Results in Physics 19 (2020) 103480

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Study of the dynamical nonlinear modified Korteweg–de Vries equation arising in plasma physics and its analytical wave solutions

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ARTICLE INFO

Keywords:
Auxiliary equation mapping method
2-D modified Korteweg–de Vries (mK–dV) equation
Ion-acoustic waves
Solitary type waves
Physical representation

ABSTRACT

In this article we have discussed the analytical analysis of two dimensional modified Korteweg–de Vries (mK–dV) equation arising in plasma physics that governs the ion-acoustic solitary waves for their asymptotic behavior because of the trapping of electrons using auxiliary equation mapping method. By using this technique we have obtained some quite general and new variety of exact traveling wave solutions which are collecting some kind of semi half bright, bright, dark, semi half dark, doubly periodic, combined, periodic, half hark and half bright via three parametric values which is the primary key point of difference of our technique. These results are highly applicable to develop new theories of quantum mechanics, biomedical problems, soliton dynamics, plasma physics, nuclear physics, optical physics, fluid dynamics, electromagnetism, industrial studies, mathematical physics, biomedical problems, and in many other natural and physical sciences. For detailed physical dynamical representation of our results we have shown them with graphs in different dimensions via Mathematica 10.4 to get more understanding of different new dynamical shapes of solutions.

1. Introduction and problem formulation

In 1895 Korteweg and de Vries introduced a well known and classical Kdv model during the study of shallow water waves Kdv is a well famous governing model derived for the propagation of shallow water waves [1]. Although many researchers have presented modified and generalized Kdv equations with potential applications in different branches of applied physics and natural sciences [2–13]. After the invention of classical Kdv model Kdv theory has many applications for example it has an important role to study the problems of blood appearing in compressible fluids in fluid mechanics [6], to study the properties of electron plasma present in a cylindrical plasma, to examine the properties of oceanic water waves [2], to study bubble liquid type mixture for the detail investigation of pressure waves, to examine chemical compounds for investigation of mass transports problems [4], use to study dusty plasma [5], to read different properties of nonlinear solitary structures, for the study of surface gravity waves, to describe electrical transmission lies and in the study of many other nonlinear phenomena, its quite hard to list its all those applications over here. Other than above mentioned applications, Kdv theory has a significant role in quantum plasma for the investigation of electrostatic waves via setting different parameters appearing in plasma is one of the excited and fascinated field of research both for mathematicians and physicians from many years. To examine different properties of electrostatic waves is particularly important due to their potential applications in the development of new theories of chemical physics, space environments, plasma physics, fluid dynamics, astrophysics, optical physics, nuclear physics, geophysics, dusty plasma, fluid mechanics and different other fields of applied physics [13–19]. In recent years, to study electrostatic waves specially to discuss different properties of solitary waves is the field of soliton dynamics has played a significant role for many researchers and have received a considerable attention of them. Hence the present research work is a motivation to extract exact solutions of a generalized model of Kdv equation called as modified Korteweg–de Vries (mk–dv) equation introduced by Schamel in 1973 first time during the study of ion-acoustic nonlinear solitary waves because of the trapping of electrons [13,14]. He investigated the dependence of asymptotic behaviors of ion-acoustic waves of small with finite amplitude on the number of resonant electrons, in the result of which

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https://doi.org/10.1016/j.rinp.2020.103480
Received 26 July 2020; Received in revised form 21 September 2020; Accepted 3 October 2020
Available online 17 October 2020

he presented after some derivation a Kdv type equation with a strong nonlinearity given as in dimension less form.

$$q_t + (1 + bq^{\frac{1}{2}})q_x + \frac{1}{2}q_{xxx} = 0. \tag{1}$$

Above "q(x,t)" is a real valued wave function that governs the ion-acoustic solitary waves for their asymptotic behavior because of the trapping of electrons and here "b" as any arbitrary constant. He presented a brief theoretical comparison of those waves which are satisfying new Kdv type equation with those which are satisfying simple classical type Kdv equation given as

$$q_t + qq_x + \sigma^{-2}q_{xxx} = 0. \tag{2}$$

In above equation "q(x,t)" is the representation of 1-dimensional weakly dispersive waves governing the time asymptotic behavior of small with finite amplitude dispersive waves with σ as an arbitrary constant. He said that a wave equation with satisfying equation type (1) is same in qualitative manner as that of Kdv type equation satisfying (2). According to his detail studies a great number of solutions which are of stationary type arise on a very short period of time which propagate with high speeds and taking small widths. In 1991 after that Nejoh [15,16] and other two researchers Das and Sen in (1994) [17] have introduced a unidirectional nonlinear Kdv type wave equation governing the non-isothermal plasma with cold ions and electrons is presented in dimensionless form in the following,

$$q_t + F(q)q_x + q_{xxx} = 0. \tag{3}$$

In plasma physics above equation is used to define different properties of solitary type waves which leads us towards bursting or spiky solitary waves appearing in plasma physics. Later they developed a more generalized Kdv type 2-dimensional model named as Kadomtsev–Petviashvili (KP) equation, that governs the impact of nonlinear acoustic waves due to the trapping of electrons appearing in plasma and finally they proved the existence of some different solitary type waves in relation to the space plasma and laboratory. In a short summary, many researchers have given us the proof of the existence of bursting solitary waves on the basis of their observations which they have done using different plasma parameters, all the observations are actually related to those observations which are encountered in interplanetary space plasma.

Hence this is the best time to study the extraction of exact solutions of such models because their analytical study have own unique importance in literature to understand a complex dynamical process in a more efficient manner. Another important point of exact solutions is quite important to note that they are useful in the study of comparison of accuracy of numerical solution, also to check the stability analysis of them, in the development of new wide range of different mathematical softwares. In this work we have studied the analytical analysis of modified Kdv equation using our introduced method named as auxiliary equation mapping method. In the result of which we have obtained a variety of new more general families of exact solution in a more compact form [18–35].

Here section first presenting a brief introduction of ($mK{-}dV$) model. And section second is presenting the implementation of our method for algorithm and details of this method see Ref. [18] to extract solitons of the above model to check computational reliability and efficiency of the method. While section three is presenting the physical description of our results with respect to graphs in different dimensions. At the last the discussion of our solutions is given. And section four explaining us some of important concluding remarks.

2. Modified generalized Korteweg–de Vries ($mK--dV$)I equation with mixed nonlinearity

Here we will apply our method see Ref. [18] to extract the variety of solitons of a well known governing model modified generalized Korteweg–de Vries($mK - dV$) equation [13,14]. The ($mK - dV$) model in dimensionless form is given by

$$q_t + (1 + bq^{\frac{1}{2}})q_x + \frac{1}{2}q_{xxx} = 0. \tag{4}$$

where $q(x,t)$ is a real valued wave function that governs ion-acoustic solitary waves while b as any arbitrary constant. While x and t is the representation of associated partial derivatives. Eq. (4) is transformed into $NLODE$ using the below linear wave transformation:

$$q(x,t) = Q(\xi) \qquad \xi = (x - \upsilon t). \tag{5}$$

Here υ is the representation of wave frequency, using this linear wave transformation equation (4) is transformed into the following $NLODE$

$$-\upsilon Q' + (Q + bQ^{\frac{1}{2}})Q' + \frac{1}{2}U''' = 0. \tag{6}$$

By integrating equation (6) and setting constant of integration equal to zero then we obtain

$$-\upsilon Q + \frac{1}{2}Q^2 + b\frac{2}{3}Q^{\frac{3}{2}} + \frac{1}{2}Q'' = 0. \tag{7}$$

Further by putting $Q^{\frac{1}{2}} = W$, we have the following ODE

$$-6\upsilon W^2 + 3W^4 + 4bW^3 + 6((W')^2 + WW'') = 0. \tag{8}$$

Next with the help of "homogeneous principal" taking a quite balance between the highest order partial derivative WW'' and highest order nonlinearity term W^4 we will get $n = 1$, now next using auxiliary equation mapping method the equation 8 is supposed to consider the general form of solution as mentioned below in the form of a series:

$$W(\xi) = \sum_{j=0}^{n} a_j F^j(\xi) + \sum_{j=-1}^{-n} b_{-j} F^j(\xi) + \sum_{j=2}^{n} c_j F^{j-2}(\xi)F'(\xi) + \sum_{j=-1}^{-n} d_{-j} F^j(\xi)F'(\xi). \tag{9}$$

where the a_j, b_j, c_j, d_j are constants which will be calculated later, in above equation $F(\xi)$ satisfy the below mentioned auxiliary ordinary differential equation with its associated partial derivatives:

$$F'^2 = (\frac{dF}{d\xi})^2 = pF^2(\xi) + qF^3(\xi) + rF^4(\xi); \tag{10}$$

$$F''(\xi) = pF(\xi) + \frac{3}{2}qF^2(\xi) + 2rF^3(\xi); \tag{11}$$

$$F'''(\xi) = (p + 3qF(\xi) + 6rF^2(\xi))F'(\xi). \tag{12}$$

For $n = 1$, the generalized solution of Eq. (8) takes the following form:

$$W(\xi) = a_0 + a_1 F(\xi) + \frac{b_1}{F(\xi)} + d_1 \frac{F'(\xi)}{F(\xi)}, \tag{13}$$

By putting Eq. (13) with the help of (10) into Eq. (8), with collecting all those coefficients of same powers of $F'^k(\xi)F^j(\xi)$ ($\varepsilon k = 0, 1\varepsilon.\ \ \varepsilon j = 0, 1, 2, 3, 4, 5, 6, \dots \dots n\varepsilon$) and setting them equal to zero yields a system of algebraic equations, via any suitable mathematical tool for example Matlab, Mathematica 10.4, or Maple, different new and more general families of exact solutions associated with different values of constants a_0, a_1, b_1, d_1, and frequency are obtained, by putting them into Eq. (13) we will obtain the variety of new solitons of (4) which are listed below.

Family 1:

$$a_0 = \frac{-4b}{5},\ a_1 = \frac{15q}{4b},\ b_1 = d_1 = 0,\ p = \frac{-8b^2}{75},\ r = \frac{-75q^2}{32b^2},\ \upsilon = \frac{-16b^2}{75}. \tag{14}$$

Next by substituting all values mentioned above into Eq. (13) with the help of Eq. (10), we will obtain in this family the following solutions

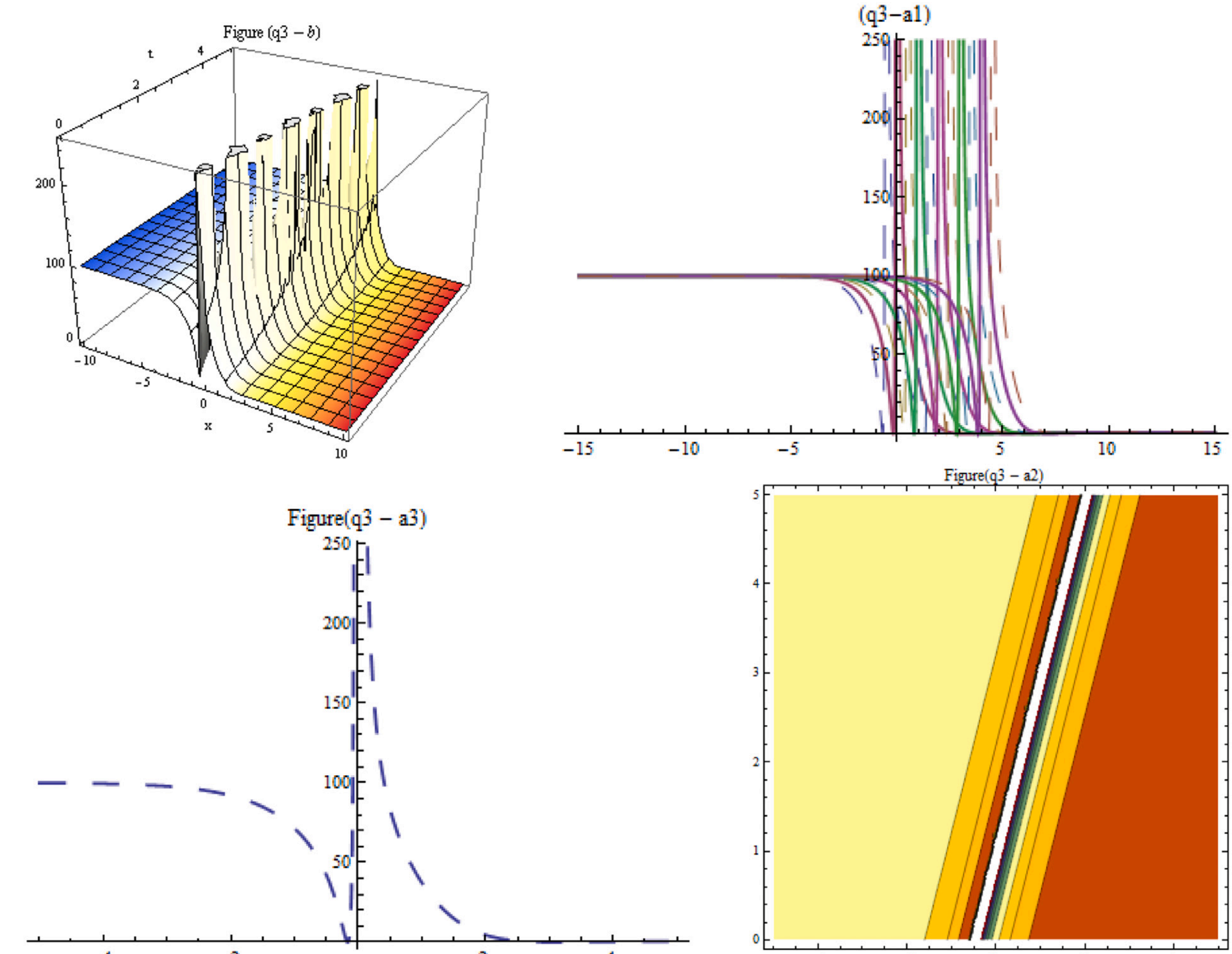

Fig. 1. The physical plotting of (17) is given in different dimensions. Here (q3-b) shows 3-dimensional plotting of (17) as bright soliton with parametric values $\xi_0 = 0.7$, $\rho = 1.3$, $p = 0.8$, $q = 1.7$, $b = .5$, $r = 0.5$, $s = 0.8$ in intervals (0,5), (−10,10). And (q3-a1) is the 2-dimensional plotting of (17) with same parametric values as used above with in intervals (−15,15), (0,10). While (q3-a2) is the contour plotting of (17) with the same parametric values as mentioned above with in intervals (−5,5), (0,5). And (q3-a3) shows 1-dimensional plotting of (17) bright soliton using interval (−5,5) of (17) with parametric values $\xi_0 = 0.9$, $\rho = 1.3$, $p = 0.8$, $q = 1.8$, $b = .5$, $r = 0.5$, $s = 0.9$.

using our method, so the solutions in this family for Eq. (4) are obtained as below.

$$q_1(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{4b}{5} - \frac{15p(1 + s\ tanh[\frac{\sqrt{p}}{2}(x-\upsilon t)+\xi_0])}{4b}\right)^2, \quad p > 0. \tag{15}$$

$$q_2(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{4b}{5} + \frac{15q}{8b}\sqrt{\frac{p}{r}}\left(\coprod\right)\right)^2, \qquad p > 0. \tag{16}$$

$$q_3(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{4b}{5} - \frac{15p\left(1 + \frac{s\ (\rho\sqrt{1+\sigma^2} + cosh\sqrt{p}(x-\upsilon t))}{\sigma + sinh\sqrt{p}(x-\upsilon t)}\right)}{4b}\right)^2, \quad p > 0. \tag{17}$$

While above $\coprod = \left(1 + \frac{s\ sinh\sqrt{p}(x-\upsilon t)}{\rho + cosh\sqrt{p}(x-\upsilon t)}\right)$.

Family 2:

$$a_0 = 0,\ a_1 = -\frac{15q}{4b},\quad b_1 = d_1 = 0,\ p = \frac{\upsilon}{2},\ r = \frac{-75q^2}{32b^2}. \tag{18}$$

Next by substituting all values mentioned above into Eq. (13) with the help of Eq. (10), we will obtain in this family the following solutions using our method, so the solutions in this family for Eq. (4) are obtained as below.

$$q_4(x,t) = Q(\xi) = (W(\xi))^2 = \left(\frac{15p(1 + s\ tanh[\frac{\sqrt{p}}{2}(x-\upsilon t)+\xi_0])}{4b}\right)^2, \quad p > 0. \tag{19}$$

$$q_5(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{15q}{8b}\sqrt{\frac{p}{r}}\left(\coprod\right)\right)^2, \qquad p > 0. \tag{20}$$

$$q_6(x,t) = Q(\xi) = (W(\xi))^2 = \left(\frac{15p\left(1 + \frac{s\ (\rho\sqrt{1+\sigma^2} + cosh\sqrt{p}(x-\upsilon t))}{\sigma + sinh\sqrt{p}(x-\upsilon t)}\right)}{4b}\right)^2, \quad p > 0. \tag{21}$$

While above $\coprod = \left(1 + \frac{s\ sinh\sqrt{p}(x-\upsilon t)}{\rho + cosh\sqrt{p}(x-\upsilon t)}\right)$.

Family 3:

$$a_0 = 0,\ a_1 = \pm i\sqrt{6r},\ b_1 = d_1 = 0,\ q = \pm\ ib\frac{4}{5}\sqrt{\frac{2}{3}}\sqrt{r},\ \upsilon = 2p. \tag{22}$$

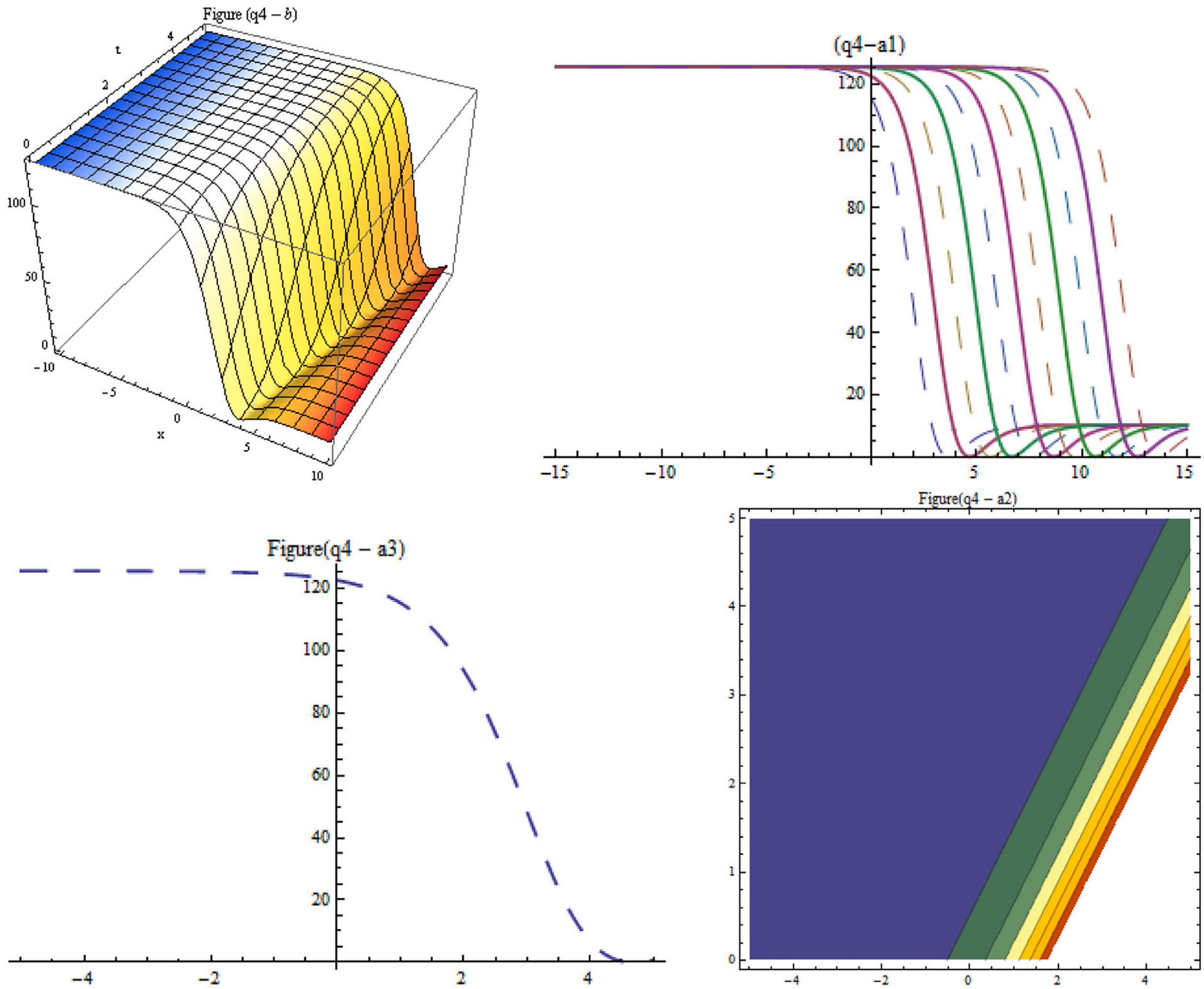

Fig. 2. The physical plotting of (19) is given in different dimensions. Here (q4-b) shows 3-dimensional plotting of (19) as anti kink type soliton with parametric values $\xi_0 = 0.7$, $\rho = 1.3$, $p = 0.8$, $q = 1.7$, $b = .5$, $r = 0.5$, $s = 0.8$ in intervals (0,5), (−10,10). And (q4-a1) is the 2-dimensional plotting of (19) with same parametric values as used above with in intervals (−15,15), (0,10). While (q4-a2) is the contour plotting of (19) with the same parametric values as mentioned above with in intervals (−5,5), (0,5). And (q4-a3) shows 1-dimensional plotting of (19) anti kink type soliton using interval (−5,5) of (19) with parametric values $\xi_0 = 1$, $\rho = 1.5$, $p = 0.8$, $q = 0.8$, $b = .5$, $r = 0.5$, $s = 1$.

Next by substituting all values mentioned above into Eq. (13) with the help of Eq. (10), we will obtain in this family the following solutions using our method, so the solutions in this family for Eq. (4) are obtained as below.

$$q_7(x,t) = Q(\xi) = (W(\xi))^2 = \left(\pm ip\frac{\sqrt{6r}(1+s\ tanh[\frac{\sqrt{p}}{2}(x-\upsilon t)+\xi_0])}{q}\right)^2, \quad r<0. \tag{23}$$

$$q_8(x,t) = Q(\xi) = (W(\xi))^2 = \left(\pm i\sqrt{r}\sqrt{\frac{3p}{2r}}\left(\coprod\right)\right)^2, \quad r<0. \tag{24}$$

$$q_9(x,t) = Q(\xi) = (W(\xi))^2 = \left(\pm i\frac{p\sqrt{6r}\ \prod}{q}\right)^2, \quad r<0. \tag{25}$$

While above $\coprod = \left(1 + \frac{s\ sinh\sqrt{p}(x-\upsilon t)}{\rho+cosh\sqrt{p}(x-\upsilon t)}\right)$ and $\prod = \left(1 + \frac{s\ (\rho\sqrt{1+\sigma^2}+cosh\sqrt{p}(x-\upsilon t))}{\sigma+sinh\sqrt{p}(x-\upsilon t)}\right)$.

Family 4:

$$a_0 = -\frac{4b}{5},\ a_1 = \pm i\sqrt{6r},\ \ b_1 = d_1 = 0,\ q = \pm ib\frac{4}{5}\sqrt{\frac{2}{3}}\sqrt{r},\ p = \frac{-8b^2}{75},\ \upsilon = -\frac{16b^2}{75}. \tag{26}$$

Next by substituting all values mentioned above into Eq. (13) with the help of Eq. (10), we will obtain in this family the following solutions using our method, so the solutions in this family for Eq. (4) are obtained as below.

$$q_{10}(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{4b}{5} \pm ip\frac{\sqrt{6r}(1+s\ tanh[\frac{\sqrt{p}}{2}(x-\upsilon t)+\xi_0])}{q}\right)^2, \quad r<0. \tag{27}$$

$$q_{11}(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{4b}{5} \pm i\sqrt{r}\sqrt{\frac{3p}{2r}}\left(\coprod\right)\right)^2, \quad r<0. \tag{28}$$

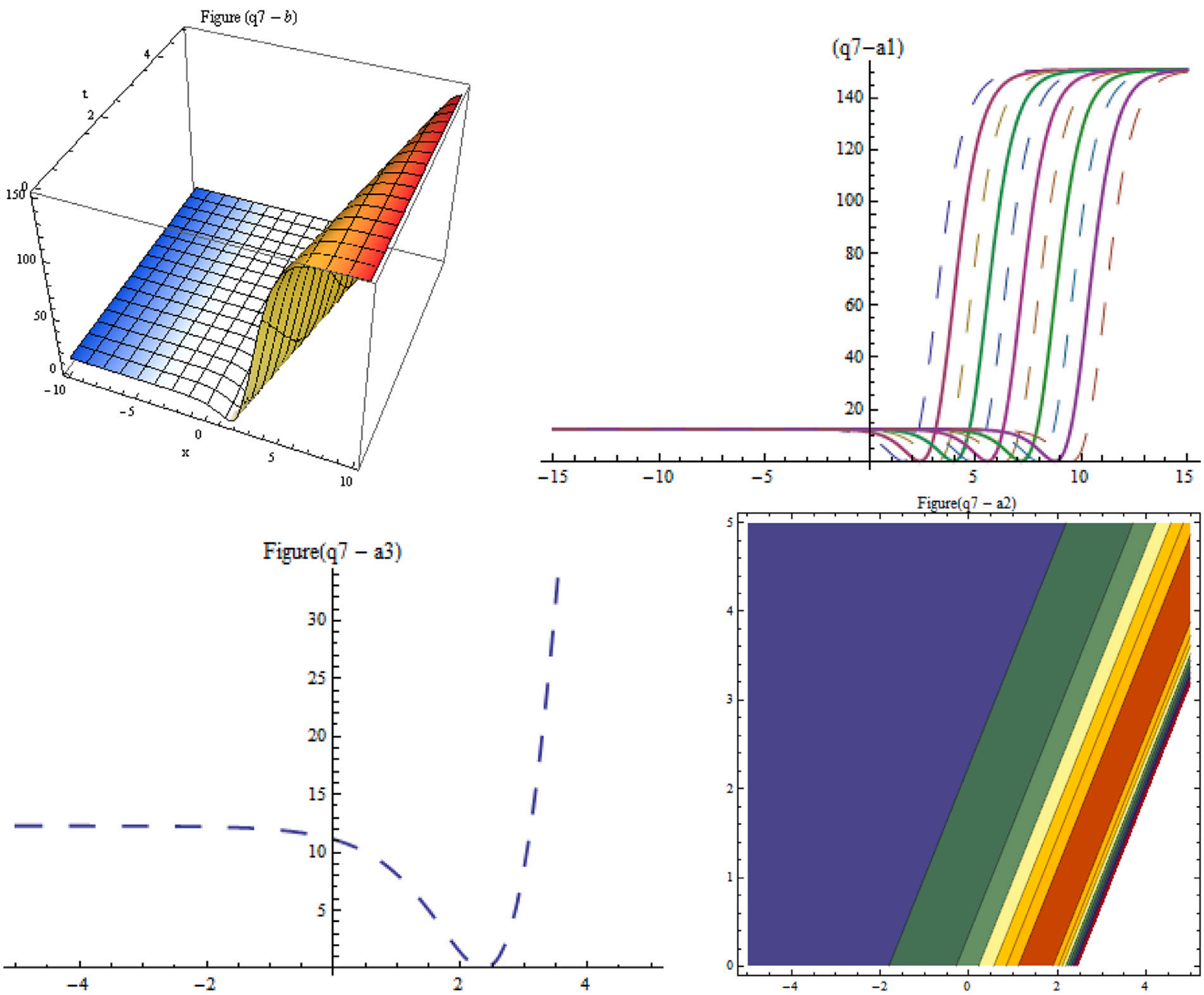

Fig. 3. The physical plotting of (23) is given in different dimensions. Here (q7-b) shows 3-dimensional plotting of (23) with parametric values $\xi_0 = 0.7$, $\rho = 1.3$, $p = 0.9$, $q = 1.9$, $b = .5$, $r = -0.5$, $s = 0.8$ in intervals (0,5), (−10,10). And (q7-a1) is the 2-dimensional plotting of (23) with same parametric values as used above with in intervals (−15,15), (0,10). While (q7-a2) is the contour plotting of (23) with the same parametric values as mentioned above with in intervals (−5,5), (0,5). And (q7-a3) shows 1-dimensional plotting of (23) using interval (−5,5) of (23) with parametric values $\xi_0 = 1.1$, $\rho = 1.5$, $p = 1.0$, $q = 0.8$, $b = .5$, $r = 0.5$, $s = 1.1$.

$$q_{12}(x,t) = Q(\xi) = (W(\xi))^2 = \left(-\frac{4b}{5} \pm i\frac{p\sqrt{6r}\,\prod}{q}\right)^2, \qquad r < 0. \tag{29}$$

While above $\coprod = \left(1 + \frac{s\ sinh\sqrt{p}(x-vt)}{\rho+cosh\sqrt{p}(x-vt)}\right)$ and $\prod = \left(1 + \frac{s\ (\rho\sqrt{1+\sigma^2}+cosh\sqrt{p}(x-vt))}{\sigma+sinh\sqrt{p}(x-vt)}\right)$.

Family 5:

$$a_0 = \frac{\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}}}{6},\ d_1 = \frac{\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}}}{6\sqrt{p}},$$
$$b_1 = a_1 = 0,$$
$$v = \frac{-9p+2\sqrt{b^4-9b^2p}+2b(b+2\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}})}{18},\ q = r = 0. \tag{30}$$

Next by substituting all values mentioned above into Eq. (13) with the help of Eq. (10), we will obtain in this family the following solutions using our method, so the solutions in this family for Eq. (4) are obtained as below.

$$q_{13} = \left(\frac{\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}}}{6} + \frac{\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}}}{6\sqrt{p}} \times \left(\frac{\frac{(-p^{\frac{3}{2}} s\ sech(\frac{\sqrt{p}(x-vt)}{2}+\xi_0)^2)}{2q}}{-\frac{p}{q}(1+s\ tanh(\frac{\sqrt{p}(x-vt)}{2}+\xi_0))}\right)\right)^2. \tag{31}$$

$$q_{14} = \left(\frac{\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}}}{6} + \frac{\sqrt{2b^2-9p+2\sqrt{b^4-9b^2p}}}{6\sqrt{p}} \times \left(\frac{\frac{1}{2}\sqrt{\frac{p}{r}}\left(\frac{\sqrt{p}s\ cosh\sqrt{p}(x-vt)}{\rho+cosh\sqrt{p}(x-vt)} - \frac{\sqrt{p}s\ sinh(\sqrt{p}(x-vt))^2}{\rho+cosh(\sqrt{p}(x-vt))^2}\right)}{\sqrt{\frac{p}{4r}}(1+\frac{s\ sinh\sqrt{p}(x-vt)}{\rho+cosh\sqrt{p}(x-vt)})}\right)\right)^2. \tag{32}$$

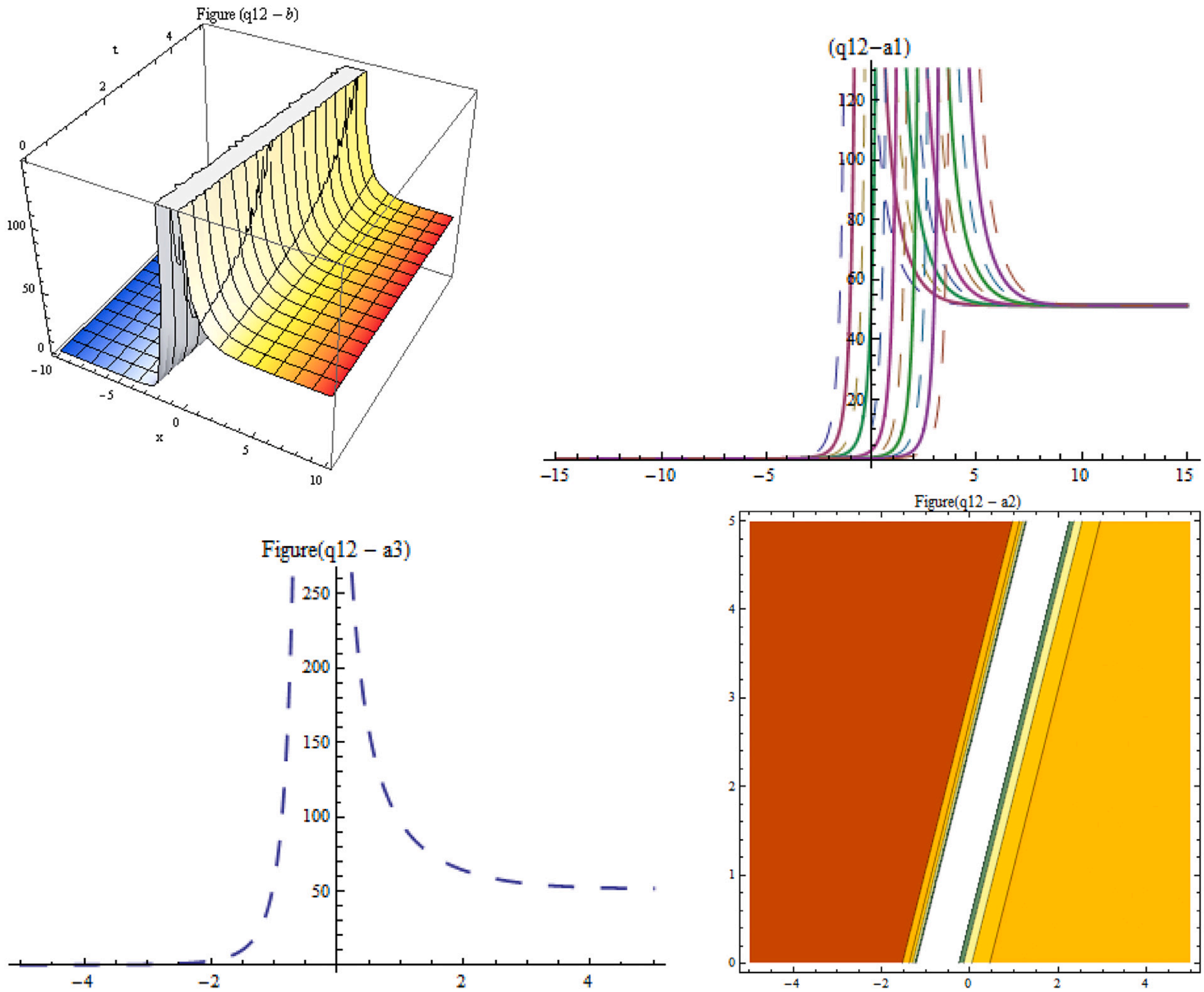

Fig. 4. The physical plotting of (29) is given in different dimensions. Here (q12-b) shows 3-dimensional plotting of (29) with parametric values $\xi_0 = 0.7$, $\rho = 1.3$, $p = 6$, $q = 3$, $b = .5$, $r = -0.5$, $s = 0.8$ in intervals (0,5), (−10,10). And (q12-a1) is the 2-dimensional plotting of (29) with same parametric values as used above with in intervals (−15,15), (0,10). While (q12-a2) is the contour plotting of (29) with the same parametric values as mentioned above with in intervals (−5,5), (0,5). And (q12-a3) shows 1-dimensional plotting of (29) using interval (−5,5) of (29) with parametric values $\xi_0 = 0.7$, $\rho = 1.3$, $p = 6$, $q = 3.1$, $b = .5$, $r = -0.5$, $s = 1.1$.

$$q_{15} = \left(\frac{\sqrt{2b^2 - 9p + 2\sqrt{b^4 - 9b^2 p}}}{6} + \frac{\sqrt{2b^2 - 9p + 2\sqrt{b^4 - 9b^2 p}}}{6\sqrt{p}} \times \left(\frac{\left(\frac{p^{3/2} s(1+\rho\sqrt{1+\sigma^2} cosh\sqrt{p}(x-\upsilon t) - \sigma sinh\sqrt{p}(x-\upsilon t))}{q(\sigma + sinh\sqrt{p}(x-\upsilon t))^2}\right)}{-\frac{p}{q}(1 + \frac{s\ cosh\sqrt{p}(x-\upsilon t)+\rho\sqrt{1+\sigma^2}}{sinh\sqrt{p}(x-\upsilon t)+\sigma})} \right) \right)^2. \tag{33}$$

Family 6:

$$a_0 = -\frac{b^2 + \sqrt{b^4 - 9b^2 p}}{6b}, \quad d_1 = \frac{\sqrt{2b^2 - 9p + 2\sqrt{b^4 - 9b^2 p}}}{6\sqrt{p}},$$
$$b_1 = a_1 = 0, \; q = r = 0, \quad \upsilon = \frac{-2b^2 - 9p - 2\sqrt{b^4 - 9b^2 p}}{18}. \tag{34}$$

Family 7:

$$a_0 = -\frac{2b}{3}, \qquad d_1 = \frac{\sqrt{-9 + \frac{2b^2}{p} - \frac{2\sqrt{b^2(b^2-9p)}}{p}}}{6}, \; b_1 = a_1 = 0,$$
$$q = r = 0, \; \upsilon = \frac{-9p - 14b^2 - 2\sqrt{b^2(b^2 - 9p)}}{72}, \tag{35}$$

One can also construct new solitary wave solutions using the values of the constants given in family (6, 7) for (4).

3. Physical representation of the solutions

Here we have listed the physical representation or plotting of our newly obtained solutions in different dimensions in detail which are including trigonometric type functions, hyperbolic type functions, rational type functions, combined functions in different shapes to get more understanding of the physical structures of modified generalized Korteweg–de Vries ($K--dV$) Equation with mixed nonlinearity using Mathematica 10.4 (see Figs. 1–4).

Discussion

The purpose of this section is to differentiate the differences and the similarities of our obtained new families of solutions which are obtained using our technique with all those results which are available already in the literature using some other different old mathematical methods for the same dynamical system.

- First: The primary key point of difference of our technique to obtain new families of solutions is the uniqueness of structural body of our proposed method (9), which is quite new and different structure using only three parameters that is of main significance point to mention here.
- Second: To get more understanding about the dynamics of our newly found results their graphical structures in different dimensions using different sets of values of constants a_j, b_j, c_j, d_j are given using any mathematical software for example Mathematica 10.4, Maple, or Matlab.
- It is important to mention here that Eq. (10) gives us different new and more general forms of exact solutions including trigonometric functions, combined functions, rational type functions and hyperbolic functions.

So, from above comparison we can say that the solutions found by our method are quite reliable, straight forward with less computational time, more simplified and helpful in a sufficient manner to get more understanding about the analytical analysis of other complicated nonlinear dynamical partial differential equations arising in many other branches of physical and natural sciences.

Conclusion

The focus of our work was to discuss the analytical treatment of $(mK - dV)$ model as a result of which we have found some new and more general families of exact solutions which have potential applications to read the qualitative analysis of many nonlinear wave phenomena in a more exact manner, further these results have a high impact to develop the theories of soliton dynamics, adiabatic parameter dynamics and in quantum mechanics. These solutions are highly useful to introduce new mathematical softwares in the market which are highly helpful in numerical analysis of other complex nonlinear partial differential equations and also to get more understanding about the comparison of analytical solutions.

So, the solutions found in this article are quite general and new with potential applications in the development of theoretical quantum mechanics, plasma physics, nonlinear optics, nano-technology, and hydrodynamics with a richer taste than those solutions which are obtained in the literature earlier on the topic of extraction exact solutions of this powerful dynamical system. Next to show a detailed dynamical understanding of our obtained results in different dimensions we have presented the graphical structures of our solutions in different dimensions using Mathematica 10.4 which are helpful to learn more clearly about the dynamics of solutions. In the last its important to mention here that the less computational time and efficiency of our technique actually explains the straightforwardness, reliability, accuracy, and simplicity of the technique. Hence the presented method is highly useful to study analytical treatment of other nonlinear complex dynamical partial differential equations(NLPDES) with the help of three parameters.

CRediT authorship contribution statement

Nadia Cheemaa: Data curation, Writing - original draft, Software, Validation, Writing - review & editing. **Aly R. Seadawy:** Conceptualization, Methodology, Software, Visualization, Investigation, Supervision. **Taghreed G. Sugati:** Writing - review & editing. **Dumitru Baleanu:** Software.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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