Computers, Materials & Continua
DOI:10.32604/cmc.2021.012301
Article

Optimal Control Model for the Transmission of Novel COVID-19

Isa Abdullahi Baba[1,*], Bashir Ahmad Nasidi[1] and Dumitru Baleanu[2,3,4]

[1]Department of Mathematical Sciences, Bayero University Kano, Nigeria
[2]Department of Mathematics, Cankaya University, Ankara, 06530, Turkey
[3]Institute of Space Sciences, Bucharest, Romania
[4]Department of Medical Research, China Medical University Hospital, Taichung, Taiwan
[*]Corresponding Author: Isa Abdullahi Baba. Email: iababa.mth@buk.edu.ng
Received: 24 June 2020; Accepted: 06 October 2020

Abstract: As the corona virus (COVID-19) pandemic ravages socio-economic activities in addition to devastating infectious and fatal consequences, optimal control strategy is an effective measure that neutralizes the scourge to its lowest ebb. In this paper, we present a mathematical model for the dynamics of COVID-19, and then we added an optimal control function to the model in order to effectively control the outbreak. We incorporate three main control efforts (isolation, quarantine and hospitalization) into the model aimed at controlling the spread of the pandemic. These efforts are further subdivided into five functions; $u_1(t)$ (isolation of the susceptible communities), $u_2(t)$ (contact track measure by which susceptible individuals with contact history are quarantined), $u_3(t)$ (contact track measure by which infected individualsare quarantined), $u_4(t)$ (control effort of hospitalizing the infected I_1) and $u_5(t)$ (control effort of hospitalizing the infected I_2). We establish the existence of the optimal control and also its characterization by applying Pontryaging maximum principle. The disease free equilibrium solution (DFE) is found to be locally asymptotically stable and subsequently we used it to obtain the key parameter; basic reproduction number. We constructed Lyapunov function to which global stability of the solutions is established. Numerical simulations show how adopting the available control measures optimally, will drastically reduce the infectious populations.

Keywords: COVID-19; optimal control; Pontryaging maximum principle; mathematical model; existence of control; stability analysis

1 Introduction

The novel coronavirus pneumonia which was officially named as Corona Virus Disease 2019 (COVID-19) by World Health Organization (WHO) was reported first in late December 2019, in Wuhan, China [1]. The source of the virus is not yet known, but genetic investigation revealed that COVID-19 virus has the same genetic characteristics with SARS-CoV2 (which was likely to be originated from bats) [2]. It is also found to be significantly less severe than the other two coronaviruses; Severe Acute Respiratory Syndrome (SARS-COV) and Middle East Respiratory Syndrome (MERS-COV) that caused an outbreak in 2002 and 2008 respectively [3]. The most important routes of human to human transmission of COVID-19 are respiratory droplets and contact transmission [4]. After the incubation period which is generally 2–14 days, the mild symptoms may persist from high degree fever, cough and shortness of breath to being severely ill and subsequently death [4].

As scientists all over the world are busy trying to develop a cure and vaccine, all hands must be put together to support and comply with the standard recommendations that can lower the transmissions of the disease. This is why, the following measures must be taken; social distancing, self-isolation, use of personal protective equipment (such as face mask, hand globes, overall gown, etc.), regular hand washing using soap or sanitizer, avoid having contact with person showing the symptoms and report any suspected case. Moreover, relevant authorities must engage in widely public orientation exercise for sensitization and enlightenment, banning of social (or religious) gathering and local (or international) trip, contact tracing and isolation of infected individuals, providing sanitizers at public domains like markets and car parks, fumigating exercise, and to the large extent imposing lockdown.

The scourge does not only cause apocalyptic proportion in terms of infection, morbidity and fatality, but also socio-economic consequences. To control the above mentioned problems, there is need to have better understanding on the transmission dynamics of the disease. This could be achieved by developing mathematical model that optimizes the possible control measures.

Optimal control is considered as an effective mathematical tool used to optimize the control problems arising in different field including epidemiology, aeronautic engineering, economics, finance, robotics, etc [5]. Mathematical model offers an insight in to the transmission and control of infectious disease [6–13]. Zhao et al. [14] developed a Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected (SUQC) model to characterize the dynamics of COVID-19 and explicitly parameterize the intervention effects of control measures. Song et al. [15] established deterministic mathematical model (SEIHR) to suit Korean outbreak, in which he estimated the reproduction number and the effect of preventive measures.

Tahir et al. [16] developed a mathematical model (for MERS) in form of nonlinear system of differential equations, in which he considered a camel to be the source of the infection. The virus is then spread to human population, then human to human transmission, then human to clinic center and then human to care center. They used Lyapunov function to investigate the global stability analyses of the equilibrium solutions and subsequently obtained the basic reproduction number or roughly, a key parameter describing transmission of the infection.

Yang et al. [17] proposed a mathematical model to investigate the current outbreak of the coronavirus disease (COVID-19) in Wuhan, China. The model described the multiple transmission pathways in the infection dynamics, and emphasized the role of environmental reservoir in the transmission and spread of the disease. However, the model employed non-constant transmission rates which change with the epidemiological status and environmental conditions and which reflect the impact of the ongoing disease control measures.

Chen et al. [18] modeled (based on SEIR) the outbreak in Wuhan with individual reaction and governmental action (holiday extension, city lockdown, hospitalization and quarantine) in which they estimated the preliminary magnitude of different effect of individual reaction and governmental action. Sunhwa et al. [19] developed a Bats-Hosts-Reservoir-People (BHRP) transmission network model for the potential transmission from the infection source (probably bats) to the human, which focuses on calculating R_0. Elhia et al. [20] developed a mathematical model based on the epidemiology of COVID-19, incorporating the isolation of healthy people, confirmed cases and close contacts.

Most of these models have a general shortcoming of not taking into consideration time dependent control strategies. For the model to be more realistic, it has to be time dependent [21–26]. Here, we modified the work of Elhia et al. [20], by incorporating control functions with the aim of deriving optimal control that drastically minimizes the spread of the infection.

The paper is arranged in the following order: Chapter 1 gives the introduction, Chapter 2 gives preliminary definitions and theorems, Chapter 3 is the model formulation, Chapter 4 discusses the formulation and analysis of optimal control, Chapter 5 presents local and global stability analyses of the solutions of the model and the derivation of the reproduction number and lastly Chapter 6 gives numerical simulation results and then the discussion follows.

2 Preliminary Definitions and Theorem

Definition 1 (Optimal Control) [27]: A fairly general continuous time optimal control problem can be defined as follows:

Problem i: To find the control vector trajectory $u\colon [t_0, t_f] \in \mathbb{R} \to \mathbb{R}^n$ that minimizes the performance index:

$$J(\boldsymbol{u}) = \varphi\left(\boldsymbol{x}\left(t_f\right)\right) + \int_{t_0}^{t_f} L\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), t\right) dt. \tag{1}$$

Subject to:

$$\dot{\boldsymbol{x}}\left(t\right) = \boldsymbol{f}\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), t\right), \quad \boldsymbol{x}\left(t_0\right) = \boldsymbol{x}_0. \tag{2}$$

where $\boldsymbol{x} = (x_1, x_2, x_3, \ldots, x_n)^{\boldsymbol{T}}$, $\boldsymbol{f} = (f_1, f_2, f_3, \ldots, f_n)^{\boldsymbol{T}}$ and $\varphi\colon \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is a terminal cost function.

Problem ii: Find t_f and $\boldsymbol{u}(t)$ to minimize:

$$J = \int_{t_0}^{t_f} 1 dt = t_f - t_0.$$

Subject to:

$$\dot{\boldsymbol{x}}\left(t\right) = \boldsymbol{f}\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), t\right), \quad \boldsymbol{x}\left(t_0\right) = \boldsymbol{x}_0.$$

This special type of optimal control problem is called the minimum time problem.

Definition 2 (Hamiltonian): A time varying Lagrange's multiplier function $\lambda\colon [t_0, t_f] \to \mathbb{R}$, also known as the co-state define Hamiltonian function H as:

$$H\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), \lambda\left(t\right), t\right) = L\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), t\right) + \lambda\left(t\right)^T \boldsymbol{f}\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), t\right), \tag{3}$$

such that

$$J(\boldsymbol{u}) = \varphi\left(\boldsymbol{x}\left(t_f\right)\right) + \int_{t_0}^{t_f} \left\{H\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), \lambda\left(t\right), t\right) - \lambda^T\left(t\right)\boldsymbol{x}\right\} dt. \tag{4}$$

Theorem 1 (Pontryagin Maximum Principle): If $\boldsymbol{u}^*(t), \boldsymbol{x}^*(t)\left(t \in \left[t_0, t_f\right]\right)$ is a solution of the optimal control problem Eqs. (1) and (2) then there exists a non-zero absolutely continuous function $\lambda(t)$ such that $\lambda(t), \boldsymbol{x}^*(t), \boldsymbol{u}^*(t)$ satisfy the system

$$\frac{dx}{dt} = \frac{\partial H}{\partial \lambda}, \quad \frac{d\lambda}{dt} = -\frac{\partial H}{\partial \boldsymbol{x}}, \tag{5}$$

such that, for almost all $t \in \left[t_0, t_f\right]$ the function in Eq. (3) attains its maximum:

$$H\left(\lambda(t), \boldsymbol{x}^*(t), \boldsymbol{u}^*(t)\right) = M(\lambda, x),$$

$$M\left(\lambda\left(t_f\right), \boldsymbol{x}^*\left(t_f\right)\right) = \sup\{H(\lambda, x, u) : u \in \boldsymbol{\mathcal{U}}\}. \tag{6}$$

and such that at terminal time t_f the conditions

$M\left(\lambda\left(t_f\right), \boldsymbol{x}^*\left(t_f\right)\right) = 0, \quad \lambda_0(t_f) \leq,$ are satisfied.

If the functions $\lambda(t), x(t), u(t)$ satisfy the relation Eqs. (5) and (6) (i.e., $x(t), u(t)$ are Portryagin extremals), then the condition

$\boldsymbol{\mathcal{M}}(t) = M(\lambda(t), x(t)) = const., \quad \lambda_0(t) = const$ holds.

Remark 1: Becerra states that for a minimum, it is necessary for the stationary (optimality) condition to give:

$$\frac{\partial H^T}{\partial \boldsymbol{u}} = 0. \tag{7}$$

3 Model Formulation

We know that most people are susceptible to COVID-19 and the patients in the incubation period can infect healthy people. We denote the population of susceptible people with S, the patients in the incubation period and the patients that are yet to be diagnosed by I, patients in the hospital by H, removed people by R, respectively. Here the infectivity of the patients in the incubation period and the patients that are yet to be diagnosed are assumed to be the same.

After the outbreak of COVID-19, susceptible people are advised to lock themselves down at home, and all close contacts of infected individuals tracked are quarantined. Therefore, we divide the population of susceptible people into; susceptible people (S_1), the quarantined susceptible people (by close contacts tracked measure) (S_2) and general isolated susceptible people (due to community lockdown) (S_3). Infected people population is divided into general infected people, including the patients in the incubation period and the infected people that are yet to be diagnosed (I_1) and infected people that are quarantined (I_2). Here we assume that all susceptible people isolated at home cannot be infected and all infected people isolated at home cannot infect healthy people. Thus, we establish the transmission dynamics of the disease as in Fig. 1.

The transmission dynamics can be described by the nonlinear system of first order differential equations as follows:

$$\frac{dS_1}{dt} = \alpha_1 S_2 - \frac{\beta S_1 I_1}{N - S_2 - S_3 - I_2 - H} - q_1 - \mu S_1 + \alpha_2 S_3, \tag{8}$$

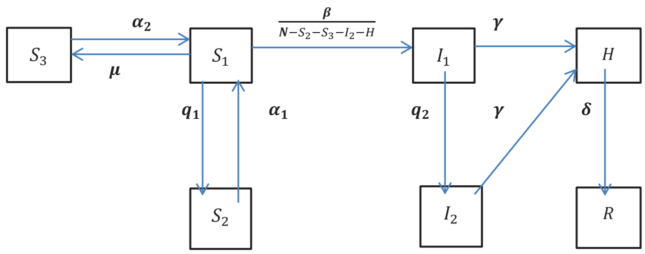

Figure 1: Transmission dynamics of the disease

$$\frac{dS_2}{dt} = q_1 - \alpha_1 S_2, \tag{9}$$

$$\frac{dS_3}{dt} = \mu S_1 - \alpha_2 S_3, \tag{10}$$

$$\frac{dI_1}{dt} = \frac{\beta S_1 I_1}{N - S_2 - S_3 - I_2 - H} - q_2 - \gamma I_1, \tag{11}$$

$$\frac{dI_2}{dt} = q_2 - \gamma I_2, \tag{12}$$

$$\frac{dH}{dt} = \gamma I_1 + \gamma I_2 - \delta H, \tag{13}$$

$$\frac{dR}{dt} = \delta H, \tag{14}$$

where,

$$q_1 = \min\left\{S_1, \gamma b \frac{(I_1 + I_2)}{S_2 + I_2} S_2\right\} \text{ and } q_2 = \min\left\{I_1, \gamma b \frac{(I_1 + I_2)}{S_2 + I_2} I_2\right\}.$$

$$S_1(0) \geq 0, \quad S_2(0) \geq 0, \quad S_3(0) \geq 0, \quad I_1(0) \geq 0, \quad I_2(0) \geq 0, \quad H(0) \geq 0, \quad R(0) \geq 0.$$

When patients go to hospital and are diagnosed ($I_1 + I_2$), by the close contacts tracked measure, susceptible people (q_1) and infected people (q_2) are quarantined by the proposition b. Here the number of quarantined susceptible people (q_1) and quarantined infected people (q_2) are less than the number of susceptible people (S_1) and infected people (I_1) respectively. Then $q_1 = \min\left\{S_1, \gamma b \frac{(I_1 + I_2)}{S_2 + I_2} S_2\right\}$ and $q_2 = \min\left\{I_1, \gamma b \frac{(I_1 + I_2)}{S_2 + I_2} I_2\right\}$.

After the isolation of 14 days $\left(\frac{1}{\alpha_1}\right)$, the quarantined susceptible individuals become susceptible. When quarantined infected people have symptoms, they are hospitalized and diagnosed (I_2). After the time of treatment $\left(\frac{1}{\delta}\right)$, they are removed from the hospital. The communities are isolated and healthy people are also advised to isolate themselves at home unless they have something urgent to deal with. Then μS_1 and $\alpha_2 S_3$ denote the weak movements of population from susceptible to isolated susceptible and from isolated susceptible to susceptible, respectively.

4 Optimal Control

Here the detail formulation and analysis of the optimal control problem with respect to the model Eqs. (8)–(14) is given.

4.1 Formation of an Optimal Control

The aim of the control strategy is to prevent the susceptible population from becoming infected and reduce the infected population by increasing hospitalization which eventually reduces the number of new cases.

Let the control functions

$u_1(t) \in [0, u_1(t)_{max}]$ be the rate at which susceptible communities are isolated.

$u_2(t) \in [0, u_2(t)_{max}]$ be the contact track measure by which susceptible individuals with contact history are quarantined.

$u_3(t) \in [0, u_3(t)_{max}]$ be the contact track measure by which infected individuals are quarantined.

$u_4(t) \in [0, u_4(t)_{max}]$ be the control effort of hospitalizing the infected I_1.

$u_5(t) \in [0, u_5(t)_{max}]$ be the control effort of hospitalizing the infected I_2.

The dynamics of control system can be described by the following system of nonlinear ODE;

$$\frac{dS_1}{dt} = \alpha_1 S_2 - \frac{\beta S_1 I_1}{N - S_2 - S_3 - I_2 - H} - u_2(t) q_1 - (\mu + u_1(t)) S_1 + \alpha_2 S_3, \tag{15}$$

$$\frac{dS_2}{dt} = u_2(t) q_1 - \alpha_1 S_2, \tag{16}$$

$$\frac{dS_3}{dt} = (\mu + u_1(t)) S_1 - \alpha_2 S_3, \tag{17}$$

$$\frac{dI_1}{dt} = \frac{\beta S_1 I_1}{N - S_2 - S_3 - I_2 - H} - u_3(t) q_2 - (\gamma + u_4(t)) I_1, \tag{18}$$

$$\frac{dI_2}{dt} = u_3(t) q_2 - (\gamma + u_5(t)) I_2, \tag{19}$$

$$\frac{dH}{dt} = (\gamma + u_4(t)) I_1 + (\gamma + u_5(t)) I_2 - \delta H, \tag{20}$$

$$\frac{dR}{dt} = \delta H. \tag{21}$$

For a fixed terminal time t_f, the problem to minimize the objective functional associated to system Eq. (15) through Eq. (21) is

$$J(u(t)) = \int_{t_0}^{t_f} [A_1 S_1 + A_2 q_1 + A_3 q_2 + A_4 I_1 + A_5 I_2 + \frac{A_6}{2} {u_1}^2(t) + \frac{A_7}{2} {u_2}^2(t) + \frac{A_8}{2} {u_3}^2(t) + \frac{A_9}{2} {u_4}^2(t) + \frac{A_{10}}{2} {u_5}^2(t)] dt. \tag{22}$$

where,

$A_i \geq 0, i = 1, 2, \ldots, 10$ denote the weights parameters that balanced the size of the terms.

We seek for optimal control u^* such that

$$J\left(u^*\right) = \min \{J(u) : u \in \mathcal{U}\},$$

where

$\mathcal{U}$ is the set of admissible controls defined by

$$\mathcal{U} = \{u_i(t) : 0 \leq u_i(t) \leq 1, \quad i = 1, 2, \ldots, 10, \quad u_i(t) \ \textit{is Lebesgue measurable}\}.$$

4.2 Existence of Optimal Control

The system of nonlinear ODE Eqs. (15)–(21) can be written as,

$$F(x(t), u(t)) = A\boldsymbol{x}(t) + \boldsymbol{u}_{145}(t)\, \boldsymbol{x}(t) + \boldsymbol{u}_{23}(t)\, \boldsymbol{q}(\boldsymbol{x}(t)) + g(\boldsymbol{x}(t)), \tag{23}$$

where,

$$\boldsymbol{x}(t) = (S_1(t), S_2(t), S_3(t), I_1(t), I_2(t), H(t), R(t))^T,$$

$$A = \begin{bmatrix} -\mu & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \mu & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma & 0 & 0 \\ 0 & 0 & 0 & \gamma & \gamma & -\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{u}_{145}(t) = \begin{bmatrix} -u_1(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u_1(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -u_4(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_5(t) & 0 & 0 \\ 0 & 0 & 0 & u_4(t) & u_5(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{u}_{23}(t) = \begin{bmatrix} 0 & -u_2(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & u_2(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -u_3(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & u_3(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{q}(\boldsymbol{x}(t)) = \begin{bmatrix} 0 \\ \gamma b \dfrac{(I_1 + I_2)}{S_2 + I_2} S_2 \\ 0 \\ \gamma b \dfrac{(I_1 + I_2)}{S_2 + I_2} I_2 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$g\left(\boldsymbol{x}(t)\right)=\begin{bmatrix}-\dfrac{\beta S_1 I_1}{N-S_2-S_3-I_2-H}\\0\\0\\\dfrac{\beta S_1 I_1}{N-S_2-S_3-I_2-H}\\0\\0\\0\end{bmatrix}.$$

Theorem 3: The optimal control system Eq. (23) is Lipschitz continuous.

Proof

$$\left|F\left(\boldsymbol{x}(t),u(t)\right)-F\left(\boldsymbol{x}^*(t),u(t)\right)\right|$$

$$=\left|\left(A+\boldsymbol{u}_{145}(t)\right)\left(\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right)+\boldsymbol{u}_{23}(t)\left(\boldsymbol{q}\left(\boldsymbol{x}(t)\right)-\boldsymbol{q}\left(\boldsymbol{x}^*(t)\right)\right)+g\left(\boldsymbol{x}(t)\right)-g\left(\boldsymbol{x}^*(t)\right)\right|$$

$$\leq\left|A+\boldsymbol{u}_{145}(t)\right|\left|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right|+\left|\boldsymbol{q}\left(\boldsymbol{x}(t)\right)-\boldsymbol{q}\left(\boldsymbol{x}^*(t)\right)\right|+\left|g\left(\boldsymbol{x}(t)\right)-g\left(\boldsymbol{x}^*(t)\right)\right|$$

$$\leq\left(\|A\|+\max_{[t_0,t_f]}\left|\boldsymbol{u}_{145}(t)\right|\right)\left|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right|+\max_{[t_0,t_f]}\left|\boldsymbol{u}_{23}(t)\right|\left|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right|+\left|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right|$$

$$\leq\left(\|A\|+\max_{[t_0,t_f]}\left|\boldsymbol{u}_{145}(t)\right|+\max_{[t_0,t_f]}\left|\boldsymbol{u}_{23}(t)\right|+1\right)\left\|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right\|\ \left\|F\left(\boldsymbol{x}(t),u(t)\right)-F\left(\boldsymbol{x}^*(t),u(t)\right)\right\|$$

$$\leq M\left\|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\right\|,$$

where,

$$M=\|A\|+\max_{[t_0,t_f]}\left|\boldsymbol{u}_{145}(t)\right|+\max_{[t_0,t_f]}\left|\boldsymbol{u}_{23}(t)\right|+1<\infty.$$

4.3 Characterization of Optimal Control

To formulate the optimal control strategy, we define the Hamiltonian as:

$$H=A_1S_1+A_2q_1+A_3q_2+A_4I_1+A_5I_2+\frac{A_6}{2}{u_1}^2(t)+\frac{A_7}{2}{u_2}^2(t)+\frac{A_8}{2}{u_3}^2(t)$$
$$+\frac{A_9}{2}{u_4}^2(t)+\frac{A_{10}}{2}{u_5}^2(t)+\sum_{i=1}^{7}\lambda_i f_i$$

$$H=A_1S_1+A_2q_1+A_3q_2+A_4I_1+A_5I_2+\frac{A_6}{2}{u_1}^2(t)+\frac{A_7}{2}{u_2}^2(t)+\frac{A_8}{2}{u_3}^2(t)+\frac{A_9}{2}{u_4}^2(t)$$
$$+\frac{A_{10}}{2}{u_5}^2(t)+\lambda_1\left(\alpha_1S_2-\frac{\beta S_1I_1}{N-S_2-S_3-I_2-H}-u_2(t)\,q_1-\left(\mu+u_1(t)\right)S_1+\alpha_2S_3\right)$$

$$+\lambda_2\left(u_2(t)q_1-\alpha_1 S_2\right)+\lambda_3\left(\left(\mu+u_1(t)\right)S_1-\alpha_2 S_3\right)+\lambda_4\left(\frac{\beta S_1 I_1}{N-S_2-S_3-I_2-H}-u_3(t)q_2\right.$$
$$\left.-\left(\gamma+u_4(t)\right)I_1\right)+\lambda_5\left(u_3(t)q_2-\left(\gamma+u_5(t)\right)I_2\right)+\lambda_6\left(\left(\gamma+u_4(t)\right)I_1\right.$$
$$\left.+\left(\gamma+u_5(t)\right)I_2-\delta H\right)+\lambda_7\delta H. \tag{24}$$

Theorem 4: Let $\boldsymbol{x}(t)=\left(S_1(t),S_2(t),S_3(t),I_1(t),I_2(t),H(t),R(t)\right)$ with associated optimal control variables u_1,u_2,u_3,u_4,u_5, then there exists a co-state variable satisfying:

$$\dot{\lambda}_i=-\frac{\partial H}{\partial \boldsymbol{x}},\quad i=1,2,\ldots,7.$$

Proof:

Applying the co-state (adjoint) condition of Eq. (5) yield

$$\dot{\lambda}_1=-\frac{\partial H}{\partial S_1}$$
$$=\left(\frac{\beta I_1}{N-S_2-S_3-I_2-H}+\mu+u_1(t)\right)\lambda_1-\left(\mu+u_1(t)\right)\lambda_3-\frac{\beta I_1}{N-S_2-S_3-I_2-H}\lambda_4-A_1, \tag{25}$$

$$\dot{\lambda}_2=-\frac{\partial H}{\partial S_2}$$
$$=\left(\frac{u_2(t)q_2}{S_2+I_2}+\frac{\beta S_1 I_1}{\left(N-S_2-S_3-I_2-H\right)^2}-\alpha_1\right)\lambda_1+\left(\alpha_1-\frac{u_2(t)q_2}{S_2+I_2}\right)\lambda_2$$
$$-\left(\frac{\beta S_1 I_1}{\left(N-S_2-S_3-I_2-H\right)^2}+\frac{u_3(t)q_2}{S_2+I_2}\right)\lambda_4+\frac{u_3(t)q_2}{S_2+I_2}\lambda_5, \tag{26}$$

$$\dot{\lambda}_3=-\frac{\partial H}{\partial S_3}$$
$$=\left(\frac{\beta S_1 I_1}{\left(N-S_2-S_3-I_2-H\right)^2}-\alpha_2\right)\lambda_1+\alpha_2\lambda_3-\frac{\beta S_1 I_1}{\left(N-S_2-S_3-I_2-H\right)^2}\lambda_4, \tag{27}$$

$$\dot{\lambda}_4=-\frac{\partial H}{\partial I_1}$$
$$=\left(\frac{\beta S_1}{N-S_2-S_3-I_2-H}+\gamma b\frac{u_2(t)S_2}{S_2+I_2}\right)\lambda_1-\gamma b\frac{u_2(t)S_2}{S_2+I_2}\lambda_2+\left(\gamma b\frac{u_3(t)I_2}{S_2+I_2}-\frac{\beta S_1}{N-S_2-S_3-I_2-H}\right.$$
$$\left.+\gamma+u_4(t)\right)\lambda_4-\gamma b\frac{u_3(t)I_2}{S_2+I_2}\lambda_5-\left(\gamma+u_4(t)\right)\lambda_6-\frac{\gamma b S_2}{S_2+I_2}-\frac{\gamma b I_2}{S_2+I_2}-A_4, \tag{28}$$

$$\dot{\lambda_5} = -\frac{\partial H}{\partial I_2}$$

$$= \left(\frac{\beta S_1 I_1}{(N - S_2 - S_3 - I_2 - H)^2} + \frac{\gamma b u_2(t)(S_2 - I_2)}{(S_2 + I_2)^2} S_2\right)\lambda_1 - \frac{\gamma b u_2(t)(S_2 - I_2)}{(S_2 + I_2)^2} S_2 \lambda_2$$

$$+ \left(\frac{\gamma b u_3(t)[(S_2 + I_2)(I_1 + 2I_2) - (I_1 + I_2) I_2]}{(S_2 + I_2)^2} - \frac{\beta S_1 I_1}{(N - S_2 - S_3 - I_2 - H)^2}\right)\lambda_4$$

$$- \left(\frac{\gamma b u_3(t)[(S_2 + I_2)(I_1 + 2I_2) - (I_1 + I_2) I_2]}{(S_2 + I_2)^2} - \gamma - u_5(t)\right)\lambda_5$$

$$- (\gamma + u_5(t))\lambda_6 - \frac{\gamma b[(S_2 - I_2) S_2 + (S_2 + I_2)(I_1 + 2I_2) - (I_1 + I_2) I_2]}{(S_2 + I_2)^2} - A_5, \tag{29}$$

$$\dot{\lambda_6} = -\frac{\partial H}{\partial \boldsymbol{H}} = \frac{\beta S_1 I_1}{(N - S_2 - S_3 - I_2 - H)^2}\lambda_1 - \frac{\beta S_1 I_1}{(N - S_2 - S_3 - I_2 - H)^2}\lambda_4 + \delta\lambda_6 - \delta\lambda_7, \tag{30}$$

$$\dot{\lambda_7} = -\frac{\partial H}{\partial R} = 0, \tag{31}$$

subject to the following transversality conditions;

$$\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f) = \lambda_6(t_f) = \lambda_7(t_f) = 0. \tag{32}$$

Applying the optimality conditions, we get

$$\frac{\partial H}{\partial u_1} = A_6 u_1(t) - \lambda_1 S_1 + \lambda_3 S_1 = 0, \quad \Longrightarrow u_1(t) = \frac{(\lambda_1 - \lambda_3) S_1}{A_6}, \tag{33}$$

$$\frac{\partial H}{\partial u_2} = A_7 u_2(t) - \lambda_1 q_1 + \lambda_2 q_1 = 0, \quad \Longrightarrow u_2(t) = \frac{(\lambda_1 - \lambda_2) q_1}{A_7}, \tag{34}$$

$$\frac{\partial H}{\partial u_3} = A_8 u_3(t) - \lambda_4 q_2 + \lambda_5 q_2 = 0, \quad \Longrightarrow u_3(t) = \frac{(\lambda_4 - \lambda_5) q_2}{A_8}, \tag{35}$$

$$\frac{\partial H}{\partial u_4} = A_9 u_4(t) - \lambda_4 I_1 + \lambda_6 I_1 = 0, \quad \Longrightarrow u_4(t) = \frac{(\lambda_4 - \lambda_6) I_1}{A_9}, \tag{36}$$

$$\frac{\partial H}{\partial u_5} = A_{10} u_5(t) - \lambda_5 I_2 + \lambda_6 I_2 = 0, \quad \Longrightarrow u_5(t) = \frac{(\lambda_5 - \lambda_6) I_2}{A_{10}}, \tag{37}$$

$$u_1^*(t) = \min\left\{1, \max\left(0, \frac{(\lambda_1 - \lambda_3) S_1}{A_6}\right)\right\}, \tag{38}$$

$$u_2^*(t) = \min\left\{1, \max\left(0, \frac{(\lambda_1 - \lambda_2) q_1}{A_7}\right)\right\}, \tag{39}$$

$$u_3^*(t) = \min\left\{1, \max\left(0, \frac{(\lambda_4 - \lambda_5) q_2}{A_8}\right)\right\}, \tag{40}$$

$$u_4{}^*(t) = \min\left\{1, \max\left(0, \frac{(\lambda_4 - \lambda_6)I_1}{A_9}\right)\right\}, \tag{41}$$

$$u_5{}^*(t) = \min\left\{1, \max\left(0, \frac{(\lambda_5 - \lambda_6)I_2}{A_{10}}\right)\right\}. \tag{42}$$

Solving the optimality system requires initial and transversality conditions together with characterization obtained in Eqs. (38)–(42), in addition, from the Largrangian equation

$$L = A_1S_1 + A_2q_1 + A_3q_2 + A_4I_1 + A_5I_2 + \frac{A_6}{2}u_1{}^2(t) + \frac{A_7}{2}u_2{}^2(t) + \frac{A_8}{2}u_3{}^2(t) + \frac{A_9}{2}u_4{}^2(t) + \frac{A_{10}}{2}u_5{}^2(t),$$

we can see that the second derivative with respect to $u_1, u_2, u_3, u_4, and\ u_5$ is positive. This shows that the optimal control problem is minimum at controls $u_1, u_2, u_3, u_4, and\ u_5$ respectively.

Now by substituting Eqs. (38)–(42) into the system Eqs. (15)–(21) we have;

$$\frac{dS_1}{dt} = \alpha_1S_2 - \frac{\beta S_1I_1}{N - S_2 - S_3 - I_2 - H} - \min\left\{1, \max\left(0, \frac{(\lambda_1 - \lambda_2)q_1}{A_7}\right)\right\}q_1 - \left(\mu + \min\left\{1, \max\left(0, \frac{(\lambda_1 - \lambda_3)S_1}{A_6}\right)\right\}\right)S_1 + \alpha_2S_3, \tag{43}$$

$$\frac{dS_2}{dt} = \min\left\{1, \max\left(0, \frac{(\lambda_1 - \lambda_2)q_1}{A_7}\right)\right\}q_1 - \alpha_1S_2, \tag{44}$$

$$\frac{dS_3}{dt} = \left(\mu + \min\left\{1, \max\left(0, \frac{(\lambda_1 - \lambda_3)S_1}{A_6}\right)\right\}\right)S_1 - \alpha_2S_3, \tag{45}$$

$$\frac{dI_1}{dt} = \frac{\beta S_1I_1}{N - S_2 - S_3 - I_2 - H} - \min\left\{1, \max\left(0, \frac{(\lambda_4 - \lambda_5)q_2}{A_8}\right)\right\}q_2 - \left(\gamma + \min\left\{1, \max\left(0, \frac{(\lambda_4 - \lambda_6)I_1}{A_9}\right)\right\}\right)I_1, \tag{46}$$

$$\frac{dI_2}{dt} = \min\left\{1, \max\left(0, \frac{(\lambda_4 - \lambda_5)q_2}{A_8}\right)\right\}q_2 - (\gamma + u_5(t))I_2, \tag{47}$$

$$\frac{dH}{dt} = \left(\gamma + \min\left\{1, \max\left(0, \frac{(\lambda_4 - \lambda_6)I_1}{A_9}\right)\right\}\right)I_1 + \left(\gamma + \min\left\{1, \max\left(0, \frac{(\lambda_5 - \lambda_6)I_2}{A_{10}}\right)\right\}\right)I_2 - \delta H, \tag{48}$$

$$\frac{dR}{dt} = \delta H. \tag{49}$$

5 Stability Analysis

In this chapter, two equilibrium points; Disease Free and Endemic Equilibria are found. Basic reproduction ratio is obtained. Global stability analyses of the equilibrium solutions are carried out.

5.1 Equilibria

Since there does not appear the state variable R in Eq. (15) through Eq. (21), it suffices to analyze the system Eq. (15) through Eq. (20).

Disease free equilibrium E_0 is obtained by substituting $I_1 = I_2 = H = 0$ into Eqs. (15)–(20), thus we have

$$E_0 = \left(S_1^0, S_2^0, S_3^0, I_1^0, I_2^0, H\right) = \left(S_1^0, 0, \frac{\mu + u_1}{\alpha_2} S_1^0, 0, 0, 0\right).$$

The endemic equilibrium $E^* = \left(S_1^*, S_2^*, S_3^*, I_1^*, I_2^*, H^*\right)$ is obtained when $I_1 \neq 0, I_2 \neq 0$, $H \neq 0$, thus

$$S_3^* = \frac{\mu + u_1}{\alpha_2} S_1^*, \tag{50}$$

$$I_2^* = \frac{\alpha_1 \gamma b u_3 - (\gamma + u_5)(\gamma b u_2 + \gamma + u_4)}{\alpha_1 u_3 (\gamma + u_5) + \gamma b u_3 [u_2 (\gamma + u_5) - \alpha_1 u_3] + (\gamma + u_5)^2 (u_3 - u_2)} u_3 I_1^*. \tag{51}$$

Since the endemic equilibrium is positive, then $I_2^* > 0$ i.e.,

$$\alpha_1 \gamma b u_3 - (\gamma + u_5)(\gamma b u_2 + \gamma + u_4) > 0,$$

or

$$\alpha_1 \gamma b u_3 > (\gamma + u_5)(\gamma b u_2 + \gamma + u_4).$$

$$S_2^* = \gamma b \frac{u_3 (t) \left(I_1^* + I_2^*\right)}{\gamma + u_5} - I_2^*,$$

$$S_2^* = \left[\frac{\gamma b u_3}{\gamma + u_5} + \left(\frac{\gamma b u_3}{\gamma + u_5} - 1\right) \frac{\alpha_1 \gamma b u_3 - (\gamma + u_5)(\gamma b u_2 + \gamma + u_4)}{\alpha_1 u_3 (\gamma + u_5) + \gamma b u_3 [u_2 (\gamma + u_5) - \alpha_1 u_3] + (\gamma + u_5)^2 (u_3 - u_2)} u_3\right] I_1^*. \tag{52}$$

Also

$$\frac{\gamma b u_3}{\gamma + u_5} - 1 > 0 \ \ or \ \ \frac{\gamma b u_3}{\gamma + u_5} > 1.$$

$$H^* = \frac{1}{\delta} \left[(\gamma + u_4) I_1^* + (\gamma + u_5) I_2^*\right],$$

$$H^* = \frac{1}{\delta} \left[\gamma + u_4 + \frac{\alpha_1 \gamma b u_3 - (\gamma + u_5)(\gamma b u_2 + \gamma + u_4)}{\alpha_1 u_3 (\gamma + u_5) + \gamma b u_3 [u_2 (\gamma + u_5) - \alpha_1 u_3] + (\gamma + u_5)^2 (u_3 - u_2)} (\gamma + u_5) u_3\right] I_1^*. \tag{53}$$

5.2 Local Stability of the Equilibria

We construct the Jacobian matrix from Eqs. (15)–(20) as:

$$J=\begin{bmatrix}-D_1-\mu-u_1 & \alpha_1-D_2-u_2D_3 & -D_2+\alpha_2 & -D_4-u_2D_5 & -D_2-u_2D_7 & -D_2\\ 0 & u_2D_3-\alpha_1 & 0 & u_2D_5 & u_2D_7 & 0\\ \mu+u_1 & 0 & -\alpha_1 & 0 & 0 & 0\\ D_1 & D_2+u_3D_3 & D_2 & D_4-u_3D_6-\gamma-u_4 & D_2-u_3D_8 & D_2\\ 0 & -u_3D_3 & 0 & u_3D_6 & u_3D_8-\gamma-u_5 & 0\\ 0 & 0 & 0 & \gamma+u_4 & \gamma+u_5 & -\delta\end{bmatrix}$$

where,

$$D_1=\frac{\beta I_1}{N-S_2-S_3-I_2-H},\quad D_2=\frac{\beta S_1 I_1}{(N-S_2-S_3-I_2-H)^2},\quad D_3=\frac{\gamma b\,(I_1+I_2)\,I_2}{(S_2+I_2)^2},$$

$$D_4=\frac{\beta S_1}{N-S_2-S_3-I_2-H},\quad D_5=\frac{\gamma b S_2}{S_2+I_2},\quad D_6=\frac{\gamma b I_2}{S_2+I_2},$$

$$D_7=\frac{\gamma b\,(S_2-I_1)\,S_2}{(S_2+I_2)^2},\quad D_8=\frac{(S_2+I_2)\,(I_1+2I_2)-(I_1+I_2)\,I_2}{(S_2+I_2)^2}.$$

Theorem 5: The disease free equilibrium E_0 is locally asymptotically stable.

Proof:

$$J_{E_0}=\begin{bmatrix}-\mu-u_1 & \alpha_1 & \alpha_2 & -\beta S_1(0)/[N-S_3(0)] & 0 & 0\\ 0 & -\alpha_1 & 0 & 0 & 0 & 0\\ \mu+u_1 & 0 & -\alpha_1 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta S_1(0)/[N-S_3(0)]-\gamma-u_4 & 0 & 0\\ 0 & 0 & 0 & 0 & -\gamma-u_5 & 0\\ 0 & 0 & 0 & \gamma+u_4 & \gamma+u_5 & -\delta\end{bmatrix}$$

The eigenvalue is obtained from;

$det\left|J_{E_0}-KI\right|=0.$

This implies;

$$\begin{bmatrix}-\mu-u_1-K & \alpha_1 & \alpha_2 & -\beta S_1(0)/[N-S_3(0)] & 0 & 0\\ 0 & -\alpha_1-K & 0 & 0 & 0 & 0\\ \mu+u_1 & 0 & -\alpha_1-K & 0 & 0 & 0\\ 0 & 0 & 0 & \beta S_1(0)/[N-S_3(0)]-\gamma-u_4-K & 0 & 0\\ 0 & 0 & 0 & 0 & -\gamma-u_5-K & 0\\ 0 & 0 & 0 & \gamma+u_4 & \gamma+u_5 & -\delta-K\end{bmatrix}=0$$

$K_1=-\mu-u_1,$

$K_2=-\alpha_1,$

$K_3=-\alpha_2-\mu-u_1,$

$$K_4 = \frac{\beta S_1(0)}{N - S_3(0)} - \gamma - u_4,$$

$$K_5 = -\delta, K_6 = 0.$$

5.3 Basic Reproduction Number

For the DFE to be locally asymptotically stable, the eigenvalue K_4 must be negative. That is:

$$\frac{\beta S_1(0)}{N - S_3(0)} - \gamma - u_4 < 0,$$

or

$$\frac{\beta S_1(0)}{(\gamma + u_4)(N - S_3(0))} < 1.$$

Now, define the basic reproduction ratio (R_0) to be:

$$R_0 = \frac{\beta S_1(0)}{(\gamma + u_4)(N - S_3(0))}. \tag{54}$$

5.4 Global Stability Analysis

Here the global stability analyses of the two equilibrium points are carried out.

Theorem 6: The disease free equilibrium is globally asymptotically stable.

Proof:

Let the Lyapunov candidate function be,

$$V(S_1, S_2, S_3, I_1, I_2, H) = \frac{1}{2}[(S_1 - S_1{}^*) + S_2 + (S_3 - S_3{}^*) + I_1 + I_2 + H]^2.$$

Clearly the above function $V(S_1, S_2, S_3, I_1, I_2, H) > 0$.

Also $V(S_1, S_2, S_3, I_1, I_2, H) = 0$, if $(S_1, S_2, S_3, I_1, I_2, H,) = \left(S_1, 0, \frac{\mu + u_1}{\alpha_2} S_1, 0, 0, 0\right)$.

$$\frac{dV}{dt} = \left[(S_1 - S_1{}^*) + S_2 + (S_3 - S_3{}^*) + I_1 + I_2 + H\right]\left[\frac{dS_1}{dt} + \frac{dS_2}{dt} + \frac{dS_3}{dt} + \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dH}{dt}\right],$$

$$\frac{dV}{dt} = -\delta H\left[(S_1 - S_1{}^*) + S_2 + (S_3 - S_3{}^*) + I_1 + I_2 + H\right].$$

Clearly,

$$\frac{dV}{dt} = 0 \textit{ if } (S_1, S_2, S_3, I_1, I_2, H,) = \left(S_1, 0, \frac{\mu + u_1}{\alpha_2} S_1, 0, 0, 0\right),$$

$$\frac{dV}{dt} < 0 \textit{ if } (S_1, S_2, S_3, I_1, I_2, H,) \neq \left(S_1, 0, \frac{\mu + u_1}{\alpha_2} S_1, 0, 0, 0\right).$$

Theorem 7: The endemic equilibrium is globally asymptotically stable.

Proof:

Let the Lyapunov candidate function be,

$$W(S_1,S_2,S_3,I_1,I_2,H)=\frac{1}{2}\left[(S_1-S_1{}^*)+(S_2-S_2{}^*)+(S_3-S_3{}^*)+(I_1-I_1{}^*)+(I_2-I_2{}^*)+(H-H^*)\right]^2.$$

Clearly, $W(S_1,S_2,S_3,I_1,I_2,H)\geq 0$.

Also $W(S_1,S_2,S_3,I_1,I_2,H)=0$, if $(S_1,S_2,S_3,I_1,I_2,H)=(S_1{}^*,S_2{}^*,S_3{}^*,I_1{}^*,I_2{}^*,H^*)$.

$$\begin{aligned}\frac{dW}{dt}&=\left[(S_1-S_1{}^*)+(S_2-S_2{}^*)+(S_3-S_3{}^*)+(I_1-I_1{}^*)+(I_2-I_2{}^*)+(H-H^*)\right]\\&\quad\times\left[\frac{dS_1}{dt}+\frac{dS_2}{dt}+\frac{dS_3}{dt}+\frac{dI_1}{dt}+\frac{dI_2}{dt}+\frac{dH}{dt}\right]\\&=-\delta H\left[(S_1-S_1{}^*)+(S_2-S_2{}^*)+(S_3-S_3{}^*)+(I_1-I_1{}^*)+(I_2-I_2{}^*)+(H-H^*)\right].\end{aligned}$$

Clearly,

$$\frac{dW}{dt}=0, if\ (S_1,S_2,S_3,I_1,I_2,H,)=(S_1{}^*,S_2{}^*,S_3{}^*,I_1{}^*,I_2{}^*,H^*),$$

$$\frac{dW}{dt}<0, if\ (S_1,S_2,S_3,I_1,I_2,H,)\neq(S_1{}^*,S_2{}^*,S_3{}^*,I_1{}^*,I_2{}^*,H^*)$$

6 Numerical Simulations

In this chapter numerical simulations are carried out to support the analytic results and to show the significance of the controller. Most of the data used in the simulation for the parameters and the variables is from china as in [7]. The values can be found in Tabs. 1 and 2 below.

Table 1: Model variables, descriptions and values

Model variables	Descriptions	Mean value
N	Total population	1.3362×10^9
$S_1(0)$	Initial susceptible population	2.6723×10^8
$S_2(0)$	Initial quarantine susceptible population	3762
$S_3(0)$	Initial isolated susceptible population	1.069×10^9
$I_1(0)$	Initial infected population	4101
$I_2(0)$	Initial quarantine infected population	700
$H(0)$	Initial hospitalized population	3886
$R(0)$	Initial removed individuals	64

Table 2: Model parameters, descriptions and values

Model parameters	Description	Value
α_1	time of isolation at home for susceptible population	$1/14$
α_2	Transfer rate from isolated susceptible population	5.05×10^{-6}
β	Transmission rate of COVID-19	0.3567
μ	Transfer rate from susceptible to isolated susceptible population	1.76×10^{-4}
γ	Hospitalization rate of infected population	0.1429
δ	Discharge rate from hospital	0.0949
b	Isolation coefficient	12

It can be seen from Fig. 2, that when no any control measure is observed and people were allowed to behave as usual the number of infected individuals will escalate. On the other hand if the control measures were observed optimally, that is; susceptible communities are isolated, susceptible individuals that have contact with infected individuals are quarantined, asymptomatic individuals are quarantined, and infected individuals are traced and hospitalized, then the number of infected individuals will drastically be reduced as shown in Fig. 3.

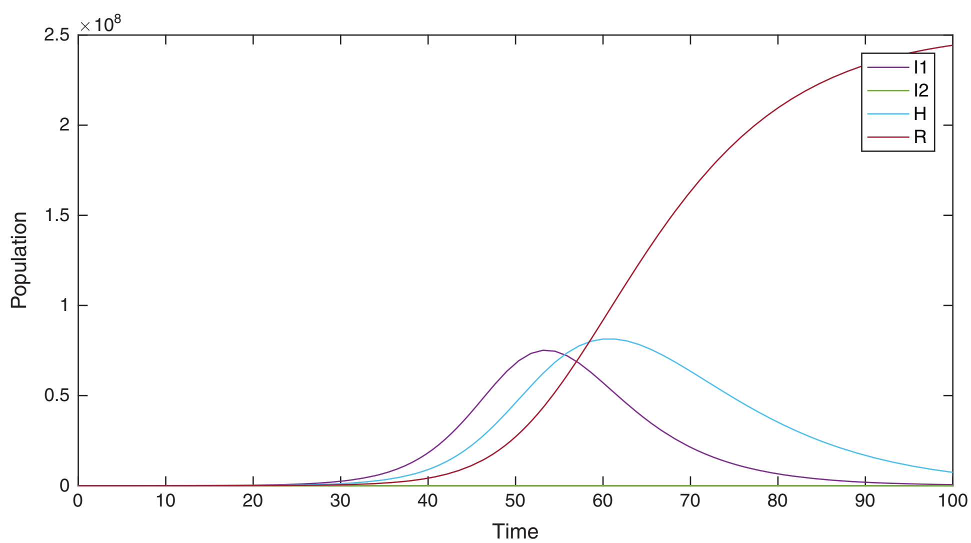

Figure 2: Dynamics of the infected population when there is no control

Although these control measures aren't easy to be observed but their significance can easily be seen from the above graphs. It is clearly shown that when individuals and governments at various levels put hands together the spread of the disease will be curbed. From the above two graphs it can be seen that the number of people that will be removed from the population (either by death or by natural recovery) will be reduced from about 2.5×10^8 when there is no control to less than 9000 people when control is observed optimally.

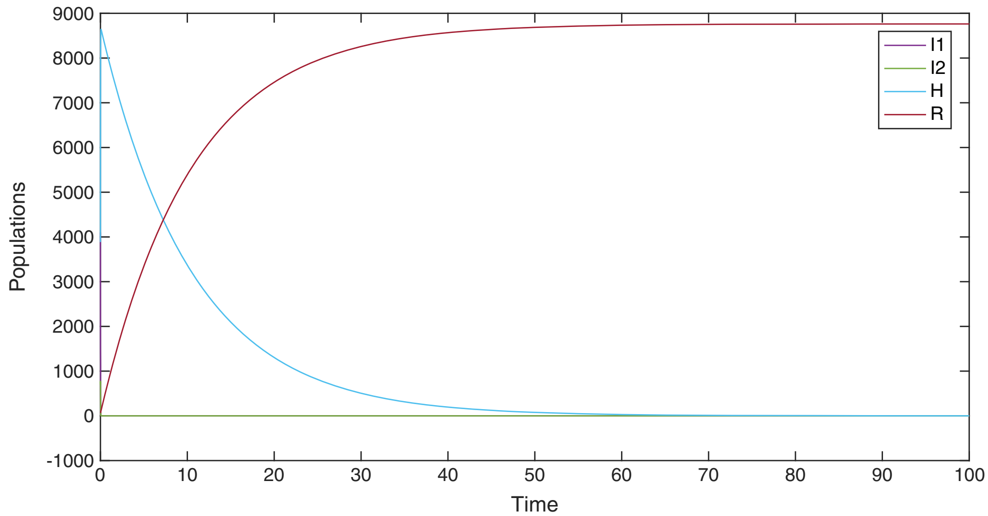

Figure 3: Dynamics of the infected population when all control measures are optimally observed

Acknowledgement: We thank the reviewers for their valuable contributions.

Funding Statement: The author(s) received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] World Health Organization (WHO), "Novel coronavirus-China," 2020. [Online]. Available: https://www.who.int/csr/don/12-january-2020-novel-coronavirus-china/en/.

[2] N. Zhu, D. Y. Zhang, W. L. Wang, X. W. Li, Y. Bo *et al.,* "A novel coronavirus from patients with pneumonia in China 2019," *New England Journal of Medicine,* vol. 382*,* no. 8*,* pp. 727–733, 2020.

[3] I. I. Bogoch, A. Watts, A. Thomas-Bachli, C. Huber, M. U. G. Kraemer *et al.,* "Pneumonia of unknown etiology in China: Potential of international spread via commercial air travel," *Journal of Travel Medicine,* vol. 27*,* no. 2*,* taaa008, 2020.

[4] Centers for Disease Control and Prevention, "Coronavirus disease 2019 (COVID-19)," 2019. [Online]. Available: https://www.cdc.gov/coronavirus/2019-ncov/index.html.

[5] A. Ivan, F. Ndairou, J. J. Nieto, C. J. Silva and D. F. M. Torres, "Ebola model and optimal control with vaccination constraints," *Journal of Industrial and Management Optimaisation,* vol. 14*,* no. 2*,* pp. 427–446, 2018.

[6] B. A. Baba, P. Ismaili and I. A. Baba, "Optimal control approach to study two strain malaria model," *AIP Conference Proceedings,* vol. 2183*,* no. 1*,* pp. 070004, 2019.

[7] I. A. Baba and B. Ghanbari, "Existence and uniqueness of solution of a fractional order tuberculosis model," *European Physical Journal Plus,* vol. 134*,* no. 10*,* pp. 398, 2019.

[8] I. A. Baba, "A fractional order bladder cancer model with BCG treatment effect," *Computational and Applied Mathematics,* vol. 38*,* no. 37*,* pp. 542, 2019.

[9] H. Kademi, F. T. Saad, B. Ulusoy, I. A. Baba and C. Hecer, "Mathematical model for aflatoxins risk mitigation in food," *Journal of Food Engineering,* vol. 263*,* no. 2*,* pp. 25–29, 2019.

[10] I. A. Baba, E. Hincal and S. H. K. Alsaadi, "Global stability analysis of a two strain epidemic model with awareness," *Advances in Differential Equations and Control Processes,* vol. 19*,* no. 2*,* pp. 83–100, 2018.

[11] I. A. Baba, E. Hincal and B. Kaymakamzade, “Two-strain epidemic model with two vaccinations,” *Chaos Solitons & Fractals,* vol. 106, no. 1, pp. 342–348, 2018.

[12] I. A. Baba and E. Hincal, “A model for Influenza with vaccination and awareness,” *Chaos Solitons & Fractals,* vol. 106, no. 1, pp. 49–55, 2018.

[13] I. A. Baba and E. Hincal, “Global stability analysis of two-strain epidemic model with bilinear and non-monotonic incidence rates,” *European Physical Journal Plus,* vol. 132, no. 5, pp. 1–7, 2017.

[14] S. Zhao and H. Chen, “Modeling the epidemic dynamics and control of COVID-19 in China,” *Quantitative Biology,* vol. 11, no. 1, pp. 1–9, 2020.

[15] H. Song, F. Liu, F. Li, X. Cao, H. Wang *et al.,* “The impact of isolation on the transmission of COVID-19 and estimation of potential second epidemic in China,” 2020. [Online]. Available: www.preprints.org.

[16] M. Tahir, S. I. A. Shah, G. Zamzn and T. Khan, “Stability behavior of mathematical model of MERS Corona virus spread in population,” *Filomat,* vol. 33, no. 12, pp. 3947–3960, 2019.

[17] C. Yang and J. Wang, “A mathematical model for the novel Coronavirus epidemic in Wuhan, China,” *Mathematical Biosciences and Engineering,* vol. 17, no. 3, pp. 2708–2724, 2020.

[18] T. M. Chen, J. Rui, Q. P. Wang, Z. Y. Zhao, J. A. Cui *et al.*, “A mathematical model for simulating the phased-based transmissibility of a novel coronavirus,” *Infectious Disease of Poverty,* vol. 9, no. 24, pp. 1–7, 2020.

[19] C. C. Sunhwa and K. Moran, “Estimating the reproductive number and the outbreak size of novel coronavirus (COVID-19) using mathematical model republic of Korea,” *Journal of Epidemiology and Health,* vol. 42, no. 1, pp. 1–7, 2020.

[20] M. Elhia, M. Rachik and E. Benlahmar, “Optimal control of an SIR model with delay in state and control variables,” *International Scholarly Research Notices,* vol. 2013, pp. 1–7, 2013.

[21] I. A. Baba, R. A. Abdulkadir and P. Esmaili, “Analysis of tuberculosis model with saturated incidence rate and optimal control physica A,” *Statistical Mechanics and its Applications,* vol. 540, no. 1, 123237, 2020.

[22] Y. S. Luo, K. Yang, Q. Tang, J. M. Zhang, P. Li *et al.*, “An optimal data service providing framework in cloud radio access network,” *EURASIP Journal on Wireless Communications and Networking,* vol. 2016, no. 1, pp. 62, 2016.

[23] L. G. Liu, Y. X. Peng and W. Q. Xu, “To converge more quickly and effectively: Mean field annealing based optimal path selection in WMN,” *Information Sciences,* vol. 294, pp. 216–226, 2015.

[24] W. W. Liu, Y. Tang, F. Yang, Y. Dou and J. Wang, “A multi-objective decision-making approach for the optimal location of electric vehicle charging facilities,” *Computers Materials & Continua,* vol. 60, no. 2, pp. 813–834, 2019.

[25] F. Wang, L. L. Zhang, S. W. Zhou and Y. Y. Huang, “Neural network-based finite-time control of quantized stochastic nonlinear systems,” *Neurocomputing,* vol. 362, pp. 195–202, 2019.

[26] S. M. He, K. Xie, K. X. Xie, C. Xu and J. Wang, “Interference-aware multisource transmission in multiradio and multichannel wireless network,” *IEEE Systems Journal,* vol. 13, no. 3, pp. 2507–2518, 2019.

[27] M. Long and X. Xiao, “Outage performance of double-relay cooperative transmission network with energy harvesting,” *Physical Communication,* vol. 29, pp. 261–267, 2018.