



# Specific wave structures of a fifth-order nonlinear water wave equation



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## ABSTRACT

Investigated in the present paper is a fifth-order nonlinear evolution (FONLE) equation, known as a nonlinear water wave (NLWW) equation, with applications in the applied sciences. More precisely, a traveling wave hypothesis is firstly applied that reduces the FONLE equation to a 1D domain. The Kudryashov methods (KMs) are then adopted as leading techniques to construct specific wave structures of the governing model which are classified as *W*-shaped and other solitons. In the end, the effect of changing the coefficients of nonlinear terms on the dynamical features of *W*-shaped and other solitons is investigated in detail for diverse groups of the involved parameters.

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## 1. Introduction

In recent decades, a lot of interest has been drawn in searching for solitons of NLE equations, since many phenomena from science to engineering are described by such a class of equations. Nonlinear water wave equations are a widespread class of NLE equations that have achieved great attention in the last few decades. Many scholars have considered the NLWW equations in their papers and have derived their solitons. For example, Ali and Seadawy [1] obtained solitons of a shallow WW system using the simple equation method. Helal et al. [2] employed a direct method to find solitons of the modified Degasperis–Procesi equation. Shen and Tian [3] acquired solitons of a generalized NLWW equation using the Hirota method. Tian [4] applied the Lie symmetry method to seek solitons of a generalized Boussinesq equation. For more investigations regarding the NLE equations and their solitons, see [5–25].

The main concern of the present research is to consider the following NLWW equation [26–30]

$$u_t(x, t) = -u_x(x, t) - c_1 u(x, t) u_x(x, t) - c_2 u_{xxx}(x, t) - c_3 u_x(x, t) u_{xx}(x, t) - c_4 u(x, t) u_{xxx}(x, t) - c_5 u_{xxxxx}(x, t), \quad (1)$$

and explore the dynamics of its solitons through applying the Kudryashov methods. It is noteworthy that the governing model (1) was established by Olver [26] as an evolutionary form of the classical Boussinesq model. The authors of [27] derived solitons of Eq. (1) using a straightforward method. Solitons of Eq. (1) were obtained in [28] through the modified tanh method. In [29], the authors tried to find the exact solutions of Eq. (1) using the invariant subspace method. Very newly, Hosseini et al. [30] found valid approximations of Eq. (1) involving the Caputo time-fractional derivative using the homotopy analysis transform method.

Kudryashov methods [31–45] are known as effective techniques in retrieving solitons of NLE equations. Both methods are able to handle NLE equations of high-order easily and the resulting systems in these methods are not very huge. For the information of enthusiasts, the efficiency of Kudryashov methods has been demonstrated many times in research papers. In this respect, solitons of a nonlinear Schrödinger (NLS) equation involving the cubic and quartic terms were acquired in [35] using the KM I. The authors in [36] gained solitons of the Zoomeron equation through employing the KM I. The KM II was applied in [44] to obtain soli-

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tons of a NLS equation of high-order. Solitons of the Heisenberg equation were achieved in [45] using the KM II.

Other sections of this paper are organized as follows: In Section 2, Kudryashov methods I and II are described comprehensively. In Section 3, a traveling wave hypothesis is applied that reduces the FONLE equation to a 1D domain. In Section 4, Kudryashov methods I and II are adopted to retrieve solitons of the FONLE equation that are classified as *W*-shaped and other solitons. Moreover, Section 4 represents the dynamical features of *W*-shaped and other solitons for diverse groups of the involved parameters. The study ends by presenting the outcomes.

## 2. Kudryashov methods

In the present section, the authors address the ideas of Kudryashov methods I and II. The KM I introduces the following series

$$U(\epsilon) = a_0 + a_1K(\epsilon) + a_2K^2(\epsilon) + \dots + a_NK^N(\epsilon), \quad a_N \neq 0, \quad (2)$$

which is recruited as the solution of

$$O(U(\epsilon), U'(\epsilon), U''(\epsilon), \dots) = 0. \quad (3)$$

In (2),  $a_i, i = 0, 1, \dots, N$  are unknowns,  $N$  is acquired by the balance approach, and  $K(\epsilon)$  is

$$K(\epsilon) = \frac{1}{1 + d\alpha\epsilon},$$

which satisfies

$$K'(\epsilon) = K(\epsilon)(K(\epsilon) - 1)\ln(a).$$

Owing to Eqs. (2) and (3), a nonlinear algebraic system is derived which its solution results in solitons of Eq. (3).

The KM II uses a series similar to the KM I while it applies

$$K(\epsilon) = \frac{1}{(A - B) \sinh(\epsilon) + (A + B) \cosh(\epsilon)},$$

as the solution of the equation

$$(K'(\epsilon))^2 = K^2(\epsilon)(1 - 4ABK^2(\epsilon)).$$

A process similar to what was done in the KM I yields solitons of Eq. (3).

## 3. Reduction of the FONLE equation

To reduce the FONLE equation to a 1D domain, a traveling wave hypothesis is applied as follows

$$u(x, t) = U(x - wt),$$

such that its substitution into Eq. (1) results in

$$(1 - w) \frac{dU(\epsilon)}{d\epsilon} + c_1U(\epsilon) \frac{dU(\epsilon)}{d\epsilon} + c_2 \frac{d^3U(\epsilon)}{d\epsilon^3} + c_3 \frac{dU(\epsilon)}{d\epsilon} \frac{d^2U(\epsilon)}{d\epsilon^2} + c_4U(\epsilon) \frac{d^3U(\epsilon)}{d\epsilon^3} + c_5 \frac{d^5U(\epsilon)}{d\epsilon^5} = 0, \quad (4)$$

where  $w$  is the speed of the soliton. Integrating Eq. (4) once w.r.t.  $\epsilon$  gives

$$\frac{1}{2}(c_3 - c_4) \left( \frac{dU(\epsilon)}{d\epsilon} \right)^2 + c_4U(\epsilon) \frac{d^2U(\epsilon)}{d\epsilon^2} + c_2 \frac{d^2U(\epsilon)}{d\epsilon^2} + \frac{1}{2}c_1U^2(\epsilon) + (1 - w)U(\epsilon) + c_5 \frac{d^4U(\epsilon)}{d\epsilon^4} = 0, \quad (5)$$

where the integration constant is zero. From  $U^{(5)}(\epsilon)$  and  $U'(\epsilon)U''(\epsilon)$  in Eq. (4), the balance number is found as

$$N + 5 = 2N + 3 \Rightarrow N = 2.$$

Furthermore, by considering  $U^{(4)}(\epsilon)$  and  $U(\epsilon)U''(\epsilon)$  in Eq. (5), the balance number is obtained as

$$N + 4 = 2N + 2 \Rightarrow N = 2.$$

## 4. The FONLE equation and its solitons

In the current section, Kudryashov methods I and II are adopted to retrieve solitons of the FONLE equation that are classified as *W*-shaped and other solitons. Moreover, the dynamical features of *W*-shaped and other solitons for diverse groups of the involved parameters are represented herein.

### 4.1. KM I and its application

To retrieve solitons of the FONLE equation using the KM I, the following series

$$U(\epsilon) = a_0 + a_1K(\epsilon) + a_2K^2(\epsilon), \quad a_2 \neq 0,$$

is inserted into Eq. (4). After some operations, a nonlinear algebraic system is derived as

$$\begin{aligned} & -(\ln(a))^4 a_1 c_5 - (\ln(a))^2 a_0 a_1 c_4 - (\ln(a))^2 a_1 c_2 - a_0 a_1 c_1 + w a_1 - a_1 = 0, \\ & 31(\ln(a))^4 a_1 c_5 - 32(\ln(a))^4 a_2 c_5 + 7(\ln(a))^2 a_0 a_1 c_4 \\ & - 8(\ln(a))^2 a_0 a_2 c_4 - (\ln(a))^2 a_1^2 c_3 - (\ln(a))^2 a_1^2 c_4 + 7(\ln(a))^2 a_1 c_2 \\ & - 8(\ln(a))^2 a_2 c_2 + a_0 a_1 c_1 - 2a_0 a_2 c_1 - a_1^2 c_1 - w a_1 + 2w a_2 \\ & + a_1 - 2a_2 = 0, \\ & -180(\ln(a))^4 a_1 c_5 + 422(\ln(a))^4 a_2 c_5 - 12(\ln(a))^2 a_0 a_1 c_4 \\ & + 38(\ln(a))^2 a_0 a_2 c_4 + 4(\ln(a))^2 a_1^2 c_3 + 7(\ln(a))^2 a_1^2 c_4 \\ & - 6(\ln(a))^2 a_1 a_2 c_3 - 9(\ln(a))^2 a_1 a_2 c_4 - 12(\ln(a))^2 a_1 c_2 \\ & + 38(\ln(a))^2 a_2 c_2 + 2a_0 a_2 c_1 + a_1^2 c_1 - 3a_1 a_2 c_1 - 2w a_2 + 2a_2 = 0, \\ & 390(\ln(a))^4 a_1 c_5 - 1710(\ln(a))^4 a_2 c_5 + 6(\ln(a))^2 a_0 a_1 c_4 \\ & - 54(\ln(a))^2 a_0 a_2 c_4 - 5(\ln(a))^2 a_1^2 c_3 - 12(\ln(a))^2 a_1^2 c_4 \\ & + 22(\ln(a))^2 a_1 a_2 c_3 + 45(\ln(a))^2 a_1 a_2 c_4 - 8(\ln(a))^2 a_2^2 c_3 \\ & - 8(\ln(a))^2 a_2^2 c_4 + 6(\ln(a))^2 a_1 c_2 - 54(\ln(a))^2 a_2 c_2 + 3a_1 a_2 c_1 - 2a_2^2 c_1 = 0, \\ & -360(\ln(a))^4 a_1 c_5 + 3000(\ln(a))^4 a_2 c_5 + 24(\ln(a))^2 a_0 a_2 c_4 + 2(\ln(a))^2 a_1^2 c_3 \\ & + 6(\ln(a))^2 a_1^2 c_4 - 26(\ln(a))^2 a_1 a_2 c_3 - 66(\ln(a))^2 a_1 a_2 c_4 + 28(\ln(a))^2 a_2^2 c_3 \\ & + 38(\ln(a))^2 a_2^2 c_4 + 24(\ln(a))^2 a_2 c_2 + 2a_2^2 c_1 = 0, \\ & 120(\ln(a))^4 a_1 c_5 - 2400(\ln(a))^4 a_2 c_5 + 10(\ln(a))^2 a_1 a_2 c_3 + 30(\ln(a))^2 a_1 a_2 c_4 \\ & - 32(\ln(a))^2 a_2^2 c_3 - 54(\ln(a))^2 a_2^2 c_4 = 0, \\ & 720(\ln(a))^4 a_2 c_5 + 12(\ln(a))^2 a_2^2 c_3 + 24(\ln(a))^2 a_2^2 c_4 = 0. \end{aligned}$$

Through employing the Maple package, we derive

$$a_0 = -\frac{5(\ln(a))^2 c_5 c_4 - 5c_5 c_1 + c_2 c_3 + 2c_2 c_4}{c_4(c_3 + 2c_4)},$$

$$a_1 = \frac{60(\ln(a))^2 c_5}{c_3 + 2c_4},$$

$$a_2 = -\frac{60(\ln(a))^2 c_5}{c_3 + 2c_4},$$

$$w = \frac{(\ln(a))^4 c_5 c_4 c_3 - 3(\ln(a))^4 c_5 c_4^2 + 5c_1^2 c_5 - c_1 c_2 c_3 - 2c_1 c_2 c_4 + c_3 c_4 + 2c_4^2}{c_4(c_3 + 2c_4)}.$$

Therefore, the following soliton to the FONLE equation can be gained

$$u(x, t) = -\frac{5(\ln(a))^2 c_5 c_4 - 5c_5 c_1 + c_2 c_3 + 2c_2 c_4}{c_4(c_3 + 2c_4)}$$

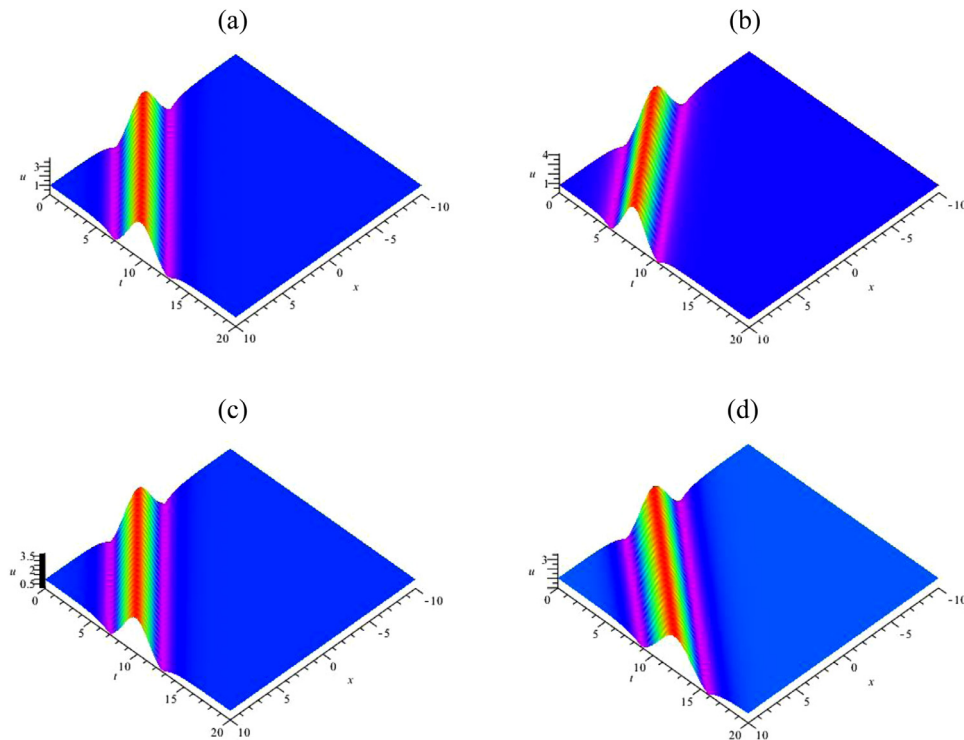


Fig. 1. (a) *W*-shaped soliton  $u(x, t)$  for group 1; (b) *W*-shaped soliton  $u(x, t)$  for group 2; (c) *W*-shaped soliton  $u(x, t)$  for group 3; (d) *W*-shaped soliton  $u(x, t)$  for group 4.

$$+ \frac{60(\ln(a))^2 c_5}{c_3 + 2c_4} \frac{1}{1 + da^{x-wt}} - \frac{60(\ln(a))^2 c_5}{c_3 + 2c_4} \left( \frac{1}{1 + da^{x-wt}} \right)^2,$$

where the soliton speed is

$$w = \frac{(\ln(a))^4 c_5 c_4 c_3 - 3(\ln(a))^4 c_5 c_4^2 + 5c_1^2 c_5 - c_1 c_2 c_3 - 2c_1 c_2 c_4 + c_3 c_4 + 2c_4^2}{c_4 (c_3 + 2c_4)}.$$

To show the dynamical features of the above soliton, we select diverse groups of the involved parameters as follows

- Group 1:  $\{c_1 = 1, c_2 = 1, c_3 = 1, c_4 = 1, c_5 = 1, a = 2.7, d = 1\}$ ,
- Group 2:  $\{c_1 = 1.1, c_2 = 1, c_3 = 1, c_4 = 1, c_5 = 1, a = 2.7, d = 1\}$ ,
- Group 3:  $\{c_1 = 1, c_2 = 1, c_3 = 1.1, c_4 = 1, c_5 = 1, a = 2.7, d = 1\}$ ,
- Group 4:  $\{c_1 = 1, c_2 = 1, c_3 = 1, c_4 = 1.1, c_5 = 1, a = 2.7, d = 1\}$ .

In Fig. 1, the authors want to analyze the effect of changing the coefficients of nonlinear terms on the dynamics of the *W*-shaped soliton.

It is notable that when the values of  $c_1$  and  $c_3$  increase from 1 to 1.1, the amplitude of the *W*-shaped soliton increases. Additionally, by changing  $c_4$  from 1 to 1.1, the amplitude of the *W*-shaped soliton decreases.

#### 4.2. KM II and its application

Now, the authors are interested in retrieving other solitons of the FONLE equation using the KM II. To this end, the following series

$$U(\epsilon) = a_0 + a_1 K(\epsilon) + a_2 K^2(\epsilon), \quad a_2 \neq 0,$$

is substituted into Eq. (5). After some operations, a nonlinear algebraic system is obtained as

$$1920A^2 B^2 a_2 c_5 - 8ABa_2^2 c_3 - 16ABa_2^2 c_4 = 0,$$

$$384A^2 B^2 a_1 c_5 - 8ABa_1 a_2 c_3 - 24ABa_1 a_2 c_4 = 0,$$

$$\frac{1}{2} a_2^2 c_1 + 2a_2^2 c_4 + 2a_2^2 c_3 - 24ABa_0 a_2 c_4 - 2ABa_1^2 c_3$$

$$- 6ABa_1^2 c_4 - 24ABa_2 c_2 - 480ABa_2 c_5 = 0,$$

$$- 8ABa_0 a_1 c_4 - 8ABa_1 c_2 - 80ABa_1 c_5 + a_1 a_2 c_1 + 2a_1 a_2 c_3 + 3a_1 a_2 c_4 = 0,$$

$$16a_2 c_5 + \frac{1}{2} a_1^2 c_3 - wa_2 + \frac{1}{2} a_1^2 c_4 + \frac{1}{2} a_1^2 c_1 + 4a_2 c_2 + 4a_0 a_2 c_4 + a_0 a_2 c_1 + a_2 = 0,$$

$$a_0 a_1 c_1 + a_0 a_1 c_4 - wa_1 + a_1 c_2 + a_1 c_5 + a_1 = 0,$$

$$a_0 + \frac{1}{2} a_0^2 c_1 - wa_0 = 0.$$

Through applying the Maple package, we acquire Case 1:

$$a_0 = - \frac{8(4c_3 + 5c_1 - 12c_4)c_5}{c_1(c_3 + 2c_4)},$$

$$a_1 = 0,$$

$$a_2 = \frac{240ABC_5}{c_3 + 2c_4},$$

$$w = - \frac{20c_1 c_5 + 16c_3 c_5 - 48c_4 c_5 - c_3 - 2c_4}{c_3 + 2c_4},$$

$$c_2 = \frac{c_5(5c_1^2 + 20c_1 c_4 + 32c_3 c_4 - 96c_4^2)}{c_1(c_3 + 2c_4)}.$$

Therefore, the following soliton to the FONLE equation can be acquired

$$u_1(x, t) = - \frac{8(4c_3 + 5c_1 - 12c_4)c_5}{c_1(c_3 + 2c_4)}$$

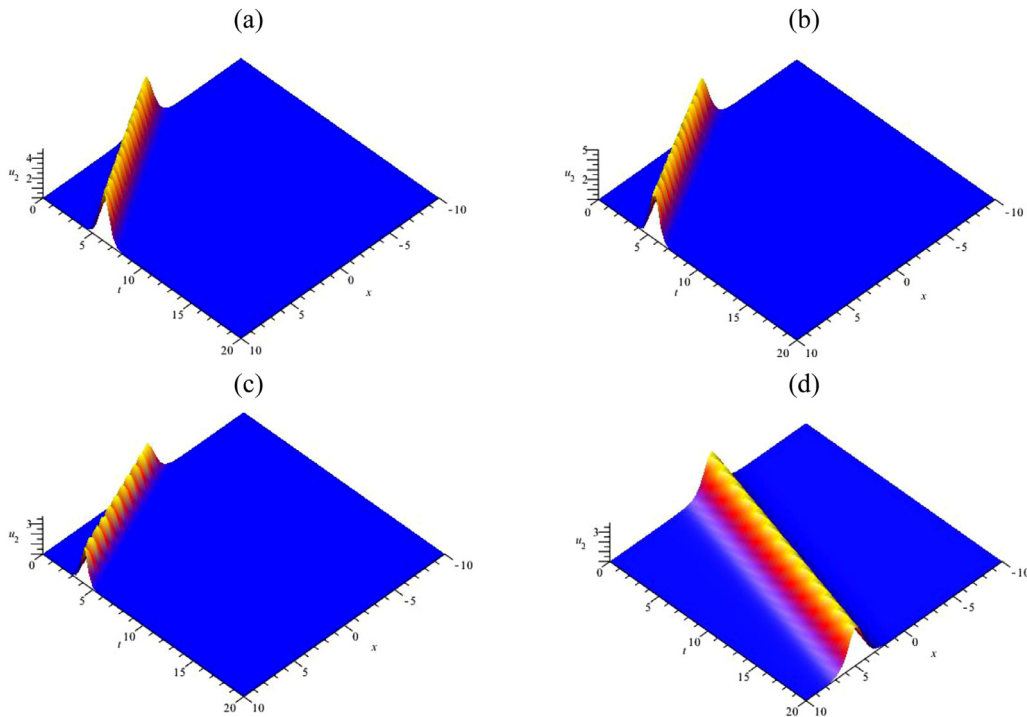


Fig. 2. (a) Bright soliton  $u_2(x, t)$  for group 1; (b) Bright soliton  $u_2(x, t)$  for group 2; (c) Bright soliton  $u_2(x, t)$  for group 3; (d) Bright soliton  $u_2(x, t)$  for group 4.

$$+ \frac{240ABC_5}{c_3 + 2c_4} \left( \frac{1}{(A - B) \sinh(x - wt) + (A + B) \cosh(x - wt)} \right)^2,$$

where

$$w = -\frac{20c_1c_5 + 16c_3c_5 - 48c_4c_5 - c_3 - 2c_4}{c_3 + 2c_4},$$

$$c_2 = \frac{c_5(5c_1^2 + 20c_1c_4 + 32c_3c_4 - 96c_4^2)}{c_1(c_3 + 2c_4)}.$$

Case 2:

$$a_0 = 0,$$

$$a_1 = 0,$$

$$a_2 = \frac{240ABC_5}{c_3 + 2c_4},$$

$$w = \frac{20c_1c_5 + 16c_3c_5 - 48c_4c_5 + c_3 + 2c_4}{c_3 + 2c_4},$$

$$c_2 = \frac{5c_5(c_1 - 4c_4)}{c_3 + 2c_4}.$$

Consequently, the following soliton to the FONLE equation can be derived

$$u_2(x, t) = \frac{240ABC_5}{c_3 + 2c_4} \left( \frac{1}{(A - B) \sinh(x - wt) + (A + B) \cosh(x - wt)} \right)^2,$$

where

$$w = \frac{20c_1c_5 + 16c_3c_5 - 48c_4c_5 + c_3 + 2c_4}{c_3 + 2c_4}, \quad c_2 = \frac{5c_5(c_1 - 4c_4)}{c_3 + 2c_4}.$$

To demonstrate the dynamical features of the second soliton, diverse groups of the involved parameters are considered as

Group 1:  $\{c_1 = 2, c_3 = 1, c_4 = 1, c_5 = 0.25, A = 2, B = 1\}$ ,

Group 2:  $\{c_1 = 2.1, c_3 = 1, c_4 = 1, c_5 = 0.25, A = 2, B = 1\}$ ,

Group 3:  $\{c_1 = 2, c_3 = 2, c_4 = 1, c_5 = 0.25, A = 2, B = 1\}$ ,

Group 4:  $\{c_1 = 2, c_3 = 1, c_4 = 1.4, c_5 = 0.25, A = 2, B = 1\}$ .

In Fig. 2, the authors try to examine the effect of changing the coefficients of nonlinear terms on the dynamics of the bright soliton.

It is remarkable that when the values of  $c_3$  and  $c_4$  increase from 1 to 2 and from 1 to 1.4 respectively, the amplitude of the bright soliton decreases. Moreover, by changing  $c_1$  from 2 to 2.1, the amplitude of the bright soliton increases.

**Remark** The authors believe that their results are new and have been achieved for the first time.

### 5. Conclusion

The current paper explored a fifth-order nonlinear evolution equation in the applied sciences which is referred to as a nonlinear water wave equation. In this regard, the authors first adopted a traveling wave hypothesis to arrive at the reduced form of the FONLE equation in a 1D domain. The Kudryashov methods were then adopted as robust techniques to establish W-shaped and other solitons of the governing model. In the end, the effect of changing the coefficients of nonlinear terms on the dynamical features of W-shaped and bright solitons was investigated in detail for diverse groups of the involved parameters. Based on the authors' knowledge of previous studies, the outcomes of the present paper are new and have been achieved for the first time.

### Declaration of Competing Interest

No conflict of interest exists in the submission of this manuscript, and the manuscript is approved by all authors for publication.

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