Research Article

Junaid Akhtar, Aly R. Seadawy*, Kalim U. Tariq, and Dumitru Baleanu

On some novel exact solutions to the time fractional (2 + 1) dimensional Konopelchenko–Dubrovsky system arising in physical science

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Abstract: The purpose of this article is to construct some novel exact travelling and solitary wave solutions of the time fractional (2 + 1) dimensional Konopelchenko–Dubrovsky equation, and two different forms of integration schemes have been utilized in this context. As a result, a variety of bright and dark solitons, kink- and antikink-type solitons, hyperbolic functions, trigonometric functions, elliptic functions, periodic solitary wave solutions and travelling wave solutions are obtained, and the sufficient conditions for the existence of solution are also discussed. Moreover, some of the obtained solutions are illustrated as two- and three-dimensional graphical images by using computational software Mathematica. These types of solutions have a wide range of applications in applied sciences and mathematical physics. The proposed methods are very useful for solving nonlinear partial differential equations arising in physical science and engineering.

Keywords: Fractional Konopelchenko–Dubrovsky equation, Jumarie's modified Riemann–Liouville, unified Riccati equation expansion, modified extended auxiliary equation mapping method

1 Introduction

Over the last few decades, nonlinear phenomena have been observed to have fascinating characteristics in mathematical physics and engineering. The phenomena of nonlinear evaluation equations (NLEEs) have attracted much attention and have become one of the most interesting fields of research. These types of equations are broadly utilized to explain complex physical phenomena arising in fluid mechanics, plasma wave, optical fibre telecommunication, biophysics, soliton theory and atmospheric science [1–3]. Nowadays, for the constructions of the travelling wave solutions of these types of coupled equations have been one of the most attractive areas of research, exact solutions of coupled nonlinear equations can be useful for better understanding rather than numerical solutions. Therefore, it is necessary for mathematicians and physicists to construct the exact solutions of these NLEEs for aiming this, and many effective and powerful techniques have been established such as the inverse scattering transformation [4,5], the Backlund transformation technique [6], the auxiliary equation method [7,8], the extended direct algebraic method [9,10], the Darboux transformation method [11], the homotopy perturbation method [12,13] and many others [14–16]. Recently, many scientists and researchers have agreed that we cannot neglect space and time-fractional evaluation for exploring the many physical problems due to the presence of nonlocality or nonconservative systems in real-world problem. For this purpose, scientist community have denoted their energy for finding new solutions of NLEEs by using various types of fractional derivatives such as conformable fractional [17], beta-fractional [18], M-truncated fractional [19] and time-fractional depending upon the requirement of the dynamical system.

In our present work, the unified Riccati equation expansion method and the modified extended auxiliary equation mapping method are successfully employed to construct a variety of new travelling wave solutions to a coupled time fractional (2 + 1) dimensional Konopelchenko–Dubrovsky system [20,21]:

$$\begin{aligned} &D_t^{\alpha}\Upsilon - \Upsilon_{xxx} - 6\lambda\Upsilon\Upsilon_x + \frac{3}{2}\rho\Upsilon^2\Upsilon_x \\ &- 3\Xi_y + 3\rho\Xi\Upsilon_x = 0, \quad 0 < \alpha \le 0 \\ &\Xi_x = \Upsilon_y, \end{aligned} \tag{1}$$

* **Corresponding author: Aly R. Seadawy,** Mathematics Department, Faculty of science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia, e-mail: aly742001@yahoo.com
Junaid Akhtar, Kalim U. Tariq: Department of Mathematics, Mirpur University of Science and Technology, Mirpur, 10250 (AJK), Pakistan
Dumitru Baleanu: Department of Mathematics, Cankaya University, Ankara, Turkey; Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan; Department of Mathematics, Institute of Space Sciences, 077125 Magurele, Romania

where $\Upsilon = \Upsilon(x, y, t)$, $\Xi = \Xi(x, y, t)$ are complex valued functions and ρ, λ are real constants while D_t^{α} represents the conformable derivative operator of order α. The independent variables t, x and y are temporal and spatial variables. The Konopelchenko–Dubrovsky equation (KDE) system is widely used by many authors in different forms such as the coupled system equation (1) will give Kadomtsev–Petviashvili [22] and modified Kadomtsev–Petviashvili [23] for $\rho = 0$ and $\lambda = 0$ and many more [24–34].

Fractional calculus has been found to have a revolutionary effect in many physical phenomena to overcome the limitations found in classical integers and have many applications such as signal processing, acoustic waves, systems identifications, biomechanics and many others [35–37]. Due to abundant features of fractional differential equations (FDEs), it has become one of the most interesting fields of research. For this purpose, various techniques have been developed to formulate exact and travelling wave solutions of FDEs. Recently, a new definition of fractional calculus has been introduced by Jumarie's modified Riemann–Liouville (mRL) of order α as [38,39]:

$$p^{(\alpha)}(t) = \lim_{q\to 0}\left[\frac{\sum_{n=0}^{\infty}(-1)^n\binom{\alpha}{n}p\{t + (\alpha - n)q\}}{q^{\alpha}}\right], \tag{2}$$
$$\alpha \in \Re, \quad 0 < \alpha \le 0.$$

This paper is arranged as follows: in Section 2, the mathematical analysis of (1) is discussed, and the unified Riccati equation expansion and the modified extended auxiliary equation mapping method are applied to extract a variety of novel solutions in Sections 3 and 4. Finally, in Section 5 concluding remarks are presented.

2 Mathematical analysis

In this section, consider the travelling wave transformation in equation (1) as

$$\Upsilon(x, y, t) = \Psi(\theta), \quad \Xi(x, y, t) = \Phi(\theta),$$
$$\theta = \mu_1 x + \mu_2 y + \frac{\mu_3 t^{\alpha}}{\Gamma(\alpha + 1)}, \tag{3}$$

where μ_1, μ_2 and μ_3 and α are constants to be determined. By utilizing the aforementioned transformation, we acquire the following ordinary differential equation of the couple system as

$$\mu_3\Psi' - \mu_1^3\Psi''' - 6\lambda\mu_1\Psi\Psi' + \frac{3}{2}\rho\mu_1\Psi^2\Psi' - 3\mu_2\Phi' + 3\rho\mu_1\Psi'\Phi = 0, \tag{4}$$

$$\mu_1\Phi' = \mu_2\Psi'. \tag{5}$$

By integrating (5) once with respect to θ, we acquire

$$\Phi = \frac{\mu_2\Psi}{\mu_1} + c, \tag{6}$$

where c is the constant of integration, by inserting equation (6) into equation (4), we obtain

$$-2\mu_1^4\Psi''' + 3\rho\mu_1^2\Psi^2\Psi' + 6\mu_1(\rho\mu_2 - 2\lambda\mu_1)\Psi\Psi' + 2(3c\rho\mu_1^2 + \mu_1\mu_3 - 3\mu_2^2)\Psi' = 0. \tag{7}$$

2.1 The unified Riccati equation expansion method

Let us suppose the solution of equation (7) will be of the form:

$$\Psi(\theta) = b_0 + b_1\chi(\theta), \tag{8}$$

where b_0 and b_1 are parameters to be determined, such that $b_1 \neq 0$, while $\chi(\theta)$ satisfies the Riccati equation:

$$\chi'(\theta) = f_0 + f_1\chi(\theta) + f_2\chi^2(\theta), \tag{9}$$

where f_i $(i = 0, 1, 2)$ are constants to be determined, such that $f_2 \neq 0$. Now, equation (9) has the formal rational solution of the form:

$$\chi(\theta) = -\frac{f_1}{2f_2} - \frac{\sqrt{\Delta}}{2f_2}\left[\frac{k_1\tanh\left(\frac{\sqrt{\Delta}}{2}\theta\right) + k_2}{k_1 + k_2\tanh\left(\frac{\sqrt{\Delta}}{2}\theta\right)}\right], \tag{10}$$
$$\Delta > 0, (k_1^2 + k_2^2) \neq 0,$$

$$\chi(\theta) = -\frac{f_1}{2f_2} + \frac{\sqrt{-\Delta}}{2f_2}\left[\frac{k_3\tan\left(\frac{\sqrt{-\Delta}}{2}\theta\right) - k_4}{k_3 + k_4\tan\left(\frac{\sqrt{-\Delta}}{2}\theta\right)}\right], \tag{11}$$
$$\Delta < 0, (k_3^2 + k_4^2) \neq 0,$$

and

$$\chi(\theta) = -\frac{f_1}{2f_2} - \frac{1}{f_2\theta + c}, \quad \Delta = 0, \tag{12}$$

where $\Delta = f_2^2 - 4f_0f_1$ and k_i $(i = 1, 2, 3, 4)$ are arbitrary constants and c is the integration constant. Now, putting equation (8) along with (9) into equation (7), and then collecting each coefficient of all terms with same powers of $f^i(\theta)$ $(i = 0, 1, 2, 3, 4)$ to zero, we get a system of algebraic equations for f_0, f_1, f_2, b_0 and b_1.

$$\begin{aligned}
&-b_1f_0[3\mu_1^2(4b_0\lambda + b_0^2\rho - 2c\rho) - 2\mu_1(3b_0\mu_2\rho + \mu_3)\\
&\quad + 2(f_1^2 + 2f_0f_2)\mu_1^4 + 6\mu_2^2] = 0,\\
&-b_1[f_1(3\mu_1^2(4b_0\lambda + b_0^2\rho - 2c\rho) - 2\mu_1(3b_0\mu_2\rho + \mu_3)\\
&\quad + 2(f_1^2 + 8f_0f_2)\mu_1^4 + 6\mu_2^2)\\
&\quad + 6b_1f_0\mu_1(\mu_1(b_0\rho + 2\lambda) - \mu_2\rho)] = 0,\\
&-b_1[f_2(3\mu_1^2(4b_0\lambda + b_0^2\rho - 2c\rho) - 2\mu_1(3b_0\mu_2\rho + \mu_3)\\
&\quad + 2(7f_1^2 + 8f_0f_2)\mu_1^4 + 6\mu_2^2)\\
&\quad + 6b_1f_1\mu_1(\mu_1(b_0\rho + 2\lambda) - \mu_2\rho) + 3b_1^2f_0\mu_1^2\rho] = 0,\\
&-3b_1\mu_1[2b_1f_2(\mu_1(b_0\rho + 2\lambda) - \mu_2\rho)\\
&\quad + b_1^2f_1\mu_1\rho + 8f_1f_2^2\mu_1^3] = 0,\\
&-3b_1[b_1^2f_2\mu_1^2\rho + 4f_2^3\mu_1^4].
\end{aligned} \tag{13}$$

Solving this system of algebraic equations, with the aid of *Mathematica* yields the following results of the form:

Case I:

$$\begin{aligned}
b_0 &= \pm\frac{if_1\mu_1^2\sqrt{\rho} - 2\lambda\mu_1 + \mu_2\rho}{\mu_1\rho},\\
b_1 &= \pm\frac{2if_2\mu_1}{\sqrt{\rho}}, \quad f_1 = f_1,\ f_2 = f_2,\\
f_0 &= \frac{6\mu_1^2(c\rho^2 + 2\lambda^2) + f_1^2\mu_1^4\rho + 2\mu_1\rho(\mu_3 - 6\lambda\mu_2) + 3\mu_2^2(\rho - 2)\rho}{4f_2\mu_1^4\rho}.
\end{aligned} \tag{14}$$

From equations (3), (6), (7), (8), (10) and (14), for only the positive value of b_1, the solutions of equation (1) can be acquired as follows:

$$\begin{aligned}
\Psi_1(x, y, t) &= \frac{\mu_2\rho - 2\lambda\mu_1 - i\sqrt{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}}{\mu_1\rho}\\
&\quad \left[\frac{f_1\tanh\left(\sqrt{\frac{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}{4\mu_1^4\rho}}(\theta) + f_2\right)}{f_1 + f_2\tanh\left(\sqrt{\frac{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}{4\mu_1^4\rho}}(\theta)\right)}\right],\\
\Phi_1(x, y, t) &= \frac{\mu_2\Psi_1}{\mu_1} + c, \quad \theta = \mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha + 1)}.
\end{aligned} \tag{15}$$

Case II:
If we set $f_1 = 0$ and $f_2 \neq 0$ in equation (15), then we have singular soliton solution:

$$\begin{aligned}
\Psi_2(x, y, t) &= \frac{\mu_2\rho - 2\lambda\mu_1 - i\sqrt{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}}{\mu_1\rho}\\
&\quad \left[\coth\left(\sqrt{\frac{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}{4\mu_1^4\rho}}(\theta)\right)\right],\\
\Phi_2(x, y, t) &= \frac{\mu_2\Psi_2}{\mu_1} + c, \quad \theta = \mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha + 1)}.
\end{aligned}$$

Case III:
If we set $f_1 \neq 0$ and $f_2 = 0$ in equation (15), then we have the dark soliton solution:

$$\begin{aligned}
\Psi_3(x, y, t) &= \frac{\mu_2\rho - 2\lambda\mu_1 - i\sqrt{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}}{\mu_1\rho}\\
&\quad \left[\tanh\left(\sqrt{\frac{2\mu_1\rho(6\lambda\mu_2 - \mu_3) - 6\mu_1^2(c\rho^2 + 2\lambda^2) - 3\mu_2^2(\rho - 2)\rho}{4\mu_1^4\rho}}(\theta)\right)\right],\\
\Phi_3(x, y, t) &= \frac{\mu_2\Psi_3}{\mu_1} + c, \quad \theta = \mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha + 1)}.
\end{aligned}$$

These results are valid for

$$\mu_1^4\rho(-6\mu_1^2(c\rho^2+2\lambda^2)+2\mu_1\rho(6\lambda\mu_2-\mu_3)-3\mu_2^2(\rho-2)\rho)<0.$$

Case IV:
From equations (3), (6)–(8), (11) and (14), we have the following new solitary solutions of equation (1):

$$\Psi_4(x,y,t)=\frac{\mu_2\rho-2\lambda\mu_1+i\sqrt{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}}{\mu_1\rho}\left[\frac{f_1\tan\left(\sqrt{\frac{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}{4\mu_1^4\rho}}(\theta)+f_2\right)}{f_1+f_2\tan\left(\sqrt{\frac{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}{4\mu_1^4\rho}}(\theta)\right)}\right],$$

$$\Phi_4(x,y,t)=\frac{\mu_2\Psi_4}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.$$

Case V:
Similarly, for $f_3=0$ and $f_4\neq0$ in equation (29), we have the periodic solution:

$$\Psi_5(x,y,t)=\frac{\mu_2\rho-2\lambda\mu_1+i\sqrt{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}}{\mu_1\rho}\left[\cot\left(\sqrt{\frac{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}{4\mu_1^4\rho}})(\theta)\right)\right],$$

$$\Phi_5(x,y,t)=\frac{\mu_2\Psi_5}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.$$

Case VI:
When $f_3\neq0$ and $f_4=0$ in equation (29), we have the periodic solution:

$$\Psi_6(x,y,t)=\frac{\mu_2\rho-2\lambda\mu_1+i\sqrt{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}}{\mu_1\rho}\left[\tan\left(\sqrt{\frac{6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho}{4\mu_1^4\rho}}(\theta)\right)\right],$$

$$\Phi_6(x,y,t)=\frac{\mu_2\Psi_6}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.$$

These results are valid for

$$\mu_1^4\rho(6\mu_1^2(c\rho^2+2\lambda^2)-2\mu_1\rho(6\lambda\mu_2-\mu_3)+3\mu_2^2(\rho-2)\rho)>0.$$

Case VII:
From equations (3), (6)–(8), (12) and (14), we have the rational solution of equation (1) as:

$$\Psi_7(x,y,t)=\frac{\mu_2}{\mu_1}-\frac{2\lambda}{\rho}-\frac{2if_2\mu_1}{\sqrt{\rho}(c+f_2\theta)},\qquad(21)$$

$$\Phi_7(x,y,t)=\frac{\mu_2\Psi_7}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.\qquad(22)$$

2.2 Modified extended auxiliary equation mapping method

In this section, the modified extended auxiliary equation mapping method is employed to the time fractional (2 + 1) dimensional KDE to compute the families of travelling and solitary wave solutions. The formal solutions of the couple system of KDE have a series of the following form:

$$\Psi(\theta) = \sum_{i=0}^{m} f_i\chi(\theta)^i + \sum_{i=-1}^{-m} f_{-i}\chi(\theta)^i + \sum_{i=2}^{m} f_i\chi(\theta)^{i-2}\chi'(\theta) + \sum_{i=1}^{m} f_{i+2}\left(\frac{\chi'(\theta)}{\chi(\theta)}\right)^i, \tag{23}$$

$$\Phi(\theta) = \sum_{j=0}^{n} f_i\chi(\theta)^i + \sum_{j=-1}^{-n} f_{-j}\chi(\theta)^j + \sum_{j=2}^{n} f_j\chi(\theta)^{j-2}\chi'(\theta) + \sum_{j=1}^{n} f_{j+2}\left(\frac{\chi'(\theta)}{\chi(\theta)}\right)^j, \tag{24}$$

where $f_0, f_1, \ldots, f_n$ are constants and m, n are the non-negative integers, the values of $\chi(\theta)$ and $\chi'(\theta)$ satisfy the following equations:

$$\chi'^2(\theta) = \left(\frac{d\chi}{d\theta}\right)^2 = \gamma_1\chi^2(\theta) + \gamma_2\chi^3(\theta) + \gamma_3\chi^4(\theta), \tag{25}$$

$$\chi''(\theta) = \gamma_1\chi(\theta) + \frac{2}{3}\gamma_2\chi^2(\theta) + 2\gamma_3\chi^3(\theta), \tag{26}$$

$$\chi'''(\theta) = (\gamma_1 + 3\gamma_2\chi(\theta) + 6\gamma_3\chi^2(\theta))\chi'(\theta), \tag{27}$$

where γ_1, γ_2 and γ_3 are arbitrary constants. Balancing Ψ''' and $\Psi^2\Psi'$ in equation (7), we acquire $N = 1$. The formal solutions of equation (7) will be of the form:

$$\Psi(\theta) = f_0 + f_1\chi(\theta) + f_2\frac{1}{\chi(\theta)} + f_3\frac{\chi'(\theta)}{\chi(\theta)}. \tag{28}$$

Inserting equation (28) into equation (7) and by equating the coefficients of all terms of $\chi'^i\chi^j$ $(i = 0, 1; j = 0, 1, 2, 3, 4, \ldots, n)$ to zero, we have a system of algebraic equations. Then by solving this system of equation by utilizing any computational solver software such as Mathematica, Maple and MATLAB, we determine the values of f_0, f_1, f_2, f_3.

Case I:

$$\begin{aligned} f_0 &= \frac{2\mu_2\rho\sqrt{\gamma_3} - 4\lambda\mu_1\sqrt{\gamma_3} + i\gamma_2\mu_1^2\sqrt{\rho}}{2\mu_1\rho\sqrt{\gamma_3}}, \\ f_1 &= \pm 2i\mu_1\sqrt{\frac{\gamma_3}{\rho}},\ f_2 = f_3 = 0, \\ \mu_3 &= \mu_1\left(-3c\rho - \frac{6\lambda^2}{\rho}\right) + \mu_1^3\left(\rho\gamma_1 - \frac{3\gamma_2^2}{8\gamma_3}\right) + 6\lambda\mu_2 - \frac{3\mu_2^2(\rho-2)}{2\mu_1}. \end{aligned} \tag{29}$$

Inserting equation (29) into equation (28), for only the positive value of f_1, the solutions of equation (1) can be acquired as follows:

$$\Psi_{1,1}(x,y,t) = \frac{2\mu_2\rho\sqrt{\gamma_3} - 4\lambda\mu_1\sqrt{\gamma_3} + i\gamma_2\mu_1^2\sqrt{\rho}}{2\mu_1\rho\sqrt{\gamma_3}} - \frac{2i\mu_1\gamma_1\sqrt{\gamma_3}\left(1 + \epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}\left(\mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}\right)\right]\right)}{\gamma_2\sqrt{\rho}}, \tag{30}$$

$$\Phi_{1,1}(x,y,t) = \frac{\mu_2\Psi_{1,1}}{\mu_1} + c, \tag{31}$$

$$\Psi_{1,2}(x,y,t) = \frac{2\mu_2\rho\sqrt{\gamma_3} - 4\lambda\mu_1\sqrt{\gamma_3} + i\gamma_2\mu_1^2\sqrt{\rho}}{2\mu_1\rho\sqrt{\gamma_3}} - \frac{i\mu_1\sqrt{\frac{\gamma_1}{\gamma_3}}\sqrt{\gamma_3}\left(1 + \frac{\epsilon_0\sinh\left[\sqrt{\gamma_1}\left(\mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}\right)\right]}{\delta_0 + \cosh\left[\sqrt{\gamma_1}(\mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha+1)})\right]}\right)}{\sqrt{\rho}}, \tag{32}$$

$$\Phi_{1,2}(x,y,t) = \frac{\mu_2\Psi_{1,2}}{\mu_1} + c, \tag{33}$$

$$\Psi_{1,3}(x,y,t) = \frac{2\mu_2\rho\sqrt{\gamma_3} - 4\lambda\mu_1\sqrt{\gamma_3} + i\gamma_2\mu_1^2\sqrt{\rho}}{2\mu_1\rho\sqrt{\gamma_3}} + \frac{2i\mu_1\sqrt{\gamma_3}\left(-\frac{\epsilon_0\left(inh\left[\sqrt{\gamma_1}\left(\mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}\right)\right] + q\right)}{\delta_0\sqrt{1+q^2} + \cosh\left[\sqrt{\gamma_1}\left(\mu_1x + \mu_2y + \frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}\right)\right]} - 1\right)}{\sqrt{\rho}}, \tag{34}$$

$$\Phi_{1,3}(x,y,t) = \frac{\mu_2\Psi_{1,3}}{\mu_1} + c. \tag{35}$$

Case II:

$$\begin{aligned} &f_0 = \frac{\rho\mu_2 - 2\lambda\mu_1}{\mu_1\rho},\quad f_1 = \pm i\mu_1\sqrt{\frac{\gamma_3}{\rho}},\quad f_2 = 0,\quad f_3 = \pm\frac{i\mu_1}{\sqrt{\rho}}, \\ &\mu_3 = -\frac{6\mu_1^2(c\rho^2 + 2\lambda^2) - 12\lambda\mu_2\mu_1\rho + \mu_1^4\rho\rho\gamma_1 + 3\mu_2^2(\rho-2)\rho}{2\mu_1\rho}. \end{aligned} \tag{36}$$

Inserting equation (36) into equation (28), for only the positive values of f_1 and f_3, the solutions of equation (1) can be acquired as follows:

$$\Psi_{2,1}(x,y,t) = \frac{\rho\mu_2 - 2\lambda\mu_1}{\mu_1\rho} - \frac{i\mu_1\gamma_1\sqrt{\gamma_3}\left(1 + \epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}(\theta)\right]\right)}{\gamma_2\sqrt{\rho}} - \frac{i\mu_1\sqrt{\gamma_1}\epsilon_0\operatorname{csch}\left[\frac{\sqrt{\gamma_1}}{2}(\theta)\right]^2}{\sqrt{\rho}\left(2 + 2\epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}(\theta)\right]\right)}, \tag{37}$$

$$\Phi_{2,1}(x,y,t)=\frac{\mu_2\Psi_{2,1}}{\mu_1}+c,\quad \theta=\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}, \tag{38}$$

$$\Psi_{2,2}(x,y,t)=\frac{\rho\mu_2-2\lambda\mu_1}{\mu_1\rho}-\frac{i\mu_1\sqrt{\frac{\gamma_1}{\gamma_3}}\sqrt{\gamma_3}\left(1+\frac{\epsilon_0\sinh[\sqrt{\gamma_1}(\theta)]}{\delta_0+\cosh[\sqrt{\gamma_1}(\theta)]}\right)}{2\sqrt{\rho}}+\frac{i\mu_1\sqrt{\gamma_1}\epsilon_0(\delta_0\cosh[\sqrt{\gamma_1}(\theta)]+1)}{\sqrt{\rho}(\delta_0+\cosh[\sqrt{\gamma_1}(\theta)](\delta_0+\cosh[\sqrt{\gamma_1}(\theta)]+\epsilon_0\sinh[\sqrt{\gamma_1}(\theta)])},$$

$$\Phi_{2,2}(x,y,t)=\frac{\mu_2\Psi_{2,2}}{\mu_1}+c,\quad \theta=\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}, \tag{39}$$

$$\Psi_{2,3}(x,y,t)=\frac{\rho\mu_2-2\lambda\mu_1}{\mu_1\rho}-\frac{i\mu_1\gamma_1\sqrt{\gamma_3}\left(-\frac{\epsilon_0(\sinh[\sqrt{\gamma_1}(\theta)]+\delta_0)}{\delta_0\sqrt{1+q^2}+\cosh[\sqrt{\gamma_1}(\theta)]}-1\right)}{\gamma_2\sqrt{\rho}}+\frac{(\delta_0\sqrt{1+q^2}\cosh[\sqrt{\gamma_1}(\theta)]-q\sinh[\sqrt{\gamma_1}(\theta)]+1)}{(\varepsilon_0(\sinh[\sqrt{\gamma_1}(\theta)]+q)+\cosh[\sqrt{\gamma_1}(\theta)]+\delta_0\sqrt{1+q^2})}\times\frac{i\mu_1}{\sqrt{\rho}(\cosh[\sqrt{\gamma_1}(\theta)]+\delta_0\sqrt{1+q^2})}, \tag{40}$$

$$\Phi_{2,3}(x,y,t))=\frac{\mu_2\Psi_{2,3}}{\mu_1}+c,\quad \theta=\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}. \tag{41}$$

Case III:

$$f_0=0,\ f_1=\pm 2\mu_1^2\sqrt{\frac{3c\gamma_3}{\mu_1\mu_3-3\mu_2^2-\rho\gamma_1\mu_1^4}},\quad f_2=f_3=0,\quad \rho=-\frac{\mu_1\mu_3-3\mu_2^2-\rho\gamma_1\mu_1^4}{3c\mu_1^2},\quad \gamma_2=\frac{2(3\mu_2^3-\mu_1\mu_2\mu_3-6c\lambda\mu_1^3+\rho\gamma_1\mu_2\mu_1^4)}{\mu_1^3}\times\sqrt{\frac{\gamma_3}{3c(\mu_1\mu_3-3\mu_2^2-\rho\gamma_1\mu_1^4)}}. \tag{42}$$

Inserting equation (42) into equation (28), for only the positive value of f_1, the solutions of equation (1) can be acquired as follows:

$$\Psi_{3,1}(x,y,t)=-\frac{2\mu_1^2\gamma_1\sqrt{3c\gamma_3}\left(1+\epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]\right)}{\gamma_2\sqrt{\mu_1\mu_3-3\mu_2^2-\rho\gamma_1\mu_1^4}}, \tag{43}$$

$$\Phi_{3,1}(x,y,t)=\frac{\mu_2\Psi_{3,1}}{\mu_1}+c, \tag{44}$$

$$\Psi_{3,2}(x,y,t)=-\frac{\mu_1^2\sqrt{3c\frac{\gamma_1}{\gamma_3}}\sqrt{\gamma_3}\left(1+\frac{\epsilon_0\sinh\left[\sqrt{\gamma_1}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]}{\delta_0+\cosh\left[\sqrt{\gamma_1}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]}\right)}{\sqrt{\mu_1\mu_3-3\mu_2^2-\rho\gamma_1\mu_1^4}}, \tag{45}$$

$$\Phi_{3,2}(x,y,t)=\frac{\mu_2\Psi_{3,2}}{\mu_1}+c, \tag{46}$$

$$\Psi_{3,3}(x,y,t)=\frac{2\mu_1^2\gamma_1\sqrt{3c\gamma_3}\left(-\frac{\epsilon_0\left(\sinh\left[\sqrt{\gamma_1}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]+q\right)}{\delta_0\sqrt{1+q^2}+\cosh\left[\sqrt{\gamma_1}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]}-1\right)}{\gamma_2\sqrt{\mu_1\mu_3-3\mu_2^2-\rho\gamma_1\mu_1^4}}, \tag{47}$$

$$\Phi_{3,3}(x,y,t)=\frac{\mu_2\Psi_{3,3}}{\mu_1}+c. \tag{48}$$

Case IV:

$$f_0=f_1=f_2=0,\quad f_3=\pm\frac{2i\mu_1}{\sqrt{\rho}},\quad \lambda=\frac{\rho\mu_2}{2\mu_1},\quad \gamma_1=\frac{3\mu_2^2-\mu_1\mu_3-3c\rho\mu_1^2}{2\mu_1^4}. \tag{49}$$

Inserting equation (49) into equation (28), for only the positive value of f_3, the solutions of equation (1) can be acquired as follows:

$$\Psi_{4,1}(x,y,t)=-\frac{i\mu_1\sqrt{\gamma_1}\epsilon_0\ \operatorname{csch}\left[\frac{\sqrt{\gamma_1}}{2}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]^2}{\sqrt{\rho}1+\epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}\left(\mu_1 x+\mu_2 y+\frac{\mu_3 t^\alpha}{\Gamma(\alpha+1)}\right)\right]}, \tag{50}$$

$$\Phi_{4,1}(x,y,t)=\frac{\mu_2\Psi_{4,1}}{\mu_1}+c, \tag{51}$$

$$\Psi_{4,2}(\theta)=\frac{2i\mu_1\sqrt{\gamma_1}\epsilon_0(\delta_0\cosh[\gamma_1(\theta)]+1)}{\sqrt{\rho}(\delta_0+\cosh[\gamma_1(\theta)](\delta_0+\cosh[\gamma_1(\theta)]+\epsilon_0\sinh[\gamma_1(\theta)])}, \tag{52}$$

$$\Phi_{4,2}(x,y,t)=\frac{\mu_2\Psi_{4,2}}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)},\tag{53}$$

$$\Psi_{4,3}(x,y,t)=\frac{(\delta_0\sqrt{1+q^2}\cosh[\gamma_1(\theta)]-q\sinh[\gamma_1(\theta)]+1)}{(\epsilon_0(\sinh[\gamma_1(\theta)]+q)+\cosh[\gamma_1(\theta)]+\delta_0\sqrt{1+q^2})}\times\frac{2i\mu_1}{\sqrt{\rho}\,(\cosh[\gamma_1(\theta)]+\delta_0\sqrt{1+q^2})},\tag{54}$$

$$\Phi_{4,3}(x,y,t)=\frac{\mu_2\Psi_{4,3}}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.\tag{55}$$

Case V:

$$\begin{aligned}&f_0=f_2=0,\\ &f_1=\pm\frac{\mu_1^2\sqrt{6c\gamma_3}}{\sqrt{\mu_1^4\rho\gamma_1+2\mu_3\mu_1-6\mu_2^2}},\\ &f_3=\pm\frac{\mu_1^2\sqrt{6c\gamma_3}}{\sqrt{\mu_1^4\rho\gamma_1+2\mu_3\mu_1-6\mu_2^2}},\\ &\rho=\frac{6\mu_2^2-\rho\gamma_1\mu_1^4-2\mu_3\mu_1}{6c\mu_1^2},\\ &\lambda=\frac{6\mu_2^3-\mu_2\mu_1^4\rho\gamma_1-2\mu_2\mu_3\mu_1}{12c\mu_1^3}.\end{aligned}\tag{56}$$

Inserting equation (56) into equation (28), for only the positive value of f_1 and f_3, the solutions of equation (1) can be acquired as follows:

$$\Psi_{5,1}(x,y,t)=-\frac{\mu_1^2\sqrt{6c\gamma_3}}{\sqrt{\mu_1^4\rho\gamma_1+2\mu_3\mu_1-6\mu_2^2}}\times\left(\frac{\gamma_1\left(1+\epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}(\theta)\right]\right)}{\gamma_2}+\frac{\sqrt{\gamma_1}\epsilon_0\mathrm{csch}\left[\frac{\sqrt{\gamma_1}}{2}(\theta)\right]^2}{2+2\epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}(\theta)\right]}\right),\tag{57}$$

$$\Phi_{5,1}(x,y,t)=\frac{\mu_2\Psi_{5,1}}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)},\tag{58}$$

$$\Psi_{5,2}(x,y,t)=-\frac{\mu_1^2\sqrt{6c\gamma_3}}{\sqrt{\mu_1^4\rho\gamma_1+2\mu_3\mu_1-6\mu_2^2}}\times\left(\frac{\sqrt{\frac{\gamma_1}{\gamma_3}}\sqrt{\gamma_3}\left(1+\frac{\epsilon_0\sinh[\sqrt{\gamma_1}(\theta)]}{\delta_0+\cosh[\sqrt{\gamma_1}(\theta)]}\right)}{2}-\frac{\sqrt{\gamma_1}\epsilon_0(\delta_0\cosh[\gamma_1(\theta)]+1)}{(\delta_0+\cosh[\gamma_1(\theta)](\delta_0+\cosh[\gamma_1(\theta)]+\epsilon_0\sinh[\gamma_1(\theta)])}\right),\tag{59}$$

$$\Phi_{5,2}(x,y,t)=\frac{\mu_2\Psi_{5,2}}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)},\tag{60}$$

$$\Psi_{5,3}(x,y,t)=\frac{\mu_1^2\sqrt{6c\gamma_3}}{\sqrt{\mu_1^4\rho\gamma_1+2\mu_3\mu_1-6\mu_2^2}}\times\left(\frac{\gamma_1\left(-\frac{\epsilon_0(\sinh[\sqrt{\gamma_1}(\theta)]+\delta_0)}{\delta_0\sqrt{1+q^2}+\cosh[\sqrt{\gamma_1}(\theta)]}-1\right)}{\gamma_2}+\frac{(\delta_0\sqrt{1+q^2}\cosh[\gamma_1(\theta)]-q\sinh[\gamma_1(\theta)]+1)}{(\epsilon_0(\sinh[\gamma_1(\theta)]+q)+\cosh[\gamma_1(\theta)]+\delta_0\sqrt{1+q^2})}\times\frac{1}{\cosh[\sqrt{\gamma_1}(\theta)]+\delta_0\sqrt{1+q^2}}\right),\tag{61}$$

$$\Phi_{5,3}(x,y,t)=\frac{\mu_2\Psi_{5,3}}{\mu_1}+c,\quad \theta=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.\tag{62}$$

Case VI:

$$\begin{aligned}&f_0=\pm\sqrt{\frac{6\mu_2^2-\mu_1(6c\mu_1\rho+\gamma_1\mu_1^3+2\mu_3)}{3\rho\mu_1^2}},\\ &f_1=f_2=0,\quad f_3=\pm\frac{i\mu_1}{\sqrt{\rho}},\ \gamma_3=0,\\ &\lambda=\frac{3\rho\mu_2-\sqrt{3\rho(6\mu_2^2-\mu_1(6c\mu_1\rho+\gamma_1\mu_1^3+2\mu_3))}}{6\mu_1}.\end{aligned}\tag{63}$$

Inserting equation (63) into equation (28), for only the positive value of f_0 and f_3, the solutions of equation (1) can be acquired as follows:

$$\Psi_{6,1}(x,y,t)=\sqrt{\frac{6\mu_2^2-\mu_1(6c\mu_1\rho+\gamma_1\mu_1^3+2\mu_3)}{3\rho\mu_1^2}}-\frac{i\mu_1\sqrt{\gamma_1}\epsilon_0\mathrm{csch}\left[\frac{\sqrt{\gamma_1}}{2}\left(\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}\right)\right]^2}{\sqrt{\rho}2+2\epsilon_0\coth\left[\frac{\sqrt{\gamma_1}}{2}\left(\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}\right)\right]},\tag{64}$$

$$\Phi_{6,1}(x,y,t)=\frac{\mu_2\Psi_{6,1}}{\mu_1}+c, \tag{65}$$

$$\begin{aligned}\Psi_{6,2}(x,y,t)\\ =\sqrt{\frac{6\mu_2^2-\mu_1(6c\mu_1\rho+\gamma_1\mu_1^3+2\mu_3)}{3\rho\mu_1^2}}\\ +\frac{i\mu_1\sqrt{\gamma_1}\,\epsilon_0(\delta_0\cosh[\sqrt{\gamma_1}(\theta)]+1)}{\sqrt{\rho}\,(\delta_0+\cosh[\sqrt{\gamma_1}(\theta)])(\delta_0+\cosh[\gamma_1(\theta)]+\epsilon_0\sin[[\sqrt{\gamma_1}(\theta)])},\end{aligned} \tag{66}$$

$$\begin{aligned}\Phi_{6,2}(x,y,t)&=\frac{\mu_2\Psi_{6,2}}{\mu_1}+c,\\ \theta&=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)},\end{aligned} \tag{67}$$

$$\begin{aligned}\Psi_{6,3}(x,y,t)\\ =\sqrt{\frac{6\mu_2^2-\mu_1(6c\mu_1\rho+\gamma_1\mu_1^3+2\mu_3)}{3\rho\mu_1^2}}\\ +\frac{(\delta_0\sqrt{1+q^2}\cosh[\sqrt{\gamma_1}(\theta)]-q\sinh[\sqrt{\gamma_1}(\theta)]+1)}{(\epsilon_0(\sinh[\sqrt{\gamma_1}(\theta)]+q)+\cosh[\sqrt{\gamma_1}(\theta)]+\delta_0\sqrt{1+q^2})}\\ \times\frac{i\mu_1}{\sqrt{\rho}\,(\cosh[\sqrt{\gamma_1}(\theta)]+\delta_0\sqrt{1+q^2})},\end{aligned} \tag{68}$$

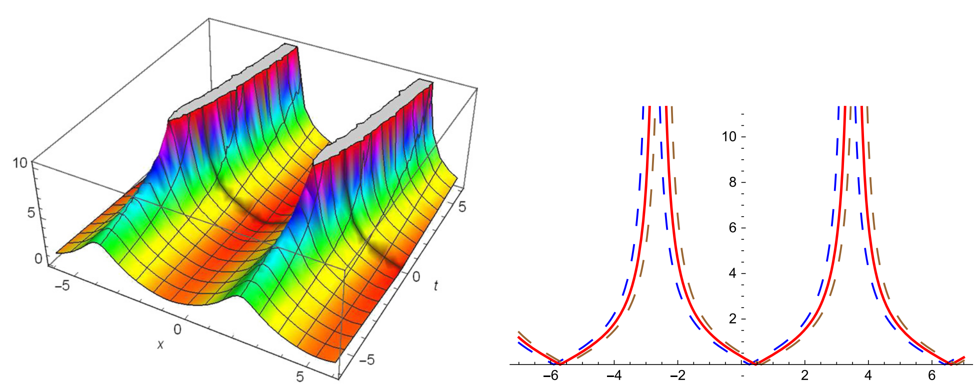
$$\begin{aligned}\Phi_{6,3}(x,y,t)&=\frac{\mu_2\Psi_{6,3}}{\mu_1}+c,\\ \theta&=\mu_1x+\mu_2y+\frac{\mu_3t^\alpha}{\Gamma(\alpha+1)}.\end{aligned} \tag{69}$$

3 Physical description of the solutions

To visualize the behaviour of model (1), Mathematica 11.0 is employed for some selected parameters. A collection of bright, dark, singular kink- and antikink-type solitons, hyperbolic functions, trigonometric functions, elliptic functions and periodic solitary wave solutions have been plotted to investigate the phenomenon of some novel travelling wave solutions corresponding to various constraints.

4 Concluding remarks

In this study, we have introduced two interesting algorithms for the extraction of travelling and solitary wave solutions of NLEE, which demonstrate a wide range of applications in mathematical physics, plasma wave chemical physics, particularly in fluid mechanics and many other nonlinear sciences. To aiming this, we have successfully applied two interesting algorithms that are unified Riccati equation expansion and modified extended auxiliary equation mapping method to compute the exact travelling and solitary wave solutions of the time fractional (2 + 1) dimensional coupled KDE. This system of coupled KDE describes the evolution of nonlinear wave, which is the extension of Kadomtsev–Petviashvili



(a) (b)

Figure 1: (a) 3D and (b) 2D dark soliton solutions of absolute value of $\Psi_3(x,y,t)$ when $\lambda=0.45$, $\rho=1.50$, $\mu_1=-1.75$, $\mu_2=0.15$, $\mu_3=0.90$, $\alpha=0.75$, $\Gamma=1$, $y=-0.50$, $c=0$.

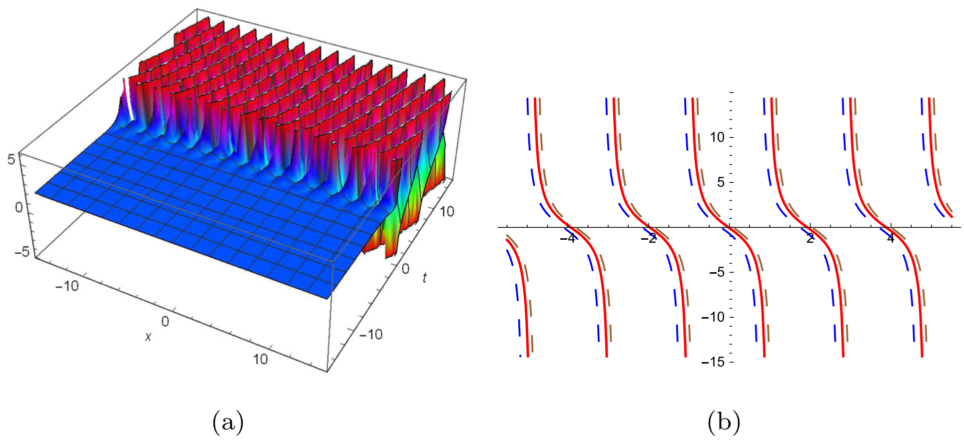


Figure 2: (a) 3D and (b) 2D periodic solutions of imaginary value of $\Psi_5(x, y, t)$ when $\lambda = 0.45$, $\rho = 2.50$, $\mu_1 = -0.75$, $\mu_2 = 1.15$, $\mu_3 = 0.90$, $\alpha = 0.70$, $\Gamma = 1$, $y = 0.50$, $c = 0$.

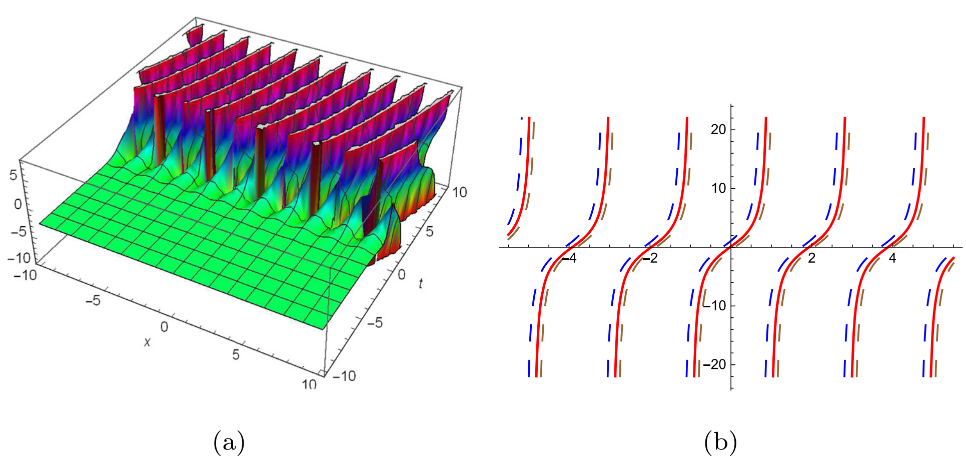


Figure 3: (a) 3D and (b) 2D periodic solutions of imaginary value of $\Phi_5(x, y, t)$ when $\lambda = 0.45$, $\rho = 2.50$, $\mu_1 = -0.75$, $\mu_2 = 1.15$, $\mu_3 = 0.90$, $\alpha = 0.70$, $\Gamma = 1$, $y = 0.50$, $c = 0$.

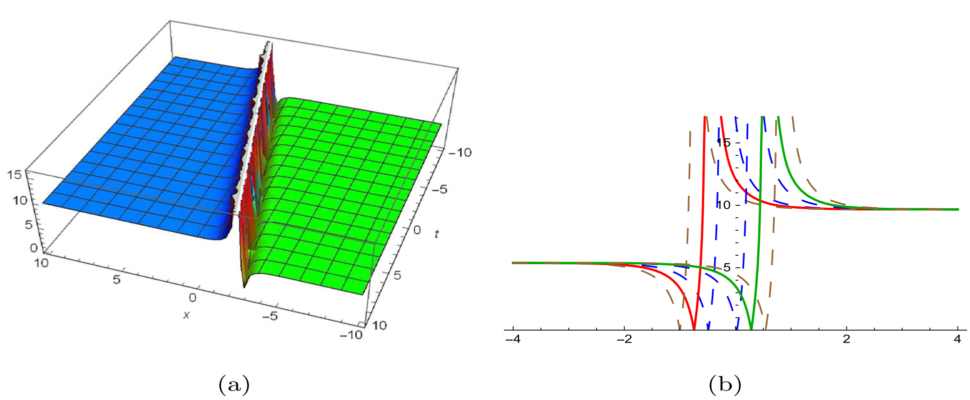
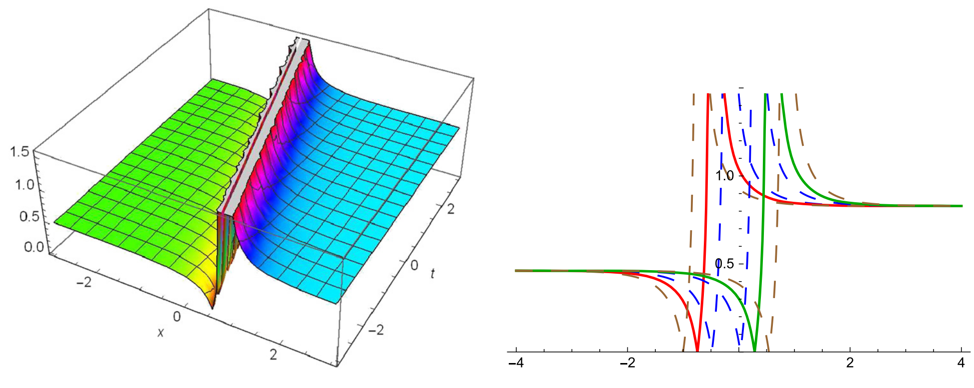
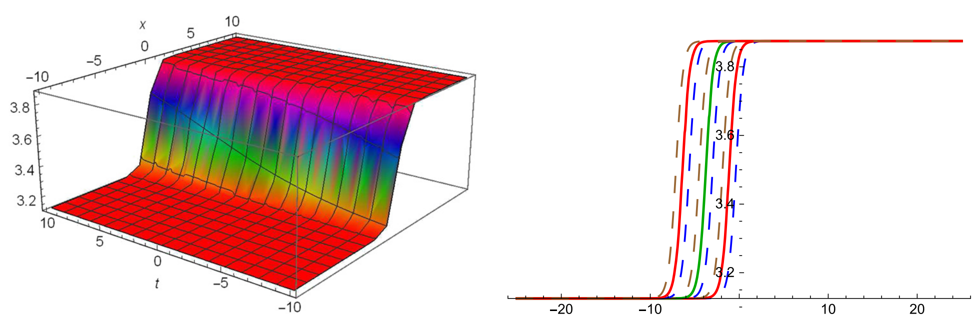


Figure 4: (a) 3D and (b) 2D solitary wave solutions of absolute value of $\Psi_{1,1}(x, y, t)$ when $\lambda = 0.45$, $\rho = -1.50$, $\mu_1 = 1.75$, $\mu_2 = 0.15$, $\mu_3 = 0.90$, $\gamma_1 = 1.50$, $\gamma_2 = 0.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = -0.50$, $c = 0$, $\epsilon_0 = 0.25$.



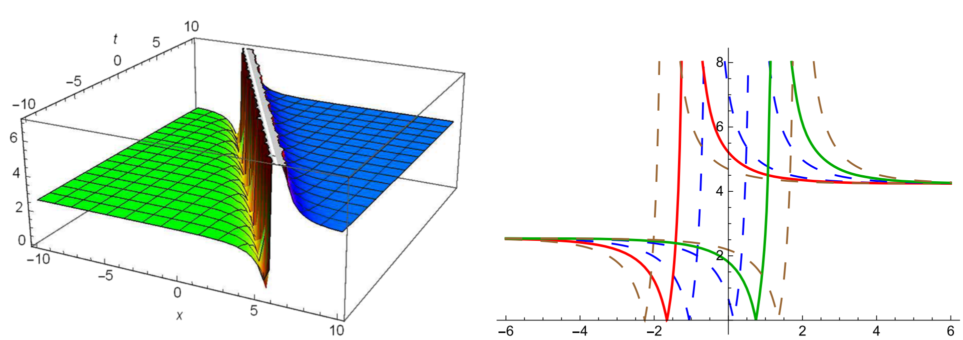
(a) (b)

Figure 5: (a) 3D and (b) 2D solitary wave solutions of absolute value of $\Phi_{1,1}(x, y, t)$ when $\lambda = 0.45$, $\rho = -1.50$, $\mu_1 = 1.75$, $\mu_2 = 0.15$, $\mu_3 = 0.90$, $\gamma_1 = 1.50$, $\gamma_2 = 0.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = -0.50$, $c = 0$, $\epsilon_0 = 0.25$.



(a) (b)

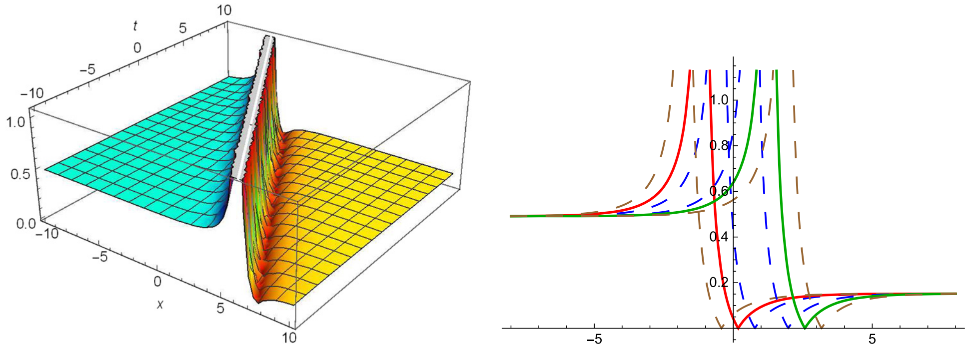
Figure 6: (a) 3D and (b) 2D solitary wave solutions of real value of $\Psi_{1,2}(x, y, t)$ when $\lambda = 2$, $\rho = -1$, $\mu_1 = 1$, $\mu_2 = 0.75$, $\mu_3 = 0.30$, $\gamma_1 = 1.50$, $\gamma_2 = 0.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 2$, $y = -0.50$, $\epsilon_0 = -0.25$.



(a) (b)

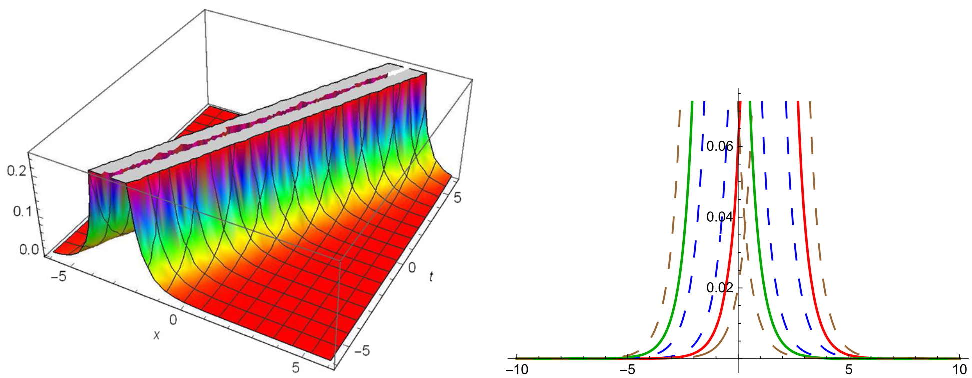
Figure 7: (a) 3D and (b) 2D solitary wave solutions of real value of $\Psi_{3,1}(x, y, t)$ when $\lambda = 1.45$, $\rho = -1.50$, $\mu_1 = 0.75$, $\mu_2 = 0.15$, $\mu_3 = 0.90$, $\gamma_1 = 1.50$, $\gamma_2 = 0.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = -0.50$, $c = 1$, $\epsilon_0 = 0.25$.

and modified Kadomtsev–Petviashvili. As a result, new families of traveling and solitary wave solutions are recovered in the form of bright and dark solitons, kink- and antikink-type solitons, hyperbolic functions, trigonometric functions and elliptic functions, and for details see Figures 1–13. The obtained solutions in this work will be useful for



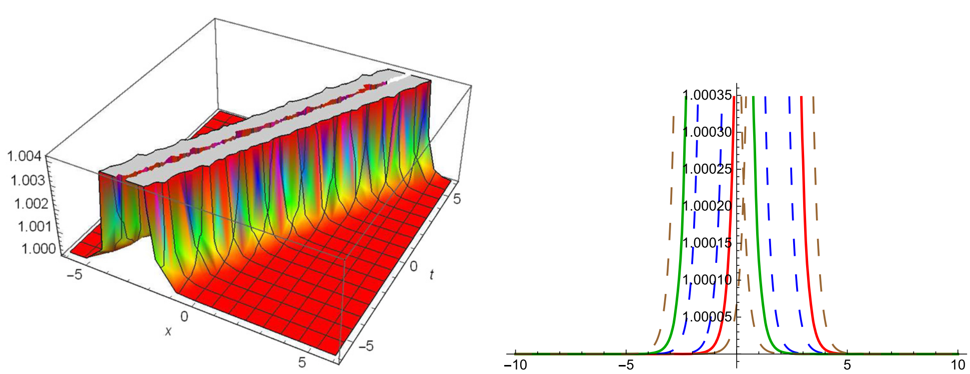
(a) (b)

Figure 8: (a) 3D and (b) 2D solitary wave solutions of real value of $\Phi_{3,1}(x, y, t)$ when $\lambda = 1.45$, $\rho = -1.50$, $\mu_1 = 0.75$, $\mu_2 = 0.15$, $\mu_3 = 0.90$, $\gamma_1 = 1.50$, $\gamma_2 = 0.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = -0.50$, $c = 1$, $\epsilon_0 = 0.25$.



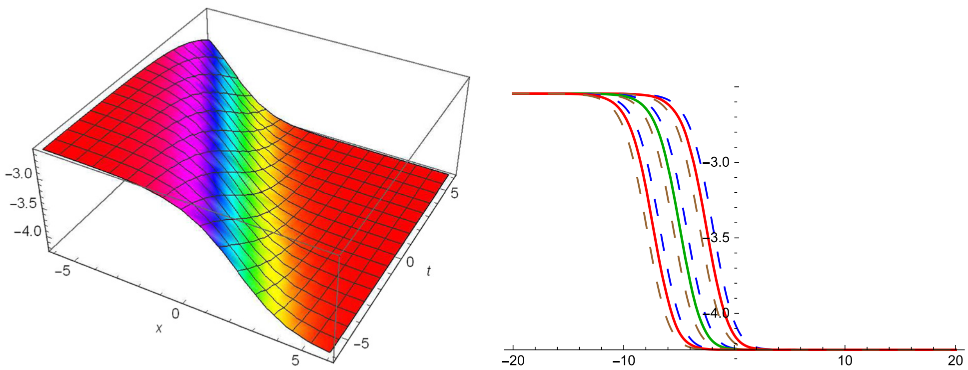
(a) (b)

Figure 9: (a) 3D and (b) 2D solitary wave solutions of absolute value of $\Phi_{4,1}(x, y, t)$ when $\lambda = 1$, $\rho = -1.50$, $\mu_1 = -1.75$, $\mu_2 = 1$, $\mu_3 = 1.90$, $\gamma_1 = 1$, $\gamma_2 = 1.50$, $\gamma_3 = 1.50$, $\alpha = 1$, $\Gamma = 1$, $y = 0.50$, $c = 1$, $\epsilon_0 = 0.05$.



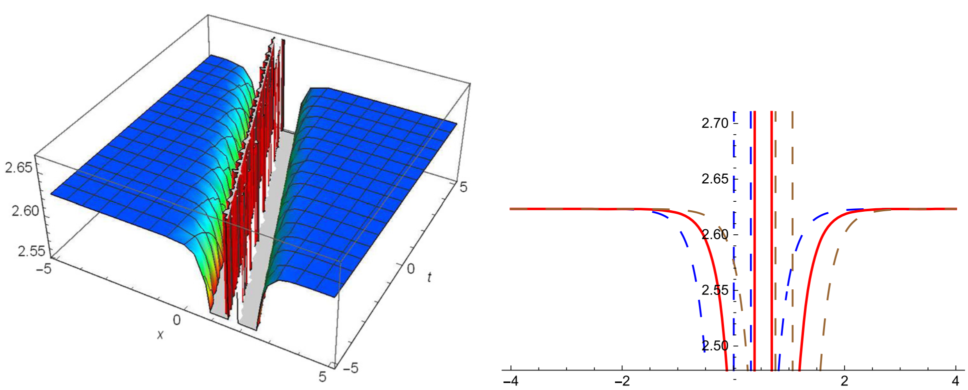
(a) (b)

Figure 10: (a) 3D and (b) 2D solitary wave solutions of absolute value of $\Psi_{4,1}(x, y, t)$ when $\lambda = 1$, $\rho = -1.50$, $\mu_1 = -1.75$, $\mu_2 = 1$, $\mu_3 = 1.90$, $\gamma_1 = 1$, $\gamma_2 = 1.50$, $\gamma_3 = 1.50$, $\alpha = 1$, $\Gamma = 1$, $y = 0.50$, $c = 1$, $\epsilon_0 = 0.05$.



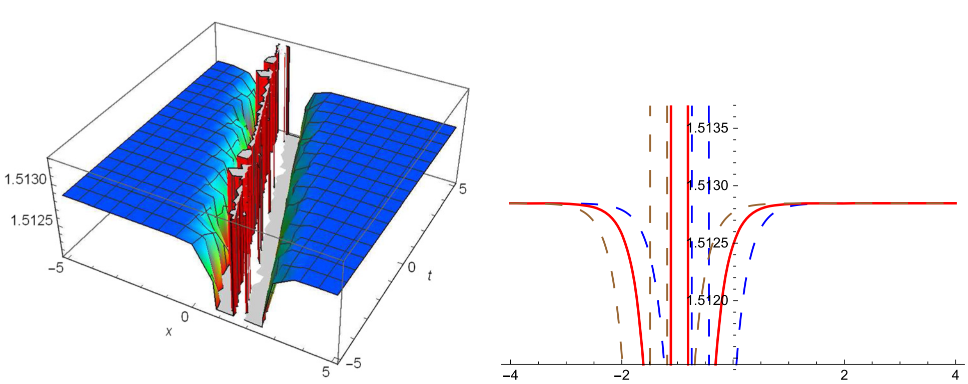
(a) (b)

Figure 11: (a) 3D and (b) 2D solitary wave solutions of real value of $\Phi_{3,3}(x,y,t)$ when $\lambda = 1.45$, $\rho = -1.50$, $\mu_1 = 0.75$, $\mu_2 = 0.15$, $\mu_3 = 0.90$, $\gamma_1 = 1.50$, $\gamma_2 = 0.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = -0.50$, $q = 1.90$, $c = 1$, $\epsilon_0 = 0.25$, $\delta_0 = 0.90$.



(a) (b)

Figure 12: (a) 3D and (b) 2D solitary wave solutions of absolute value of $\Psi_{6,1}(x,y,t)$ when $\lambda = 1$, $\rho = 1.50$, $\mu_1 = 2$, $\mu_2 = 2$, $\mu_3 = 1.50$, $\gamma_1 = 4$, $\gamma_2 = 1.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = 2$, $q = 1.90$, $c = 1.50$, $\epsilon_0 = 0.25$, $\delta_0 = 0.90$.



(a) (b)

Figure 13: (a) 3D and (b) 2D solitary wave solutions of absolute value of $\Phi_{6,1}(x,y,t)$ when $\lambda = 1$, $\rho = 1.50$, $\mu_1 = 2$, $\mu_2 = 2$, $\mu_3 = 1.50$, $\gamma_1 = 4$, $\gamma_2 = 1.50$, $\gamma_3 = 1$, $\alpha = 1$, $\Gamma = 1$, $y = 2$, $q = 1.90$, $c = 1.50$, $\epsilon_0 = 0.25$, $\delta_0 = 0.90$.

a better understanding of many physical phenomena that occur in nature. Furthermore, the effectiveness, capability and reliability of the proposed methods can be extended to extract the exact solutions of many NLEEs.

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