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Two-wave, breather wave solutions and stability analysis to the (2 + 1)-dimensional Ito equation

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The current study employs the novel Hirota bilinear scheme to investigate the nonlinear model. Thus, we acquire some two-wave and breather wave solutions to the governing equation. Breathers are pulsating localized structures that have been used to mimic extreme waves in a variety of nonlinear dispersive media with a narrow banded starting process. Several recent investigations, on the other hand, imply that breathers can survive in more complex habitats, such as random seas, despite the attributed physical restrictions. The authenticity and genuineness of all the acquired solutions agreed with the original equation. In order to shed more light on these novel solutions, we plot the 3-dimensional and contour graphs to the reported solutions with some suitable values. The governing model is also stable because of the idea of linear stability. The study's findings may help explain the physics behind some of the challenges facing ocean engineers.

1. Introduction

In the form of nonlinear evolution model, various complicated nonlinear integrable system can be depicted. Such system could be utilised to express multifarious physical nonlinear phenomena [1–11]. The aspect of soliton solutions is one of the relevant and captivating subjects for examining the propagation of soliton via nonlinear optical fibres, water waves, rogue waves [14]. Increasing attention has been paid to nonlinear model, which are crucial in the fields of physics, mathematics, and engineering science Using several effective approaches, many particular solutions to the nonlinear evolution model are found [12,13,15–37]. Lump solutions are a type of rational solution that arises in many of the above-mentioned nonlinear physical characteristics. Another significant and intriguing aspect of many physical disciplines is the interplay between lump solutions with other soliton solutions. Collisions involving interactions between a lump and a soliton might be elastic or completely non-elastic [38]. There are several studies in this direction [39–49].

However, in this study the approach by Hirota [50] is used in constructing some new two-wave and breather wave solutions to the model [51].

The model is provided by Yang et al. [51]

$$\phi_{tt} + \phi_{xxxt} + 6\phi_x\phi_t + 3\phi\phi_{xt} + 3\phi_{xx}\chi_t + \sigma\phi_{yt} + \delta\phi_{xt} = 0, \tag{1}$$

and $\chi_x = \phi$ and σ, δ indicate not nonzero constants.

The model has been derived for small-amplitude surface waves in a channel or strait with slowly changing depth and breadth and non-vanishing vorticity [52].

Eq. (1) may be equivalently written as

$$\chi_{xtt} + \chi_{xxxxt} + 6\chi_{xx}\chi_{xt} + 3\chi_x\chi_{xxt} + 3\chi_{xxx}\chi_{xt} + \sigma\chi_{xyt} + \delta\chi_{xxt} = 0. \tag{2}$$

Using the Cole–Hopf transformations [50]

$$\phi = 2(lnf)_{xx} \text{ and } \chi = 2(lnf)_x, \tag{3}$$

on Eqs. (1) and (2), yields [51,53,54]:

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$$2(f_{tt}f - f_t^2 + f_{xxxt}f - 3f_{xxt}f_x + 3f_{xt}f_{xx} - f_{xxx}f_t + \sigma(f_{yt}f - f_yf_t) + \delta(f_{xt}f - f_xf_t)) = 0. \quad (4)$$

In Hirota bilinear operator, Eq. (4) is given by Yang et al. [51]

$$2(D_t^2 + D_x^3D_t + \sigma D_yD_t + \delta D_xD_t)f \cdot f = 0. \quad (5)$$

2. Two-wave solutions

To search for the two-wave solutions, the form below for Eq. (1) is written as:

$$f(x, y, t) = a_1e^{\eta_1} + a_2e^{-\eta_1} + a_3\sin(\eta_2) + a_4\sinh(\eta_3), \quad (6)$$

where $\eta_1 = \varrho_1x + \varrho_2y + \varrho_3t$, $\eta_2 = \varrho_4x + \varrho_5y + \varrho_6t$, $\eta_3 = \varrho_7x + \varrho_8y + \varrho_9t$, and $a_1, a_2, a_3, a_4, \varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6, \varrho_7, \varrho_8, \varrho_9$ are constants to be resolved later.

Inserting Eq. (6) into Eq. (4), provides a polynomial in exponential and trigonometric functions. A system of equations is generated by considering the coefficients of the exponential and trigonometric functions of the same power and equating each collection to zero. To determine the values of the parameters, the gathered system of equations is solved. Using Eqs. (6) and (3), yields the novel two-wave solutions to the governing equation.

Set-1: When

$$a_1 = 0\, a_2 = 0,\ a_3 = 0,\ a_4 = a_4,\ \varrho_1 = -\varrho_7,\ \varrho_2 = -\frac{-4\varrho_7^3 - \delta\varrho_7 + \varrho_3}{\sigma},\ \varrho_3 = \varrho_3,\ \varrho_4 = \varrho_4,$$
$$\varrho_5 = \varrho_5,\ \varrho_6 = \varrho_6,\ \varrho_7 = \varrho_7,\ \varrho_8 = \varrho_8,\ \varrho_9 = -4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7,$$

we have

$$f_1(x, y, t) = a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t), \quad (7)$$

$$\phi_1 = 2\Bigg(\varrho_7^2\Big(a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t)\Big) \times \Big(a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t)\Big) - \varrho_7\Big(a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} + a_4\cosh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t)\Big)^2\Bigg) \Big/ \Big(a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t)\Big)^2, \quad (8)$$

$$\chi_1 = \frac{2\varrho_7\left(a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} + a_4\cosh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t)\right)}{a_2e^{\varrho_7x + \frac{(-4\varrho_7^3 - \delta\varrho_7 + \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh(-\varrho_3x - \varrho_8y + (4\varrho_7^3 + \delta\varrho_8 + \sigma\varrho_7)t)}. \quad (9)$$

Set-2: When

$$a_1 = a_1\, a_2 = a_2,\ a_3 = 0,\ a_4 = a_4,\ \varrho_1 = -\varrho_7,\ \varrho_2 = \frac{4\varrho_7^3 + \delta\varrho_7 - \varrho_3}{\sigma},\ \varrho_3 = \varrho_3,\ \varrho_4 = \varrho_4,$$
$$\varrho_5 = \varrho_5,\ \varrho_6 = \varrho_6,\ \varrho_7 = \varrho_7,\ \varrho_8 = \varrho_8,\ \varrho_9 = -4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7,$$

we have

$$f_2(x, y, t) = a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big), \quad (10)$$

$$\phi_2 = 2\Bigg(\varrho_7^2\Big(a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big)\Big) \times \Big(a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big)\Big) - \varrho_7\Big(-a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} + a_4\cosh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big)\Big)^2\Bigg) \Big/ \Big(a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big)\Big)^2, \quad (11)$$

$$\chi_2 = 2\varrho_7\Big(-a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} + a_4\cosh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big)\Big) \Big/ \quad (12)$$
$$a_1e^{-\varrho_7x + \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y + \varrho_3t} + a_2e^{\varrho_7x - \frac{(4\varrho_7^3 + \delta\varrho_7 - \varrho_3)}{\sigma}y - \varrho_3t} - a_4\sinh\big(-\varrho_7x - \varrho_8y - (-4\varrho_7^3 - \sigma\varrho_8 - \delta\varrho_7)t\big).$$

Set-3: When

$$a_1 = -\frac{a_4^2}{4a_2},\ a_2 = a_2,\ a_3 = 0,\ a_4 = a_4,\ \varrho_1 = \varrho_7,\ \varrho_2 = -\frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma},$$
$$\varrho_3 = \varrho_3,\ \varrho_4 = \varrho_4,\ \varrho_5 = \varrho_5,\ \varrho_6 = \varrho_6,\ \varrho_7 = \varrho_7,\ \varrho_8 = \varrho_8,\ \varrho_9 = \varrho_9,$$

we have

$$f_3(x, y, t) = -\frac{a_4^2e^{\varrho_7x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y + \varrho_9t}}{4a_2} + a_4\sinh(\varrho_7x + \varrho_8y + \varrho_9t) + a_2e^{-\varrho_7x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y - \varrho_9t}, \quad (13)$$

$$\phi_3 = 2\Bigg(\varrho_7^2\Big(-\frac{a_4^2e^{\varrho_7x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y + \varrho_9t}}{4a_2} + a_4\sinh(\varrho_7x + \varrho_8y + \varrho_9t) + a_2e^{-\varrho_7x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y - \varrho_9t}\Big) \times \Big(-\frac{a_4^2e^{\varrho_7x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y + \varrho_9t}}{4a_2} + a_4\sinh(\varrho_7x + \varrho_8y + \varrho_9t) + a_2e^{-\varrho_7x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y - \varrho_9t}\Big) - \varrho_7\Big(-\frac{a_4^2e^{\varrho_7x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y + \varrho_9t}}{4a_2} + a_4\cosh(\varrho_7x + \varrho_8y + \varrho_9t) - a_2e^{-\varrho_7x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y - \varrho_9t}\Big)^2\Bigg) \Big/ \Big(-\frac{a_4^2e^{\varrho_7x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y + \varrho_9t}}{4a_2} + a_4\sinh(\varrho_7x + \varrho_8y + \varrho_9t) + a_2e^{-\varrho_7x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma}y - \varrho_9t}\Big)^2, \quad (14)$$

$$\chi_3 = 2\varrho_7\left(- \frac{a_4^2 e^{\varrho_7 x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma} y + \varrho_9 t}}{4a_2} + a_4\cosh(\varrho_7 x + \varrho_8 y + \varrho_9 t) - a_2 e^{-\varrho_7 x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma} y - \varrho_9 t}\right) \Bigg/ - \frac{a_4^2 e^{\varrho_7 x - \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma} y + \varrho_9 t}}{4a_2} + a_4\sinh(\varrho_7 x + \varrho_8 y + \varrho_9 t) + a_2 e^{-\varrho_7 x + \frac{(8\varrho_7^3 + \sigma\varrho_8 + 2\delta\varrho_7 + 2\varrho_9)}{\sigma} y - \varrho_9 t}, \tag{15}$$

Set-4: When

$$a_1 = 0,\ a_2 = a_2,\ a_3 = a_3,\ a_4 = a_4,\ \varrho_1 = -\varrho_7,\ \varrho_2 = \frac{8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3}{\sigma},\ \varrho_3 = \varrho_3,\ \varrho_4 = \varrho_4,\ \varrho_5 = -\frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4},$$
$$\varrho_6 = \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4},\ \varrho_7 = \varrho_7,\ \varrho_8 = \frac{8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3}{\sigma},\ \varrho_9 = \varrho_3$$

we have

$$f_4(x,y,t) = a_2 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} + a_3\sin\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\sinh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right), \tag{16}$$

$$\begin{aligned}\phi_4 = 2\Bigg(\Bigg(&\left(a_2\varrho_7^2 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} - a_3\varrho_4^2\sin\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\varrho_7^2\sinh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right)\right)\\
&\times\left(a_2 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} + a_3\sin\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\sinh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right)\right)\\
&-\left(a_2\varrho_7 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} + a_3\varrho_4\cos\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\varrho_7\cosh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right)\right)^2\Bigg)\\
\Bigg/&\left(a_2 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} + a_3\sin\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\sinh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right)\right)^2,\end{aligned} \tag{17}$$

$$\begin{aligned}\chi_4 = 2\Bigg(&a_2\varrho_7 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} + a_3\varrho_4\cos\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\varrho_7\cosh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right)\Bigg)\\
\Bigg/\ &a_2 e^{\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 - \varrho_3)}{\sigma} y - \varrho_3 t} + a_3\sin\left(\varrho_4 x - \frac{(3a_3^2\varrho_4^2\varrho_7^2 - a_3^2\varrho_4^4 + \delta a_3^2\varrho_4^2 + a_4^2\varrho_3\varrho_7)}{\sigma a_3^2\varrho_4} y + \frac{a_4^2\varrho_3\varrho_7}{\varrho_3^2\varrho_4} t\right) + a_4\sinh\left(\varrho_7 x - \frac{(8\varrho_7^3 - 3\varrho_4^2\varrho_7 + \delta\varrho_7 + \varrho_3)}{\sigma} y + \varrho_3 t\right).\end{aligned} \tag{18}$$

3. The breather wave solution

Here, the breather wave solution to the model are constructed. To establish for this solution, Eq. (1) is supposed as

$$f(x,y,t) = e^{-r_1(x + a_0 y + b_0 t)} + a_1\cos(r_0(x + c_0 y + d_0 t)) + a_2 e^{r_1(x + a_0 y + b_0 t)}, \tag{19}$$

where $r_0,\ r_1,\ a_0,\ a_1,\ a_2,\ b_0,\ c_0,\ d_0$ are constants to be resolved later.

Inserting Eq. (19) into Eq. (4), provides a polynomial in trigonometric and exponential functions. Taking the coefficients of the same-power exponential and trigonometric functions and equating each sum to zero, generates an equations. The attained equations are solved to evaluate the parameters values. Using the attained values in Eq. (19), and Eq. (3), yields the novel interaction solutions to the governing equation.

Set-1: When

$$a_0 = -\frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2},$$
$$r_0 = r_0,$$
$$b_0 = \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2},\ c_0 = c_0,\ d_0 = -\sigma c_0 + r_0^2 - 3r_1^2 - \delta,$$
$$a_1 = a_1,\ a_2 = a_2,$$

$r_1 = r_1$, we have

$$\begin{aligned}f_1(x,y,t) &= e^{-r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\\
&+ a_1\cos\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right)\\
&+ a_2 e^{r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)},\end{aligned} \tag{20}$$

$$\begin{aligned}\phi_1 = 2\Bigg\{&\Bigg(r_1^2 e^{-r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\\
&- a_1 r_0^2\cos\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right)\\
&+ a_2 r_1^2 e^{r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\Bigg)\\
&\times\Bigg(e^{-r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\\
&+ a_1\cos\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right)\\
&+ a_2 e^{r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\Bigg)\\
&-\Bigg(-r_1 e^{-r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\\
&- a_1 r_0\sin\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right)\\
&+ a_2 r_1 e^{r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\Bigg)^2\Bigg\}\\
\Bigg/&\Bigg(e^{-r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\\
&+ a_1\cos\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right)\\
&+ a_2 e^{r_1\left(x - \frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2} y + \frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2} t\right)}\Bigg)^2,\end{aligned} \tag{21}$$

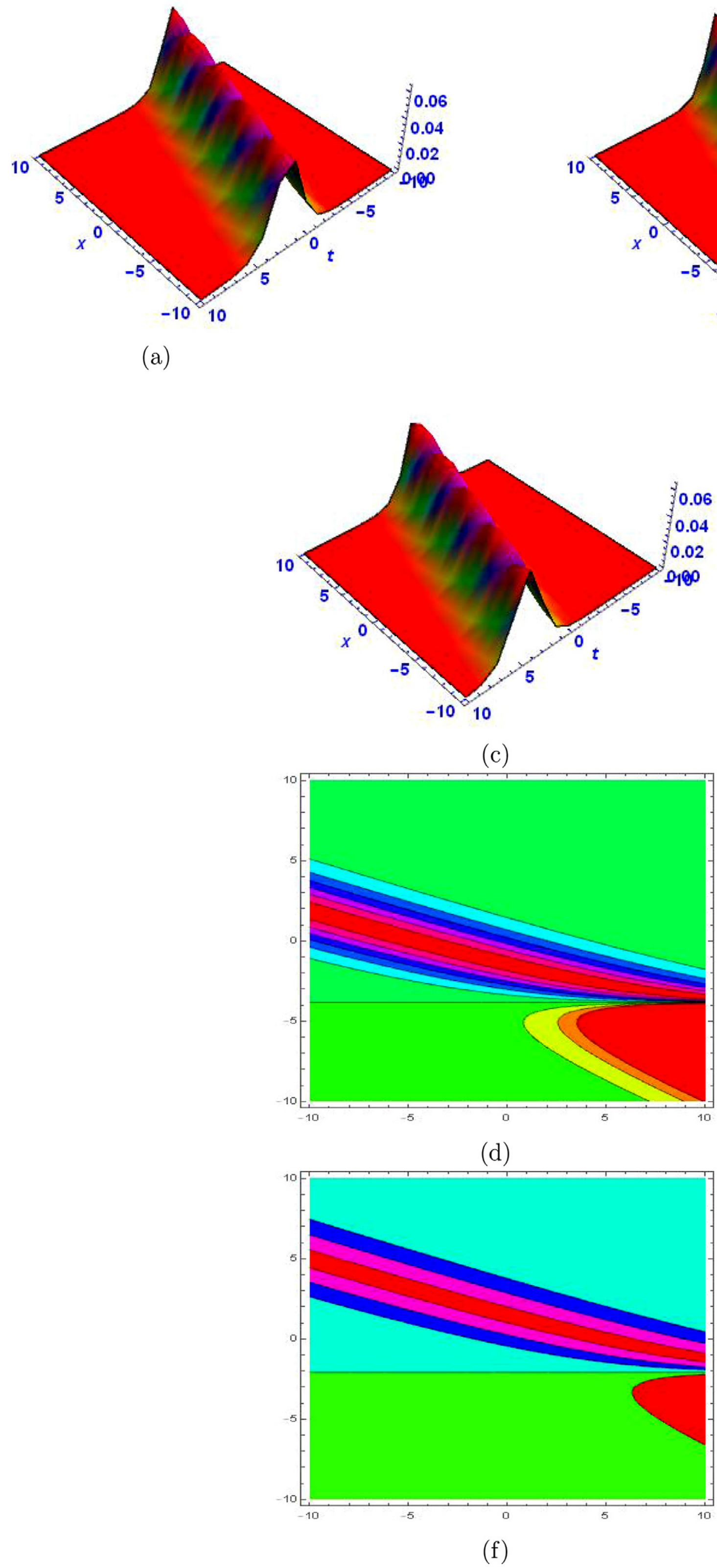

Fig. 1. Features of Eq. (8) at (a,d) $y = -2$, (b,e) $y = 0$, (c,f) $y = 2$.

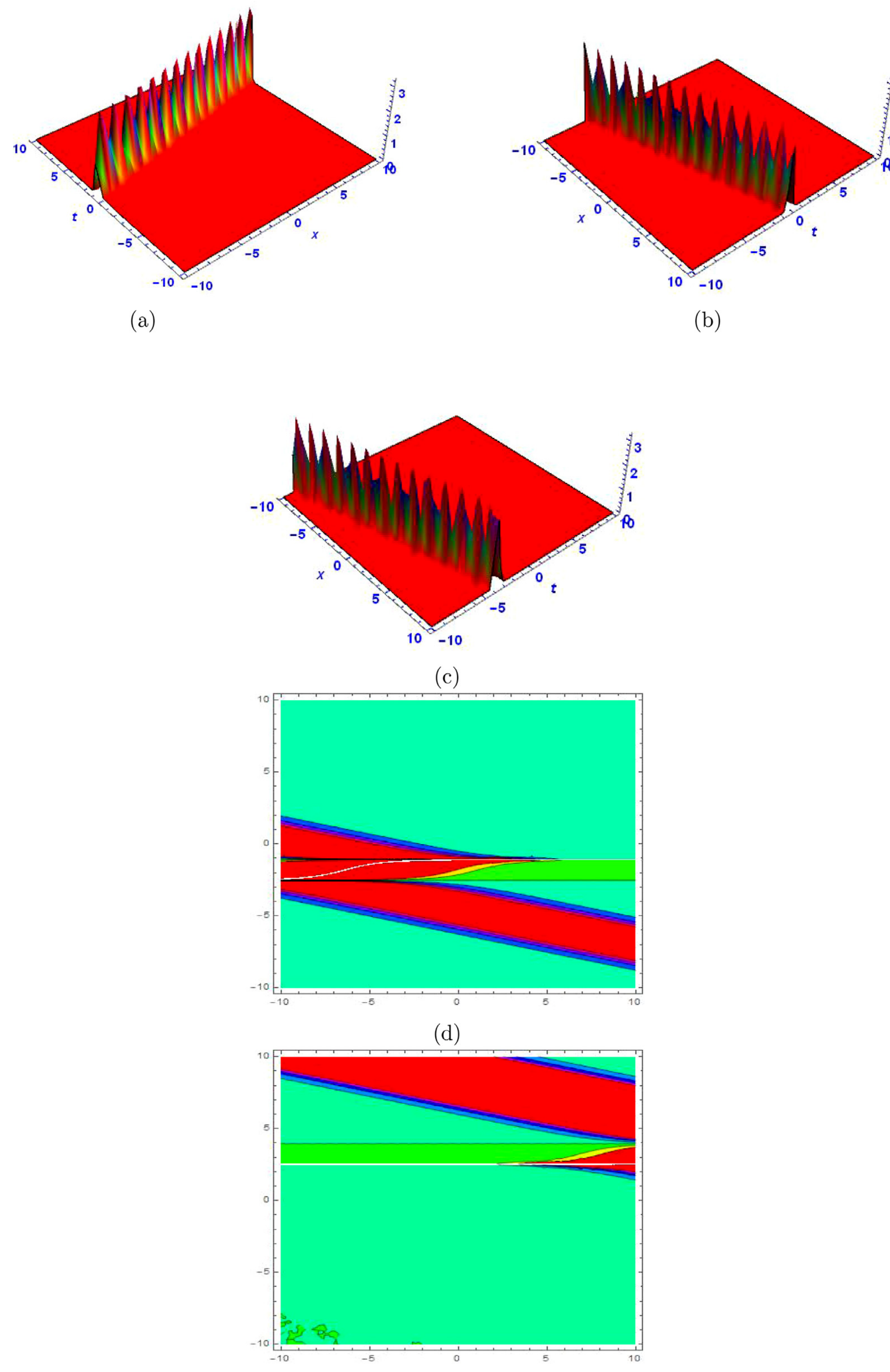

Fig. 2. Features of Eq. (9) at (a,d) $y = -2$, (b,e) $y = 0$, (c,f) $y = 2$.

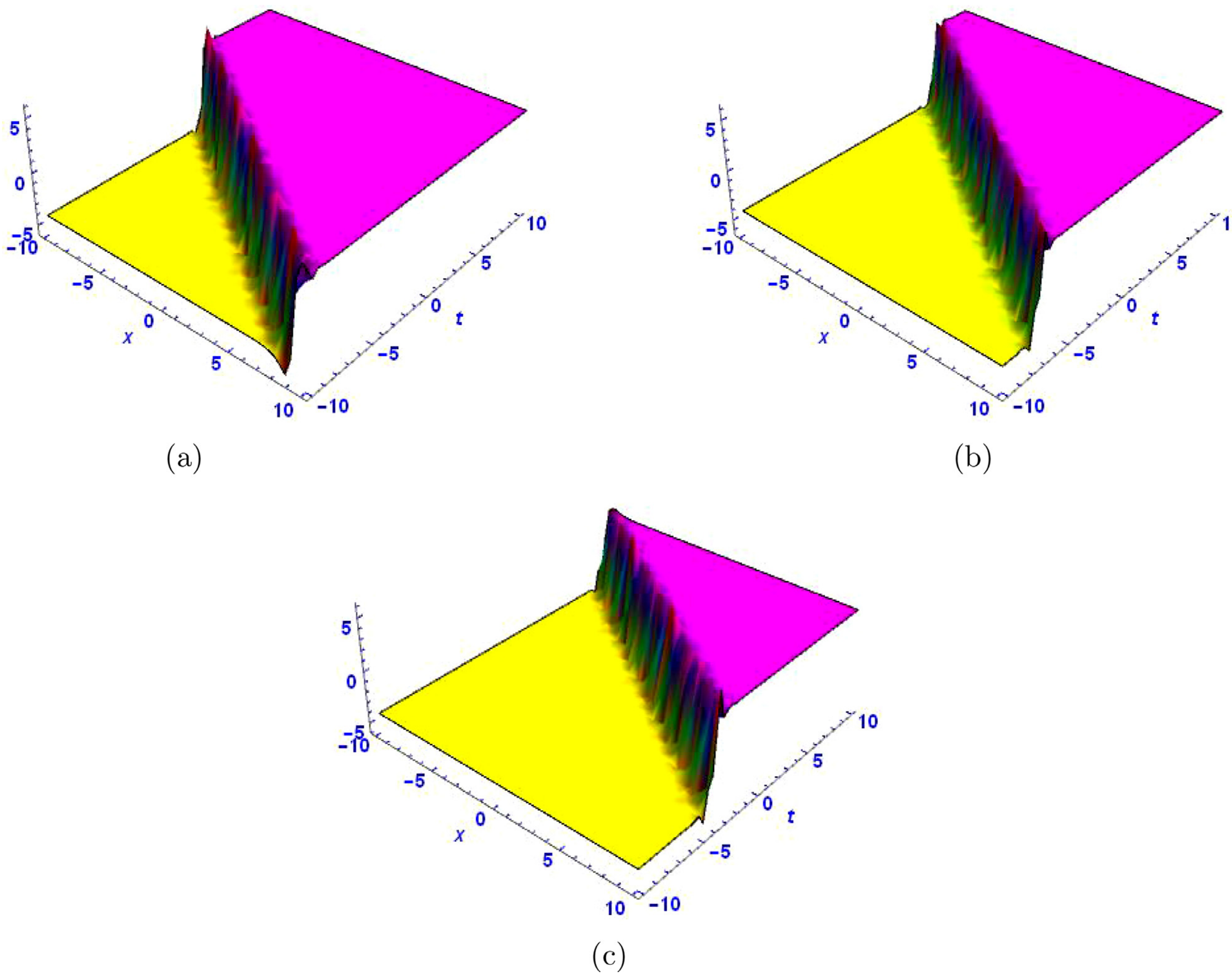

Fig. 3. Features of Eq. (21) at (a) $y = -2$, (b) $y = 0$, (c) $y = 2$.

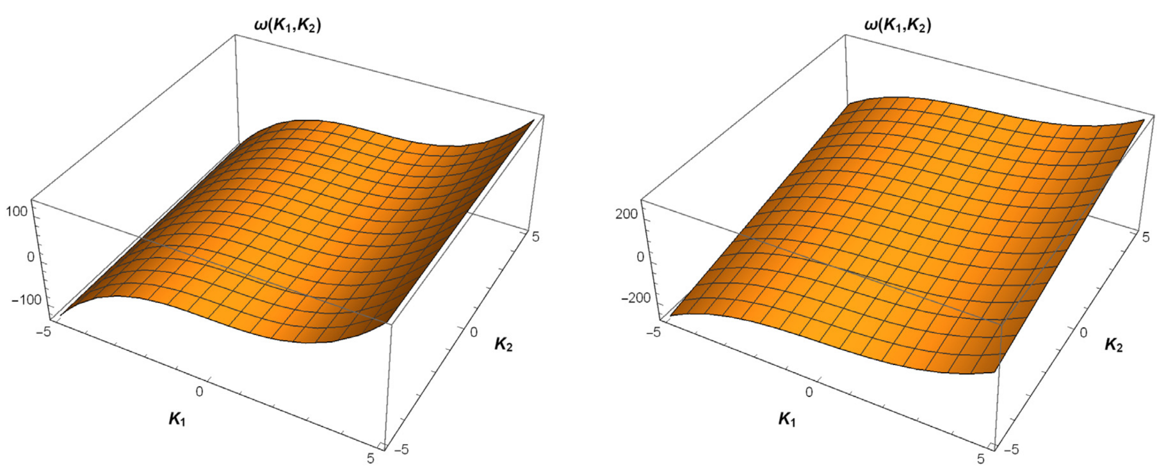

Fig. 4. The 3D profile for the Eq. (28) when (a) $p = 5, \epsilon = 0.1, \sigma_1 = 5, \delta = 1k_2 = 0$, (b) $p = 0.5, \epsilon = 0.1, \sigma_1 = 0.5, \delta = 0.01, k_2 = 0.5$.

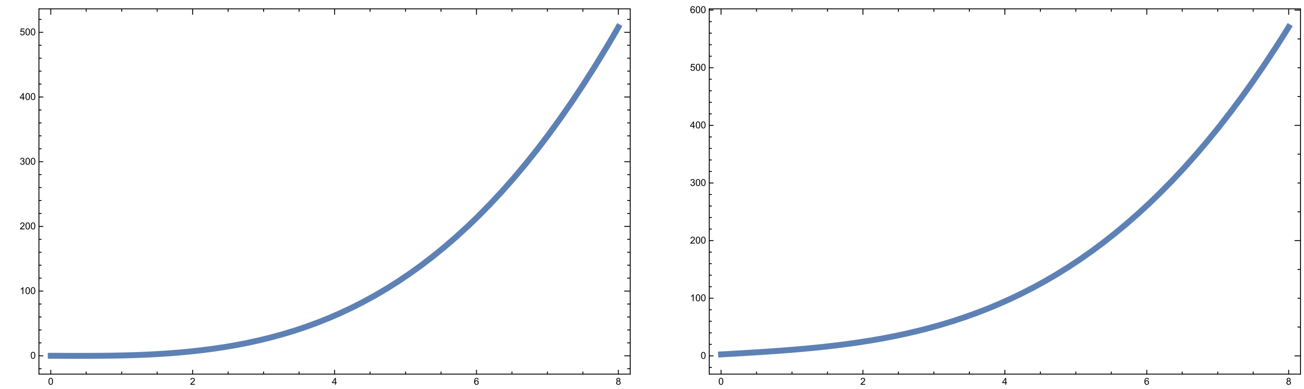

Fig. 5. Frequency of perturbation versus wave number K_1 when (a) $p = 5, \epsilon = 0.1, \sigma_1 = 5, \delta = 1$, (b) $p = 0.5, \epsilon = 0.1, \sigma_1 = 0.5, \delta = 0.01$.

$$\begin{aligned}\chi_1 = 2\Bigg\{\Bigg(& -r_1 e^{-r_1\left(x-\frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2}y+\frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2}t\right)} \\ & - a_1 r_0 \sin\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right) \\ & + a_2 r_1 e^{r_1\left(x-\frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2}y+\frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2}t\right)}\Bigg)\Bigg\} \\ & \Bigg/ e^{-r_1\left(x-\frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2}y+\frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2}t\right)} \\ & + a_1 \cos\left(r_0(x + c_0 y + (-\sigma c_0 + r_0^2 - 3r_1^2 - \delta)t)\right) \\ & + a_2 e^{r_1\left(x-\frac{(\sigma c_0 a_1^2 r_0^2 - a_1^2 r_0^4 + 3a_1^2 r_0^2 r_1^2 + \delta a_1^2 r_0^2 - 12a_2 r_0^2 r_1^2 + 4a_2 r_1^4 + 4\delta a_2 r_1^2)}{4\sigma a_2 r_1^2}y+\frac{a_1^2 r_0^2(\sigma c_0 - r_0^2 + 3r_1^2 + \delta)}{4r_1^2 a_2}t\right)}.\end{aligned} \tag{22}$$

4. Stability analysis for the model

The stability analysis for the equation under consideration is examined herein [55–57].

4.1. Dispersion relations

Surmising that we have a function $u(x, t, y) \in \mathbb{R}$ which holds for nonlinear and linear PDEs with constant coefficient. Take into account the solution as comes next

$$u(x, t, y) = e^{k_1 x + k_2 y - \omega t}. \tag{23}$$

Plugging Eq. (23) into Eq. (2), yields $\omega = \omega(k_1, k_2)$ which depicts the dispersion relation (DR) $e^{k_1 x + k_2 y}$.

4.2. Stability analysis

Deducing that Eq. (2) has a perturbed solution expressed by

$$u(x, t, y) = \sqrt{P} + \Xi(x, t, y) e^{P(k_1 + P k_2 \epsilon)t}, \tag{24}$$

it can easily be shown that any constant P generates a steady state solution for Eq. (2). Plugging Eq. (24) into Eq. (2), yields

$$\begin{aligned} & k_1^2 P^2 \Xi_x + 2k_1 k_2 P^3 \epsilon \Xi_x + 9k_2 P^2 \epsilon \Xi_x \Xi_{xx} e^{k_1 P t + k_2 P^2 t \epsilon} \\ & \quad + 3k_2 P^2 \epsilon \Xi_x \Xi_{xxx} e^{k_1 P t + k_2 P^2 t \epsilon} \\ & \quad + 9k_1 P \Xi_x \Xi_{xx} e^{k_1 P t + k_2 P^2 t \epsilon} + 3k_1 P \Xi_x \Xi_{xxx} e^{k_1 P t + k_2 P^2 t \epsilon} \\ & \quad + 6 \Xi_{xx} \Xi_{xt} e^{k_1 P t + k_2 P^2 t \epsilon} \\ & \quad + 3 \Xi_x \Xi_{xxt} e^{k_1 P t + k_2 P^2 t \epsilon} + 3 \Xi_{xxx} \Xi_{xt} e^{k_1 P t + k_2 P^2 t \epsilon} \\ & \quad + 2k_1 P \Xi_{xt} + \delta k_1 P \Xi_{xx} + k_1 P \sigma \Xi_{xy} \\ & \quad + k_1 P \Xi_{xxxx} + k_2^2 P^4 \epsilon^2 \Xi_x + 2k_2 P^2 \epsilon \Xi_{xt} + \delta k_2 P^2 \epsilon \Xi_{xx} \\ & \quad + k_2 P^2 \sigma \epsilon \Xi_{xy} + k_2 P^2 \epsilon \Xi_{xxxx} \\ & \quad + \delta \Xi_{xxt} + \sigma \Xi_{xty} + \Xi_{xtt} + \Xi_{xxxxt} = 0. \end{aligned} \tag{25}$$

By linearizing the above equation, we attain

$$\begin{aligned} & k_1^2 P^2 \Xi_x + 2k_1 k_2 P^3 \epsilon \Xi_x + 2k_1 P \Xi_{xt} + \delta k_1 P \Xi_{xx} \\ & \quad + k_1 P \sigma \Xi_{xy} + k_1 P \Xi_{xxxx} \\ & \quad + k_2^2 P^4 \epsilon^2 \Xi_x + 2k_2 P^2 \epsilon \Xi_{xt} + \delta k_2 P^2 \epsilon \Xi_{xx} \\ & \quad + k_2 P^2 \sigma \epsilon \Xi_{xy} + k_2 P^2 \epsilon \Xi_{xxxx} \\ & \quad + \delta \Xi_{xxt} + \sigma \Xi_{xty} + \Xi_{xtt} + \Xi_{xxxxt} = 0. \end{aligned} \tag{26}$$

It surmises that the above equation leads to

$$\Xi(x, t, y) = \beta e^{k_1 x + k_2 y - \omega t}, \tag{27}$$

the normalized number of wave is represented by k_1 and k_2.

Plugging Eq. (27) into Eq. (26), we get

$$\omega = k_1 P + k_2 P^2 \epsilon \text{ or } \omega = k_1^3 + \delta k_1 + k_1 P + k_2 P^2 \epsilon + k_2 \sigma. \tag{28}$$

By stimulating Raman scattering and modulation of the self-phase, the stage of stability is attained (Figs. 1–3).

5. Conclusions

Using the Hirota bilinear technique, two-wave and breather wave solutions to the model were developed. As a result of this, various types of solutions have been established. To the best of our knowledge, these findings are unique and have never been published before for the model. Additionally, we examined the stability analysis for the model under consideration. See Figs. 4 and 5. The findings of this investigation definitely added new information and outcomes to the model. The findings of this study may be useful in understanding the physical features of various maritime engineering issues.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

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