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ORIGINAL ARTICLE

Solitary wave solution for a generalized Hirota-Satsuma coupled KdV and MKdV equations: A semi-analytical approach

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Abstract Nonlinear fractional differential equations (NFDEs) offer an effective model of numerous phenomena in applied sciences such as ocean engineering, fluid mechanics, quantum mechanics, plasma physics, nonlinear optics. Some studies in control theory, biology, economy, and electrodynamics, etc. demonstrate that NFDEs play the primary role in explaining various phenomena arising in real-life. Now-a-day NFDEs in various scientific fields in particular optical fibers, chemical physics, solid-state physics, and so forth have the most important subjects for study. Finding exact responses to these equations will help us to a better understanding of our environmental nonlinear physical phenomena. In this regard, in the present study, we have applied fractional reduced differential transform method (FRDTM) to obtain the solution of nonlinear time-fractional Hirota-Satsuma coupled KdV and MKdV equations. The novelty of the FRDTM is that it does not require any discretization, transformation, perturbation, or any restrictive conditions. Moreover, this method requires less computation compared to other methods. Computed results are compared with the existing results for the special cases of integer order. The present results are in good agreement with the existing solutions. Here, the fractional derivatives are considered in the Caputo sense. The presented method is a semi-analytical method based on the generalized Taylor series expansion and yields an analytical solution in the form of a polynomial.

1. Introduction

In current years, it has turned out that various physical phenomena in engineering, physics, chemistry, and other branches of sciences can be portrayed efficiently employing models, the use of mathematical tools from fractional calculus, which is the

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idea of derivatives and integrals of non-integer order. For instance, European and Vanilla option pricing may be demonstrated by using fractional derivatives [1]. The vibration analysis of damped beams and large membrane may be modeled by fractional derivative [2,3]. Similarly, the primary equation of fluid mechanics viz. Navier-Stokes equations may be well-defined by fractional derivatives [4]. It is sometimes challenging to obtain the solution of fractional differential equations. Various fractional differential equations (FDE) do not have exact analytical solutions, so approximate and numerical techniques have to be used in order to get the desired results. So one may require efficient computational methods for the solution of FDEs. Some important works on fractional calculus have been studied in the past couple of years, and different books have been written by various authors namely Baleanu et al. [5,6], Miller and Ross [7], Kilbas et al. [8], Podlubny [9]. An extensive analysis of fractional calculus is included in these books, which may help the researchers for understating the basic ideas of fractional calculus. As such, several semi-analytical and numerical techniques have been established for the solution of such types of physical model problems viz. homotopy perturbation method [10], conformal decomposition method [11], Adomian decomposition method [12,13], modified decomposition method [14], etc. Some other researches can be found in [15–23] relating to the complex study of fractional calculus and various methods.

In this investigation, the solution of generalized time-fractional Hirota–Satsuma coupled KdV, and MKdV equations with proper initial conditions are discussed. The generalized Hirota-Satsuma coupled KdV and MKdv systems are the essential nonlinear equations in mathematics and physics. Hirota-Satsuma coupled KdV equation occurs as a specific case of the Toda lattice equation, a very well-known soliton equation in one space and one-time dimension that is used to model the interaction of neighboring particles of equal weight in a crystal lattice formation. In many nonlinear science fields, these models have many applications. These systems can be used to define generic characteristics of string dynamics in constant curvature space for strings and multi-strings. These equations also investigate the interaction of two long waves with different dispersion relationships. In addition, these models are used in the study of shallow-water waves to describe wave propagation.

The time-fractional Hirota–Satsuma coupled KdV which are represented by a system of partial FDES of the form:

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} = \frac{1}{2}\psi_{xxx} - 3\psi\psi_x + 3(\xi\zeta)_x,$$

$$\frac{\partial^\alpha \xi}{\partial t^\alpha} = -\xi_{xxx} + 3\psi\xi_x, \quad \text{where } 0 < \alpha \leqslant 1 \tag{1.1}$$

$$\frac{\partial^\alpha \zeta}{\partial t^\alpha} = -\zeta_{xxx} + 3\psi\zeta_x,$$

with initial conditions:

$$\psi(x,0) = \frac{\beta - 2m^2}{3} + 2m^2\tan^2 h\,(mx),$$

$$\xi(x,0) = \frac{-4m^2c_0(\beta+m^2)}{3c_1^2} + 4m^2\frac{(\beta+m^2)}{3c_1}\tanh\,(mx), \tag{1.2}$$

$$\zeta(x,0) = c_0 + c_1\tanh\,(mx),$$

where $m, c_0, c_1 \neq 0$ and β are arbitrary constant.

And a new coupled MKdV equation is as follows:

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} = \frac{1}{2}\psi_{xxx} - 3\psi^2\psi_x + \frac{3}{2}\xi_{xx} + 3\psi\xi_x + 3\psi_x\xi - 3\lambda\psi_x,$$

$$\frac{\partial^\alpha \xi}{\partial t^\alpha} = -\xi_{xxx} - 3\xi\xi_x - 3\psi_x\xi_x + 3\psi^2\xi_x + 3\lambda\xi_x, \quad \text{where } 0 < \alpha \leqslant 1 \tag{1.3}$$

subject to two initial conditions:

Case I: $\psi(x,0) = m\tanh(mx)$,

$$\xi(x,0) = \frac{1}{2}\left(4m^2 + \lambda\right) - 2m^2\tanh^2(mx). \tag{1.4}$$

Case II: $\psi(x,0) = \frac{b_1}{2m} + m\tanh(mx)$,

$$\xi(x,0) = \frac{\lambda}{2}\left(1 + \frac{m}{b_1}\right) + b_1\tanh(mx) \tag{1.5}$$

Solitary solutions of various nonlinear wave equations have been discussed by various methods which may be problem-specific that is a particular type of problems are solved using these methods. In this regard, many authors have investigated this nonlinear wave equation using different methods. Yu et al. [24], and Yong and Zhang [25] used the Jacobi elliptic function method and projective Riccati equations method respectively to solve a generalized Hirota–Satsuma coupled KdV equations. The algebraic method, variational iteration method, Adomian decomposition method, extended tanh-function method, and homotopy perturbation method (HPM) have been applied to solve generalized Hirota–Satsuma coupled KdV equations by many researchers [26–31]. All these above-mentioned authors have solved integer-order generalized Hirota–Satsuma coupled KdV equations using different techniques. Various authors also discussed non-integer order generalized Hirota–Satsuma coupled KdV equation using various approaches. Ganji et al. [32] used HPM to solve the time-fractional Hirota–Satsuma coupled KdV equation. Similarly, the differential transform method and fractional iterative method have also been applied by Merdan et al. [33] and Shateri and Ganji [34] to solve time-fractional Hirota–Satsuma coupled KdV equation. In this present study, we have applied FRDTM to solve time-fractional generalized Hirota–Satsuma coupled KdV and MKdV equations.

The other parts of the manuscript are arranged as follows: In Section 2 we have presented the basic notation and definitions of fractional calculus. Methodology and theorems of FRDTM are discussed in Section 3. Implementation of the present method for solving generalized time-fractional Hirota–Satsuma coupled KdV and MKdV equations in Section 4 and 5, respectively. Lastly, a conclusion section is given in Section 6.

2. Preliminaries

There are various ways of defining fractional derivatives. However, here two commonly used fractional operators are discussed viz. Caputo and Reimann-Liouville differential and integral operator. For more details, one may refer to [5–9].

Definition 2.1. The Riemann-Liouville (R-L) fractional differential operator $D^{\alpha}$of order $\alpha$is described as

$$D^{\alpha}\xi(x)=\begin{cases}\frac{d^m}{dx^m}\xi(x), & \alpha=m,\\ \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dx^m}\int_0^x\frac{\xi(t)}{(x-t)^{\alpha-m+1}}dt, & m-1<\alpha<m,\end{cases} \tag{2.1}$$

where $m\in Z^{+}$,$\alpha\in R^{+}$ and

$$D^{-\alpha}\xi(x)=\frac{1}{\Gamma(\alpha)}\int_0^x(x-t)^{\alpha-1}\xi(t)\,dt,\quad 0<\alpha\leqslant 1. \tag{2.2}$$

From Podlubny [9], we have

$$J^{\alpha}t^{n}=\frac{\Gamma(n+1)}{\Gamma(n+\alpha+1)}t^{n+\alpha}, \tag{2.3}$$

and

$$D^{\alpha}t^{n}=\frac{\Gamma(n+1)}{\Gamma(n-\alpha+1)}t^{n-\alpha}. \tag{2.4}$$

Definition 2.2. The Caputo fractional differential operator D^{α} of order α is written as follows:

$${}^{C}D^{\alpha}\xi(x)=\begin{cases}\frac{1}{\Gamma(m-\alpha)}\int_0^x\frac{\xi^{m}(t)}{(x-t)^{\alpha-m+1}}dt, & m-1<\alpha<m,\\ \frac{d^m}{dt^m}\xi(x), & \alpha=m.\end{cases} \tag{2.5}$$

Definition 2.3.

(a) $D_t^{\alpha}J_t^{\alpha}\xi(t)=\xi(t)$,

(b) $J_t^{\alpha}D_t^{\alpha}\xi(t)=\xi(t)-\sum_{k=0}^{m}\xi^{(k)}(0^{+})\frac{t^k}{k!}$, for $t>0$ and $m-1<\alpha\leqslant m$ (2.6)

3. Fractional reduced differential transform method (FRDTM)

Let us consider a function $\xi(x,t)$ that is analytic and k-times continuously differentiable. Assuming this function may be represented as a product of two single-variable functions as $\xi(x,t)=a(x)b(t)$. From the differential transform method (DTM) [35], the function may be written as follows

$$\xi(x,t)=\left(\sum_{m=0}^{\infty}A(m)x^{m}\right)\left(\sum_{n=0}^{\infty}B(n)t^{n}\right)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}P(m,n)\,x^{m}t^{n}, \tag{3.1}$$

where $P(m,n)=A(m)B(n)$ is the spectrum of $\xi(x,t)$.

Lemma 3.1. Fractional reduced differential transform (FRDT) of an analytic function $\xi(x,t)$ is defined by

$$\xi_k(x)=\frac{1}{\Gamma(\alpha k+1)}\left[D_t^{\alpha k}\xi(x,t)\right]_{t=t_0}\text{ for } k=0,1,2,\ldots. \tag{3.2}$$

Inverse transform of $\xi_k(x)$ is defined as follows:

$$\xi(x,t)=\sum_{k=0}^{\infty}\xi_k(x)(t-t_0)^{\alpha k}. \tag{3.3}$$

From Eqs. (3.2) and (3.3), we obtain

$$\xi(x,t)=\sum_{k=0}^{\infty}\frac{1}{\Gamma(\alpha k+1)}\left[D_t^{\alpha k}\xi(x,t)\right]_{t=t_0}(t-t_0)^{\alpha k}. \tag{3.4}$$

In particular, when $t_0=0$, Eq. (3.4) reduces to the following equation

$$\xi(x,t)=\sum_{k=0}^{\infty}\xi_k(x)\,t^{\alpha k}=\sum_{k=0}^{\infty}\left(\frac{1}{\Gamma(\alpha k+1)}\right)\left\{D_t^{\alpha k}\xi(x,t)\right\}_{t=0}t^{\alpha k}. \tag{3.5}$$

Theorem 1 (*[36–37]*). If $\psi(x,t)$, $\xi(x,t)$ and $\zeta(x,t)$ are the functions such that $\psi(x,t)=R_D^{-1}[\psi_k(x)]$, $\xi(x,t)=R_D^{-1}[\xi_k(x)]$ and $\zeta(x,t)=R_D^{-1}[\zeta_k(x)]$ then the following results are determined:

R1. If $\psi(x,t)=c_1\xi(x,t)\pm c_2\zeta(x,t)$, then $\psi_k(x)=c_1\xi_k(x)\pm c_2\zeta_k(x)$, where $c_1$and c_1 are constants.
R2. If $\psi(x,t)=a\,\xi(x,t)$, then $\psi_k(x)=a\,\xi_k(x)$.
R3. If $\psi(x,t)=x^{m}t^{n}$, then $\psi_k(x)=x^{m}\,\delta(k-n)$ where $\delta(k)=\begin{cases}1, & k=0\\ 0, & k\neq 0\end{cases}$.
R4. If $\psi(x,t)=x^{m}t^{n}\xi(x,t)$, then $\psi_k(x)=x^{m}\xi_{k-n}(x)$.
R5. If $\psi(x,t)=\xi(x,t)\zeta(x,t)$, then $\psi_k(x)=\sum_{i=0}^{j}\xi_i(x)\zeta_{j-i}(x)=\sum_{i=0}^{j}\zeta_i(x)\xi_{j-i}(x)$.
R6. If $\psi(x,t)=\xi(x,t)\zeta(x,t)\eta(x,t)$, then $\psi_k(x)=\sum_{j=0}^{k}\sum_{i=0}^{j}\xi_i(x)\zeta_{j-i}(x)\eta_{k-j}(x)$.
R7. If $\psi(x,t)=\frac{\partial^m}{\partial x^m}\xi(x,t)$, then $\psi_k(x)=\frac{\partial^m}{\partial x^m}\xi_k(x)$.
R8. If $\psi(x,t)=\frac{\partial^{n\alpha}}{\partial t^{n\alpha}}\xi(x,t)$, then $\psi_k(x)=\frac{\Gamma(1+(k+n)\alpha)}{(1+k\alpha)}\xi_{k+n}(x)$.

The interested authors may follow Refs. [35–39] to know more details about the present technique, including their various applications in a variety of fractional differential equations.

4. Implementation of FRDTM to Hirota-Satsuma coupled KdV

Choosing the proper result R1-R8 and applying FRDTM to Eq. (1.1), the following expressions are obtained:

$$\left.\begin{array}{l}\psi_{k+1}(x)=\frac{\Gamma(1+\alpha k)}{\Gamma(1+\alpha k+\alpha)}\left(\begin{array}{c}\frac{1}{2}\frac{\partial^3}{\partial x^3}\psi_k(x)-\\3\sum_{i=0}^{k}\psi_i(x)\,\psi_{k-i}(x)+3\left(\sum_{i=0}^{k}\xi_i(x)\frac{\partial}{\partial x}\zeta_{k-i}(x)+\sum_{i=0}^{k}\zeta_{k-i}(x)\frac{\partial}{\partial x}\xi_i(x)\right)\end{array}\right)\\ \xi_{k+1}(x)=\frac{\Gamma(1+\alpha k)}{\Gamma(1+\alpha k+\alpha)}\left(-\frac{\partial^3}{\partial x^3}\xi_k(x)+3\sum_{i=0}^{k}\psi_i(x)\frac{\partial}{\partial x}\xi_{k-i}(x)\right),\\ \zeta_{k+1}(x)=\frac{\Gamma(1+\alpha k)}{\Gamma(1+\alpha k+\alpha)}\left(-\frac{\partial^3}{\partial x^3}\zeta_k(x)+3\sum_{i=0}^{k}\psi_i(x)\frac{\partial}{\partial x}\zeta_{k-i}(x)\right),\quad \text{for } k=0,\,1,\ldots\end{array}\right\}\tag{4.1}$$

Now, using FRDTM to initial conditions Eq. (1.2), we have

$$\left.\begin{array}{r}\psi_0(x)=\frac{\beta-2m^2}{3}+2m^2\tan^2h\,(mx),\\ \xi_0(x)=\frac{-4m^2c_0\left(\beta+m^2\right)}{3c_1^2}+4m^2\frac{\left(\beta+m^2\right)}{3c_1}\tanh\,(mx),\\ \zeta_0(x)=c_0+c_1\tanh\,(mx),\end{array}\right\}\tag{4.2}$$

Using Eq. (4.2) into Eq. (4.1), the following values of $\psi_k(x), \xi_k(x), \zeta_k(x)$ for $k=1, 2\ldots$are evaluated:

$$\psi_1=\frac{8m^5\left(\cosh(mx)^2-3\right)\sinh(mx)}{\Gamma(1+\alpha)\cosh(mx)^5},$$

$$\xi_1=\frac{-8m^5\left(m^2+\beta\right)\left(2\cosh(mx)^2-3\right)}{3c_1\Gamma(1+\alpha)\cosh(mx)^4},$$

$$\zeta_1=\frac{-2m^3c_1\left(2\cosh(mx)^2-3\right)}{\Gamma(1+\alpha)\cosh(mx)^4},$$

$$\psi_2=\frac{-8m^8\left(4\cosh(mx)^6-126\cosh(mx)^4+420\cosh(mx)^2-315\right)}{\Gamma(1+2\alpha)\cosh(mx)^8},$$

$$\xi_2=\frac{-128m^8\left(\cosh(mx)^4-15\cosh(mx)^2+\frac{45}{2}\right)\left(m^2+\beta\right)\sinh(mx)}{3c_1\Gamma(1+2\alpha)\cosh(mx)^7},$$

$$\zeta_2=\frac{-32m^6\left(\cosh(mx)^4-15\cosh(mx)^2+\frac{45}{2}\right)c_1\sinh(mx)}{\Gamma(1+2\alpha)\cosh(mx)^7},$$

Now, using inverse FRDT, we get

$$\psi(x,t)=\sum_{k=0}^{\infty}\psi_k(x)\,t^{\alpha k},$$

$$\psi(x,t)=\frac{\beta-2m^2}{3}+2m^2\tan^2h\,(mx)+\frac{8m^5\left(\cosh(mx)^2-3\right)\sinh(mx)}{\Gamma(1+\alpha)\cosh(mx)^5}t^{\alpha}+\frac{-8m^8\left(4\cosh(mx)^6-126\cosh(mx)^4+420\cosh(mx)^2-315\right)}{\Gamma(1+2\alpha)\cosh(mx)^8}t^{2\alpha}+\ldots.,\tag{4.3}$$

$$\xi(x,t)=\sum_{k=0}^{\infty}\xi_k(x)\,t^{\alpha k},$$

$$\xi(x,t)=\frac{-4m^2c_0\left(\beta+m^2\right)}{3c_1^2}+4m^2\frac{\left(\beta+m^2\right)}{3c_1}\tanh\,(mx)+\frac{-8m^5\left(m^2+\beta\right)\left(2\cosh(mx)^2-3\right)}{3c_1\Gamma(1+\alpha)\cosh(mx)^4}t^{\alpha}+\frac{-128m^8\left(\cosh(mx)^4-15\cosh(mx)^2+\frac{45}{2}\right)\left(m^2+\beta\right)\sinh(mx)}{3c_1\Gamma(1+2\alpha)\cosh(mx)^7}t^{2\alpha}+\ldots..,\tag{4.4}$$

$$\zeta(x,t)=\sum_{k=0}^{\infty}\zeta_k(x)\,t^{\alpha k},$$

$$\zeta(x,t)=c_0+c_1\tanh\,(mx)+\frac{-2m^3c_1\left(2\cosh(mx)^2-3\right)}{\Gamma(1+\alpha)\cosh(mx)^4}t^{\alpha}+\frac{-32m^6\left(\cosh(mx)^4-15\cosh(mx)^2+\frac{45}{2}\right)c_1\sinh(mx)}{\Gamma(1+2\alpha)\cosh(mx)^7}t^{2\alpha}+\ldots.,\tag{4.5}$$

Eqs. (4.3)–(4.5) are the series solution of the Hirota-Satsuma coupled KdV equation. It is noted that the present solutions are in good agreement with the results given by Raslan [30] and Ganji et al. [32] using ADM and HPM at a particular case ($\alpha=1$) with few iterations. Here, all the computations are done

$$\psi_3=\frac{64m^{11}\left(2\cosh(mx)^8-510\cosh(mx)^6+6615\cosh(mx)^4-18900\cosh(mx)^2+14175\right)\sinh(mx)}{\Gamma(1+3\alpha)\cosh(mx)^{11}},$$

$$\xi_3=\frac{-512m^{11}\left(m^2+\beta\right)\left(2\cosh(mx)^8-255\cosh(mx)^6+2205\cosh(mx)^4-4725\cosh(mx)^2+2835\right)}{3c_1\Gamma(1+3\alpha)\cosh(mx)^{10}},$$

$$\zeta_3=\frac{-128m^9c_1\left(2\cosh(mx)^8-255\cosh(mx)^6+2205\cosh(mx)^4-4725\cosh(mx)^2+2835\right)}{\Gamma(1+3\alpha)\cosh(mx)^{10}},$$

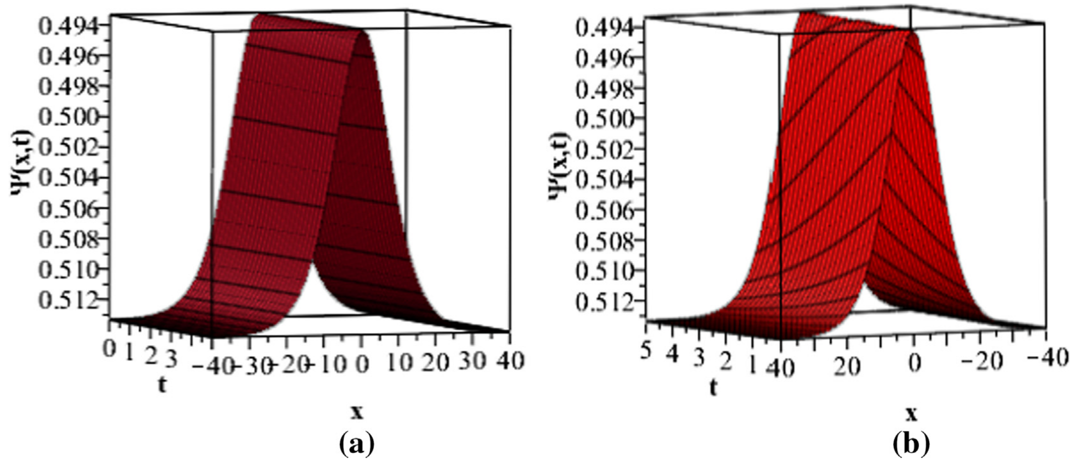

Fig. 1 Comparison plots of $\psi(x,t)$ (a) present solution (b) exact solution of Eq. (1.1) at $\alpha = 1, \beta = 1$, $c_0 = 1.5, c_1 = 0.1$ and $m = 0.1$

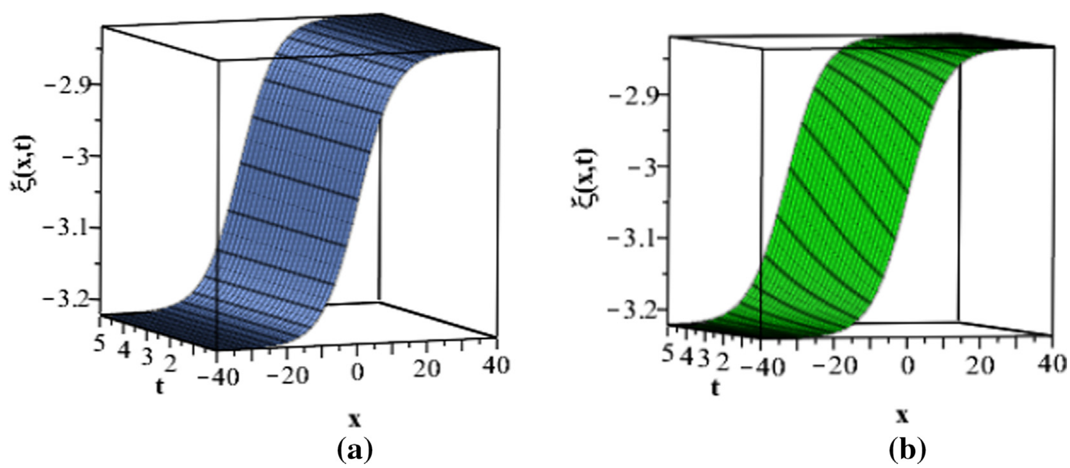

Fig. 2 Comparision plots of $\xi(x,t)$ (a) present solution (b) exact solution of Eq. (1.1) at $\alpha = 1, \beta = 1$, $c_0 = 1.5, c_1 = 0.1$ and $m = 0.1$.

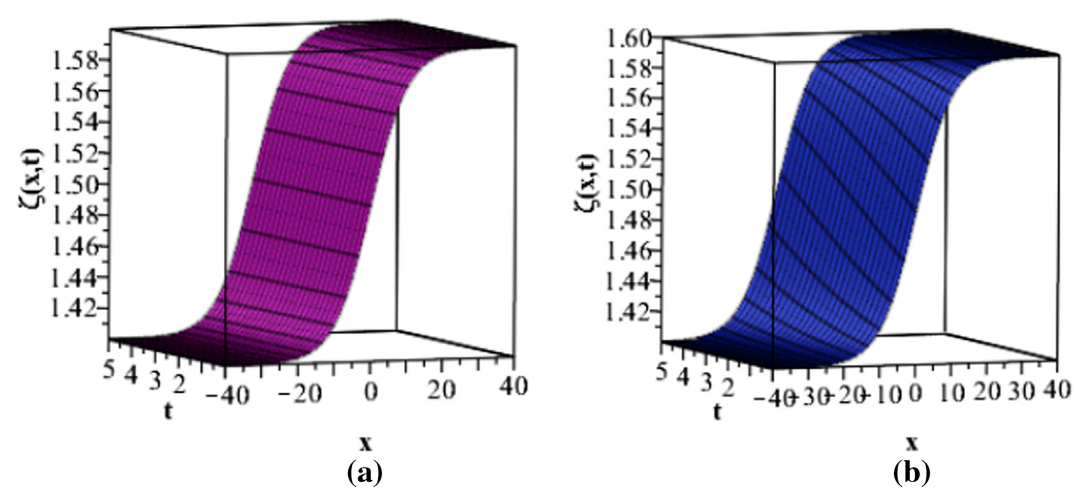

Fig. 3 Comparison plots of $\zeta(x,t)$ (a) present solution (b) exact solution of Eq. (1.1) at $\alpha = 1, \beta = 1$, $c_0 = 1.5, c_1 = 0.1$ and $m = 0.1$.

by taking the finite number of terms of solution ($n = 4$), and the values of the parameters involved in this equation are taken as $m = 0.1$, $\beta = c_0 = 1.5$and $c_1 = 0.1$. Figs. 1–3 show the comparison plots of the present solutions with exact solutions at $\alpha = 1$. Similarly, Tables 1–3 illustrates the comparison results solved by the present method with the results by HPM at various values of $\alpha\,(= 0.5,\ 0.75,\ 1.0)$. Fig. 4 depicts the solution plots of Eqs. (1.1) and (1.2) when $t \in [0,\ 50]$.

Table 1 Numerical results of Eq. (1.1) for $\psi(x,t)$ at $\beta = 1$, $c_0 = 1.5, c_1 = 0.1$ and $m = 0.1$.

t	x	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.0$		Exact
		ψ_{HPM}	ψ_{FRDTM}	ψ_{HPM}	ψ_{FRDTM}	ψ_{HPM}	ψ_{FRDTM}	
0.2	0	0.493351	0.493333	0.493351	0.493333	0.493351	0.493333	0.493351
	0.25	0.493413	0.493344	0.493406	0.493344	0.493393	0.493345	0.493393
	0.5	0.493500	0.493379	0.493485	0.493380	0.493460	0.493381	0.493460
	0.75	0.493611	0.493439	0.493588	0.493441	0.493552	0.493443	0.493552
	1.0	0.493746	0.493524	0.493716	0.493527	0.493667	0.493528	0.493667
0.4	0	0.493405	0.493334	0.493405	0.493333	0.493405	0.493333	0.493405
	0.25	0.493509	0.493343	0.493495	0.493343	0.493477	0.493344	0.493477
	0.5	0.493636	0.493378	0.493609	0.493379	0.493574	0.493380	0.493573
	0.75	0.493788	0.493437	0.493747	0.493439	0.493694	0.493440	0.493693
	1	0.493962	0.493521	0.493908	0.493523	0.493837	0.493525	0.493836
0.6	0	0.493495	0.493334	0.493495	0.493333	0.493495	0.493333	0.493494
	0.25	0.493634	0.493343	0.493617	0.493343	0.493597	0.493343	0.493595
	0.5	0.493798	0.493377	0.493763	0.493377	0.493723	0.493378	0.493720
	0.75	0.493983	0.493435	0.493931	0.493437	0.493871	0.493438	0.493868
	1	0.494191	0.493519	0.494122	0.493520	0.494042	0.493523	0.494038

Table 2 Numerical results of Eq. (1.1) for $\xi(x,t)$ at $\beta = 1$, $c_0 = 1.5, c_1 = 0.1$ and $m = 0.1$

t	x	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.0$		Exact
		ξ_{HPM}	ξ_{FRDTM}	ξ_{HPM}	ξ_{FRDTM}	ξ_{HPM}	ξ_{FRDTM}	
0.2	0	−3.009951	−3.019796	−3.011485	−3.019868	−3.013960	−3.019919	−3.013961
	0.25	−3.004930	−3.014765	−3.006462	−3.014837	−3.008936	−3.014887	−3.008937
	0.5	−2.999927	−3.009741	−3.001457	−3.009812	−3.003926	−3.009862	−3.003927
	0.75	−2.994950	−3.004730	−2.996474	−3.004800	−2.998935	−3.004849	−2.998937
	1.0	−2.990003	−2.999739	−2.991521	−2.999808	−2.993971	−2.999856	−2.993973
0.4	0	−3.001587	−3.019713	−3.004319	−3.019779	−3.007920	−3.019839	−3.007934
	0.25	−2.996584	−3.014682	−2.999314	−3.014748	−3.002913	−3.014807	−3.002927
	0.5	−2.991611	−3.009658	−2.994336	−3.009724	−2.997927	−3.009782	−2.997942
	0.75	−2.986672	−3.004649	−2.989389	−3.004713	−2.992969	−3.004771	−2.992984
	1	−2.981775	−2.999659	−2.984480	−2.999723	−2.988045	−2.999779	−2.988058
0.6	0	−2.994318	−3.019648	−2.997835	−3.019701	−3.001880	−3.019758	−3.001928
	0.25	−2.989342	−3.014618	−2.992858	−3.014670	−2.996899	−3.014727	−2.996948
	0.5	−2.984405	−3.009595	−2.987914	−3.009647	−2.991948	−3.009703	−2.991996
	0.75	−2.979512	−3.004587	−2.983009	−3.004638	−2.987031	−3.004693	−2.987078
	1	−2.974668	−2.999599	−2.978150	−2.999648	−2.982154	−2.999702	−2.982200

5. Implementation of FRDTM to Hirota-Satsuma coupled MKdV

Case I. Applying fractional reduced differential transform on both sides of Eqs. (1.3) and (1.4) and then selecting appropriate results given in Theorem 1, we obtain the following recurrence relations:

$$\left.\begin{aligned}
\psi_{k+1}(x) &= \frac{\Gamma(1+\alpha k)}{\Gamma(1+\alpha k+\alpha)}\left(\begin{aligned}&\tfrac{1}{2}\tfrac{\partial^3}{\partial x^3}\psi_k(x) - 3\sum_{j=0}^{k}\sum_{i=0}^{j}\psi_i(x)\psi_{j-i}(x)\tfrac{\partial}{\partial x}\psi_{k-j}(x) + \tfrac{3}{2}\tfrac{\partial^2}{\partial x^2}\xi_k(x)\\ &+3\left(\sum_{i=0}^{k}\psi_i(x)\tfrac{\partial}{\partial x}\xi_{k-i}(x) + \sum_{i=0}^{k}\xi_{k-i}(x)\tfrac{\partial}{\partial x}\psi_i(x)\right) - 3\lambda\tfrac{\partial}{\partial x}\psi_k(x)\end{aligned}\right),\\
\xi_{k+1}(x) &= \frac{\Gamma(1+\alpha k)}{\Gamma(1+\alpha k+\alpha)}\left(\begin{aligned}&-\tfrac{\partial^3}{\partial x^3}\xi_k(x) - 3\left(\sum_{i=0}^{k}\xi_i(x)\tfrac{\partial}{\partial x}\xi_{k-i}(x) + \sum_{i=0}^{k}\tfrac{\partial}{\partial x}\psi_i(x)\tfrac{\partial}{\partial x}\xi_{k-i}(x)\right)\\ &+3\sum_{j=0}^{k}\sum_{i=0}^{j}\psi_i(x)\psi_{j-i}(x)\tfrac{\partial}{\partial x}\xi_{k-j}(x) + 3\lambda\tfrac{\partial}{\partial x}\xi_k(x)\end{aligned}\right),
\end{aligned}\right\} \quad (5.1)$$

$$\left.\begin{aligned}
\psi_0(x) &= m\tanh(mx),\\
\xi_0(x) &= \tfrac{1}{2}(4m^2+\lambda) - 2m^2\tanh^2(mx),
\end{aligned}\right\} \quad (5.2)$$

Plugging Eq. (5.2) in Eq. (5.1) and iterating Eq. (5.1) for $k = 0, 1, 2, ...$, the following values of $\psi_k(x)$ and $\xi_k(x)$ are obtained:

Table 3 Numerical results of Eq. (1.1) for $\zeta(x,t)$ at $\beta = 1$, $c_0 = 1.5, c_1 = 0.1$ and $m = 0.1$

t	x	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.0$		Exact
		ζ_{HPM}	ζ_{FRDTM}	ζ_{HPM}	ζ_{FRDTM}	ζ_{HPM}	ζ_{FRDTM}	
0.2	0	1.504990	1.500100	1.504229	1.500065	1.503000	1.500039	1.502999
	0.25	1.507484	1.502599	1.506723	1.502564	1.505495	1.502539	1.505494
	0.5	1.509969	1.505095	1.509210	1.505060	1.507983	1.505035	1.507983
	0.75	1.512442	1.507584	1.511684	1.507549	1.510462	1.507525	1.510461
	1.0	1.514899	1.510063	1.514145	1.510029	1.512928	1.510005	1.512927
0.4	0	1.509145	1.500142	1.507788	1.500109	1.506000	1.500079	1.505993
	0.25	1.511630	1.502641	1.510274	1.502608	1.508486	1.502579	1.508479
	0.5	1.514100	1.505136	1.512746	1.505103	1.510963	1.505075	1.510956
	0.75	1.516553	1.507624	1.515204	1.507592	1.513425	1.507564	1.513418
	1	1.518986	1.510102	1.517642	1.510071	1.515871	1.510043	1.515865
0.6	0	1.512755	1.500174	1.511008	1.500148	1.509000	1.500111	1.508975
	0.25	1.515227	1.502673	1.513481	1.502647	1.511473	1.502619	1.511449
	0.5	1.517679	1.505167	1.515936	1.505142	1.513933	1.505114	1.513909
	0.75	1.520109	1.507655	1.518372	1.507630	1.516375	1.507603	1.516352
	1	1.522516	1.510133	1.520786	1.510108	1.518797	1.510081	1.518774

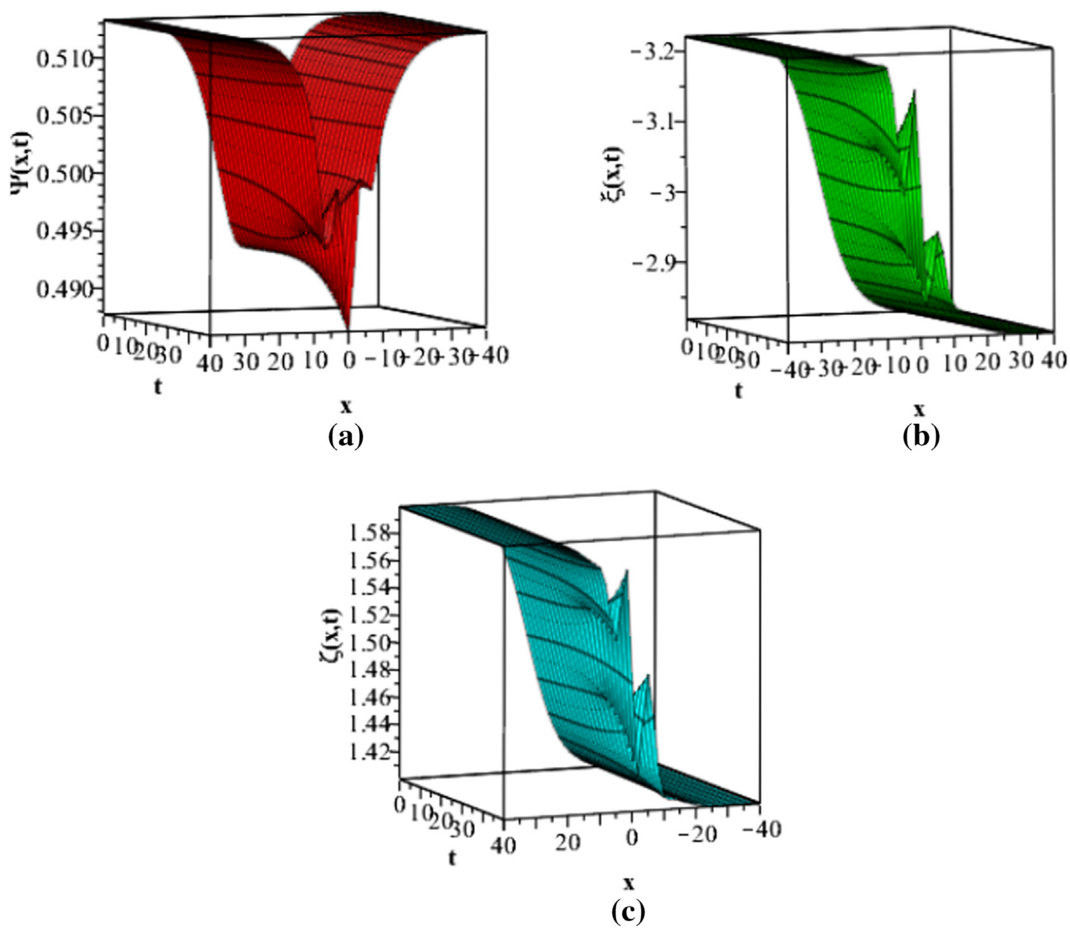

Fig. 4 The present method solution of Eqs. (1.1) and (1.2) at $\alpha = 1, \beta = 1$, $c_0 = 1.5, c_1 = 0.1$, and $m = 0.1$ when $0 \leqslant t \leqslant 50$.

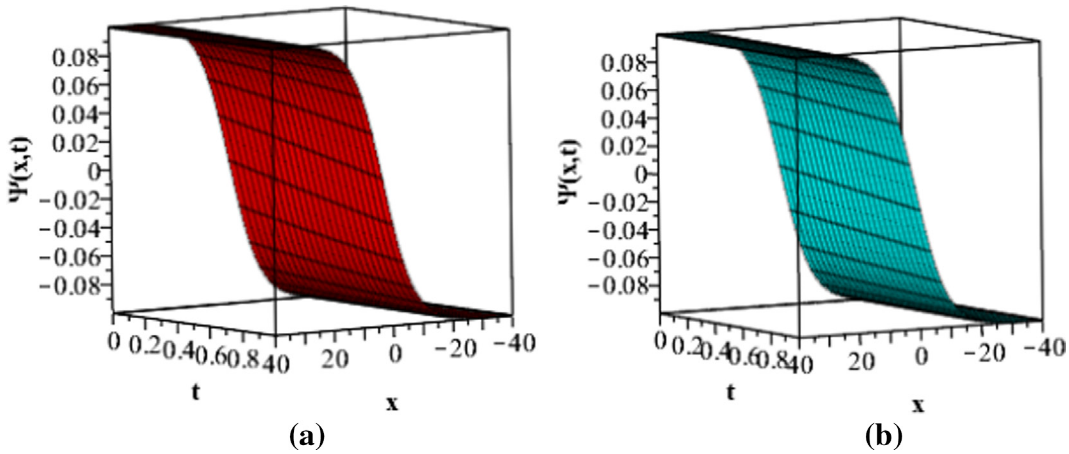

Fig. 5 Comparison plots of $\psi(x,t)$ for Case I (a) present solution (b) exact solution at $\alpha = 1, \lambda = 1$ and $m = 0.1$.

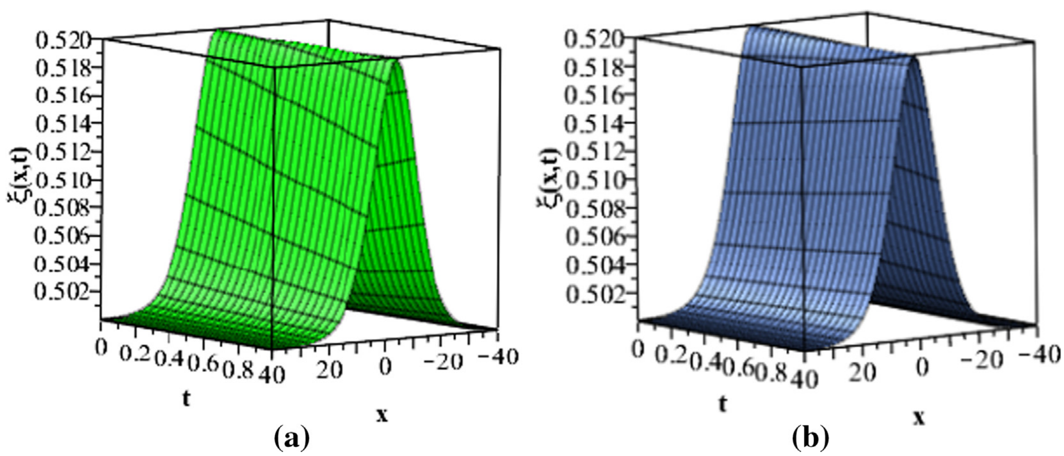

Fig. 6 Comparison plots of $\xi(x,t)$ for Case I (a) present solution (b) exact solution at $\alpha = 1, \lambda = 1$ and $m = 0.1$.

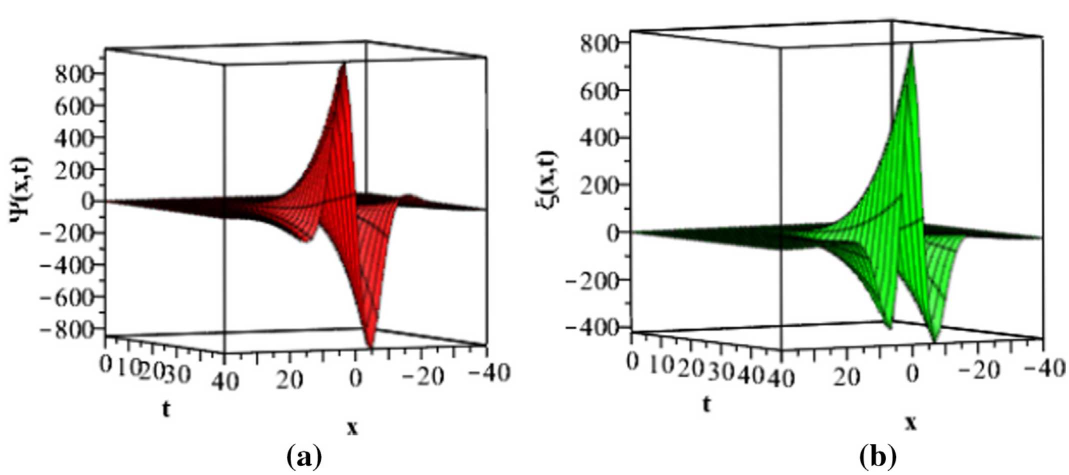

Fig. 7 The present method solution of Eqs. (1.3) and (1.4) at $\alpha = 1, \lambda = 1$ and $m = 0.1$when $0 \leqslant t \leqslant 50$.

Table 4 Comparison between the exact solution and FRDTM solution of Case I when $\alpha = 1, \lambda = 1$ and $m = 0.1$.

t	x	ψ_{FRDTM}	ψ_{Exact}	ψ_{Error}	ξ_{FRDTM}	ξ_{Exact}	ξ_{Error}
0.2	0	−0.00613	−0.003099	0.003032	0.519924	0.519980	0.000056
	0.25	−0.003637	−0.000599	0.003037	0.519850	0.519999	0.000148
	0.5	−0.001138	0.001899	0.003037	0.519753	0.519992	0.000239
	0.75	0.001363	0.004397	0.003033	0.519631	0.519961	0.000329
	1.0	0.003863	0.006889	0.003025	0.519487	0.519905	0.000418
0.4	0	−0.012215	−0.003598	0.006122	0.519699	0.519925	0.000226
	0.25	−0.009744	−0.001099	0.006145	0.519567	0.519974	0.000406
	0.5	−0.007259	0.001399	0.006159	0.519412	0.519997	0.000584
	0.75	−0.004765	0.003898	0.006165	0.519235	0.519996	0.000760
	1	−0.002264	−0.009074	0.006162	0.519037	0.519969	0.000616
0.6	0	−0.018200	−0.006590	0.009125	0.519334	0.519835	0.000501
	0.25	−0.015770	−0.004095	0.009180	0.519147	0.519913	0.000765
	0.5	−0.013319	−0.001599	0.009222	0.518940	0.519966	0.001025
	0.75	−0.010851	0.000899	0.009251	0.518714	0.519994	0.001280
	1	−0.008368	−0.001099	0.009268	0.518468	0.519998	0.001529

$$\psi_1 = \frac{(14m^4 - 3\lambda m^2)\cosh(mx)^2 - 21m^4}{\Gamma(1+\alpha)\cosh(mx)^4},$$

$$\xi_1 = \frac{16m^3\sinh(mx)\left(\left(m^2 - \frac{3}{4}\lambda\right)\cosh(mx)^2 - 3m^2\right)}{\Gamma(1+\alpha)\cosh(mx)^5},$$

$$\psi_2 = \frac{\left(2m^3\sinh(mx)\left(\begin{array}{c}-20m^4\cosh(mx)^4 - 12\cosh(mx)^4\lambda m^2 + 9\cosh(mx)^4\lambda^2 + \\ 300\cosh(mx)^2 m^4 + 36\cosh(mx)^2\lambda m^2 - 450m^4\end{array}\right)\right)}{\Gamma(1+2\alpha)\cosh(mx)^7},$$

$$\xi_2 = \frac{\left(128m^4\left(\begin{array}{c}\left(m^2 - \frac{3}{4}\lambda\right)^2\cosh(mx)^6 + \left(-\frac{63}{2}m^4 + \frac{45}{4}\lambda m^2 - \frac{27}{32}\lambda^2\right)\cosh(mx)^4 + \\ \left(105m^4 - \frac{45}{4}\lambda m^2\right)\cosh(mx)^2 - \frac{315}{4}m^4\end{array}\right)\right)}{\Gamma(1+2\alpha)\cosh(mx)^8},$$

$$\psi_3 = \frac{\left(2m^4\left(\begin{array}{c}-304m^6\cosh(mx)^8 + 504\lambda m^4\cosh(mx)^8 - 324\lambda^2 m^2\cosh(mx)^8 + \\ 38760m^6\cosh(mx)^6 + 54\lambda^3\cosh(mx)^8 - 15876\lambda m^4\cosh(mx)^6 + \\ 2430\lambda^2 m^2\cosh(mx)^6 - 335160m^6\cosh(mx)^4 - 81\lambda^3\cosh(mx)^6 + \\ 52920\cosh(mx)^4\lambda m^4 - 2430\lambda^2 m^2\cosh(mx)^4 + 718200m^6\cosh(mx)^2 - \\ 39690\cosh(mx)^2\lambda m^4 - 430920m^6\end{array}\right)\right)}{\Gamma(1+3\alpha)\cosh(mx)^{10}},$$

$$\xi_3 = \frac{\left(1024m^5\sinh(mx)\left(\begin{array}{c}\left(m^2 - \frac{3}{4}\lambda\right)^3\cosh(mx)^8 + \left(-255m^6 + \frac{567}{4}\lambda m^4 - \frac{405}{16}\lambda^2 m^2 + \frac{81}{64}\lambda^3\right) \\ \cosh(mx)^6 + \frac{6615}{2}m^2\left(m^2 - \frac{3}{28}\lambda\right)^2\cosh(mx)^4 - 9450m^4\left(m^2 - \frac{3}{40}\lambda\right) \\ \cosh(mx)^2 + \frac{14175}{2}m^6\end{array}\right)\right)}{\Gamma(1+3\alpha)\cosh(mx)^{11}},$$

Now, using inverse FRDT, we get

$$\psi(x,t) = \sum_{k=0}^{\infty}\psi_k(x)\, t^{\alpha k},$$

$$\psi(x,t) = m\tanh(mx) + \frac{\left(14m^4 - 3\lambda m^2\right)\cosh(mx)^2 - 21m^4}{\Gamma(1+\alpha)\cosh(mx)^4}t^{\alpha} + \frac{\left(2m^3\sinh(mx)\left(\begin{array}{c}-20m^4\cosh(mx)^4 - 12\cosh(mx)^4\lambda m^2 + 9\cosh(mx)^4\lambda^2 + \\ 300\cosh(mx)^2 m^4 + 36\cosh(mx)^2\lambda m^2 - 450m^4\end{array}\right)\right)}{\Gamma(1+2\alpha)\cosh(mx)^7}t^{2\alpha}, \quad (5.3)$$

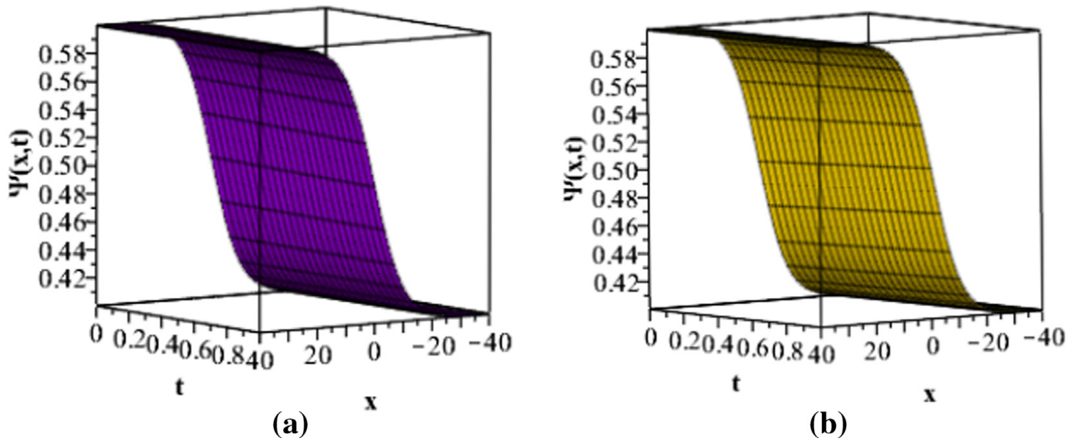

Fig. 8 Comparison plots of $\psi(x,t)$ for Case II (a) present solution (b) exact solution when $\alpha = 1, \lambda = 0.1, b_1 = 0.1$ and $m = 0.1$.

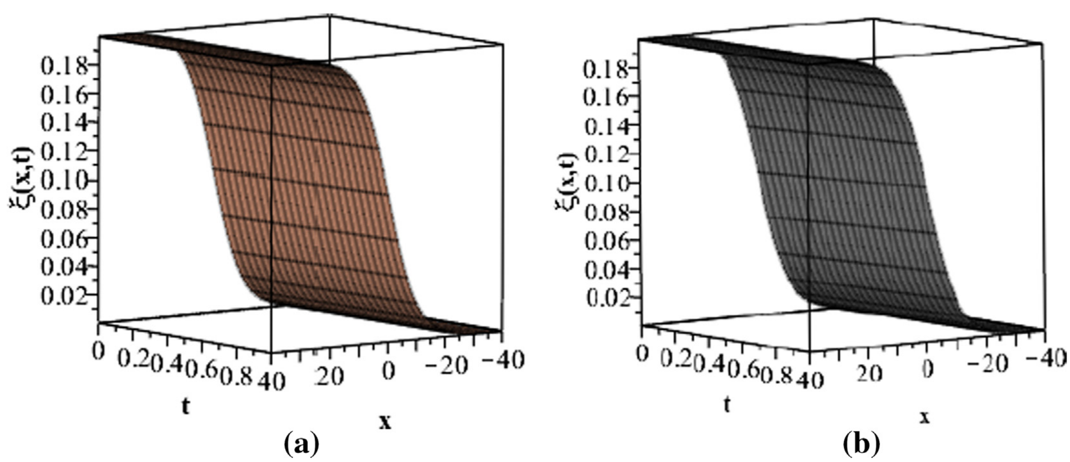

Fig. 9 Comparison plots of $\xi(x,t)$ for Case II (a) present solution (b) exact solution when $\alpha = 1, \lambda = 0.1, b_1 = 0.1$ and $m = 0.1$.

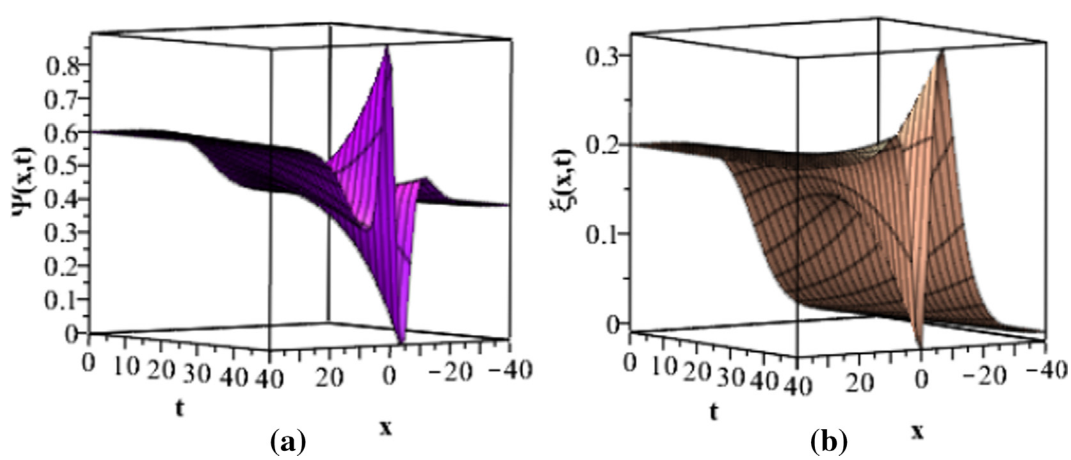

Fig. 10 Present method solution of Eqs. (1.3) and (1.5) when $\alpha = 1, \lambda = 0.1, b_1 = 0.1, m = 0.1$ and $t \in [0, 50]$.

Table 5 Comparison between the exact solution and FRDTM solution for Case II when $\alpha = 1, \lambda = b_1 = 0.1$ and $m = 0.1$.

t	x	ψ_{FRDTM}	ψ_{Exact}	ψ_{Error}	ξ_{FRDTM}	ξ_{Exact}	ξ_{Error}
0.2	0	0.509293	0.501479	0.002100	0.100639	0.101479	0.000839
	0.25	0.498759	0 0.503977	0.002113	0.103138	0.103977	0.000839
	0.5	0.501229	0.506470	0.002123	0.105633	0.106470	0.000837
	0.75	0.503697	0.508955	0.002131	0.108121	0.108955	0.000834
	1.0	0.506162	0.511429	0.002136	0.110598	0.111429	0.000831
0.4	0	0.508619	0.502959	0.004200	0.101279	0.102959	0.001679
	0.25	0.498138	0.505454	0.004225	0.103777	0.105454	0.001676
	0.5	0.500592	0.507943	0.004245	0.106270	0.107943	0.001672
	0.75	0.503047	0.510422	0.004259	0.108755	0.110422	0.001666
	1	0.508619	0.512887	0.004268	0.111229	0.112887	0.001658
0.6	0	0.505498	0.504437	0.006298	0.101919	0.104437	0.002517
	0.25	0.507943	0.506928	0.006335	0.104416	0.106928	0.002512
	0.5	0.497516	0.509412	0.006364	0.106906	0.109412	0.002505
	0.75	0.505498	0.511883	0.006384	0.109388	0.111883	0.002494
	1	0.507943	0.514340	0.006396	0.111858	0.114340	0.002482

$$\xi(x,t) = \sum_{k=0}^{\infty} \xi_k(x)\, t^{\alpha k},$$

$$\xi(x,t) = \tfrac{1}{2}(4m^2+\lambda) - 2m^2\tanh^2(mx) + \frac{16m^3\sinh(mx)\left(\left(m^2-\frac{3}{4}\lambda\right)\cosh(mx)^2-3m^2\right)}{\Gamma(1+\alpha)\cosh(mx)^5}t^{\alpha} + \frac{\left(128m^4\left(\begin{array}{c}\left(m^2-\frac{3}{4}\lambda\right)^2\cosh(mx)^6+\left(-\frac{63}{2}m^4+\frac{45}{4}\lambda m^2-\frac{27}{32}\lambda^2\right)\cosh(mx)^4+\\ \left(105m^4-\frac{45}{4}\lambda m^2\right)\cosh(mx)^2-\frac{315}{4}m^4\end{array}\right)\right)}{\Gamma(1+2\alpha)\cosh(mx)^8}t^{2\alpha}, \tag{5.4}$$

Eqs. (5.3) and (5.4) are the series solution of the Hirota-Satsuma coupled MKdV equation. It is observed that the present solutions are in good agreement with the exact results given by Fan [31] using the extended tanh-function method at a particular case ($\alpha = 1$) with few iterations that is taking $n = 4$. The values of the parameters are considered as $m = 0.1$ and $\lambda = 1.0$. Figs. 5 and 6 show the comparison plots of the present solutions with exact solutions at $\alpha = 1$. Fig. 7 illustrates the solution plots of Eqs. (1.3) and (1.4) when $0 \leqslant t \leqslant 50$. Similarly, Table 4 demonstrates the comparison results obtained by the present method with exact solutions and their absolute errors.

Case II. Using fractional reduced differential transform on both sides of Eqs. (1.3) and initial condition Eq. (1.5), Eq. (5.1) and the following recurrence relations are obtained:

$$\left.\begin{array}{l}\psi_0(x) = \frac{b_1}{2m} + m\tanh(mx),\\ \xi_0(x) = \frac{\lambda}{2}\left(1+\frac{m}{b_1}\right) + b_1\tanh(mx),\end{array}\right\} \tag{5.5}$$

Substituting Eq. (5.5) into Eq. (5.1), the following values $\psi_k(x)$ and $\xi_k(x)$ for $k = 1, 2, \ldots.$ are obtained:

$$\psi_1 = -\frac{m^2\left(-2\cosh(mx)^2m^2+3\cosh(mx)b_1\sinh(mx)+3\cosh(mx)^2\lambda+3m^2\right)}{\Gamma(1+\alpha)\cosh(mx)^4},$$

$$\xi_1 = -\frac{4b_1m\left(\left(m^2-\frac{3}{4}\lambda\right)\cosh(mx)^2m^2-\frac{3}{2}m^2\right)}{\Gamma(1+\alpha)\cosh(mx)^4},$$

$$\psi_2 = -\frac{8m^3\left(\begin{array}{c}\frac{3}{2}\cosh(mx)^5m^2b_1+\left(m^2-\frac{3}{2}\lambda\right)^2\cosh(mx)^4\sinh(mx)-\frac{45}{4}\cosh(mx)^3m^2b_1-\\ 15m^2\cosh(mx)^2\sinh(mx)\left(m^2-\frac{3}{5}\lambda\right)+\frac{45}{4}\cosh(mx)m^2b_1+\frac{45}{2}m^4\sinh(mx)\end{array}\right)}{\Gamma(1+2\alpha)\cosh(mx)^7},$$

$$\xi_2 = -\frac{32m^2b_1\sinh(mx)\left(\left(m^2-\frac{3}{4}\lambda\right)^2\cosh(mx)^4+\left(-15m^4+\frac{9}{2}\lambda m^2\right)\cosh(mx)^2+\frac{45}{2}m^4\right)}{\Gamma(1+2\alpha)\cosh(mx)^7},$$

$$\psi_3 = \frac{32m^4\left(\begin{array}{c}\left(m^2-\frac{3}{2}\lambda\right)^3\cosh(mx)^8-\frac{9}{2}b_1\sinh(mx)\cosh(mx)^7\left(m^4-\frac{3}{2}\lambda m^2+\frac{3}{4}\lambda^2\right)+\\ \cosh(mx)^6\left(-\frac{255}{2}m^6+\frac{567}{4}\lambda m^4-\frac{405}{8}\lambda^2m^2+\frac{81}{16}\lambda^3\right)+\frac{567}{2}b_1\\ \sinh(mx)\cosh(mx)^5\left(m^4-\frac{5}{14}\lambda m^2+\frac{1}{28}\lambda^2\right)+\frac{2205}{2}m^2\cosh(mx)^4\\ \left(m^2-\frac{3}{14}\lambda\right)^2-\frac{2835}{2}b_1\sinh(mx)m^2\cosh(mx)^3\left(m^2-\frac{3}{28}\lambda\right)-\\ \frac{4725}{2}m^4\cosh(mx)^2\left(m^2-\frac{3}{20}\lambda\right)+\frac{2835}{2}b_1\sinh(mx)\cosh(mx)m^4\\ +\frac{2835}{2}m^6\end{array}\right)}{\Gamma(1+3\alpha)\cosh(mx)^{10}},$$

$$\xi_3 = -\frac{256b_1m^3\left(\begin{array}{c}\left(m^2-\frac{3}{4}\lambda\right)^3\cosh(mx)^8+\cosh(mx)^6\left(\begin{array}{c}-\frac{255}{2}m^6+\frac{567}{8}\lambda m^4-\\ \frac{405}{32}\lambda^2m^2+\frac{81}{128}\lambda^3\end{array}\right)\\ +\frac{2205}{2}m^2\cosh(mx)^4\left(m^2-\frac{3}{28}\lambda\right)^2-\frac{4725}{2}m^4\cosh(mx)^2\\ \left(m^2-\frac{3}{40}\lambda\right)+\frac{2835}{2}m^6\end{array}\right)}{\Gamma(1+3\alpha)\cosh(mx)^{10}},$$

Employing inverse FRDT, we obtain

$$\psi(x,t) = \frac{b_1}{2m} + m\tanh(mx) - \frac{m^2\left(\begin{array}{c}-2\cosh(mx)^2m^2+3\cosh(mx)b_1\sinh(mx)\\ +3\cosh(mx)^2\lambda+3m^2\end{array}\right)}{\Gamma(1+\alpha)\cosh(mx)^4}t^{\alpha}$$
$$-\frac{8m^3\left(\begin{array}{c}\frac{3}{2}\cosh(mx)^5m^2b_1+\left(m^2-\frac{3}{2}\lambda\right)^2\cosh(mx)^4\sinh(mx)-\frac{45}{4}\cosh(mx)^3m^2b_1-\\ 15m^2\cosh(mx)^2\sinh(mx)\left(m^2-\frac{3}{5}\lambda\right)+\frac{45}{4}\cosh(mx)m^2b_1+\frac{45}{2}m^4\sinh(mx)\end{array}\right)}{\Gamma(1+2\alpha)\cosh(mx)^7}t^{2\alpha}, \tag{5.6}$$

$$\xi(x,t) = \frac{\lambda}{2}\left(1+\frac{m}{b_1}\right) + b_1\tanh(mx) - \frac{4b_1m\left(\left(m^2-\frac{3}{4}\lambda\right)\cosh(mx)^2m^2-\frac{3}{2}m^2\right)}{\Gamma(1+\alpha)\cosh(mx)^4}t^{\alpha}$$
$$-\frac{32m^2b_1\sinh(mx)\left(\left(m^2-\frac{3}{4}\lambda\right)^2\cosh(mx)^4+\left(-15m^4+\frac{9}{2}\lambda m^2\right)\cosh(mx)^2+\frac{45}{2}m^4\right)}{\Gamma(1+2\alpha)\cosh(mx)^7}, \tag{5.7}$$

Eqs. (5.6) and (5.7) are the series solution of the Hirota-Satsuma coupled MKdV equation with initial condition Eq. (). It is seen that the obtained solutions are the same as the exact solution resulted from Fan [31] at a particular case ($\alpha = 1$) with few iterations ($n = 4$). The values of the parameters are considered as $\lambda = 0.1, b_1 = 0.1$ and $m = 0.1$. Figs. 8 and 9 show the comparison plots of the obtained solutions with exact solutions at $\alpha = 1$. Fig. 10 shows the solution plots of Eqs. (1.3) and (1.5) when $t \in [0, 50]$. Similarly, Table 5 shows the comparison between the present solutions with exact solutions and their absolute errors.

6. Conclusion

In this article, FRDTM has been applied for obtaining the solution of time-fractional generalized Hirota–Satsuma coupled KdV and MKdV equations. It is observed that the results solved by the present method are very close to the solution of Raslan [30], Fan [31], and Ganji et al. [32] in particular cases at $\alpha = 1$. Moreover, FRDTM does not require any linearization and perturbation, which helps us to overcome the difficulties of round-off errors, high computer memory, and times. The main benefit of this method is that it requires lesser computation as compared to other perturbation methods. Obtained results demonstrate that FRDTM is a powerful and convenient technique for solving fractional PDEs.

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Declaration of Competing Interest

All authors state that they have no conflict of interest.

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