tag{2.9}$$

Subtituting (2.7), (2.8) and (2.9) into (2.1) yields

$$-(\beta h + h_t)e^{\alpha Z+\beta t} - \alpha\sigma^2(\alpha h + h_Z)e^{\alpha Z+\beta t} + \frac{\sigma^2}{2}(\alpha^2 h + 2\alpha h_Z + h_{ZZ})e^{\alpha Z+\beta t} = (-\alpha^2\sigma^2 + \alpha^2\frac{\sigma^2}{2} - \beta)he^{\alpha Z+\beta t}.$$

For simplification, we have

$$(-\beta h - h_t - \alpha^2\sigma^2 h - \alpha\sigma^2 h_Z + \frac{\sigma^2}{2}\alpha^2 h + \alpha\sigma^2 h_Z + \frac{\sigma^2}{2}h_{ZZ})e^{\alpha Z+\beta t} = (-\alpha^2\sigma^2 h + \alpha^2\frac{\sigma^2}{2}h - \beta h)e^{\alpha Z+\beta t}$$

$$\Longrightarrow -h_t + \frac{\sigma^2}{2}h_{ZZ} = 0.$$

The final result of these transformations is that the Black-Scholes equation has been reduced to the much simpler PDE, called the heat equation as follows

$$h_t = \lambda^2 h_{ZZ}.$$

So, we achieve the following heat equation

$$\begin{cases} h_t = \lambda^2 h_{ZZ}, \\ h(Z, t_0, 0) = Le^{-\alpha Z}(e^Z - e^{E^*})I_{(\delta<Z<\theta)}, \\ h(Z, t_m, m) = h(Z, t_m, m-1)I_{(0<Z<\theta)}, \quad m = 1, 2, ..., M-1. \end{cases}$$

According to [21], the analytical solution of heat equation is considered as follows

$$h(Z, t, m) = \begin{cases} \int_\delta^\theta \kappa(Z-\xi, t)Le^{-\alpha\xi}(e^\xi - e^{E^*})d\xi, \quad m = 0, \\ \\ \int_0^\theta \kappa(Z-\xi, t-t_m)h(Z, t_m, m-1)d\xi, \quad m = 1, 2, ..., M-1, \end{cases}$$

where

$$\kappa(Z, t) = \frac{1}{\sqrt{4\pi\lambda^2 t}}e^{\frac{-Z^2}{4\lambda^2 t}}.$$

For simplicity, let us formulate this analytical solution in new frame by defining $f_m(Z) = h(Z, t_m, m-1)$,

$$f_0(Z) = Le^{-\alpha Z}(e^Z - e^{E^*})I_{(\delta<Z<\theta)}, \tag{2.10}$$

$$f_1(Z) = \int_0^\theta \kappa(Z-\xi, \tau)f_0(\xi)d\xi, \tag{2.11}$$

$$f_m(Z) = \int_0^\theta \kappa(Z-\xi, \tau)f_{m-1}(\xi)d\xi, \quad m = 2, 3, ..., M. \tag{2.12}$$

3. Chebyshev polynomials approximation

In this section, we describe in more details the projection method based on the Chebyshev polynomials of the second kind. The Chebyshev polynomials of the second kind with orthogonality property in interval $[-1, 1]$ are defined as follows

$$U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x), \quad n = 2, 3, ..., \tag{3.1}$$

where, $U_0(x) = 1$ and $U_1(x) = 2x$.

The orthogonality property of these polynomials is given as follows which defines an inner product on $C[a, b]$,

$$\langle U_n(x), U_m(x)\rangle = \int_{-1}^{1} \sqrt{1-x^2}U_n(x)U_m(x)dx = \begin{cases} \frac{\pi}{2}, & m = n \neq 0, \\ \pi, & m = n = 0, \\ 0, & m \neq n. \end{cases} \tag{3.2}$$

In order to use these polynomials, the range of the variable x is the interval $[-1, 1]$. So we map the independent variable x to the variable t in the interval $[a, b]$ by transformation

$$x = \frac{2t-(a+b)}{b-a}.$$

Therefore, the relation (3.1) is led to the shifted Chebyshev polynomial of the second kind [22]. Now we set $a = 0$ and $b = \theta$ and consider the polynomial on the interval $[0, \theta]$.

A function $f(t) \in L^2[0, \theta]$, can be expressed in terms of the shifted Chebyshev polynomials as

$$f(t) = \sum_{n=0}^{\infty} A_n U_n(\frac{2t}{\theta} - 1), \tag{3.3}$$

where $\{A_n\}_{n=0}^{\infty}$ are the coefficients of the Fourier series [23]. These coefficients can be found by using orthogonality property of the Chebyshev polynomial as follows

$$\int_0^{\theta} U_n(\frac{2t}{\theta} - 1)U_m(\frac{2t}{\theta} - 1)\left(1 - (\frac{2t}{\theta} - 1)^2\right)^{\frac{1}{2}} dt = \delta_{m,n}h_m, \tag{3.4}$$

where $\delta_{m,n}$ is the Dirac function which can be defined as bellow

$$\delta_{m,n} = \begin{cases} 1, & m = n, \\ 0, & m \neq n, \end{cases}$$

and also h_m can be defined with

$$h_m = \begin{cases} \frac{\pi}{4}\theta, & m = n \neq 0, \\ \frac{\pi}{2}\theta, & m = n = 0, \\ 0, & m \neq n. \end{cases}$$

Now by defining the shifted Chebyshev polynomial of the second kind $U_n^*(t)$ as $U_n(\frac{2t}{\theta} - 1) = U_n^*(t)$ and the weight function as $W_\theta(t) = \sqrt{\theta t - t^2}$, the relation (3.4) will be change into:

$$\int_0^{\theta} U_n^*(t)U_m^*(t)W_\theta(t)dt = \delta_{m,n}\hat{h}_m, \tag{3.5}$$

where

$$\hat{h}_m = \begin{cases} \frac{\pi}{2}, & m = n \neq 0, \\ \pi, & m = n = 0, \\ 0, & m \neq n. \end{cases}$$

By multiplying $U_m^*(t)W_\theta(t)$ on the both sides of equation (3.3) and integrating from 0 to θ, we can achieve

$$\int_0^\theta f(t)U_m^*(t)W_\theta(t)dt = \sum_{n=0}^{\infty} A_N \int_0^\theta U_n^*(t)U_m^*(t)W_\theta(t)dt = A_N\delta_{m,n}\hat{h}_m.$$

Therefore the matrix form of coefficients in (3.4) can be presented as:

$$A_N = [a_0, a_1, ..., a_N]^T,$$

where

$$a_0 = \frac{1}{\pi}\int_0^\theta f(t)U_0^*(t)W_\theta(t)dt = \frac{1}{\pi}\int_0^\theta f(t)W_\theta(t)dt,$$

$$a_n = \frac{2}{\pi}\int_0^\theta f(t)U_n^*(t)W_\theta(t)dt, \quad n = 1, 2, ..., N.$$

Now, if we define

$$\tilde{U}_m^*(x) = \sqrt{\frac{W_\theta(t)}{\hat{h}_m}}U_m^*(t), \quad \text{for } \hat{h}_m \neq 0,$$

then $\{\tilde{U}_m^*(t)\}_{m=0}^{\infty}$ is as an orthogonal basis for $L^2[0, \theta]$.

4. Implementing projection method based on the Chebyshev polynomials

In this section, implementation of orthogonal projection method based on the Chebyshev polynomials is organized as follows:

- Step 1: Choose an orthogonal basis, a norm ||.|| and an inner product < . >,
- Step 2: Choose a degree of approximation (n), and define a projection operator P_n,
- Step 3: Construct a computable approximation by choosing suitable kind of orthogonal polynomials,
- Step 4: Evaluate projections involved with integration.

Theorem 4.1. *Let $f(x)$ be a polynomial of degree k or less. Then the following expansion can be presented*

$$f(x) = d_0p_0(x) + d_1p_1(x) + ... + d_kp_k(x),$$

where $d_0, d_1, ..., d_k$ are unique coefficients, $d_i = \dfrac{\langle f, p_i\rangle}{\langle p_i, p_i\rangle}$ and $\{p_i(x)\}_{i=0}^{\infty}$ are orthogonal polynomials.

Proof. Refer to [24]. □

Due to Theorem 4.1, we can define projection operator $P_n : L^2[0,\theta] \to \psi_n$, where ψ_n is the space of all polynomials of degree n or less [25], and

$$\forall f \in L^2[0,\theta], P_n(f) = \sum_{i=0}^{\infty} < f, \tilde{U}_i^*(t) > \tilde{U}_i^*(t). \tag{4.1}$$

Also, by relation (3.3) and defined operator in (4.1), we can approximate the value of f by $\tilde{f}_{m,n}$ as follows [15]:

$$\tilde{f}_{m,n} = \varphi_n^T K^{m-1} F_1. \tag{4.2}$$

The matrix form of φ_n^T, F_1 and K can be obtained as

$$\begin{aligned} \varphi_n^T &= [\tilde{U}_0^*(t), \tilde{U}_1^*(t), ..., \tilde{U}_n^*(t)], \\ F_1 &= [a_{10}, a_{11}, ..., a_{1n}]^T, \\ K &= (K_{ij})_{(n+1)(n+1)}, \end{aligned}$$

where

$$\begin{cases} a_{1i} = \int_0^\theta \int_\delta^\theta \tilde{U}_i^*(\eta)\kappa(\eta-\varepsilon,\tau)d\varepsilon d\eta, & 0 \le i \le n, \\ K_{ij} = \int_0^\theta \int_\delta^\theta \tilde{U}_{i-1}^*(\eta)\tilde{U}_{j-1}^*(\varepsilon)\kappa(\eta-\varepsilon,\tau)d\varepsilon d\eta, & 1 \le i,j \le n+1. \end{cases}$$

5. Numerical results and comparison studies

In order to verify accuracy of the presented method and demonstrate our analytical results, we prepare numerical examples with the considered value of parameters in Table 1. The all simulations are done by using MATLAB R2017a. In this section our numerical results are compared with trinomial tree [26] and projection method based on the Legendre polynomials [15] and quadrature method [13].

Table 1. The parameter values used in Examples 5.1 and 5.2.

E	S_0	r	σ	U	T
100	100	0.05	0.25	120	0.5

Example 5.1. *We consider discretely monitored knock-out double barrier option for call position. The barriers are fixed and two monitoring dates* $m = 25, 125$ *have been considered.*

Applying transformation $x_i = \frac{2t_i}{\theta} - 1$, *the maximum deviation of* $\prod_{i=0}^{n}|x - x_i|$ *from zero in the interval* $[0,\theta]$, *can be considered as follows [27],*

$$\max \prod_{i=0}^{n}|x - x_i| = |\frac{\theta}{2}|^{n+1} \max \prod_{j=0}^{n}|t - t_j| \le \frac{1}{2^n}|\frac{\theta}{2}|^{n+1}.$$

Tables 2 and 3 report the double barrier option pricing results of comparing the proposed method with other methods in weekly and daily observation modes, respectively. In Tables 4 and 5, the absolute error of employed numerical method is presented with respect to other methods. Also, these numerical

Table 2. Comparison of double barrier option prices due to Example 5.1 by weekly observations (M=25).

L	M	N	Presented Method	Trinomial	P.M-Legendre	Quad-K30
80	25	15	1.9428	1.9490	1.9420	1.9420
90	25	15	1.5362	1.5630	1.5354	1.5354
95	25	15	0.8669	0.8823	0.8668	0.8668
99	25	15	0.2933	0.3153	0.2931	0.2931

Table 3. Comparison of double barrier option prices due to Example 5.1 by daily observations (M=125).

L	M	N	Presented Method	Trinomial	P.M-Legendre	Quad-K30
80	125	15	1.6811	1.7477	1.6808	1.6808
90	125	15	1.2032	1.2370	1.2029	1.2029
95	125	15	0.5524	0.5699	0.5532	0.5532
99	125	15	0.1039	0.1201	0.1042	0.1042

Table 4. Absolute error of projection method based on the Chebyshev polynomials with respect to other methods (M=25, N=15).

L	M	Trinomial	P.M-Legendre	Quad-K30
80	25	0.0062	0.0008	0.0008
90	25	0.0268	0.0008	0.0008
95	25	0.0154	0.0001	0.0001
99	25	0.0220	0.0002	0.0002

Table 5. Absolute error of projection method based on the Chebyshev polynomials with respect to other methods (M=125, N=15).

L	M	Trinomial	P.M-Legendre	Quad-K30
80	125	0.0666	0.0003	0.0003
90	125	0.0347	0.0003	0.0009
95	125	0.0175	0.0008	0.0008
99	125	0.0162	0.0003	0.0003

results demonstrate the adaptation of our method with other benchmark methods. Tables 4 and 5 allow us to observe the effect of increasing observation dates on the computational results. These show that the computational results are not deteriorated as it usually happens with other numerical methods such as finite difference schemes and trinomial trees.

Example 5.2. *In this example, we consider knock-out double barrier option as* $L = 95.5$, $U = 120$, $r = 0.05$, $\sigma = 0.25$, $E = 100$ *and* $T = 0.5$. *In Table 6, we price knock-out double barrier option with different number of nodes* (N). *Due to these results, as we increase the number of nodes, we achieve highly accurate and reliable results. In this example, we select quadrature method (Quad-K30) as the benchmark value.*

Table 6. Double barrier option pricing of Example 5.2 with increasing number of nodes.

N(Number of nodes)	M=5	M=25	M=125
5	1.7101	0.8968	05882
10	1.7021	0.8918	0.5832
15	1.6991	0.8898	0.5762
20	1.6931	0.8858	0.5712
25	1.6881	0.8758	0.5662
30	1.6871	0.8708	0.5592
35	1.6851	0.8698	0.5572
40	1.6851	0.8698	0.5562
Benchmark value	1.6831	0.8668	0.5532

Fig. 1 shows the errors comparison for various number of monitoring date. According to Fig. 1, the approximation error tends to zero when the number of basis functions (nodes) increases. Furthermore, the obtained values of the proposed numerical method tends to the benchmark value when the number of basis functions (nodes) increases.

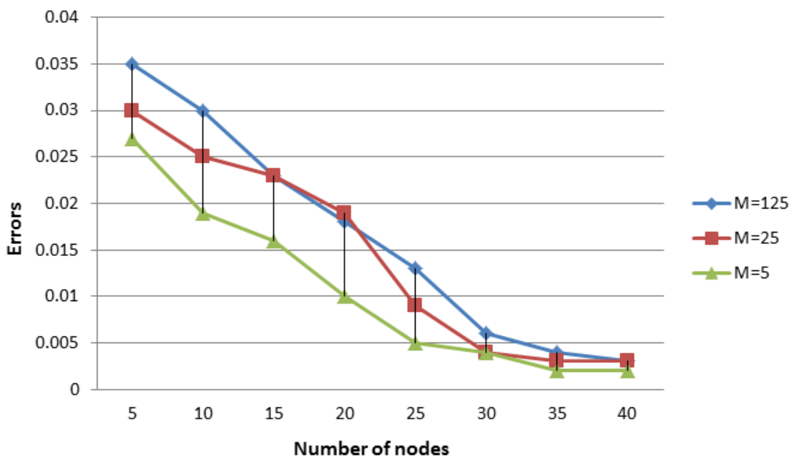

Figure 1. Errors comparison of the presented method for pricing double barrier option.

6. Conclusions

In this paper, we have implemented orthogonal projection method based on the Chebyshev polynomials of the second kind for pricing discretely monitored double barrier options. We presented numerical results and compared the performance of our method with other benchmark methods such as the trinomial tree, projection method based on the Legendre polynomials and quadrature method. These results demonstrate acceptable accordance of our method with other benchmark methods. The strength of the presented numerical method is particularly demonstrated in pricing knock-out double barrier options with highly accurate results. Future research can be devoted to extend this idea for other types of options.

Conflict of interest

The authors declared that they have no conflicts of interest to this work.

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