A MILK-RUN DISTRIBUTION SYSTEM DESIGN FOR INTEGRATING DRONES

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ABSTRACT

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FOR INTEGRATING DRONES

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The frequency and scope of use of logistics activities are increasing rapidly all over the world. As a result of the continuous development in technology, new transportation vehicles are produced and are also started to being used in logistics activities. One of these vehicle types is the unmanned aerial vehicle (UAV), also called as drone.

Milk-Run distribution is one of the most basic logistics delivery activities. In the literature, there exists studies where drones are used together with trucks in milk-run distributions; but a case where the truck makes a continuous movement during the milk-run tour and the delivery is made with only drones has not yet been analyzed.

The aim of this study is to analyze the benefits of this specific case with respect to problem specific parameters and to reveal the situations that it is superior to the classical milk-run distribution. The methodology used in this study is mathematical modeling approach, including linear and nonlinear mixed-integer programming models. The proposed model is original since 'time' is considered to be continuous rather being discrete and time is directly represented by decision variables rather than a discrete index set of decision variables in creating the required constraints. To alleviate the complexity brought by nonlinearity, the actual flight times of the drones are estimated by quadratic regression models, which are shown to produce estimations quite close to actual values. All the problem specific parameters are found from or estimated in accordance with state-of-the-art technology and real-life sources.

The proposed model was run on a numerical setting that consists of those parameters which includes two drone speeds, two flight ranges, and two cases which in terms of presence of empty drones at the customers before start of the milk-run. Our main finding is that integrating drones in a milk-run distribution is beneficial compared to the classical milk-run under any problem setting. However, the cost difference between those two milk-run types does not seem to be sufficient to make an investment for integrating drones as we propose under current cost figures. This result is also confirmed by several practitioners.

Keywords: Milk-Run Distribution, Unmanned Aerial Vehicle (UAV), Drone, Regression, Mathematical Modeling, Nonlinear Mixed-Integer Programming Model.

ÖZ

A MILK-RUN DISTRIBUTION SYSTEM DESIGN

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Lojistik faaliyetlerin kullanım sıklığı ve kapsamı tüm dünyada hızla artmaktadır. Teknolojideki sürekli gelişme neticesinde yeni taşıma araçları üretilmiş ve lojistik faaliyetlerde de kullanılmaya başlanmıştır. Bu araç türlerinden biri de insansız hava aracıdır (İHA).

Sürekli sevkiyat, en temel lojistik dağıtım faaliyetlerinden birisidir. Literatürde sürekli sevkiyatta İHA'ların kamyonlarla birlikte kullanıldığı çalışmalar bulunmaktadır; ancak sürekli sevkiyat sırasında kamyonun sürekli hareket ettiği ve teslimatın sadece İHA ile yapıldığı bir durum henüz analiz edilmemiştir.

Bu çalışmanın amacı, bu belirli durumun probleme özgü parametreler açısından faydalarını analiz etmek ve klasik sürekli sevkiyata üstün olduğu durumları ortaya çıkarmaktır. Bu çalışmada kullanılan yöntem, doğrusal olan ve doğrusal olmayan karma tamsayılı programlama modellerini içeren matematiksel modelleme yaklaşımıdır. Önerilen model orijinaldir, çünkü 'zaman' kesikli değil, sürekli olarak kabul edilir ve zaman, gerekli kısıtlamaların yaratılmasında karar değişkenlerinin ayrı bir indeks kümesi yerine karar değişkenleri tarafından doğrudan temsil edilmiştir. Doğrusal olmamanın getirdiği karmaşıklığı hafifletmek için, insansız hava araçlarının gerçek uçuş süreleri, gerçek değerlere oldukça yakın tahminler ürettiği gösterilen ikinci dereceden regresyon modelleriyle tahmin edilmiştir. Probleme özgü tüm parametreler, en son teknoloji ile üretilmiş gerçek hayat kaynaklarından bulunarak direkt olarak kullanılmış veya bunlara paralel olarak tasarlanmıştır.

Önerilen model, iki İHA hızı, iki uçuş menzili ve sürekli sevkiyat başlamadan önce müşterilerde boş İHA'ların varlığı açısından iki durum içeren parametrelerden oluşan sayısal veri setleri kullanılarak çalıştırılmıştır. Bu çalışmanın ana bulgusu, İHA'ları bir sürekli sevkiyata entegre etmenin, problemin tüm parametre ayarlarında klasik sürekli sevkiyata kıyasla daha faydalı olduğudur. Ancak, mevcut maliyetler kapsamında önerildiği gibi, bu iki sürekli sevkiyat tipi arasındaki maliyet farkı, İHA'ları entegre etmek için bir yatırım yapmak için yeterli görünmemektedir. Bu sonuç aynı zamanda birkaç uygulayıcı tarafından da doğrulanmıştır.

Anahtar Kelimeler: Sürekli Sevkiyat Dağıtımı, İnsansız Hava Aracı (İHA), İkinci Dereceden Regresyon Modeli, Doğrusal Olan ve Doğrusal Olmayan Programlama.

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NOMENCLATURE

Use	**Synonym**	**Abbr.**	**Turkish**	**Synonym**
Milk-run			Sürekli Sevkiyat	Döngüsel Hareket
Vehicle	Truck	V	Araç	Kamyon
Drone	UAV (unmanned aerial vehicles)	D	İHA (İnsansız Hava Aracı)	
Item	Parts/materials	I	Ürün	
Location	Customer	L	Müşteri	
Depot	Receiving plant		Depo	
Arrival		arr	Varış	İniş
Departure		dep	Kalkış	
Limit			Limit	Üst sınır
Setup			Kurulum	
Classic Milk-Run	CMR			
Drone Milk-Run	DMR			
Hybrid Milk-Run	HMR			
Launch	Dispatch, Take-off, Release, Send-off		Kalkış	
Land	Reconvene		İniş	
Departure			Çıkış	
Arrive			Varış	
Loaded			Yüklü	Dolu
Empty			Boş	

CHAPTER 1

INTRODUCTION

The use of logistic activities throughout the world has an exponential incline. According to IMARC (2019), the worldwide value of the logistic market has reached a fairly high value such as $ 4,000 trillion. Considering this enormous value, it is not surprising that in the logistic market there are various applied studies with the main purposes of increasing efficiency and minimizing cost. One of these applications is the **"Milk-Run Distribution"**, which is a circular transportation method. Milk-Run distribution is an approach that is first founded on the delivery and collection of milk in the northern parts of the UK. Starting from the center, while moving on the pre-determined route, the vehicles leave the milk-filled bottles at the door of each household (i.e. the customer), collect the remaining empty bottles during delivery, and then return to the center (Satoh, 2008).

The general name used for logistics of internal or external units in procurement by manufacturers and/or customers is "Milk-Run Logistic System" (Sadjadi et al., 2009). In this system, all the input and output (I/O) material requirements of manufacturers and/or customers (represented by stations) are satisfied within the milk-run route. In case the I/O volume of the stations is much less than truck capacity, the milk-run logistic system provides economical solutions (Baudin, 2005).

Meyer (2015) states the characteristics of the milk run logistics system as follows:

- The system has at least one supplier which supplies the demands of specified stations in a fixed tour, according to a fixed schedule, and with a fixed sequence of stops.
- Demands are determined daily which enables an ordering policy that aggregates the request between shipments.
- The supplier plans the milk-run.

- The constructed plan is valid for a long period of time such as a few weeks or months.
- Transshipments are allowed within the milk-run.
- The route of a milk-run system can be a circular tour that starts and ends at the depot.
- The distance and duration of the route determine the cost of the logistic system.

Some examples of milk-run logistics are given in Figure 1:

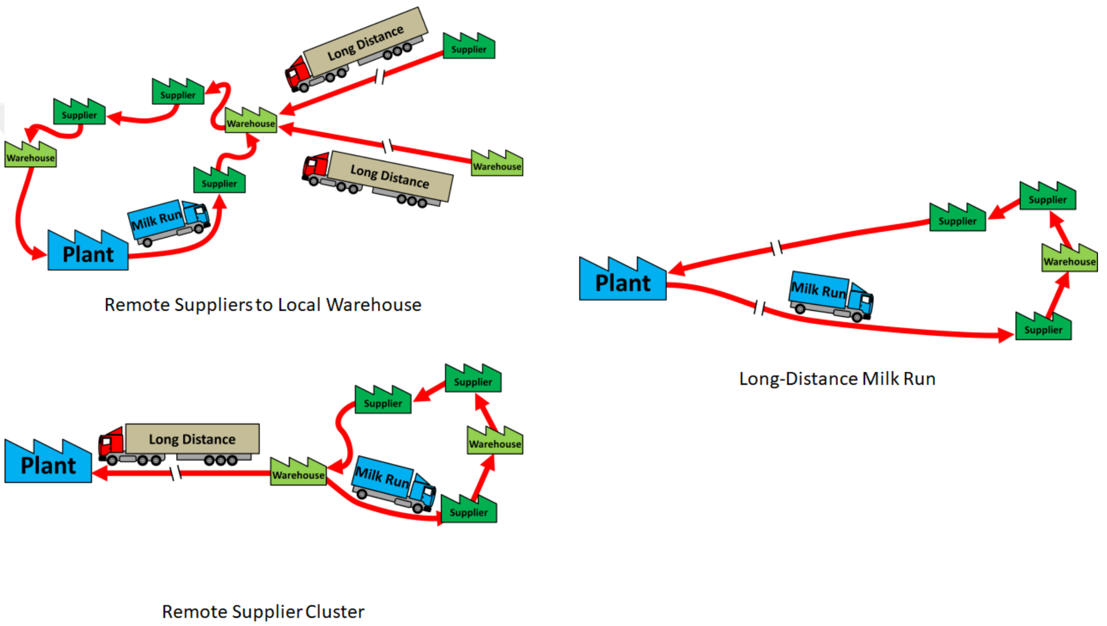


Figure 1. Examples of Milk-Run Logistics (Roser, 2019)

One of the most common uses of milk-run distribution is that manufacturers use it for collection instead of distribution. While collecting the materials from the suppliers on certain days of the week on a pre-defined route with a truck, they leave empty containers used to bring the materials to these suppliers (Sadjadi et al., 2009). To illustrate, EKOL Logistics brings the materials it regularly receives from a Turkish truck manufacturer to the production facility on certain routes determined by the manufacturer (Satır et al., 2018)

Meyer (2015) defines one of the important features of milk-run distribution as the possibility of replacing empty and full containers in a cycle that starts and ends at the center. Some of the advantages of milk-run distribution are the aggregation of loads of different manufacturers, reduction of the transportation costs, minimization of the total distance, and the increase in the utilization of the loading capacity of the vehicles (Brar & Saini, 2011).

With the ongoing improvement of technology, the transportation methods continuously adapt to this improvement and make use of new methodologies and devices developed. One of these developed devices is the **"Unmanned Aerial Vehicle"** (UAV), which is also called as drone. In general, if an aircraft does not require a human operator during flight, it is considered a UAV, which means that the vehicle is operated remotely or independently. The two main applications of UAVs are military and civilian. As might be expected, the first use of the drone was a military application, which is accomplished by the British Royal Navy in 1933. The use and development of drones in commercial and civil areas become widespread much later. But there is a rapid increase in their use and the civil use of UAVs has increased by almost threefold since 2005 (Güner, Rathnayake, & Ahmadi, 2017).

The models and features of some of the UAVs currently available on the market are given in Table 1. Ehang is a special model which is known as Dubai Taxi. It is used for human transportation, has a pretty high carrying capacity, and has a maximum speed of 100 km/h. Except for Ehang, the carriage capacities are very limited and Inspire 2 model has the highest speed of 94 km/h. The Griff 135, Griff Auteur and Griff Savior models have high carrying capacity and are suitable for material transportation. In all UAV models, the maximum flight time is around half an hour.

Table 1. UAV Models and Features (DJI, 2020, EHang, 2020 and Griff, 2020)

UAV Model	Max Flight Time (min)	Max Carriage Capacity (kg)	Max Speed (km/h)	Price
Matrice 210 RTK	32	3.27	82.8	unavailable
Matrice 210	27	2.3	61.2	$6,500
Matrice 600 Pro	38	6	65	$8,599
Inspire 2	27	3.4	94	$3,499
Prime Air	unavailable	2.7	24	unavailable
Ehang	23	100	100	$200,000
Griff 135	45	75	unavailable	unavailable
Griff Auteur	45	30	unavailable	unavailable
Griff Saviour	45	200	unavailable	unavailable

The main advantage of UAV is that it does not encounter highway transportation problems. On the other hand, it currently has many disadvantages as well. Some of these disadvantages are limits on the flight times because of battery run times, low carriage capacities, and limited maximum speed they can reach. Due to its disadvantages, the current technology of UAVs is not sufficient to be fully used for transportation in an efficient manner. On the other hand, there are two facts we want to stress about the recent development of the use of UAVs.

First of all, drones are inevitably becoming popular in every field of industry. Domino's Pizza was the first who started using UAVs in order to deliver pizza to its customers. Next, Amazon adopted the use of drones in providing faster services. Having realized the success of using drones, the cargo company UPS started using them, but for longer distances and heavier products, and a picture of these drones is given in Figure 2 (UPS, 2021). With respect to the company’s report named 'Unmanned Aerial Vehicles' 2014, there exist seven different areas of best practices. These practices are defined as (i) energy/infrastructure, (ii) agriculture/forestry, (iii) construction industry, (iv) environmental protection, (v) emergency response and police, (vi) film and (vii) photography development (Güner et al., 2017).

Figure 2. UPS Drone Delivery Service

Accordingly, despite of the fact that it is difficult for various reasons such as technology limitations, security, privacy violation, and public view usage of UAV for daily life; (Lewis, 2014; Keeney, 2016; Wang et al., 2017) it was launched by world-renowned companies such as Amazon, Google, Walmart and DHL (Hovrtek, 2018 and Butter, 2015) and today it is especially applied in the last step deliveries state that milk-run distribution started in the dairy industry and have been widely used in various fields for a long time.

One such field is retail e-commerce, which has become widespread nowadays and the sector size in the world has been very close to $ 3 trillion in 2018. Companies that make distributions to end customers, especially retailers, are researching innovative methods for distribution. One such method is "last-mile delivery". The last-mile delivery is the last step of the supply chain. It contains the delivery of relatively small purchase points. These purchase items have the characteristics of having low demands in size but being frequent. UAVs are being tested for last-mile delivery in logistics due to their suitability. The reason UAVs are suitable for this purpose is that they do not encounter road transportation problems. Therefore, they will be able to perform more efficient and faster delivery and collection from the vehicle.

Large multinational e-commerce companies such as Amazon, Google have invested substantially in the research and development (R&D) of drone technology to use drones in their last-mile deliveries. There are also various other companies worldwide that follow these big multinational companies in terms of working to develop new technology for drones in a similar manner (Erceg et al., 2017).

In the United States, the first transportation company to achieve a drone delivery has been Wing, according to the news announced on Wing's official website on 23 April 2019. The company received Air Carrier Certification from the United States Federal Aviation Administration (FAA). Chao, who is U.S. Secretary of Transportation, declared the integration of drones to transportation being an important step in the economy of the USA (Wing becomes first certified Air Carrier for drones in the US, 2019). Around six months later, another important transportation company UPS, followed its competitor Wing and started delivery of medicine ordered by its customers using drones (Bıktım, 2019).

Although the transportation companies in Turkey have not started to use drones yet, an interesting event has occurred in March 2020 at Serik, Antalya. A citizen named Hasan Kurt did his shopping from a grocery store by drone without going out to protect himself and his family from COVID-19. After this event, the grocery store owner İbrahim Çetin announced that they plan to provide four or five drones to supply customer demands via drone transportation (Demirci, 2020). This news also supports the fact that drone transportation is capable to have very important side benefits such as measures taken against the coronavirus.

It is estimated that the dollar volume of drone production throughout the world will increase from its current level of 4 billion USD per year to a level of 14 billion USD per year within the following ten years. Considering the current plans and anticipated plans in drone R&D, and its potential uses, drones have clearly performed major advances. These advances include the discovery of new platforms in which drone robustness and autonomy have increased and the development of necessary software that analyses and processes captured images. Today, the UAV market is mainly dominated by military applications with 72 percent of the overall market. It is followed by consumers with 23 percent and civilian applications with 5 percent. Although the implementation in civil sectors currently is at a relatively low level, it is growing rapidly (Erceg et al., 2017).

One of the main motivations of this research is as follows. Amazon’s CEO Jeff Bezos believes that in the future, drone delivery will be normal just like mail trucks. This shows the fact that the understanding in business logistics is changing. The related findings by Material Handling Institute showed that in future supply chains, the use of drones will be more important in parallel to the discovery of new technologies (Erceg et al., 2017).

The second fact we will consider about the recent development of the use of UAVs is that there is an increasing number of academic studies for the modelling of transportation systems which includes UAVs. The number of publications has been single-digit since 2001, exceeded the number ten for the first time in 2013, and approached the hundreds level in 2017 with an exponential increase (Otto et al., 2018). According to the study of Otto et al. (2018), distribution-related studies are discussed as vehicle routing and traveling salesman problems. However, to the best of our knowledge, a delivery where the truck makes a continuous movement during the tour and the delivery is made by only the drones has not yet been studied.

Within the scope of this thesis, we will discuss some of UAV's usage methods in the milk-run distribution and study the optimization of a specific system with mathematical programming models that we develop. The basic and the least complex method is the classical application in milk-run distribution, which is as follows. First, a vehicle is loaded at the distribution center or warehouse. Next, it follows a predetermined route on certain periods, leaves its loads which are brought by the containers (cages, special transport units, etc.) on this route, and brings the remaining empty containers back to the center.

At this point, we would like to explain the reason why the model we propose is called a milk-run model. The basic similarity between the classic milk-run model and the model we propose is as follows. In the classic milk-run, the empty containers are left at the customers served and are collected at a later time. Likewise, in the model we propose, after satisfying the customer’s demand, the empty drones are left at the customer to be taken off later. For this reason, calling the proposed model a milk-run would be consistent with the concept of milk-run. However, as might be expected, leaving the UAVs at the customers would incur a higher cost compared to the cost of the containers. In addition, there are some physical difficulties in holding the UAVs at the customers. If a UAV is to be held outside; it may be badly affected by adverse weather conditions such as snow, rain, etc. In addition, there would be security problems caused by holding the UAVs outside. On the other hand, if a storage area for UAVs were to be arranged, there would be extra costs. However, in this study, such difficulties are ignored. The assumption that such difficulties can be overcome can be attributed to the fact that the nature of this work is a futuristic one.

Besides the classical application, UAV use in milk-run distribution can be designed in many other ways, some of which are as follows. Vehicle's loads can be sent completely with UAVs, the vehicle can complete its route without going to any point, the vehicle can stop at some points that do not involve customers and meanwhile distribute to surrounding customers with UAVs, and so on. Moreover, hybrid use of vehicles and UAVs is also possible in which the vehicle serves to some customers while UAVs are serving to nearby customers. The main structures of the milk-run models with their important features are summarized in Table 2.

Table 2. The Structure of Main Milk-Run Types

Structure of Milk-Run Distribution	Classical Application	Hybrid use of UAV and truck in delivery	Only UAV delivery where truck waits for UAV	Only UAV delivery with a slower truck	Only UAV delivery with a faster truck
Does truck make delivery?	YES	YES	NO	NO	NO
Is truck speed less than UAV speed?	(no UAV)	unimportant	unimportant	YES	NO
Does the truck stop at customer points?	YES	YES	NO	NO	NO
Does the truck stop at specific points?	inapplicable	NO	YES	NO	NO
Does UAV make delivery?	inapplicable	YES	YES	YES	YES

The left-most and right-most columns in Table 2 stand for the two end structures in terms of additional assumptions added. Starting with the classical milk-run by adding, changing, or relaxing some of the model assumptions, one can obtain the other structures given in the columns to the right of the classical milk-run. In the column next to the classical application, the properties of the hybrid structure in which both the vehicle and UAVs make deliveries is given. In the hybrid structure, since the truck waits for the UAV during its delivery, it does not matter whether the truck speed is higher or lower than the UAV speed. Moreover, trucks stop at customer points since it also makes deliveries, but it does not stop at any other points and UAV also makes deliveries. These details can be followed in the remaining cells of the second column. In the structure given in the third column which is "Only UAV Delivery Where Truck Waits for UAV", the truck pauses at certain points, but not at the customer points. Finally, in the structures given in the last two columns, the truck neither makes a delivery nor waits for the UAVs. The difference between these two structures is that in the fourth column, UAV is faster whereas, in the last one, the truck is so.

The structure we will study is the one in the fifth column: "**Only UAV delivery with a faster truck**". The basic assumptions of this structure are given as follows:

i. UAV's speed is less than the truck's speed.
ii. The truck never stops and moves at a fixed speed.
iii. UAVs cannot move between customers.

The method of the study is mathematical modeling, which includes nonlinear and mixed integer models. The mathematical model that we propose for this structure of milk-run is given in detail in the "Model" section of this study. The model we propose is original compared to the models in the literature since 'time' is taken as a decision variable rather than a clustered index to be used in creating necessary constraints.

We will explain the importance of those studies involving milk-run models with UAV delivery in general. Specifically, we explain the importance of this study in three categories, which are its academic, economic, and social effects.

Due to the fact that UAVs more actively partake in daily life, the demand for UAV usage is expected to increase in all other sectors, especially in the logistics sector. Accordingly, studies on distribution models with UAVs have become widespread in recent years. Hence, studies in this area have a high academic impact and have the potential to start National / International R&D Collaborations. Companies and universities will be able to accelerate R&D activities, increase the number of researchers and enable university-industry collaborations to satisfy this increasing demand. The main academic importance of this study is that it is expected to contribute to the literature by the use of UAVs in the milk-run distribution.

When it comes to economic effects, it is important to note that, UAV production is mostly foreign originated currently. Therefore, within the midterm, it is expected that UAV production will be carried out domestically through foreign capital investment or domestic UAV production will be realized by entrepreneurs in our country. As domestic or foreign new investments are created, the number of people participating in employment will increase. With the increase in the use of UAVs, such contributions as low cost and high-speed delivery are expected to strengthen our economy. On the other hand, researches on potential application areas especially in the logistics sector and diversification of corresponding studies hereby on distribution models with UAV will increase the competitiveness of such companies as UPS, DHL, MediaMarkt, Amazon, etc. who want to apply these models.

Last but not least, we will explain the expected social effects of milk-run distribution studies with UAV. As UAVs will cover some of the distance required for delivery, trucks will make less mileage. Air pollution and carbon emission will be lessened due to the fact that UAVs' carbon-dioxide generation is almost negligible compared to vehicles working with fossil fuel because UAVs are powered by electricity. Especially in the metropoles, the traffic problem will be reduced in part and thus, the cities will become more comfortable.

CHAPTER 2

LITERATURE REVIEW

In an effort to provide faster and more cost-efficient delivery for goods ordered online, companies are looking for new technologies to bridge the last-mile to their customers. One technology-driven opportunity that has recently received much attention is the deployment of unmanned aerial vehicles or drones to support parcel delivery. An important advantage of a delivery drone as compared to a regular delivery vehicle is that it can operate without a costly human pilot. Another advantage is that a drone is fast and can fly over congested roads without delay.

Several companies, including Amazon, Alibaba, and Google, are currently running practical trails to investigate the use of drones for parcel delivery (Popper, 2014). These trails typically involve multi-propeller drones that can carry parcels of approximately 2 kilograms over a range of 20 kilometers. There are examples of drones that are already used for deliveries in practice, albeit solely in a non-urban environment. DHL Parcel, for instance, recently started operating a drone delivery service to deliver medications and other urgently needed goods to one of Germany's North Sea islands (Hern, 2014). In this example, the drone flies automated but still has to be continuously monitored. Aeronautics experts expect that drones will be able to fly autonomously and safely in urban environments within the next few years, based on rapid advances in obstacle detection and avoidance technology (Bensinger & Nicas, 2015).

Agatz et al. (2018) propose a system in which the delivery truck and the drone collaboratively serve all customers. While the delivery truck moves between different customer locations to make deliveries, the drone simultaneously serves another set of customer locations, one by one, returning to the truck after each delivery to pick up another parcel. From a transportation planning perspective, this innovative new concept gives rise to several relevant planning problems. Even for a single truck and a single drone, the problem involves both assignment decisions and routing decisions. This variant of the traveling salesman problem (TSP) is called the TSP with Drone (TSP-D). They develop a new integer programming formulation as well as several fast route first-cluster second heuristics based on local search and dynamic programming for the TSP-D. They prove worst-case approximation ratios for the heuristics and test their performance by comparing the solutions to the optimal solutions for small instances.

Ferrandez et al. (2016) examine a truck-drone team from an operational viewpoint to better understand the impact of the number and location of truck stops with regards to its effect on delivery time and energy requirements. Initially, they analyze a single drone to deliver all packages to all locations. This requires one truck stop centrally positioned among the delivery locations using K-means. The drone uses a hub configuration to egress and ingress from the truck to each delivery location and back, not constrained by range. They intend to understand the total time, cost, and energy involved in a hub configuration (star-distance) in order to contrast this configuration with truck-only delivery using a TSP route. TSP truck route is computed using a genetic algorithm to satisfy all the deliveries to all the locations. Furthermore, they use a combination of truck and drone to find the optimal number of truck stops and locations using the K-means algorithm to cluster demands in conjunction with a TSP genetic algorithm. The problem herein assumes that one or more drones and a single truck work in tandem to deliver packages to delivery locations within a given delivery space; and that the uniformly distributed delivery demands are known a priori. The drones are not constrained by range to gain a better sense of the upper/lower boundaries of time and energy. The truck is constrained to move along a TSP route while the drone is constrained to egress and ingress from the truck in hub (star) configuration to a nearby delivery location and then back to the truck.

Poikonen et al. (2017) consider a fleet of m homogeneous trucks each carries k drones with a speed of α times that of the truck. Each drone may dispatch from the top of the truck and carry a package to a customer location. The drone then returns to the top of its truck to recharge or swap batteries (we assume instantaneously). The truck itself is allowed to move and deliver packages but must be stationary at a delivery location or the depot when launching or retrieving drones. The goal is to minimize the completion time to deliver all packages and return all vehicles back to the central depot.

Ham (2018) extends the problem by considering two different types of drone tasks: drop and pickup. After a drone completes a drop, the drone can either fly back to the depot to deliver the next parcels or fly directly to another customer for pickup. Integrated scheduling of multiple depots hosting a fleet of trucks and a fleet of drones is further studied to achieve operational excellence. A constraint programming approach is proposed and tested with problem instances of m-truck, m-drone, m-depot, and hundred-customer distributed across an 8-mile square region. This paper contributes a novel application of constraint programming (CP) to multi-truck, multi-drone, and multi-depot scheduling problems constrained by time-window, drop-pickup, and multi-visit, with the objective to minimize the maximum completion time over all tasks.

Baloch & Gzara (2020) study the economic feasibility of UAV parcel delivery in terms of its impact on an e-retailer's distribution network while taking into account customer preferences, locational decisions, and regulatory and technological limitations. They consider an e-retailer offering multiple same day delivery services including a fast UAV service and develop a distribution network design formulation under service based competition where the services offered by the e-retailer not only compete with the stores (convenience, grocery, etc.) but also with each other. To solve the resulting nonlinear mathematical formulation, they develop a novel logic-based Benders decomposition approach and build a case based on NYC, carry out numerical testing, and perform sensitivity analyses over delivery charge, delivery time, government regulations, technological limitations, customer behavior, and market size. The results show that government regulations, technological limitations, and service charge decisions play a vital role in the future of UAV delivery.

Wang et al. (2017) study the vehicle routing problem with drones (VRPD). A fleet of trucks equipped with drones delivers packages to customers. Drones can be dispatched from and picked up by the trucks at the depot or any of the customer locations. The objective is to minimize the maximum duration of the routes (i.e., the completion time). Drone delivery (from trucks) would enable trucks to visit customers located centrally on the route and drones to visit farther-away customers.

In other words, trucks would get “close enough” to more distant customers and then dispatch drones. Drone delivery could reduce the number of required trucks and drivers on the road. More significantly, drones might speed up delivery. They pose several questions in order to study the maximum savings that can be obtained from using drones; then derive a number of worst-case results. The worst-case results depend on the number of drones per truck and the speed of the drones relative to the speed of the truck.

Han et al. (2019) consider the utilization of drones for delivery with the aim of increasing customer satisfaction and minimizing transportation costs per delivery in a green way. İstanbul’s Bakırköy district is selected as the implementation region, and daily drug delivery to pharmacies with drones is examined. In the first stage, a mathematical model that uses pharmacy coordinates as input is developed for clustering analysis. In the second stage, one of these clusters is selected, and the location of the drone center that will serve pharmacies in the cluster is obtained by the center of gravity method. Then, a vehicle routing model is proposed for finding the drone routes and calculating the total distance travelled within this cluster.

Cheng et al. (2018) solve a multi-trip drone routing problem, where drones' energy consumption is influenced by payload and travel distance whereas such relationships are nonlinear. To tackle the nonlinear (convex) energy function, which can be incorporated in the objective function within this particular problem, they propose two types of cuts, logical cuts, and sub-gradient cuts. This allows making an exact calculation of energy consumption, instead of using the linear approximation method as in the literature, which can fail to detect infeasible routes due to excess energy consumption. They introduce two formulations to solve the problem, one with a drone index and the other without, which are further enhanced by valid inequalities. Branch-and-cut algorithms are developed for the formulations and benchmark instances (with up to 50 customers) are first generated for this problem. Extensive numerical experiments indicate that the formulation without a drone index is superior in solving more instances to optimality and providing high quality solutions for all the generated instances. The results also indicate that even though the original model is nonlinear, the proposed approach is highly efficient, and the performance does not deteriorate so much compared to the linear approximation model in which the structure is much simpler.

Pugliese & Guerriero (2017) address the problem of managing a drone-based delivery process and consider the specific situation of a delivery company, that uses a set of trucks equipped with a given number of drones. In particular, items of limited weight and size could be delivered by using drones. A vehicle, during its trip, can launch a drone when serving a customer, the drone performs a delivery for exactly one customer and returns to the vehicle, possibly at a different customer location. Each drone can be launched several times during the vehicle's route. It is imposed a limit on the maximum distance that each drone can travel and synchronization requirements between vehicle and drone should be ensured. In particular, it is assumed that a vehicle waits for a drone for a maximum period of time. The aim is to serve all customers within their time window. The problem is modeled as a variant of the vehicle routing problem with time windows. The aim of this work is to analyze the delivery process with drones, by taking into account the total transportation cost and highlighting strategic issues, related to the use of drones. The numerical results, collected on instances generated to be very close to reality, show that the use of drones is not economically convenient in classical terms. However, when considering negative externalities related to the use of classical vehicles and quality of service requirements, the benefit of using drones becomes relevant.

Carlsson & Song (2018) study the efficiency of a delivery system with UAVs. In the system the UAVs provides service to customers and return to a truck that is moving. The UAV picks the package from the truck, deliver the product to the customer, and return to the truck that is moving for picking the next package. It is stated that the hardware for this system is ready, but it is not completely understood to what extent such an approach can provide improved quality of service. They conduct a theoretical analysis in Euclidean plane using real-time simulations on a road network. As a result, they conclude that the efficiency improvement is proportional to the square root of the ratio of the speeds of the truck and the UAV.

Drones are known for the advantage of speed, ease in delivering commodities to customers, and flexibility in services. in addition they are extremely useful for the tasks that are dangerous and dull. Chiang et al. (2019) argue whether the use of drone delivery is beneficial to the environment and results in cost savings. It is stated that drone delivery results in lower energy consumption and hence reduce CO_2 emissions and carbon footprint. They study the impact of drone delivery on CO_2 emissions and costs. They propose a binary mixed integer programming model to exploit the sustainability features of drones for parcel delivery services. As a result of computational analysis, it is shown that drones are effective in terms of cost and environmental friendliness.

Dorling et al. (2017) propose two multi-trip vehicle routing problems for drone delivery that addresses cost minimization subject to delivery time limit and overall delivery time minimization subject to budget constraint. They mathematically drive an energy consumption model for multirotor drones and validate the results experimentally. They demonstrate that energy consumption varies approximately linearly with battery weight and total payload.

Khoufi et al. (2019) propose a literature review related with UAV path optimization problems and they focus on mobility on macroscopic scale. They study the recent literature that modified the problems to the UAV context, offer a general classification and taxonomy of the problems and their formulation. In addition, they provide a synthetic overview of the resolution techniques, performance metrics and the numerical results.

Kilic et al. (2012) categorize and explain the milk-run distribution problem in the plants based on real manufacturing environment and related literature. For one main category they develop a model that minimizes the number of vehicles and the distance travelled. They also present a numerical example for a real-world application and explain the applicability of the developed models.

Marinelli et al. (2018) propose a novel approach that maximizes the use of drones in parcel delivery. The assume that a truck can pick up and deliver a drone not only at nodes of the network, but also along the route. Hence the operations of drones are not strictly related with the customer locations and hence they can serve a larger area along the truck route. They tested the proposed heuristic on test instances and state the benefits of the proposed approach.

Ponza (2016) consider the cooperation between a truck and a drone for last-mile delivery. He considers a system where the drone launches from the truck, deliver the goods to a customer and then meets with the truck. While the drone is flying, the truck delivers to other customers as long as the drone has enough battery to hover waiting for the vehicle. A mixed integer mathematical model is constructed and solved using Simulated Annealing metaheuristic. The numerical analysis show that significant savings can be obtained if trucks and drones are used together.

Otto et al. (2018) conducts a literature review on optimization approaches to civil applications of drones. They explain the most promising drone applications and summarize the characteristics of drones applicable to operations planning. More than 200 articles are reviewed and they insights are provided regarding emerging modeling approaches. they also suggest some future research directions.

CHAPTER 3

PROBLEM DEFINITION

In this chapter, we will give the problem definition using illustrative demonstrations. First, we will introduce the problem environment with a potential real-life example presented in the city map of Ankara. Via this example, both classical milk-run distribution and milk-run distribution only with drones are handled and the advantages of drone usage in the milk-run distribution in terms of time and distance are presented. The next section will be devoted to the abstraction of the problem. Via the abstraction, the problem will be mathematically tractable and suitable for modelling. In the third section, we will explain the methodology that is used in calculating the flight times of the drones, from the truck to the customers and vice versa. Finally, in the last section of this chapter, we will clarify the abstraction by two giving two examples. The first example is a classical milk-run model in which delivery is made only by the truck. The second example considers a delivery with drones while the truck follows its route. This second example contains the notation of sets and parameters of the mathematical model that we will introduce in Chapter 4 in detail.

3.1. Problem Environment

The example to be given in this chapter is not completely an application of a real-life example but the hypothetical example is designed on a real map. In particular, the problem to be modeled as milk-run with drones will be explained on a hypothetical example built on the Ankara map.

In this example, it is assumed that the demands of seven customers with fixed locations are delivered from the warehouse in the center; the default locations of the warehouse and customers are shown on the Ankara map in Figure 3. The truck, which is initially on the warehouse at point 0, will deliver the demands of the customers located at points numbered 1 to 7. In order to more clearly demonstrate the advantages of the milk-run model that will be proposed using the drone, the situation in which delivery is made with the classical milk-run will be handled first.

In Figure 3, the route of the shortest path problem is assumed to be solved for the classical milk-run is given. In this route, there will be situations just like as it travels from the second customer to the third customer where the truck uses the highway. It is also possible that the truck can choose to go into the city traffic for deliveries and continue through the city just like in the case when the truck travels from the fifth customer to the sixth customer. The length of the route of this example is 179 kilometers and the total duration of the route is given by Google Maps as 3 hours and 17 minutes, excluding pauses at customer points (11 November 2019, 15:12).

Figure 3. Classical Milk-Run Distribution Route

If the delivery in this example is made with the designed milk-run model using the drone, a route similar to that shown in Figure 4 will appear. This route is 111 kilometers long and is shorter than the classical milk-run route. Likewise, the duration of the route is completed in 1 hour and 5 minutes and is much shorter than the classical milk-run route. Compared to the milk-run distribution made with drones, the classical milk-run distribution has a 61% longer tour length and takes 203% more time.

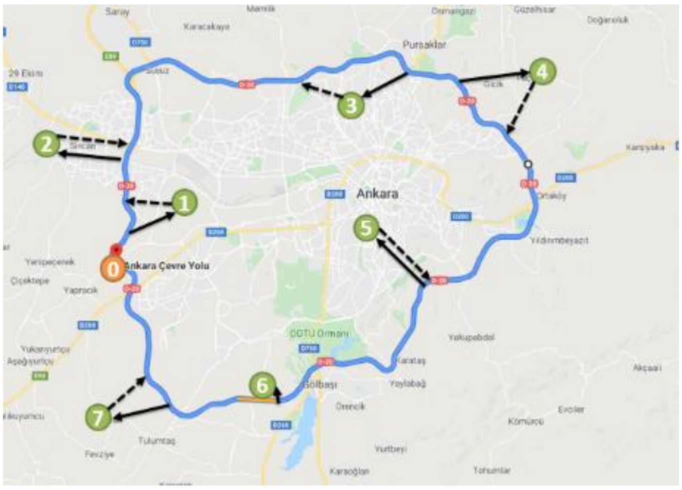


Figure 4. The Route of Milk-Run Distribution with Drones

In the milk-run distribution system with drones, the truck does not make a direct delivery to customers. As can be followed from Figure 4, when the truck approaches each customer, delivery is carried out by transporting the requested product to the customer through the drone. On the other hand, the drone, which is already empty in the customer, should move towards the truck in the right direction and correct time so that they can meet at a common point.

In Figure 4, straight arrows show the path of the drones that go from the truck to the customer, and the dashed arrows show the path of empty drones from the customer to the truck. That the truck sends the full drone to the customer and takes the empty one from the customer simultaneously is exemplified by the second customer on the map. On the other hand, the truck may pick up the empty drone from the customer before sending the full drone to the customer as exemplified by the third customer. In cases where the empty drone was taken into the truck early enough; it is also possible to load the product requested by the customer to the same drone, change its battery if necessary and then send it back to the same customer.

In comparison with the speed of a truck moving on the highway in the current technology, the speed of the UAV is expected to be lower. Therefore, after the product is delivered to the customer, it is only possible for the UAV to return to the truck if the truck waits to receive the UAV. On the contrary, most of the studies on this subject in the current literature assume that the UAVs used during delivery are faster than trucks. This assumption may be valid in cases where the truck cannot travel fast due to heavy city traffic. However, in a milk-run distribution using the highway as described here, this assumption does not seem a valid one.

When it comes to a possible investment in UAVs, someone who evaluates that UAVs are slower than trucks may ask what kind of advantage the use of UAVs will provide in an intended type of transportation. It may be thought that there are justifiable reasons for asking this question because the investment in question may include high cost details such as the infrastructure required to collect UAVs that remain at the customer point in the next day.

Despite the high investment cost that will be required initially, distribution with UAV can provide economic advantages in the long term due to three reasons. First of all, considering the improvements obtained in possible performance measures of the distribution in the imaginary sample designed on the Ankara map, it is anticipated that milk-run distribution with UAVs may have promising benefits. Even if a small cost reduction in a route is achieved compared to classical delivery, UAV transportation will become attractive for those companies whose long-term investments are at the forefront and who carry a large number of units daily. Second, slower UAVs do not always result in slower completion of the route. In cases where customers are relatively far from the route to be used in UAV transportation and thus it is difficult to reach these customers, that the truck waits to take UAVs may not extend the total time of the route. Finally, with the development of UAV technology in the future, it will be possible to produce UAVs faster than trucks and UAVs will be possibly produced at lower costs. Therefore, being able to detect the situations in which UAV transportation is more profitable will provide important advantages for the future.

This study presents a milk-run model for cities having a circular-like highway around them. We have already illustrated a hypothetical milk-run tour at Ankara. In Figure 5, we present two additional examples of such cities appropriate for the possible use of the proposed model in this study. In the figure to the left city of Eindhoven and to the right Indianapolis can be seen.

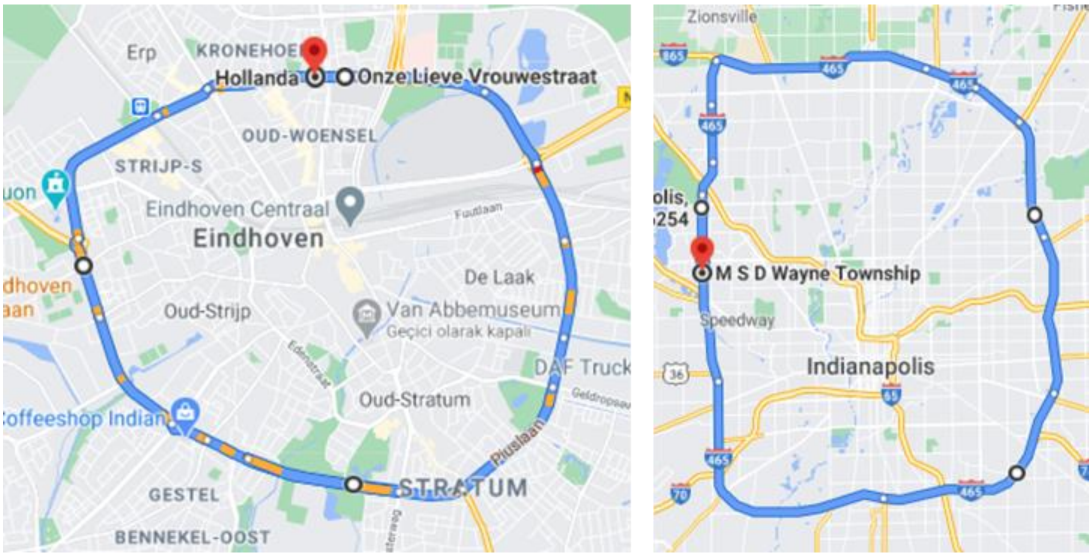


Figure 5. Examples for Circular Highways around the Cities

3.2. An Abstraction of the Problem

Before introducing the general characteristics of the milk-run model with drones, we will illustrate the abstraction of a classical milk-run model. In the classical milk-run model, the truck follows a predetermined route within the city and performs the delivery of the goods demanded by the customers. In the actual tour, the traveling times between the depot and the customers, and within the customers cannot be strictly determined due to some unexpected traffic or weather conditions. Moreover, some unexpected daily life problems like being sick can lead to delays. But in order to enable the tractability of the problem, such cases are generally neglected thinking that they do not change the overall performances significantly. Therefore, we can abstract a classical milk-run tour by assigning average speeds as constant speeds and using bird's-eye distances instead of actual distances and neglecting weather conditions or humanly delays. An illustration of such an abstraction is given in Figure 6. In this example, two customers with demands q(1) and q(2) are served by the truck using the specified route: $Depot - Customer1 - Customer2 - Depot$.

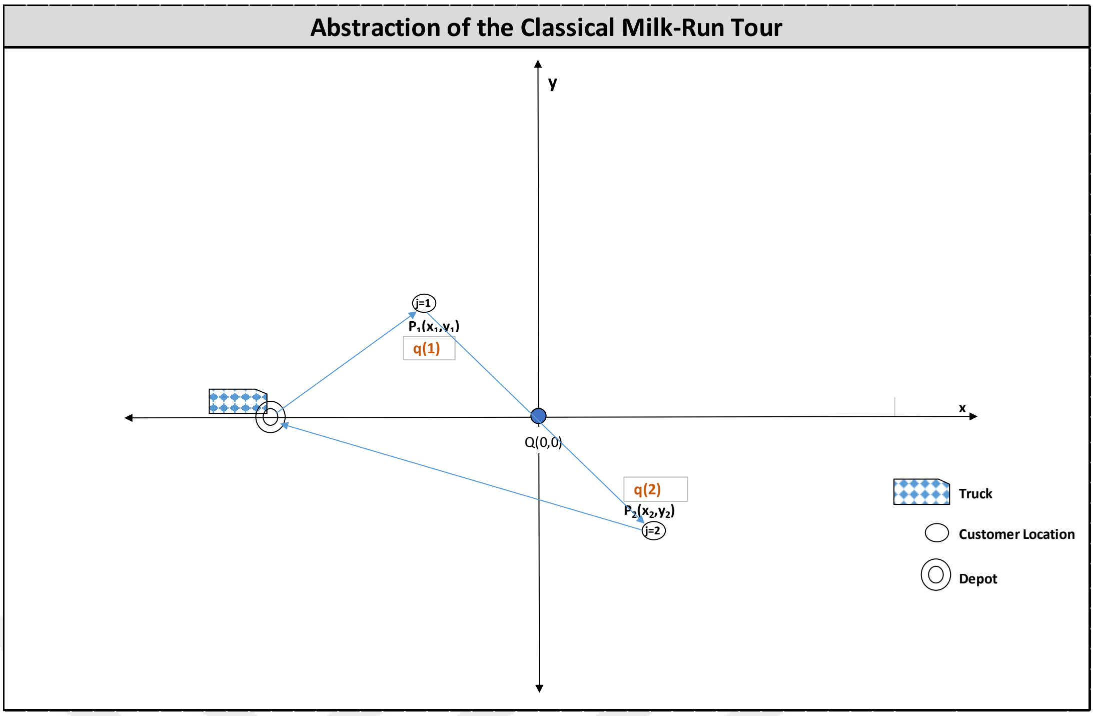


Figure 6. Abstraction of the Classical Milk-Run Tour

Now, we will introduce the general characteristics of the milk-run model with drones that we study. Via the abstraction of the real-life system, the obtained model will be mathematically tractable.

A representative circular route of the milk-run is given in Figure 7. The radius of the circle is r. Since the tour starts at the depot, the circle is centered at the location (0,0), which is shown by the blue point. The equation of the circle is (1):

$$x^2 + y^2 = r^2 \tag{1}$$

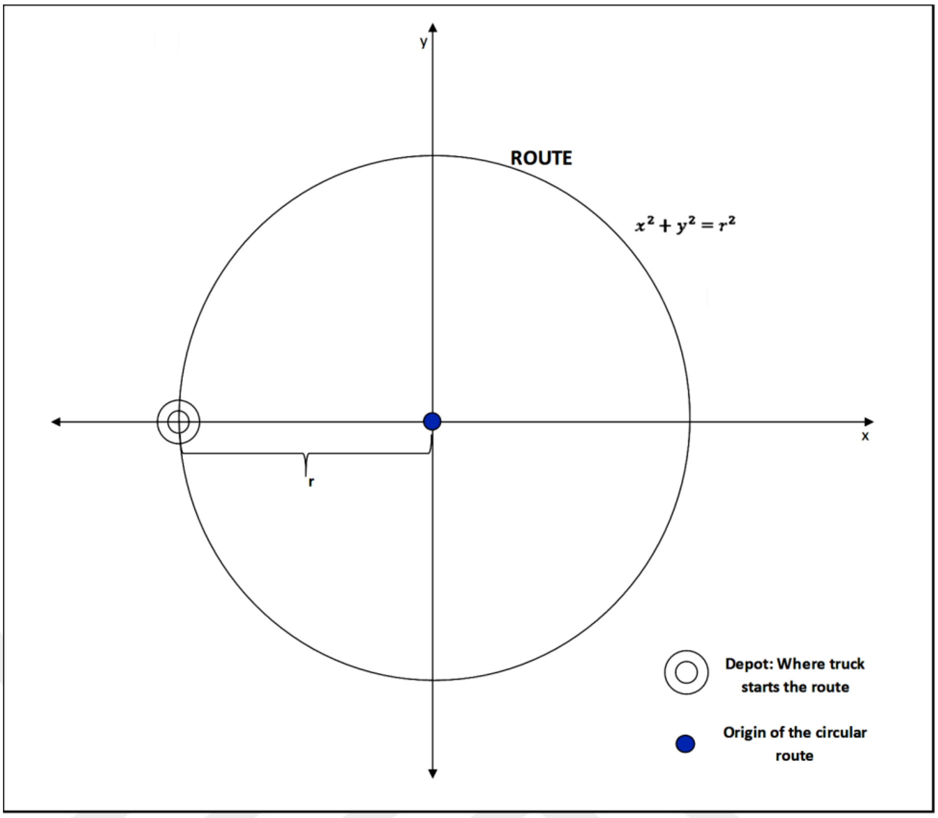


Figure 7. Route of the Milk-Run

The truck will start from the depot, located at (x, y) = (-r,0) at time t = 0. The entire route of the truck will be the tour around the circle, which is formed by tracking the 360-degree angle. In the numerical calculations, we will assume the truck's speed to be $V_{Tr} = 90km/h$ and the tour time to be 60 minutes, or equivalently 1 hour. These assumptions lead to the following calculation of the radius of the circular route (2):

$$\begin{gathered} Distance \;=\; Speed \,*\, Time \\ Circumference \;=\; 90km/h \,*\, 1h \;=\; 90\;km \;=\; 2\pi r \\ r = \frac{90km}{2\pi} = \frac{45km}{\pi} = 14.3239km \end{gathered} \tag{2}$$

The figurative representations of the model are given in Figure 8. These figures will be used to develop the explanations step by step.

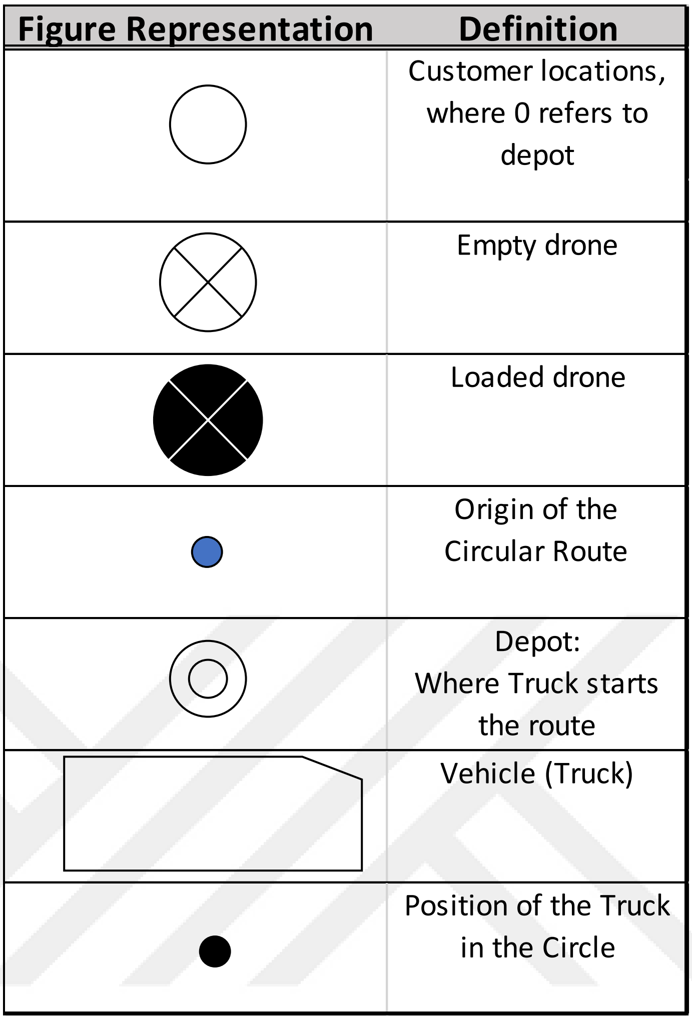


Figure 8. Figurative Representations of the Model

3.3. Calculation of Flight Times

In this section, we will explain the mathematical formulas that calculate the flight times from the truck to a specific customer location and vice versa. It will appear that the direct calculations of these equations for flight times will not be suitable to use in the mathematical milk-run model with drones, since they are highly nonlinear. Therefore, an approximation will be needed for this purpose. The approximation to be used in this study is a quadratic regression equation, which is also nonlinear, but less complex compared to actual flight time giving equations. The regression equations are customer specific, meaning that for each customer, a different quadratic regression equation is built. The response variable is flight time, which is explained by the start time of the flight.

To start with, we will explain how we calculated the earliest time and latest time that truck and customer can interact. In the first stage of calculations, we need the intersection points of the two circles: circle of milk-run route and circle of customer's range. For illustration, we will demonstrate the calculation of the two intersection points within which a loaded drone at a specified customer can be in interact with the truck. As previously mentioned, the truck route has a radius of r = 14.32 km. We will let the specified customer's coordinates to be (x_C, y_C) = (9,7), and the range of the drone to be 5.92km. The intersection points of these two circles are calculated to be $(x_E, y_E) = (6.93{,}12.53)$ and $(x_L, y_L) = (13.85{,}3.63)$. The center and intersection points for this specified customer are given in Table 3.

Table 3. Center and Intersection Points of the Customers

		Intersection Points			
X_C	**Y_C**	**x_E**	**y_E**	**x_L**	**y_L**
9	7	6.93	12.53	13.85	3.63

Corresponding intersection points are shown in Figure 9 (Luckhurst, 2018).

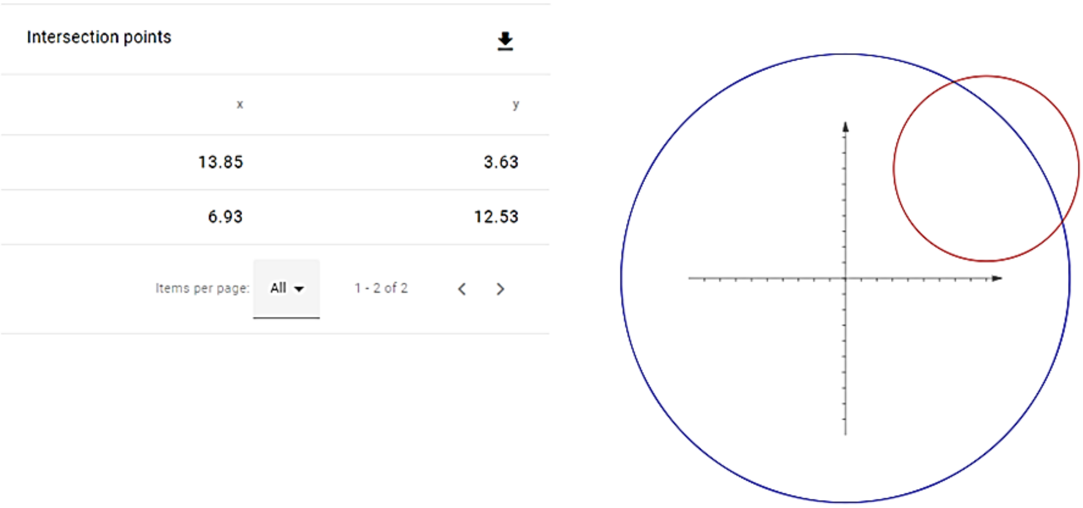


Figure 9. Points of Intersection

From these intersection points, we will calculate the earliest and latest times truck keeps interaction with drones, related to that customer. The notations that we will use for the calculations are given in Figure 10. We represented the points (l, m) on the circular route as $H_t(l, m)$. The subscript t here shows the time of the truck at which it is at the coordinate point (l, m). Hence, this point is a function of t and can be shown as $(l(t), m(t))$, which can be used interchangeably with $H_t(l, m)$. The angle that the line segment from $Q(0,0)$ to $H_t(l, m)$ makes in the clockwise direction is given by α, and the angle that the line segment from $Q(0,0)$ to $P_j(X_c, Y_c)$ makes in the clockwise direction is given by ω.

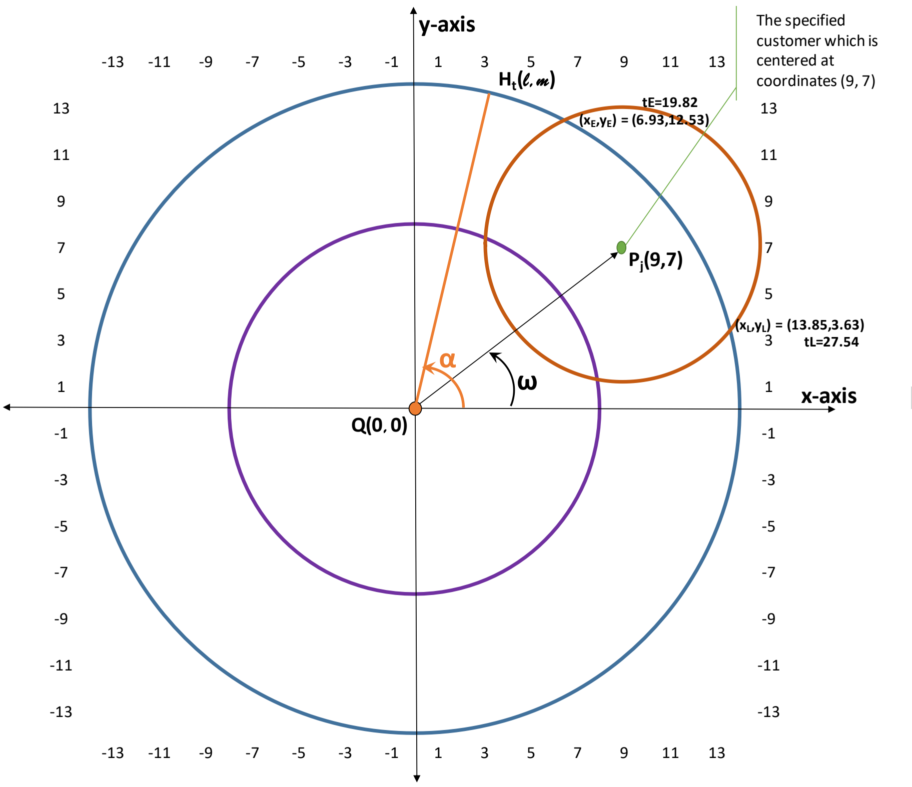


Figure 10. Distance Analysis: Intersection Points

Since the initial position of the truck is (-r,0), the initial angle of the truck is $\alpha =$ 180 degrees. Hence, during a full tour of the truck, α changes in the interval $(-180,180)$. The truck will complete the tour in 1 hour, or equivalently 60 minutes. This makes the angular speed of the truck to be $-6/minute$. Since the truck's rotation is clockwise, at each minute, its positional angle decreases by 6 degrees. Initially at $t = 0$, the truck is at the angle $\alpha = 180$, and at $t = 60$, the truck is at the angle $\alpha = -180$. The linear relationship between t and α is given in equation (3).

$$t = 30 - \frac{\alpha}{6} \quad \alpha \in (-180,180) \tag{3}$$

From equation 3, the equation of α in terms of t is found as in equation (4).

$$\alpha = 6 * (30 - t) \quad t \in (0,60) \tag{4}$$

In order to find the time from the Cartesian coordinates, we make use of the trigonometric equations given in (5) and (6).

$$l(t) = rcos(\alpha) \tag{5}$$

$$m(t) = rsin(\alpha) \tag{6}$$

When α is replaced in equations (5) and (6) using its time formula of equation (4), and considering the range properties of the inverse trigonometric functions, we obtained the time formulations, which we illustrate for the specified customer as given in Table 4. So, as given in Figure 10, the interaction of the truck with a drone related to the specified customer appears to be within the time interval $(19.82\ min, 27.54\ min)$.

Table 4. Time Formulations

X_C	9	
Y_C	7	
x_E	6.93	
y_E	12.53	
x_L	13.85	
y_L	3.63	
F_1	19.82	F_1=30-(1/6)arcCos(xE/r)
F_2	19.83	F_2=30-(1/6)arcSin(yE/r)
F_3	70.17	F_3=60-(1/6)arcSin(-yE/r)
F_4	27.54	F_4=30-(1/6)arcCos(xL/r)
F_5	27.55	F_5=30-(1/6)arcSin(yL/r)
F_6	62.45	F_6=60-(1/6)arcSin(-yL/r)
t_E	19.82	
t_L	27.54	
$AlphaDep_E$	61.07	
$AlphaDep_L$	14.78	

At this final step of calculations, we will find the flight times of the drones, from the truck to the customer and vice versa. To make the notations remembered we used so far, the coordinates of the truck around the circular route at time t is $(l(t), m(t))$, or equivalently $H_t(l,m)$, and the angle that the line segment from $Q(0,0)$ to $H_t(l,m)$ makes in the counter clockwise direction is α. We will introduce a new distance

$B = B(t)$ which corresponds to the distance of the truck to the customer at time t. For illustrative purposes, we will consider the instance at $t = 24.99$. Unless stated otherwise, distances are in kilometers and times are in minutes. At this instance, which is shown in Figure 11, the distance between the truck and the specified customer's location is given in equation (7):

$$B = B(t) = \mathrm{B}(24.99) = 3.40 \tag{7}$$

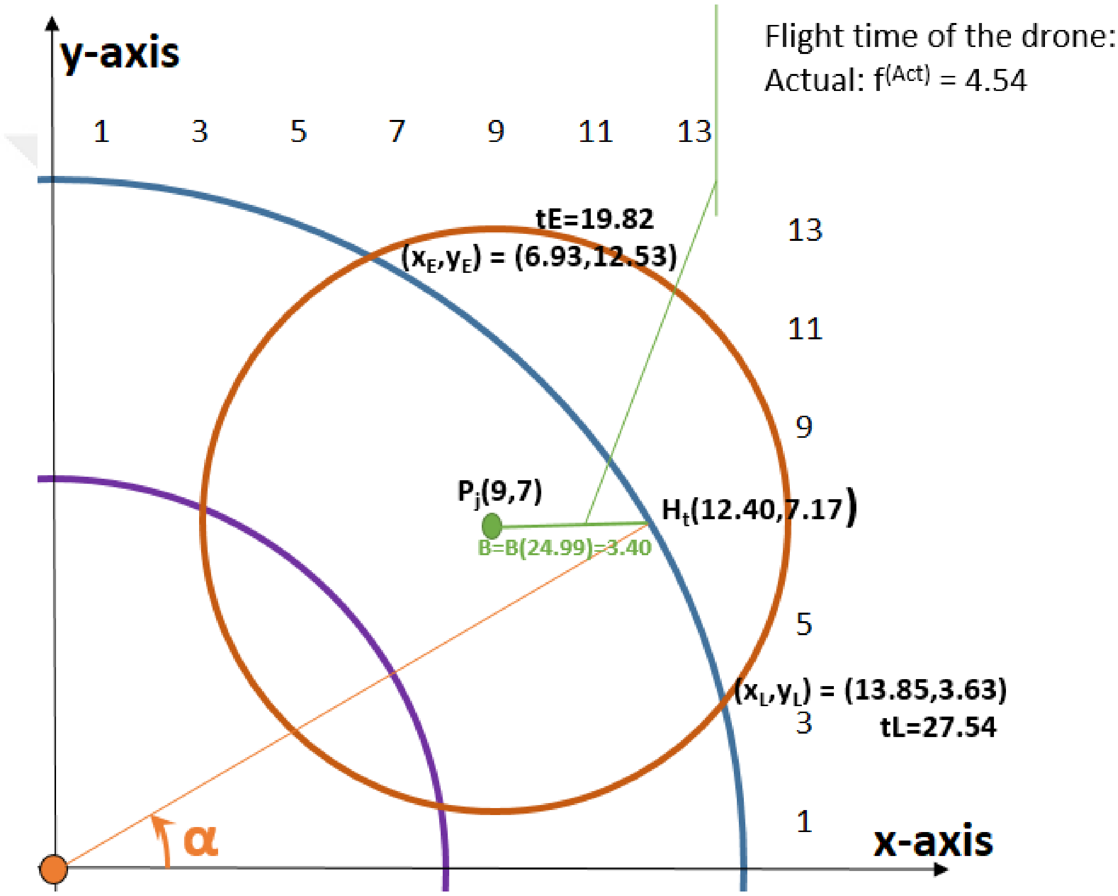


Figure 11. Distance between Drones and Customers

We will introduce the function $f_j^{Act}(t)$ for the flight time from the truck to the j[th] customer at time t and vice versa. The superscript "Act" stands for "Actual". The reason for using such a superscript is that we will later define "Estimated" flight times and use the superscript "Est" for the corresponding function. The derivation of this function is as follows. First, we will show the algebra at the time $t = 24.99$ for a one-sided flight which is from the truck to the customer and then generalize the findings. The Euclidean distance between P and H is found by the equation (8):

$$\begin{aligned} B(t) &= \left|P_{\mathrm{j}}(x_C, y_C), H_t(l, m)\right| = \sqrt{(x_C - l(t))^2 + (y_C - m(t))^2} \\ B(24.99) &= \left|P_{\mathrm{j}}(9,7), H_t(12.40,7.17)\right| \\ &= \sqrt{(9 - 12.40)^2 + (7 - 7.17)^2} = 3.40 \end{aligned} \tag{8}$$

Now, $f_{\mathrm{j}}^{Act}(24.99)$ is the actual time for the drone to take a distance of 3.40. Adding the drone's speed V_D to the notation developed so far, it becomes sufficient to find the flight time of the drone from the truck to the customer as a function of takeoff time from the truck. We will assume the drone's speed to be 45 km/h. Accordingly, the drone's speed is: Since, for this specific case under consideration $V_D = 45km/h = \frac{\pi r}{60}$, the following numerical calculations follow for this specific point:

$$\begin{gathered} distance = speed \cdot time \\ B(t) = V_D \cdot f_{\mathrm{j}}^{Act}(t) \\ B(24.99) = V_D \cdot f_{\mathrm{j}}^{Act}(24.99) \\ 3.40 = \frac{\pi r}{60} \cdot f_{\mathrm{j}}^{Act}(24.99) \\ f_{\mathrm{j}}^{Act}(24.99) = \frac{60}{14.32\pi} \cdot 3.40 = 4.54 \end{gathered} \tag{9}$$

The generalization of this finding is given as follows:

$$\begin{gathered} distance = speed \cdot time \\ B(t) = V_D \cdot f_{\mathrm{j}}^{Act}(t) \\ f_{\mathrm{j}}^{Act}(t) = B(t) \cdot \frac{1}{V_D} \\ f_{\mathrm{j}}^{Act}(t) = \sqrt{(x_C - l(t))^2 + (y_C - m(t))^2} \cdot \frac{1}{V_D} \end{gathered} \tag{10}$$

3.4. A Sample Problem

We defined a sample problem based on the abstraction of the real problem to clarify our approach. The settings for the truck route was $r = 14.32\ km$ and the time of the tour was 60 minutes. In this chapter, we have two examples for comparison purposes. These examples contain two specified customers with fixed demands. In the first example, the truck makes a TSP tour to serve two customers, and hence the truck travels within the city. The second example illustrates the milk-run distribution system design for integrating drones.

In the second example, the truck's highway speed is taken to be 90km/h as stated before. For the first example, we need a working assumption for the truck's speed within the city. The allowed maximum speed of trucks in the city is 50km/h (General Directorate of Highways, 2020). However, considering several speed lowering factors such as loss of time due to red lights and the difference between bird's eye view and travels in the city-way, we assumed the average speed of the truck to be 30km/h, or 0.5km/min. We will call this assumed speed as effective speed, which corresponds to the speed that Euclidean distance between two points is traveled by time t. Another assumption we made is about the setup times. In the first example, we assumed the delay time of unloading the item is to be 5 minutes and in the second one, we assumed the setup time of the drone to be 3 minutes.

3.4.1. Milk-Run Distribution with Truck Alone

We make the following assumptions about the first sample problem:

i. The milk-run system has two customers. The first customer has a demand for two items and the second customer has a demand for a single item.
ii. Initially, the truck is at the depot and distribution will be made using only the truck.
iii. The truck will travel within the city and use Euclidean distances between the customer locations and the depot.
iv. The effective speed of the truck is assumed to be 30km/h.
v. The load/unload time in the customers are assumed to be 5 min.

The initial conditions (when time t is zero 0) of the sample milk-run problem are given in Figure 12. Representations of the figures introduced in Figure 8 and are used in this illustration of the initial conditions.

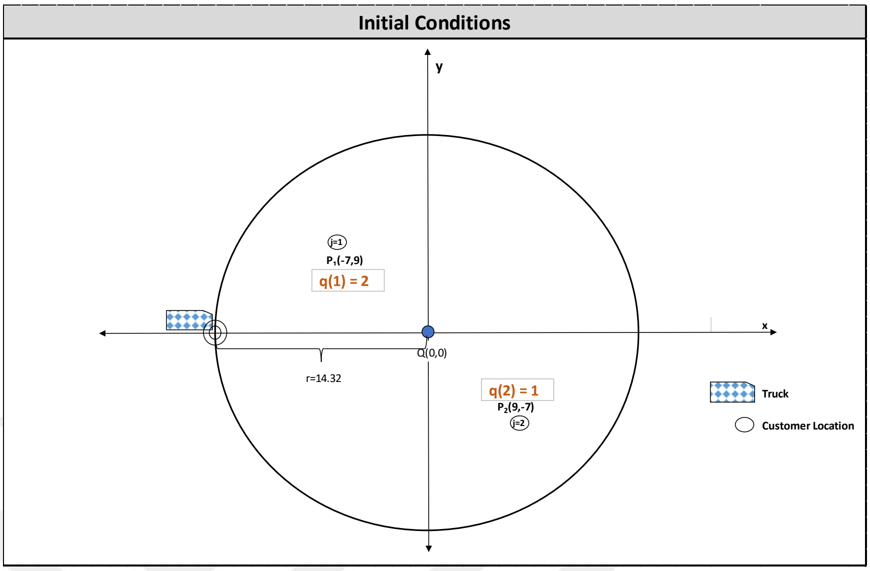


Figure 12. Initial Conditions of the Sample Problem with Truck Only

The route of the truck is given in Figure 13. For such a simple map of two customers, the best route is straightforward. For larger size problems, a TSP solution is desirable. The given distances between the three points are Euclidean distances.

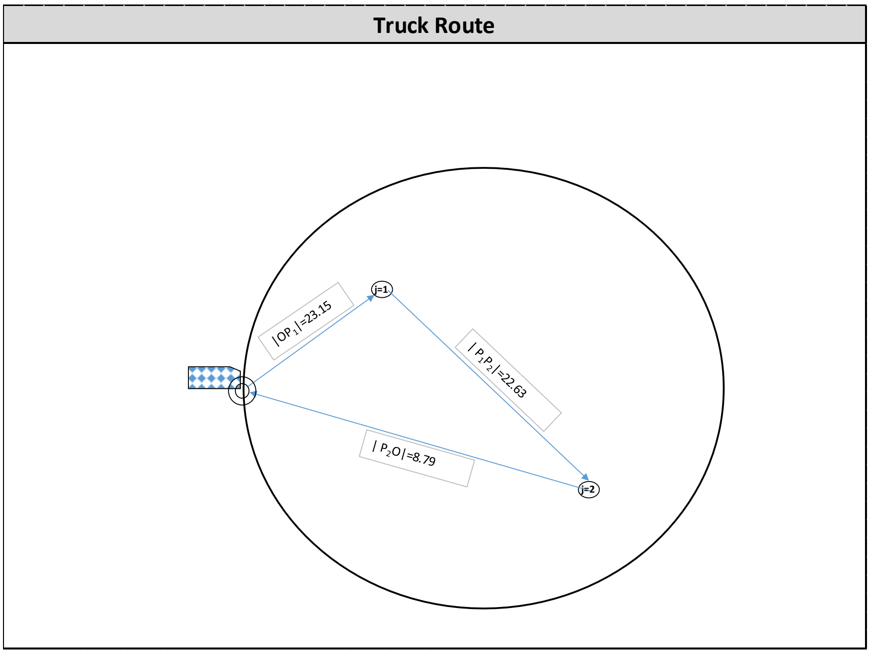


Figure 13. The Route of the Truck for the Sample Problem with Truck Only

The calculation of the Euclidean distances and corresponding driving times are given in Table 5. Since the effective speed of the truck is 30km/h, or equivalently 0.5 km/min, drive times in minutes are found by dividing the distance by 0.5.

Table 5. Driving Times Used in the Sample Problem Truck Only

From	To	Distance	Driving Time
Depot: (-14.32,0)	Customer 1: (-7,9)	$\sqrt{(-14.32+7)^2+(0-9)^2} = 23.15$	$\frac{23.15}{0.5} = 46.3$
Customer 1: (-7,9)	Customer 2: (9,-7)	$\sqrt{(-7-9)^2+(9+7)^2} = 22.63$	$\frac{22.63}{0.5} = 45.3$
Customer 2: (9,-7)	Depot: (-14.32,0)	$\sqrt{(9+14.32)^2+(-7-0)^2} = 8.79$	$\frac{8.79}{0.5} = 17.6$

Calculation of the arrival and departure times is given in Table 6. Since load/unload time is assumed to be 5 minutes, departure times from the customers are 5 more than the corresponding arrival times.

Table 6. Arrival and Departure Times Used in the Sample Problem Truck Only

From	To	Start Time	Drive Time	Arrival Time	Departure Time
Depot: (-14.32,0)	Customer 1: (-7,9)	0	46.3	$0 + 46.3 = 46.3$	$46.3 + 5 = 51.3$
Customer 1: (-7,9)	Customer 2: (9,-7)	51.3	45.3	$51.3 + 45.3 = 96.6$	$96.6 + 5 = 101.6$
Customer 2: (9,-7)	Depot: (-14.32,0)	101.6	17.6	$101.6 + 17.6 = 119.1$	$End\ of\ the\ tour$

Using the arrival and departure times calculated in Table 6, the critical times and summary of the events in the sample problem are given in Table 7.

Table 7. Critical Times and Summary of Events for the Sample Problem Truck Only

Time	Event
0.0	Departure of the Truck from Depot
46.3	Arrival to Customer 1
51.3	Departure from Customer 1
96.6	Arrival to Customer 2
101.6	Departure from Customer 2
119.1	End of the Tour

The critical events given in Table 7 will be demonstrated step by step in the following figures. To start with, Figure 14 gives the snapshot of the example's feasible solution at t=0.00. This is the time when the truck departs from the depot.

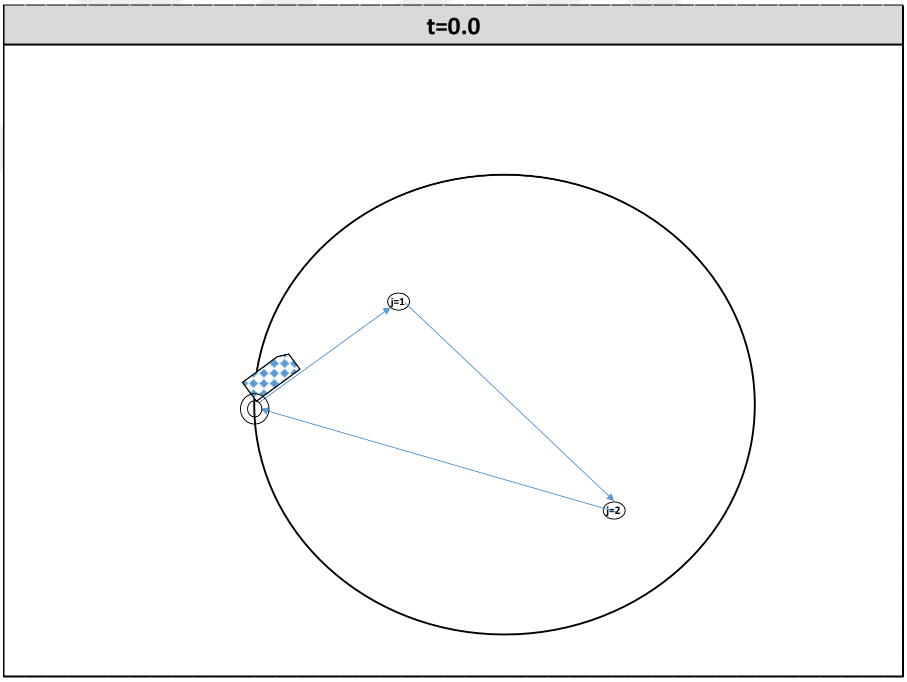


Figure 14. Snapshot at t = 0.00

Figure 15 gives the snapshot of the example's feasible solution at t=46.3. This is the time when the truck arrives at the first customer.

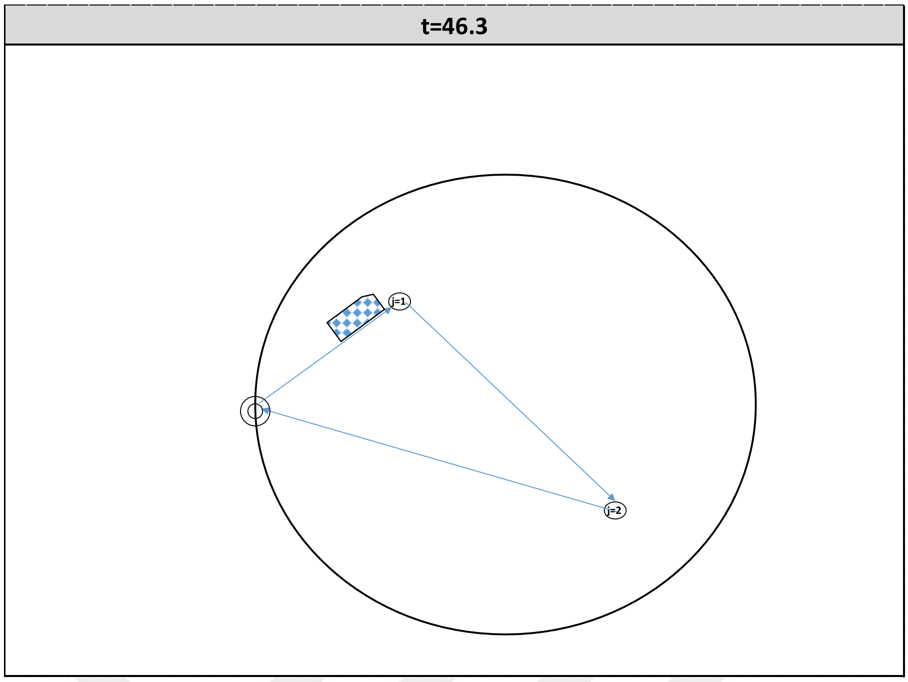


Figure 15. Snapshot at t = 46.3

Figure 16 gives the snapshot of the example's feasible solution at t=51.3. This is the time when the truck departs from the first customer.

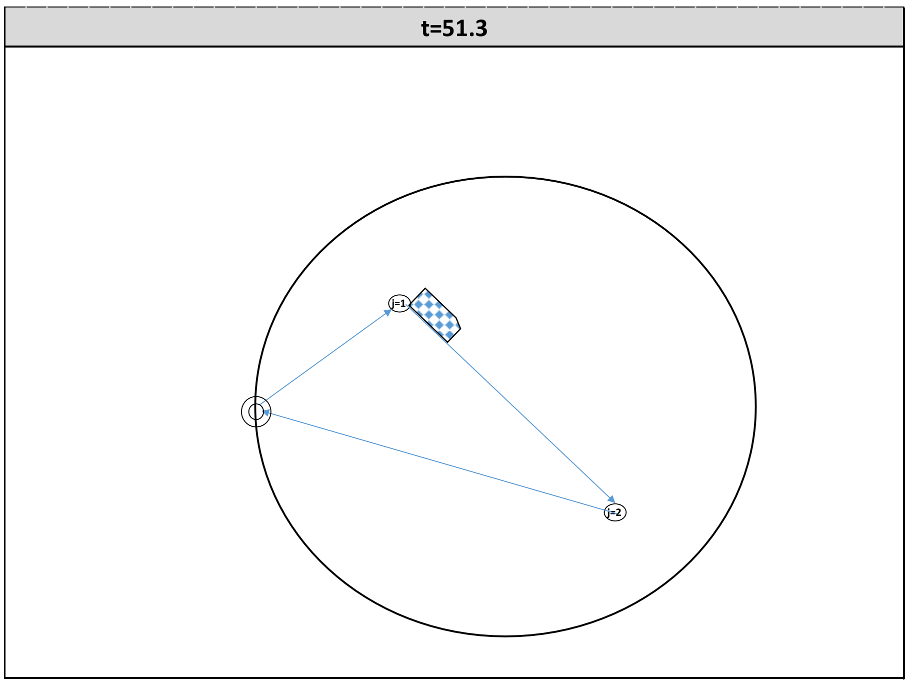


Figure 16. Snapshot at t = 51.3

Figure 17 gives the snapshot of the example's feasible solution at t=96.6. This is the time when the truck arrives at the second customer.

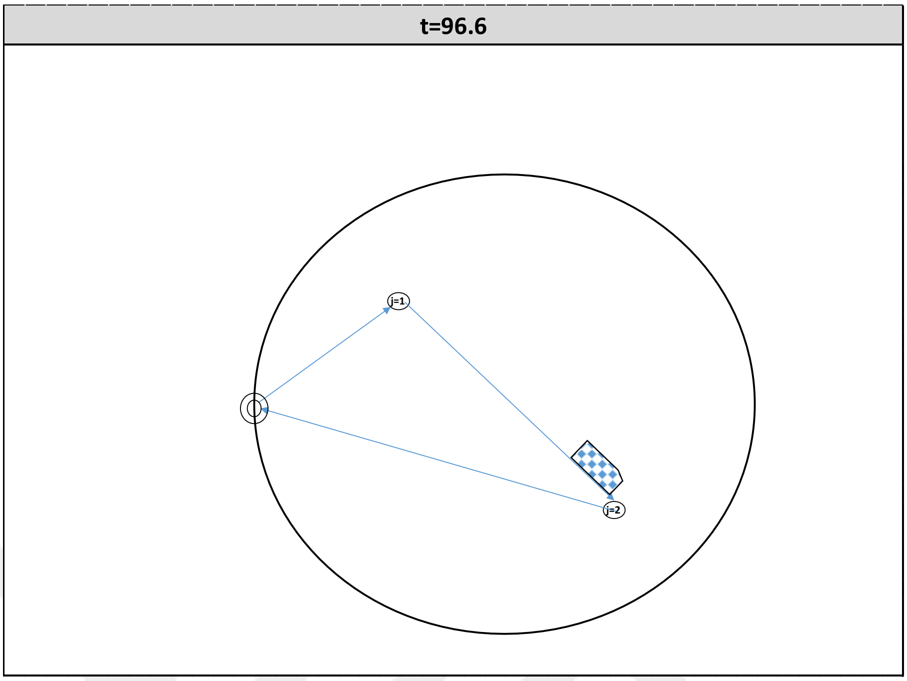


Figure 17. Snapshot at t = 96.6

Figure 18 gives the snapshot of the example's feasible solution at t=101.6. This is the time when the truck departs from the second customer.

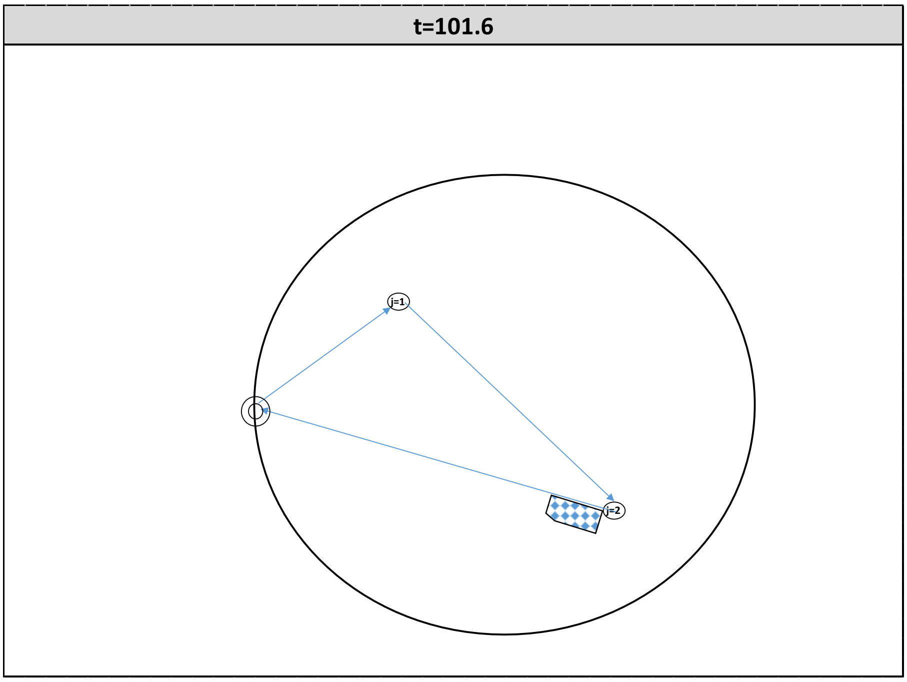


Figure 18. Snapshot at t = 101.6

Figure 19 gives the snapshot of the example's feasible solution at t=119.1. This is the time when the truck arrives back at the depot, and hence, the end of the tour.

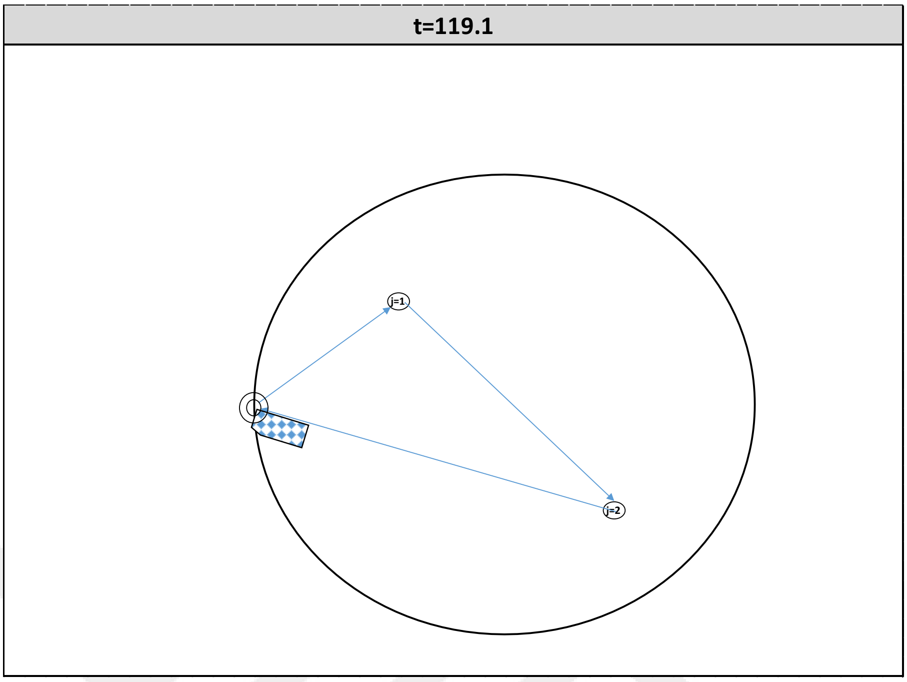


Figure 19. Snapshot at t = 119.1

3.4.2. Milk-Run Distribution with Truck and Drones

We make the following assumptions about customers, drones, and drones' initial positions about the sample problem:

i. The milk-run system has two customers and five drones.
ii. Initially, three drones are at the depot. Two of them are loaded whereas the third one is empty.
iii. The fourth drone is at the first customer and the fifth drone is at the second customer.

The initial conditions (when time *t* is zero 0) of the sample milk-run problem are given in Figure 20. Representations of the figures introduced in Figure 8 are used in this illustration of the initial conditions. As can be followed from Figure 20, the truck is located in the depot at point (-14.32,0). The two customer's locations are $P_1(-7{,}9)$ and $P_2(9, -7)$. The empty drone range is 11.84km and its loaded range is 5.92km.

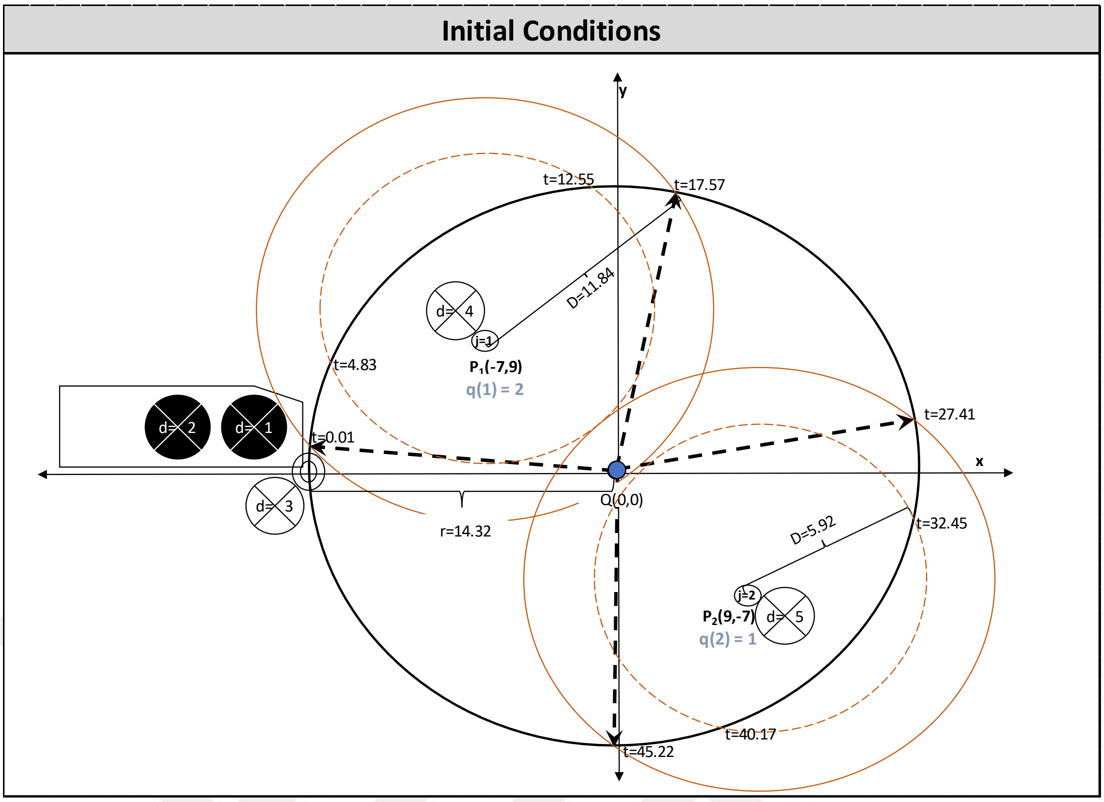


Figure 20. Initial Conditions of the Sample Problem with Truck and Drones

Of the three drones at the depot given in Figure 20, the first two drones are loaded and taken into the truck whereas the third drone is empty and not taken into the truck. The loaded drones are shown in black with a white cross inside them whereas the empty drones are shown in white with a black cross inside them. The first customer is located in the second quarter and the second customer is in the fourth quarter of the coordinate system. The fourth and fifth drones are represented near the first and second customers' locations respectively and are empty. This is a complete explanation of Figure 20.

The equation to calculate flight times where necessary is restated as follows:

$$f_j^{Act}(t) = \sqrt{(x-l)^2 + (y-m)^2} * \frac{1}{V_D} \tag{11}$$

For the first customer in the sample problem, the customer location is $(-7{,}9)$ and the speed of the drone was taken as $V_D = 45km/h = \frac{\pi r}{60}$ when loaded and $V_D =$

$60km/h = \frac{\pi r}{45}$ when empty. Therefore, for loaded and empty drones, $\frac{1}{V_D} = \frac{60}{14.32\pi}$ and $\frac{1}{V_D} = \frac{45}{14.32\pi}$ respectively. We will use the abbreviations "Arr" and "Dep" for the flight times of empty and loaded drones respectively. The reason is that drone "Arrives" at the truck when empty, and it "Departs" from the truck when loaded. Substituting $\frac{1}{V_D}$ values to equation (11), the flight times for the sample problem are found by equations (12) and (13), as follows:

$$f_1^{Act,Arr}(t) = \sqrt{(-7 - rcos(6 * (30 - t)))^2 + (9 - rsin(6 * (30 - t)))^2} * \frac{45}{14.32\pi} \tag{12}$$

$$f_1^{Act,Dep}(t) = \sqrt{(-7 - rcos(6 * (30 - t)))^2 + (9 - rsin(6 * (30 - t)))^2} * \frac{60}{14.32\pi} \tag{13}$$

Likewise, for the second customer in the sample problem, the customer location appears to be $(9, -7)$. Substituting the location of the second customer in equation (11), the flight times for the sample problem related to the second customer is found by equation (14), as follows:

$$f_2^{Act}(t) = \sqrt{(9 - rcos(6 * (30 - t)))^2 + (-7 - rsin(6 * (30 - t)))^2} * \frac{1}{V_D} \tag{14}$$

The calculations of flight times to determine critical events of the sample problem are performed using equations (12), (13), and (14). The results are given in Table 8.

Table 8. Actual Flight Times to be used in the Sample Problem

t: Time	Drone	From	To	Flight Time	Arrival Time
1.06	4	Customer 1	Truck	13.80	14.86
6.53	1 and 2	Truck	Customer 1	5.47	12.00
28.30	5	Customer 2	Truck	14.42	42.72
33.45	4	Truck	Customer 2	6.41	39.86

A feasible solution for the problem will be given with supporting explanations step by step and with the help of snapshots. A feasible solution for the example is summarized in Table 9.

Table 9. Summary of the Critical Times and Events with Actual Flight Times

Time	**Event**
0.00	The Departure of the Truck from Depot
1.06	The Departure of Drone 4 from Customer 1 towards the Truck
6.53	The Departure of Drone 1 and 2 from the Truck towards Customer 1
12.00	The Arrival of Drone 1 and 2 to Customer 1
14.86	The Arrival of Drone 4 to the Truck
28.30	The Departure of Drone 5 from Customer 2 towards the Truck
33.45	The Departure of Drone 4 from the Truck towards Customer 2
39.86	The Arrival of Drone 4 to Customer 2
42.72	The Arrival of Drone 5 to the Truck
60.00	The End of the Tour

The explanation of the critical events in chronological order is as follows. At time 0.0, the truck starts its tour and departs from the depot. Interaction range with the first customer starts at time 0.01 and approximately one minute after the range is entered, at time 1.06, the fourth drone departs from the first customer towards the truck. The flight time of this drone is found to be 13.80 by the function $f_1^{Act,Arr}(1.06)$. Next, the truck enters the range of the loaded drone at time 4.83, and at time 6.53, both of the drones in the truck departs through the first customer so that the demand of the first customer is fully satisfied. The flight time of these two drones at 6.53 is found to be 5.47 by the function $f_1^{Act,Dep}(6.53)$ and the arrival of these two drones to the first customer is at $6.53 + 5.47 \ = \ 12.00$. At time 14.86 (found by 1.06+13.80=14.86), the fourth drone arrives to the truck.

Because of the fact that the two customers in the example are located at the opposite quarters, no critical event occurs between $t = 14.86$ and $t = 28.30$. At $t = 28.30$, which is just after the starting time of the second customer's interaction region, the events related to the second customer start and the first event is the departure of the fifth drone from the second customer towards the truck. The flight time of this drone is found to be 14.42 by the function $f_2^{Act,Arr}(28.30)$ and the arrival of the fifth drone to the truck is $28.30 + 14.42 = 42.72$. At time 33.45, the fourth drone departs from the truck towards the second customer and the corresponding flight time is found to be $f_2^{Act,Dep}(33.45) = 6.41$. Accordingly, the arrival of the fourth drone to the second customer is at time $33.45 + 6.41 = 39.86$. Finally, at $t = 60.0$ the truck arrives at the depot and the tour ends.

Next, the explained events will be given step by step with corresponding snapshots. Figure 21 gives the snapshot of the example's feasible solution at t=0.00. This is the time when the truck departs from the depot.

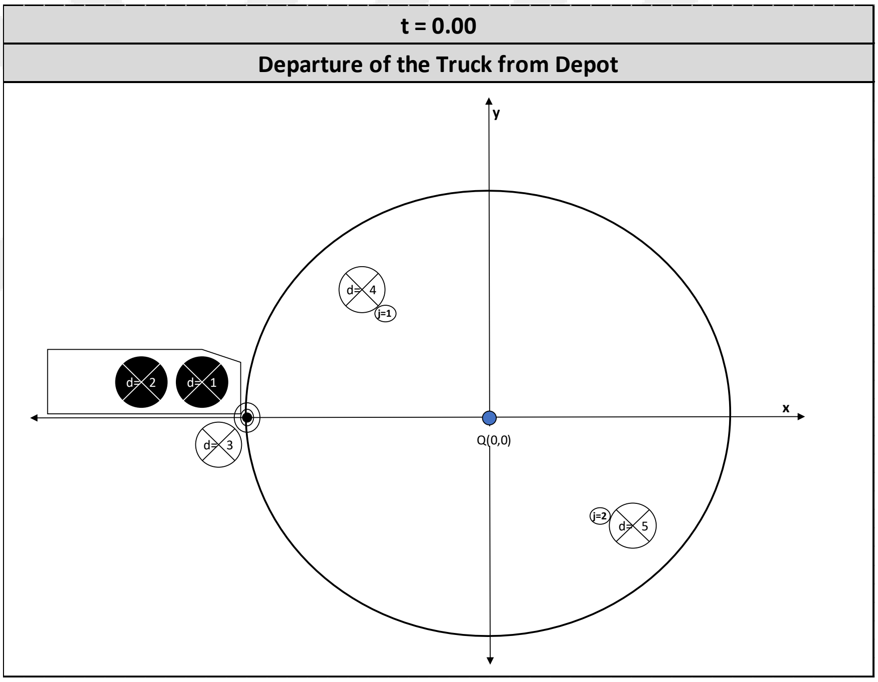


Figure 21. Snapshot at t = 0.00

Figure 22 gives the snapshot of the example's feasible solution at t=1.06. This is the time when the fourth drone departs from the second customer towards the truck.

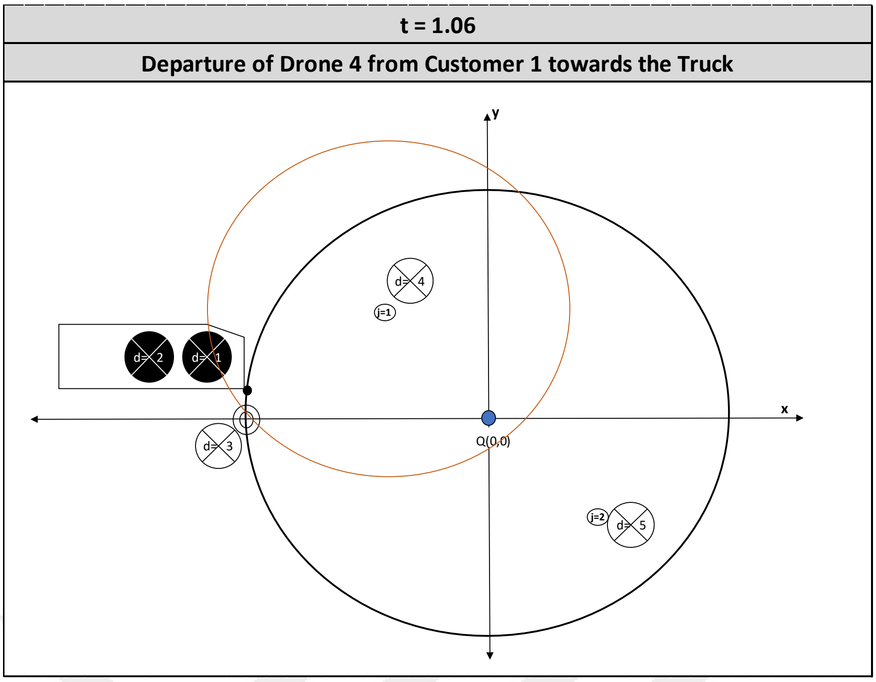


Figure 22. Snapshot at t = 1.06

Figure 23 gives the snapshot of the example's feasible solution at t=6.53. This is the time when the first two drones depart from the truck towards the first customer.

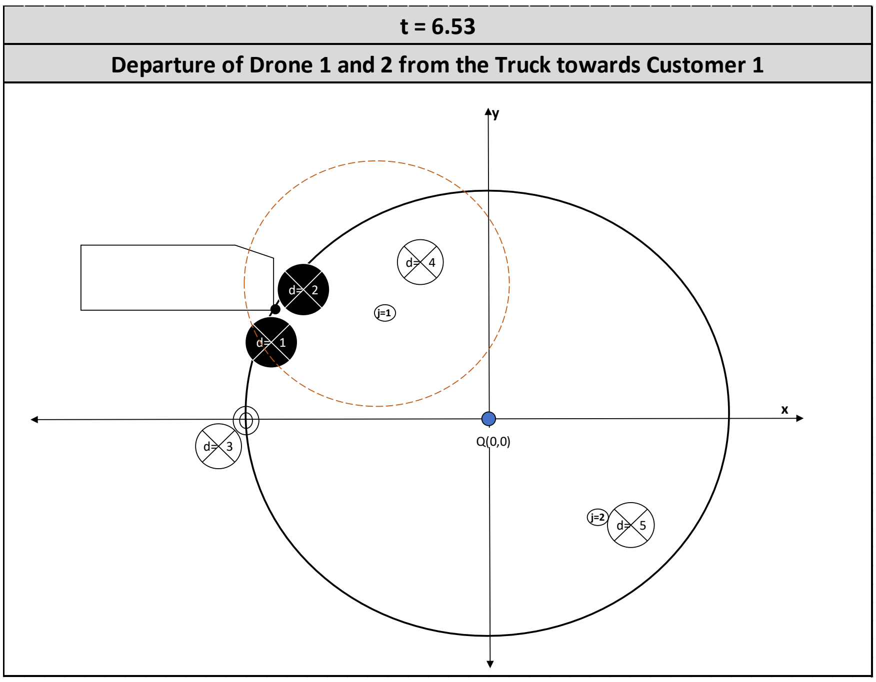


Figure 23. Snapshot at t = 6.53

Figure 24 gives the snapshot of the example's feasible solution at t=12.0. This is the time when the first two drones arrive at the first customer.

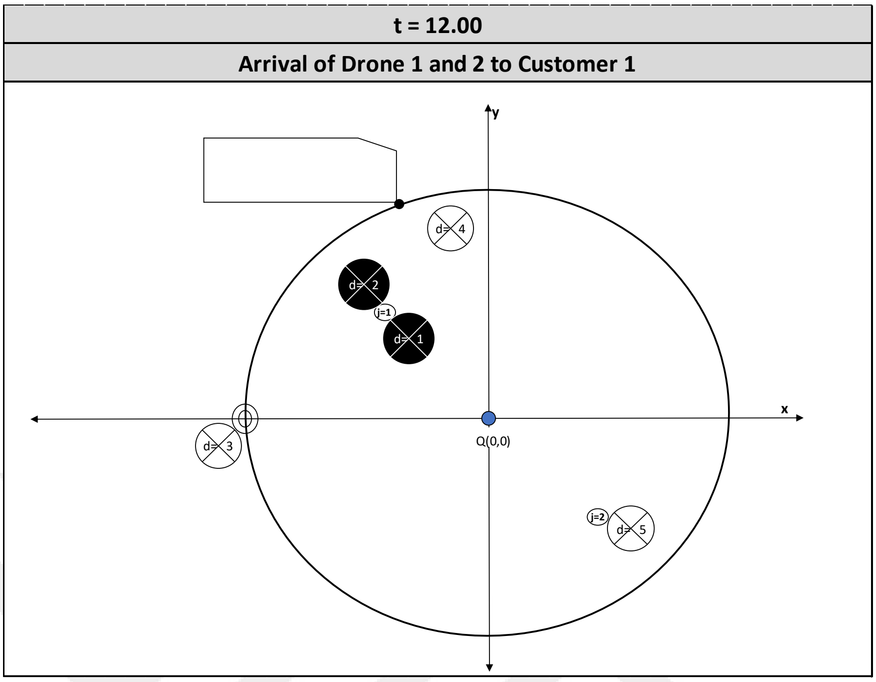


Figure 24. Snapshot at t = 12.0

Figure 25 gives the snapshot of the example's feasible solution at t=14.86. This is the time when the fourth drone arrives at the truck.

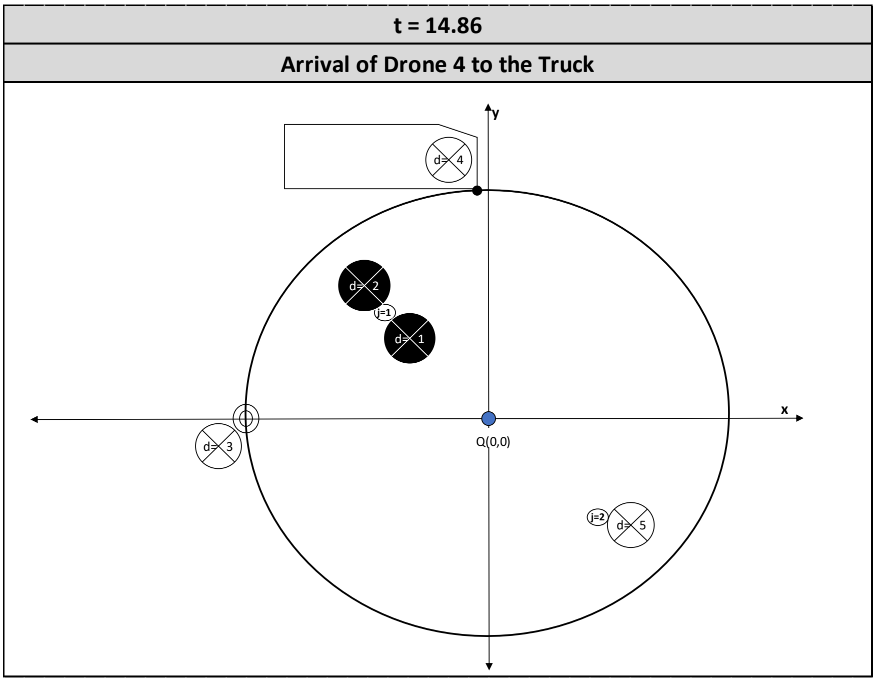


Figure 25. Snapshot at t = 14.86

Figure 26 gives the snapshot of the example's feasible solution at t=28.30. This is the time when the fifth drone departs from the second customer towards the truck.

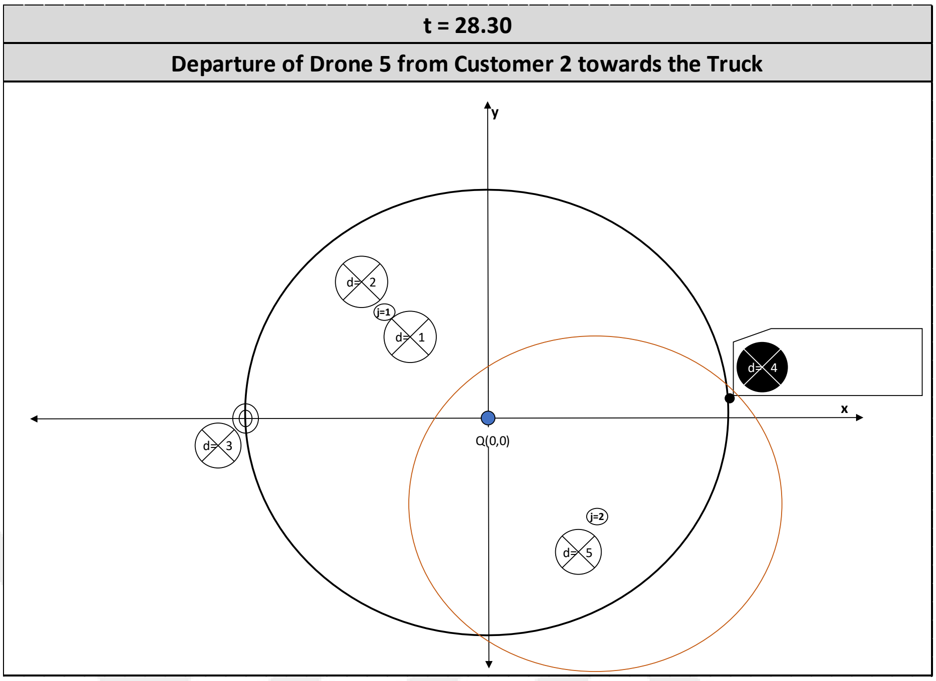


Figure 26. Snapshot at t = 28.30

Figure 27 gives the snapshot of the example's feasible solution at t=33.45. This is the time when the fourth drone departs from the truck towards the second customer.

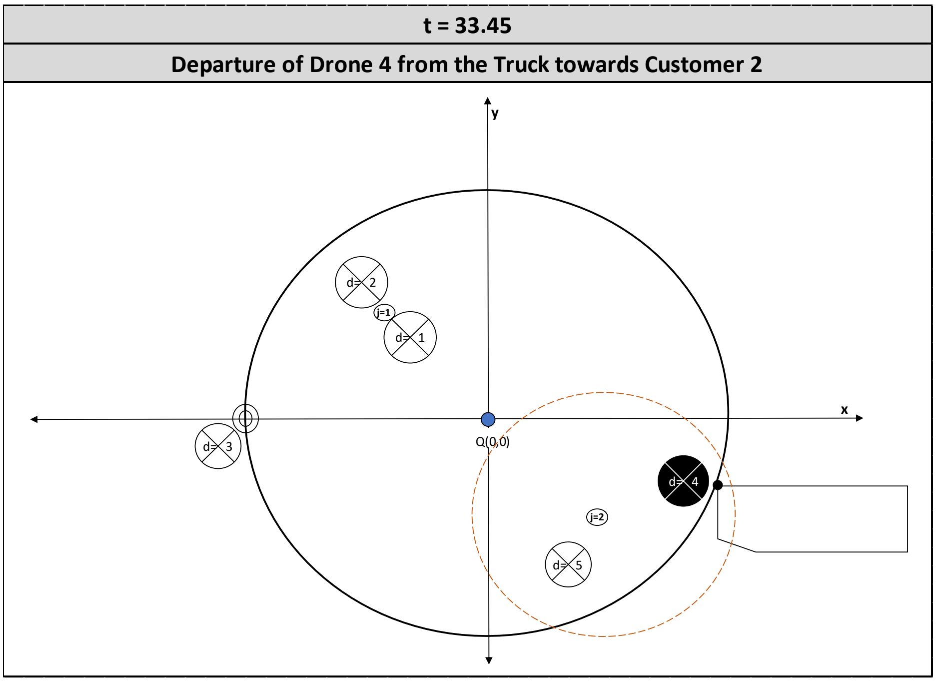


Figure 27. Snapshot at t = 33.45

Figure 28 gives the snapshot of the example's feasible solution at t=39.86. This is the time when the fourth drone arrives at the second customer.

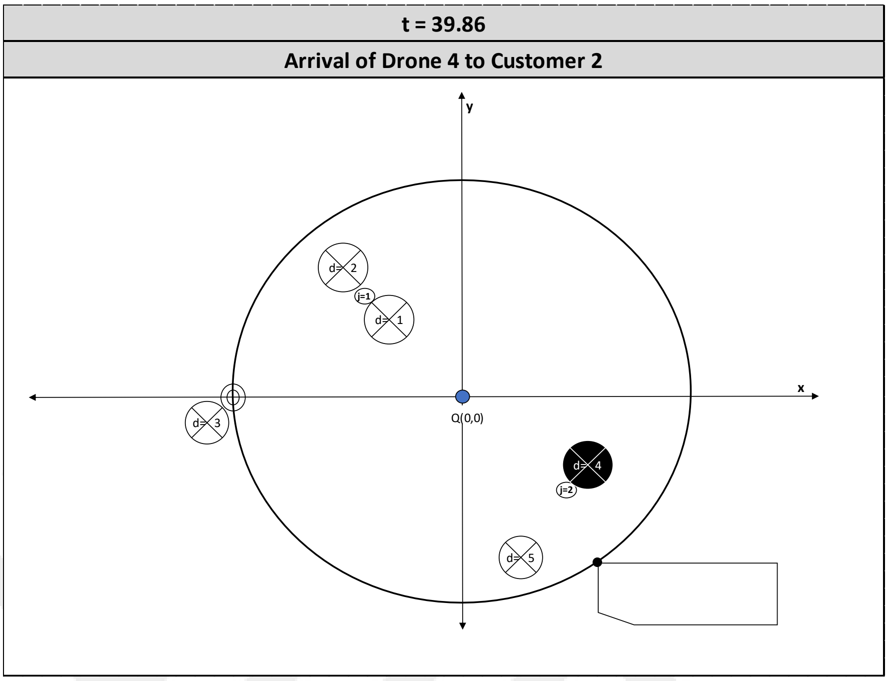


Figure 28. Snapshot at t = 39.86

Figure 29 gives the snapshot of the example's feasible solution at t=42.72. This is the time when the fifth drone arrives at the truck.

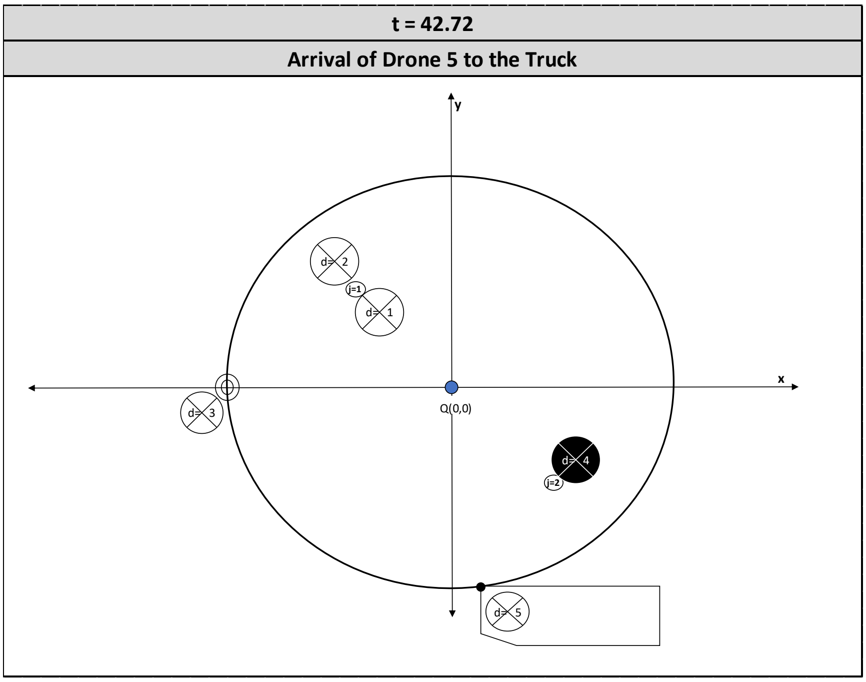


Figure 29. Snapshot at t = 42.72

Figure 30 gives the snapshot of the example's feasible solution at t=60. This is the time when the tour ends and the truck returns back to the depot.

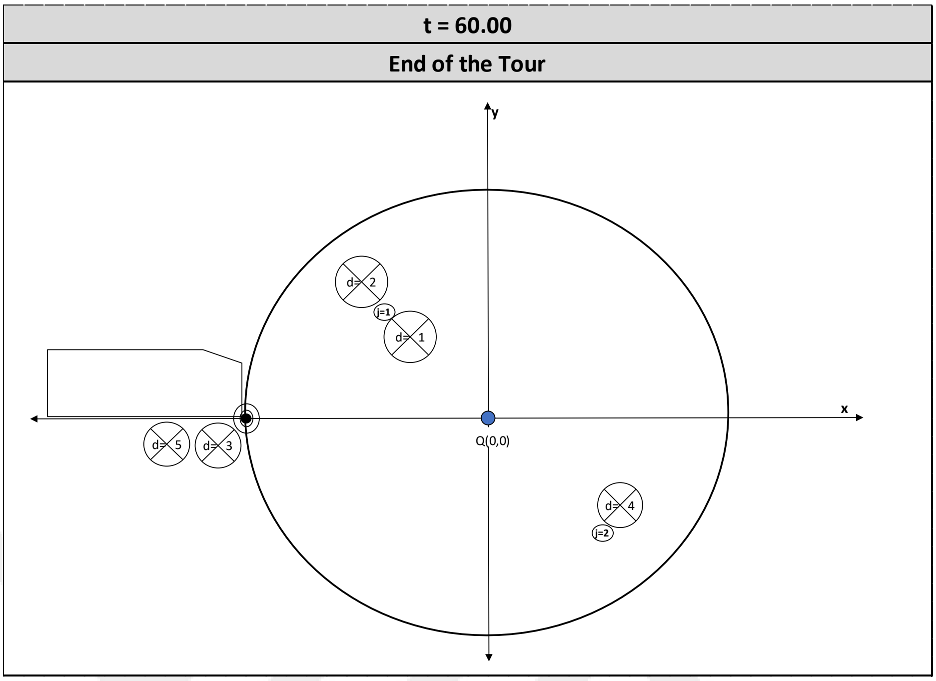


Figure 30. Snapshot at t = 60.0

CHAPTER 4

MODELING APPROACH

This chapter consists of the model that we propose for the milk-run distribution with drones. First, we will give the classical milk-run model to be used later for comparison purposes. In the next section, we will introduce our proposed model with detailed explanations for its objective function and each of the constraints. In this initial model, we will show the flight times of the drones with functions $f_{t,j}^{arr}$ and $f_{t,j}^{dep}$. Next, we fit quadratic estimation functions for flight times and show their adequacy. Then, we will introduce the model again with the flight time functions replaced with their quadratic estimation functions. Finally, we will revisit the sample problem and give two solutions for the sample problem using estimated flight time functions. The first of these solutions is an arbitrary feasible solution, shown step by step with visual figures. The second solution is the optimal solution for the sample problem found by the application of the proposed model.

4.1. The Classical Milk-Run Model

Before introducing the proposed model, we will give the classical milk-run model for comparison purposes (Winston, 2004).

It is worth mentioning the practical usage of classical milk-run at this point. In general, Classical milk-run tours are not optimized but the truck follows a pre-specified route. Although this route is not optimized by the practitioner, the considered route is generally not far away from the optimal. Moreover, the tour is updated with respect to practical changes. To illustrate, if a tour is specified to be $Depot - A - B - C - Depot$, and customer B is missing on a given day, the tour becomes $Depot - A - C - Depot$.

Due to the abstraction of the classical milk run model, we used the same tour and eliminated such adjustments. In fact, we used the TSP model to minimize the total classical milk-run tour distance. Therefore, at any rate, our findings and comparisons are valid in practice.

4.1.1. Sets

The only set of the classical milk-run model is the set of customer locations, which is given as follows:

j : $S^L = \{j: j = 0,1,2, \ldots, j, \ldots J\}$ $where\ 0\ corresponds\ to\ the\ depot$

4.1.2. Parameters

The parameters of the classical milk-run model are given as follows:

τ : *Total time of milk-run in DMR model (minutes)*

σ : *Speed of the truck on the highway (km/h)*

ϑ : *Speed ratio of the truck on the highway to in the city* ($\vartheta > 1$)

q_j : *Demand at customer location j*

c^{cty} : *Operating and fuel cost of the truck in the city ($/km)*

c^{opc} : *Opportunity cost ($/min)*

λ_{ij} : *Euclidean distance between customer location i and j (km)*

t^{wait} : *Waiting time of the truck at each customer j (min)*

4.1.3. Decision Variables

The decision variables of the classical milk-run model are given in as follows. The first decision variable set determines the optimal tour, and the last two variables are used to form the objective function.

V_{ij} : $\begin{cases} 1 & \text{if truck goes from location } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$

Z^{cty} : *The truck's total traveling cost within the city during the milk-run tour ($)*

Z^{opc} : *The cost stems from the total time difference between the classical milk-run tour and the proposed milk-run tour with drones ($)*

4.1.4. Objective Function

The objective function is the total cost of the milk-run tour which is the sum of the truck's traveling cost within the city, and the opportunity cost. Opportunity cost is the cost that stems from the total time difference between the classical milk-run tour and the proposed milk-run tour with drones. The idea is that if the tour were performed in accordance with the model proposed in this thesis, the total time of the tour would be less than the classical milk-run tour. In this time difference, the company has the opportunity of serving more customers and making more money. The objective function and definition constraints of the cost functions are given in equations (15), (16) and (17).

$$Min \sum Z^{cty} + Z^{opc} \tag{15}$$

$$Z^{cty} = c^{cty} \cdot \sum_{\forall i} \sum_{\forall j} \lambda_{ij} V_{ij} \tag{16}$$

$$Z^{opc} = c^{opc} \cdot \left[60 \frac{\sum_{\forall i} \sum_{\forall j} \lambda_{ij} V_{ij}}{\sigma / \vartheta} + t^{wait} J - \tau \right] \tag{17}$$

4.1.5. Model Constraints

The constraints of the classical milk-run model are given as follows:

$$\sum_{\forall j, i \neq j} V_{ij} = 1 \tag{18}$$

$$\sum_{\forall i, i \neq j} V_{ij} = 1 \tag{19}$$

$$\upsilon_i - \upsilon_j + J \cdot V_{ij} \leq J - 1 \tag{20}$$

Explanations of the constraints are as follows. Constraint (18) ensures that the truck enters each customer exactly once and constraint (19) confirms that the truck leaves from each customer exactly once. Constraint (20) is the sub-tour elimination constraint.

4.2. The Proposed Model

In this section, we will explain the assumptions, sets, parameters, decision variables, objective functions, and constraints of the proposed model.

4.2.1. Assumptions

In the model to be studied in this thesis, the settings regarding the milk-run distribution system using drones are as follows. The single truck follows a predefined route at a given time and it can take multiple drones within the tour, without a fixed capacity for the maximum number of drones. Having delivered the parts or materials to the customers, the truck returns back to the depot. The Milk-run system is a circular tour that starts and ends at the initial point. The system allows an exchange of full and empty drones.

When it comes to the settings about the drones, the following are noted. Each of the drones can carry only a single item, i.e., its capacity is one and drones have a limited range of flight. It is assumed that the speed of the drone is less than the truck. Accordingly, the truck may slow down for landing drone but the additional time spent for this activity is negligible. The mechanism we consider here for landing on the truck or taking off from the truck is like the landing and boarding activities of the aircraft on the aircraft carrier. As a result, both the truck and the drone are assumed to make simultaneous movements during arrival and departure activities without causing a time delay.

For the delivery of the items, only drones are used and the truck never waits for the drone, which means that they are synchronized in the delivery. Whenever two consequent use of the same drone is required, its reload time is considered as a fixed constant but the time required to charge or change its battery if necessary, is assumed to be negligible. The assumptions can be summarized as follows:

- A.1. All customer demands are known before the start of operations and all customers must be served.
- A.2. The truck never stops during its tour and launching/landing operations of drones are performed as the truck moves. Possible loss of time for these operations is negligible.

A.3. A specific drone can make at most two movements through a single tour. Either it makes no movement at all, or it makes a movement from the truck to a customer or vice versa, or it is taken from a customer to be used for service of a latter customer.

A.4. The capacity of a drone is one-unit demand.

A.5. The demand of each customer can be fulfilled only by drones; the truck is not used to satisfy demands.

A.6. In case a drone is taken from a customer to be used to satisfy a latter customer's demand, it has fixed item-loading and battery-change times before it launches.

A.7. Multiple drones can be launched / landed simultaneously.

A.8. Speed of truck and drones are fixed through their whole travel; drone speed is different if it is loaded or unloaded.

A.9. The flight range of drones is fixed and different if they are loaded or unloaded.

A.10. There is no technical problem with the drones such as drone's failure, crash, disconnection of a drone with the truck, and so on. In other words, we assume that our model is free from technical problems.

4.2.2. Sets

The nonlinear mixed integer programming milk-run model with drones is developed via the following steps.

To start with it will be useful to explain the sets and their corresponding indices used in the mathematical modelling of the problem. As is mentioned before, "time" does not correspond to indices of a set in this study. The milk-run model assumes a number of J customers whose locations are fixed within the circle, and there are D drones in the model. Some of these drones are at specified customers and the remaining are at the depot initially. The other sets and corresponding details are also given as follows:

j : $S^L = \{j: j = 0,1,2, \dots, j, \dots J\}, Customer\ locations\ (0\ means\ depot)$

d : $S^D = \{d: d = 1,2, \dots, d, \dots D\}, UAVs$

S_j^D : $(S_j^D \subset S^D), UAVs\ that\ are\ empty\ in\ customer\ j$

S_0^D : $(S_0^D \subset S^D), UAVs\ at\ the\ depot$

S_0^{D-e} : $(S_0^{D-e} \subset S^D)$, $UAVs\ empty\ in\ the\ depot$

S_0^{D-l} : $(S_0^{D-l} \subset S^D)$

$\left(S_0^D = S_0^{D-e} \cup S_0^{D-l}\right)$ and $(S^D = \bigcup_{\forall j} S_j^D \cup S_0^D)$, $UAVs\ loaded\ in\ the\ depot$

t : *Time*

(d,j) : $S^{DJ} = \left\{(d,j): d \in S_j^D \ and\ d\ is\ at\ j\right\}$, $The\ drones\ at\ the\ customers$

4.2.3. Parameters

The parameters of the model are as follows:

τ : *Total time of milk-run, minutes*

σ : *Speed of the truck on the highway, km/h*

q_j : *Demand at customer location j*

t_j^{dep-e} : *The earliest time that a drone can take-off from vehicle to location j*

t_j^{dep-l} : *The latest time that a drone can take-off from vehicle to location j*

t_j^{arr-e} : *The earliest time that a drone can land-on to the vehicle departed from location j*

t_j^{arr-l} : *The latest time that a drone can land-on to the vehicle departed from location j*

$f_{t,j}^{dep}$: *Flight time of a drone that takes-off at time t for customer location j (=0 if out of range. A nonlinear function. Proportional to the distance between vehicle location at time t and customer location j)*

$f_{t,j}^{arr}$: *Flight time of a drone that takes-off at time t from customer location j*

t^{setup} : *Setup time of a drone (land-on time & battery replacement & load payload)*

c^{hwy} : *Operating and fuel cost of the truck on the highway, \$/km*

c^{drn} : *Operating and battery usage cost of a drone, \$/min*

4.2.4. Decision Variables

In this section, the decision variables of the model are given. The decision variable U_d^B is the binary variable that takes the value of one if d^th^ drone is taken to the truck from the depot. The objective function of the model will be to minimize the total costs. The two sets of decision variables X stand for the actions of the drones from the truck to the customers whereas Y stands for the other way around. Specifically, $X_{d,j}^B$ is the binary decision variable that takes one if d^th^ drone is sent to satisfy a unit demand of the j^th^ customer, and $X_{d,j}$ is the corresponding take-off time to this sending. Likewise, $Y_{d,j}^B$ is the binary decision variable that takes one if d^th^ empty drone is sent from the j^th^ customer to the truck customer, and $Y_{d,j}$ is the corresponding take-off time to this taking. Finally, Z^{hwy} is the total operating and fuel cost of the truck during the tour on the highway, and $Z^{drn}$is total operating and battery usage cost of the drones during the tour. The objective function of the model is the sum of these last two decision variables. The decision variables of the model are given as follows:

U_d^B	:	*1 if drone d is put into the truck at the depot, 0 otherwise ($d \in S_0^D$)*
$X_{d,j}^B$	:	*1 if drone d serves location j , 0 otherwise*
$X_{d,j}$	:	*The departure time of drone d from the truck for location j (=0 means no departure of drone d for customer j)*
$Y_{d,j}^B$	:	*1 if drone d departs from location j , 0 otherwise*
$Y_{d,j}$	:	*Launch time of drone d from location j to arrive at the truck (=0 means no launch of drone d from customer j. This decision variable is valid only for drones in the set S_j^D)*
Z^{hwy}	:	*Total operating and fuel cost of the truck during the tour on the highway*
Z^{drn}	:	*Total operating and battery usage cost of the drones during the tour*

4.2.5. Objective Function and the Constraints

The objective function of the model is given as follows. The objective is to minimize the total cost of the tour, which consists of the cost incurred on the highway by the truck and the cost of the drones used in the delivery of the demanded items. In fact, the truck's highway cost is a fixed cost since the truck never stops during the tour in our proposed model. However, we need to add this fixed cost to our model so that we can compare the results of our proposed model with the classical milk-run model.

$$Min\ Z^{hwy} + Z^{drn} \tag{21}$$

$$Z^{hwy} = c^{hwy} \cdot \sigma \cdot \frac{\tau}{60} \tag{22}$$

$$Z^{drn} = c^{drn} \cdot \left[\sum_{\forall j} f_{t,j}^{arr} + \sum_{\forall j} f_{t,j}^{dep} \right] \tag{23}$$

In the objective function, a point is worth mentioning. The arrival flight times of the drones $f_{t,j}^{arr}$ takes part in the constraints whereas departure flight times of the drones $f_{t,j}^{dep}$ does not take part in the constraints. One can consider this case as a problem thinking that the optimal departure flight times would all be equal to zero in the optimum solution. However, this is not the case due to the following reason. We will update this initially proposed model by replacing the flight times with their quadratic estimation functions. These estimation functions are functions of the decision variables $X_{d,j}$ and $Y_{d,j}$. Accordingly, the proposed problem does not suffer from such a problem. The constraints of the model and their explanations are given as follows:

$$U_d^B = 0 \qquad \forall d \notin S_0^D \tag{24}$$

$$Y_{d,j}^B = 0 \qquad \forall j, d \notin S_j^D \tag{25}$$

$$Y_{d,j} \leq M \cdot Y_{d,j}^B \qquad \forall j, d, (d,j) \in S^{DJ} \tag{26}$$

$$X_{d,j'} = 0 \qquad \forall j, j' < j, d, (d,j) \in S^{DJ} \tag{27}$$

$$\sum_{\forall j} X_{d,j}^B \leq 1 \qquad \forall j, d \tag{28}$$

$$X_{d,j} \leq M \cdot X_{d,j}^B \qquad \forall j, d \tag{29}$$

$$\sum_{\forall d} X_{d,j}^{B} \geq q_j \qquad \forall j \tag{30}$$

$$\sum_{\forall j} X_{d,j}^{B} \leq U_d^B \qquad \forall j, d \in S_0^D \tag{31}$$

$$\sum_{j'=j}^{J} X_{d,j'}^{B} \leq Y_{d,j}^{B} \qquad \forall j, j' \geq j, d \in S_j^D \tag{32}$$

$$X_{d,j'} \leq M \cdot Y_{d,j}^{B} \qquad \forall j, j' \geq j, d \in S_j^D \tag{33}$$

$$Y_{d,j} + f_{t,j}^{arr} + t^{setup} \leq \sum_{j'=j}^{J} X_{d,j'} + M \cdot \left(1 - \sum_{j'=j}^{J} X_{d,j'}^{B}\right) \qquad \forall d \notin S_0^D, (d,j) \in S^{DJ} \tag{34}$$

$$Y_{d,j} + f_{t,j}^{arr} \leq t_j^{arr-l} + M \cdot \left(1 - Y_{d,j}^{B}\right) \qquad \forall j, d, (d,j) \in S^{DJ} \tag{35}$$

$$t_j^{arr-e} - M \cdot (1 - Y_{d,j}^{B}) \leq Y_{d,j} + f_{t,j}^{arr} \qquad \forall j, d, (d,j) \in S^{DJ} \tag{36}$$

$$X_{d,j} \leq X_{d,j}^{B} \cdot t_j^{dep-l} \qquad \forall j, d \tag{37}$$

$$X_{d,j}^{B} \cdot t_j^{dep-e} \leq X_{d,j} \qquad \forall j, d \tag{38}$$

$$X_{d,j} \leq \tau \qquad \forall j, d \tag{39}$$

$$Y_{d,j} + f_{t,j}^{arr} \leq \tau + M \cdot \left(1 - Y_{d,j}^{B}\right) \quad \forall j, d, (d,j) \in S^{DJ} \tag{40}$$

Constraint (24) guarantees that drones that are not initially at the depot cannot be taken into the truck. Constraint (25) ensures that drones that are not in customer j cannot be taken off from customer j. Constraint (26) enforces that a drone that is not taken off from the customer j ($Y_{d,j}^{B} = 0$) has takeoff time equal to zero ($Y_{d,j} = 0$) which indicates that it has not taken off from that customer. Constraint (27) guarantees that a drone at customer j cannot be sent to another customer which is prior to the j^{th} customer. Constraint (28) ensures that a drone can be sent to a customer at most once. Constraint (26) enforces a drone that is not taken off from the truck ($X_{d,j}^{B} = 0$) has takeoff time equal to zero ($X_{d,j} = 0$) which indicates that it has not taken off from the truck. Constraint (30) guarantees that all customers' demands must be met. Therefore, at least q_j drone must go to the customer j. Constraint (31) ensures that regarding the drones those are initially at the depot, only those drones taken from the depot can be used to satisfy a customer's demand. Constraint (32) guarantees the fact that in order to send a drone that is initially at customer j to that customer or to a subsequent customer, it is required that the drone is taken off from customer j. Constraint (33) guarantees that if a drone is initially at customer j and it will be used to satisfy the demand of that customer or to a subsequent customer, to take the drone into the truck is required. Constraint (34) states that if a drone which is initially at customer j will be used to satisfy a customer's demand, the earliest time that drone can depart from the truck is found by the addition of three components: its departure time from the customer, travel time to the truck and setup time. Constraint (35) guarantees that if a drone is to be taken from customer j to the truck, its arrival time to the truck, which is the addition of its departure time from the customer and travel time to the truck, must not exceed the latest available time that the truck can take the drone. Constraint (36) ensures that if a drone is to be taken from customer j to the truck, its arrival time to the truck, which is the addition of its departure time from the customer and travel time to the truck, must not be before the earliest time the truck can take the drone. Constraint (37) enforces that if a drone will depart from the truck towards customer j, its departure must not be later than its latest departure time. Constraint (38) guarantees that if a drone will depart from the truck towards customer j, its departure must not be before its earliest departure time. Constraint (39) ensures that all departures from the truck must be within the total time prescribed for milk-run distribution. Constraint (40) ensures that all arrivals to the truck must be within the total time prescribed for milk-run distribution.

4.3. Approximating Flight Times

The mathematical formula that calculates the flight times from the truck to the customer location was given in Chapter 3.3 with the function $f_{\mathrm{j}}^{Act}(t)$. Later, using these actual flight times, we illustrated a numerical example. In the model formulation, flight times are left with this formula. However, it appears that the direct application of this formula for flight times will not be suitable to use in the mathematical milk-run model with drones, due to its computational difficulties. For large sized problems, since a polynomial approximation is faster it will be practically feasible. Therefore, the flight times will be approximated with a polynomial function.

4.3.1. Regression for Quadratic Approximation

The approximation to be used in this study is a quadratic regression equation. The response variable of the regression is flight time, and the explanatory variable is the start time of the flight. In other words, $f_{\mathrm{j}}^{Act}(t)$ is the response variable and t is the explanatory variable. The notations to be used in this quadratic regression model is given as follows:

$B(t)$	:	$Distance\ between\ truck\ and\ customer\ at\ time\ t$
$f_j^{Act}(t)$	:	$Actual\ flight\ time\ from$ the $truck\ to\ the\ j^{th}\ customer\ at\ time\ t$
$f_j^{E\mathrm{st}}(t)$	:	$Estimated\ flight\ time\ from$ the $truck\ to\ the\ j^{th}\ customer\ at\ time\ t$

The regression model of the flight time is given by:

$$f_j^{Act}(t) = f_j^{E\mathrm{st}}(t) + \varepsilon \tag{41}$$

Estimated flight time regression is given by:

$$f_j^{E\mathrm{st}}(t) = a + bt + ct^2 \tag{42}$$

The estimates are valid within the interaction time interval of the truck with a drone related to a specific customer. We will demonstrate the regression using the same specified customer centered at (9,7) for comparison purposes. The corresponding interaction time interval is $(19.82min, 27.54min)$. The time interval has a range of $27.54 - 19.82 = 7.71min$. The angles that correspond to the earliest and latest times are given by the function:

$$\alpha = 6 * (30 - t) \quad t \in (0,60) \tag{43}$$

Application of equation (43) to the specified customer yields:

$$\alpha_E = 6 * (30 - 19.82) = 61.07$$
$$\alpha_L = 6 * (30 - 27.54) = 14.78 \tag{44}$$

The angle interval has a range of $61.07 - 14.78 = 46.29\,min$. For the regression equation estimate of each customer, we will take a sample of 100 t values. These t values are obtained by the inverse function of alpha, which is given in equation (45):

$$t = 30 - \frac{\alpha}{6} \quad \alpha \in (-180{,}180) \tag{45}$$

To take a sample of 100-time values, we started with the initial alpha value α_E. We systematically decreased this value each time with a decremented value of DecAlpha, which is one hundredth of the range of alpha. The formula for DecAlpha is given in equation (46):

$$DecAlpha = \frac{\alpha_E - \alpha_L}{100} \tag{46}$$

The calculations of the formulations explained so far for the specified customer are given in Table 10.

Table 10. Results of the Calculations

tE	tL	tRange	AlphaE	AlphaL	AlphaRange	DecAlpha
19.82	27.54	7.71	61.07	14.78	46.29	0.46

Finding B(t) values that correspond to each t value using equation (8), the regression data is obtained, and a part of the obtained values is given in Table 11.

Table 11. Departure Regression

Sample	B	time	time^2	fDep Actual	fDep Estimated
1	5.91	19.82	392.93	7.88	8.15
2	5.82	19.90	395.99	7.76	7.99
3	5.74	19.98	399.07	7.65	7.83
4	5.65	20.05	402.15	7.53	7.67
5	5.56	20.13	405.25	7.42	7.52
…	…	…	…	…	…
99	5.72	27.38	749.79	7.62	7.80
100	5.80	27.46	754.03	7.74	7.96
101	5.89	27.54	758.27	7.86	8.12

Running the regression for the specified customer's data, the summary statistics are obtained for flight time estimation function $f_j^{Est}(t)$ and are given in Table 12.

Table 12. Regression Fit for the Specified Customer's Flight Times

REGRESSION STATISTICS		
Adjusted R square		0.9912
ANOVA (OVERALL SIGNIFICANCE OF THE MODEL)		
Significance F		0.0000
REGRESSION COEFFICIENTS AND SIGNIFICANCES		
	Coefficient	p-value
Intercept	157.8335	0.0000
Time: $\boldsymbol{t}$	-12.9844	0.0000
Time Squared: $\boldsymbol{t^2}$	0.2741	0.0000

Analyzing the fitted regression given in Table 12, the following observations are obtained. The estimated regression function is:

$$f_{\mathrm{j}}^{(E)}(t) = 157.8335 - 12.9844t + 0.2741t^2 \quad (47)$$

The regression model is significant since the "Significance F" value of the ANOVA is 0.0000, and is less than any practical significance level. The individual estimated coefficients of t and t^2 are also significant since the corresponding t values are again 0.0000.

The explanatory power of the model is 99.12%, which is very close to 1. Hence, the approximation is quite a good one.

To illustrate the regression equation found, we estimate the flight time at $t = 24.99$ and compare it with its actual value. At this value of t, flight time is estimated to be:

$$f_j^{(E)}(24.99) = 157.8335 - 12.9844 * 24.99 + 0.2741 * 24.99^2 \qquad (48)$$

The actual and estimated flight times are given in Figure 31.

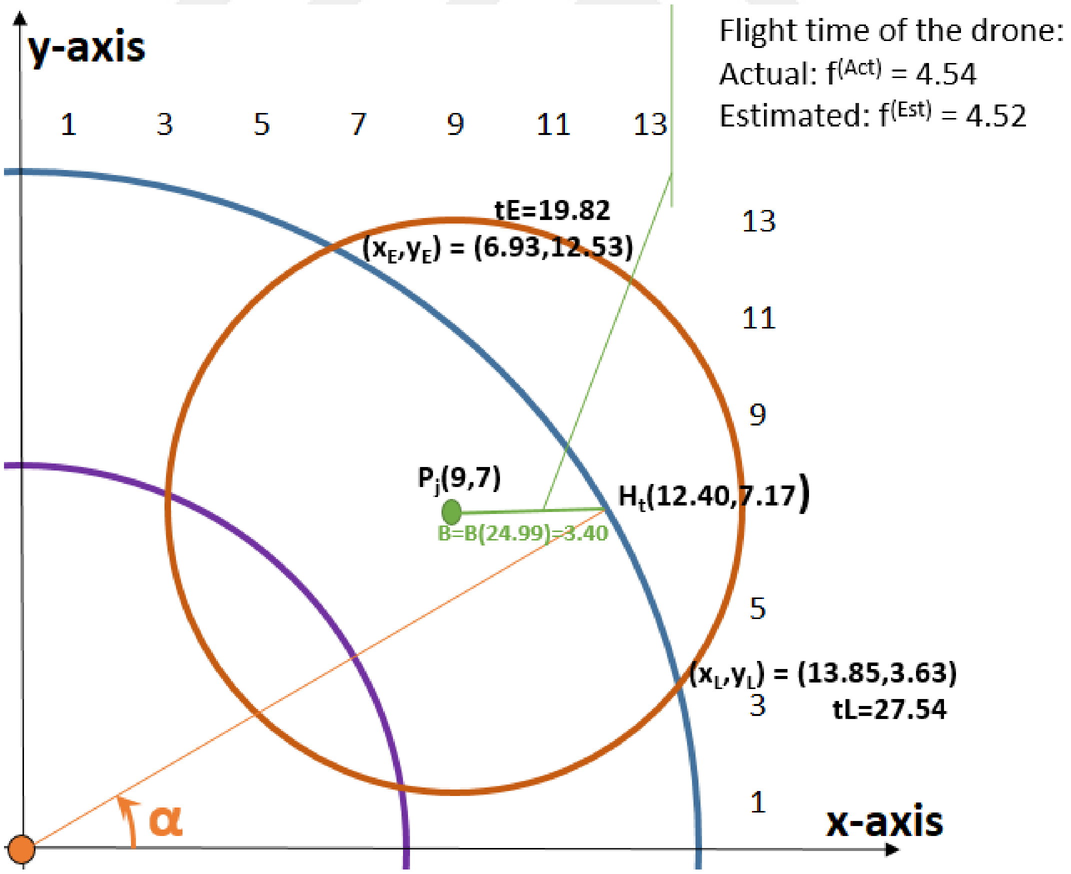


Figure 31. The Actual and Estimated Flight Times

4.3.2. Quality of Approximation

In the previous chapter, we have shown the validity of the regression equation with a significance value of 0.0000, and the explanatory power of the model is found to be 99.12%, which is very close to 1. In this section, we will give a graphical demonstration that shows the difference between actual and estimated flight times.

The customer used in previous demonstrations and also in this section is located at the point (9,7). The actual flight time for this specified customer is given by equation (49), which is the actual flight time equation introduced in chapter 3.3:

$$f_j^{Act}(t) = \sqrt{(9 - rcos(6 * (30 - t)))^2 + (7 - rsin(6 * (30 - t)))^2} * \frac{60}{14.32\pi} \quad (49)$$

The interaction of the truck with a drone related to this specified customer is calculated previously in chapter 3.3. as the time interval $(19.82min, 27.54min)$. Within this time interval, the quadratic estimated flight time function is given in equation (50).

$$f_j^{Est}(t) = 157.8335 - 12.9844t + 0.2741t^2 \quad (50)$$

We will demonstrate the quality of approximation using two graphs. The first graph is the comparison within all the time range of the route, which is $(0,60)$, and the second graph is the comparison within the interaction time interval $(19.82, 27.54)$.

The first of the comparisons is given in Figure 32. Except for the interval around the interaction range, the approximation can be considered as a bad one because, in Figure 32, the differences between orange and blue lines are quite high. In this figure, the blue line shows the actual flight times and the orange line shows the estimated flight times. However, this case is not a problem for our estimation purpose because, for each customer, we will use a different estimated flight time function within its own interaction range.

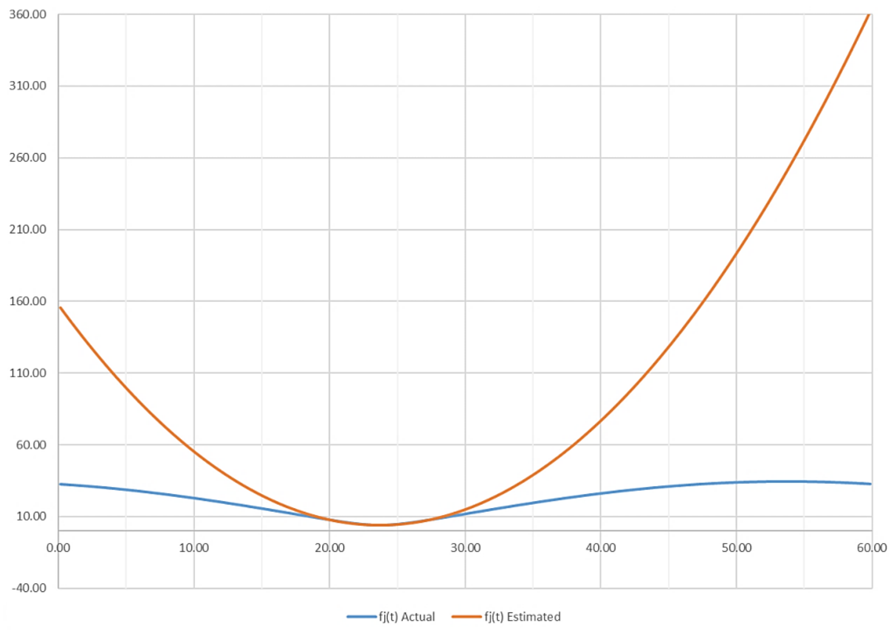


Figure 32. Regression in (0,60) Time Range

The second and the important comparison is given in Figure 33. In this figure, we observe that the estimated and the actual flight times are quite close to each other. This result supports the quality of our approximation and we will use this quadratic approximation methodology in estimating the actual flight times.

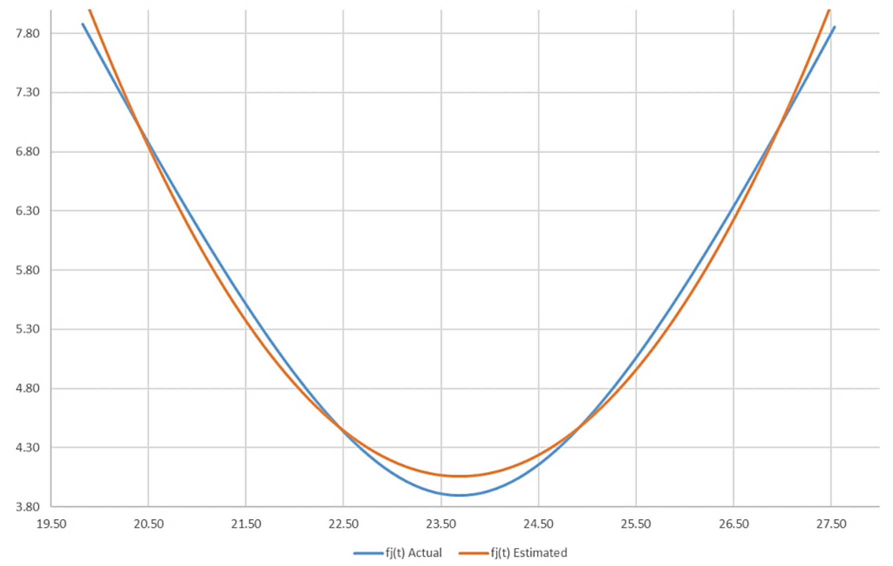


Figure 33. Regression in Customer Time Range

4.4. The Quadratic Model

The final version of the proposed model will be obtained by replacing the flight time functions with their quadratic estimation functions. The parameters of the model were given previously in Chapter 4.2.3. The estimation functions contain the estimated regression coefficients a, b, and c which are the additional parameters of the proposed model. These additional parameters are given as follows:

a_j^{dep} : *Regression a coefficient of drone departures from vehicle to location j*

b_j^{dep} : *Regression b coefficient of departures from vehicle to location j*

c_j^{dep} : *Regression c coefficient of departures from vehicle to location j*

a_j^{arr} : *Regression a coefficient of arrivals from location j to vehicle*

b_j^{arr} : *Regression b coefficient of arrivals from location j to vehicle*

c_j^{arr} : *Regression c coefficient of arrivals from location j to vehicle*

The revised model with quadratic flight times is given as follows. As previously mentioned, $f_{t,j}^{arr}$ and $f_{t,j}^{dep}$ functions are replaced in this revised model with their quadratic estimation functions which are derived in the previous section. It is worth expressing a modelling detail in replacing the original functions with their estimation counterparts. In equation (53), the constant coefficient of the quadratic regression functions a_j^{arr} and a_j^{dep} are multiplied by $Y_{d,j}^B$ and $X_{d,j}^B$ respectively. The reason is that if the flight of the drone has not occurred, we need to prevent the contribution of these coefficients to the cost. In other words, the quadratic flight time estimation function must be active only if there is such a flight. On the other hand, such a correction is not required for b and c coefficients because constraints in equations (56) and (63) force $Y_{d,j}$ and $X_{d,j}$ values to be zero when $Y_{d,j}^B = 0$ and $X_{d,j}^B = 0$ respectively. Accordingly, it appears to be that b and c coefficients are indirectly multiplied by the corresponding $Y_{d,j}^B$ and $X_{d,j}^B$ variables.

$$Min\ Z^{hwy} + Z^{drn} \tag{51}$$

$$Z^{hwy} = c^{hwy} \cdot \sigma \cdot \frac{\tau}{60} \tag{52}$$

$$Z^{drn} = c^{drn} \cdot \left[\sum_{\forall d \in S_j^D} \sum_{\forall j} \left(a_j^{arr} \cdot Y_{d,j}^B + b_j^{arr} \cdot Y_{d,j} + c_j^{arr} \cdot {Y_{d,j}}^2 \right) + \sum_{\forall d \in S_j^D} \sum_{\forall j} \left(a_j^{dep} \cdot X_{d,j}^B + b_j^{dep} \cdot X_{d,j} + c_j^{dep} \cdot {X_{d,j}}^2 \right) \right] \tag{53}$$

Subject to

$$U_d^B = 0 \qquad \forall d \notin S_0^D \tag{54}$$

$$Y_{d,j}^B = 0 \qquad \forall j, d \notin S_j^D \tag{55}$$

$$Y_{d,j} \leq M \cdot Y_{d,j}^B \qquad \forall j, d, (d,j) \in S^{DJ} \tag{56}$$

$$X_{d,j'} = 0 \qquad \forall j, j' < j, d, (d,j) \in S^{DJ} \tag{57}$$

$$\sum_{\forall j} X_{d,j}^B \leq 1 \qquad \forall j, d \tag{58}$$

$$X_{d,j} \leq M \cdot X_{d,j}^B \qquad \forall j, d \tag{59}$$

$$\sum_{\forall d} X_{d,j}^B \geq q_j \qquad \forall j \tag{60}$$

$$\sum_{\forall j} X_{d,j}^B \leq U_d^B \qquad \forall j, d \in S_0^D \tag{61}$$

$$\sum_{j'=j}^{J} X_{d,j'}^B \leq Y_{d,j}^B \qquad \forall j, j' \geq j, d \in S_j^D \tag{62}$$

$$X_{d,j'} \leq M \cdot Y_{d,j}^B \qquad \forall j, j' \geq j, d \in S_j^D \tag{63}$$

$$Y_{d,j} + \left(a_j + b_j \cdot Y_{d,j} + c_j \cdot {Y_{d,j}}^2\right) + t^{setup} \leq \sum_{j'=j}^{J} X_{d,j'} + M \cdot \left(1 - \sum_{j'=j}^{J} X_{d,j'}^B\right) \qquad \forall d \notin S_0^D, (d,j) \in S^{DJ} \tag{64}$$

$$Y_{d,j} + \left(a_j + b_j \cdot Y_{d,j} + c_j \cdot {Y_{d,j}}^2\right) \leq t_j^{arr-l} + M \cdot \left(1 - Y_{d,j}^B\right) \qquad \forall j, d, (d,j) \in S^{DJ} \tag{65}$$

$$t_j^{arr-e} - M \cdot (1 - Y_{d,j}^B) \leq Y_{d,j} + (a_j + b_j \cdot Y_{d,j} + c_j \cdot {Y_{d,j}}^2) \qquad \forall j, d, (d,j) \in S^{DJ} \tag{66}$$

$$X_{d,j} \leq X_{d,j}^B \cdot t_j^{dep-l} \qquad \forall j, d \tag{67}$$

$$X_{d,j}^B \cdot t_j^{dep-e} \leq X_{d,j} \qquad \forall j, d \tag{68}$$

$$X_{d,j} \leq \tau \qquad \forall j, d \tag{69}$$

$$Y_{d,j} + \left(a_j + b_j \cdot Y_{d,j} + c_j \cdot {Y_{d,j}}^2\right) \leq \tau + M \cdot \left(1 - Y_{d,j}^B\right) \quad \forall j, d, (d,j) \in S^{DJ} \tag{70}$$

Explanations of the constraints are as follows. Constraint (54) guarantees that drones that are not initially at the depot cannot be taken into the truck. Constraint (55) ensures that drones that are not in customer j cannot be taken off from customer j. Constraint (56) enforces that a drone that is not taken off from the customer j ($Y_{d,j}^B = 0$) has takeoff time equal to zero ($Y_{d,j} = 0$) which indicates that it has not taken off from that customer. Constraint (57) guarantees that a drone at customer j cannot be sent to another customer which is prior to the j^{th} customer. Constraint (58) ensures that a drone can be sent to a customer at most once. Constraint (59) enforces a drone that is not taken off from the truck ($X_{d,j}^B = 0$) has takeoff time equal to zero ($X_{d,j} = 0$) which indicates that it has not taken off from the truck. Constraint (60) guarantees that all customers' demands must be met. Therefore, at least q_j drone must go to the customer j. Constraint (61) ensures that regarding the drones those are initially at the depot, only those drones taken from the depot can be used to satisfy a customer’s demand. Constraint (62) guarantees the fact that in order to send a drone that is initially at customer j to that customer or to a subsequent customer, it is required that the drone is taken off from customer j. Constraint (63) guarantees that if a drone is initially at customer j and it will be used to satisfy the demand of that customer or to a subsequent customer, to take the drone into the truck is required. Constraint (64) states that if a drone which is initially at customer j will be used to satisfy a customer’s demand, the earliest time that drone can depart from the truck is found by the addition of three components: its departure time from the customer, travel time to the truck and setup time. Constraint (65) guarantees that if a drone is to be taken from customer j to the truck, its arrival time to the truck, which is the addition of its departure time from the customer and travel time to the truck, must not exceed the latest available time that the truck can take the drone. Constraint (66) ensures that if a drone is to be taken from customer j to the truck, its arrival time to the truck, which is the addition of its departure time from the customer and travel time to the truck, must not be before the earliest time the truck can take the drone. Constraint (67) enforces that if a drone will depart from the truck towards customer j, its departure must not be later than its latest departure time. Constraint (68) guarantees that if a drone will depart from the truck towards customer j, its departure must not be before its earliest departure time. Constraint (69) ensures that all departures from the truck must be within the total time prescribed for milk-run distribution. Constraint (70) ensures that all arrivals to the truck must be within the total time prescribed for milk-run distribution.

The estimation functions for flight times appear at equations (64), (65), (66), and (70) but unlike the function Z^{drn} in equation (53), their constant coefficient a_j^{arr} is not multiplied with $Y_{d,j}^{B}$. Such a correction is not required for these constraints because these constraints all contain Big-M value. Accordingly, even if the flight time seems to be the constant value a_j^{arr} due to not multiplying with $Y_{d,j}^{B}$, this constant value is much less than Big-M and hence, corresponding constraints still work properly. One can state that still multiplying with $Y_{d,j}^{B}$ is required due to readability or esthetic purposes. However, when multiplied with $Y_{d,j}^{B}$, the code works slowly and causes efficiency problems. This problem can be much more serious for larger sized problems.

4.5. Sample Problem-Revisited

In this part, we elaborate the mathematical programming model on the sample problem which we defined previously. The following section is a summary of notations and figurative representations that we use in the problem. Next, we present a feasible solution to the sample problem using the estimated flight times.

4.5.1. Quadratic Model of the Sample Problem

The example's sets, parameters, and initial status will be explained first and a feasible solution for the example will be given later.

The figurative representation of the objects and their corresponding sets are given in Figure 34. These figures were introduced before in Figure 8. In this figure, corresponding sets are added.

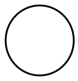
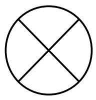
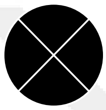
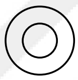
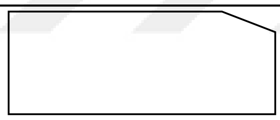
Figure Representation	Index	Set	Definition
	j	$S^j = \{j: j = 0,1,2, \dots, j, \dots J\}$	Customer locations, where 0 refers to depot
	d	$S^d = \{d: d = 1,2, \dots, d, \dots D\}$	Drones
			Empty drone
			Loaded drone
			Origin of the Circular Route
			Depot: Where Truck starts the route
			Vehicle (Truck)
			Position of the Truck in the Circle

Figure 34. Representations of the Figures with Corresponding Sets

The sets and definitions for this Sample Problem to be introduced here are given as follows. There are $J = 2$ customers and $D = 5$ drones. Initially, the fourth drone is at the first customer and the fifth drone is at the second customer. The first three drones are at the depot; first and second ones are loaded whereas the third one is empty.

j	:	$S^L = \{j: j = 0,1,2\}, Customer\ locations\ (0\ means\ depot)$
d	:	$S^D = \{d: d = 1,2,3,4,5\}, Drones$
S_j^D	:	$S_1^D = \{d: d = 4\}$ $S_2^D = \{d: d = 5\}$ $Drones\ that\ are\ empty\ in\ customer\ j$
S_0^D	:	$S_0^D = \{d: d = 1,2,3\}, Drones\ at\ the\ depot$
S_0^{D-e}	:	$S_0^{D-e} = \{d: d = 3\}, Drones\ empty\ in\ depot$
S_0^{D-l}	:	$S_0^{D-l} = \{d: d = 1,2\}, Drones\ loaded\ in\ the\ depot$
t	:	$0 \leq t \leq 60, Time$
(d,j)	:	$S^{DJ} = \{(d, j): (4,1), (5,2)\}, The\ drones\ at\ the\ customers$

The parameters of the Sample Problem are given in Table 13. The first row introduces the demands of the customers. The example's two customers have 2 and 1 demand respectively. The total time of the milk-run is 60 minutes. The third and fourth lines give the earliest and latest time a drone can take off from the vehicle, whereas the fifth and sixth lines give the earliest and latest times that a drone can land on the truck departed from the specified customer. The example's values are given in the right-most column. The seventh and eighth rows are reserved for departure and arrival flight times, which are estimated by a second-degree polynomial function. The reason for this estimation instead of using the exact values directly is for its mathematical tractability. A detailed explanation of this tractability and the parameter estimation of the corresponding regression functions will be handled in the next section. Set up time of the drone in the example is 3 minutes, as given in the last row of Table 13.

Table 13. Parameters of the Sample Problem

Parameter	Explanation	Values at Sample Problem
q_j	Demand at customer location *j* (in unit size)	$q_1 = 2\ and\ q_2 = 1$
τ	Total time of milk-run	$\tau = 60$
t_j^{dep-e}	The earliest time that a drone can take-off from the truck to location *j*	$t_1^{dep-e} = 4.83$ and $t_2^{dep-e} = 32.45$
t_j^{dep-l}	The latest time that a drone can take-off from the truck to location *j*	$t_1^{dep-l} = 12.55$ and $t_2^{dep-l} = 40.17$
t_j^{arr-e}	The earliest time that a drone can land-on to the truck departed from location *j*	$t_1^{arr-e} = 0.01$and $t_2^{arr-e} = 27.41$
t_j^{arr-l}	The latest time that a drone can land-on to the truck departed from location *j*	$t_1^{arr-l} = 17.57$and $t_2^{arr-l} = 45.22$
$f_{t,j}^{Est,Dep}$	Flight time of a drone that takes-off at time *t* from the truck towards customer location *j*	$f_{t,1}^{dep} = 24.7315 - 4.7598t + 0.2739t^2$and $f_{t,2}^{dep} = 365.2538 - 19.8942t + 0.2739t^2$
$f_{t,j}^{Est,Arr}$	Flight time of a drone that takes-off at time *t* from customer location *j* towards the truck	$f_{t,1}^{arr} = 16.7090 - 2.6984t + 0.1550t^2$and $f_{t,2}^{arr} = 206.8815 - 11.1193t + 0.1531t^2$
t^{setup}	Setup time of a drone	$t^{setup} = 3\ min$

The values at the sample problem given in Table 13 are demonstrated in Figure 35. The truck will start its tour at time t=0 and the tour will end at t=60. The first customer's demand can be satisfied within the time interval $[4.83, 12.55]$. On the other hand, the fourth drone, which is currently in the first customer can be taken to the truck in the time interval $[0.01, 17.57]$. Likewise, the second customer's demand can be satisfied within the time interval $[32.45, 40.17]$ and the fifth drone, which is currently in the second customer can be taken to the truck within the time interval $[27.41, 45.22]$.

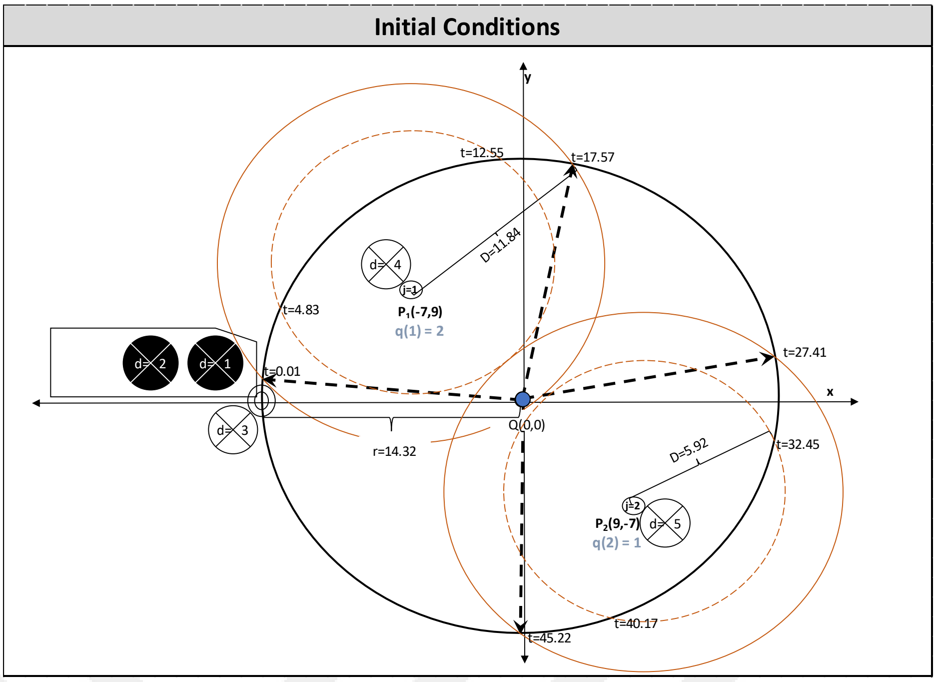


Figure 35. Initial Conditions of the Sample Problem

4.5.2. A Feasible Solution for the Sample Problem

The flight times are similar to those in the sample problem introduced in chapter 3.4.2. However, this time, calculations are performed using quadratic estimation functions, given in the 7th and 8th rows of Table 13. The resulting table is Table 14, which is the summary of estimated flight times to be used in the sample problem.

Table 14. Estimated Flight Times to be used in the Sample Problem

t: Time	Drone	From	To	Flight Time	Arrival Time
1.06	4	Customer 1	Truck	14.01	15.07
6.53	1 and 2	Truck	Customer 1	5.33	11.86
28.30	5	Customer 2	Truck	14.82	43.12
33.45	4	Truck	Customer 2	6.30	39.75

A feasible solution for the problem will be given with explanations step by step and with the help of snapshots. A feasible solution for the example is summarized in Table 15.

Table 15. Summary of the Critical Times and Events Using Estimated Flight Times

Time	**Event**
0.00	The Departure of the Truck from Depot
1.06	The Departure of Drone 4 from Customer 1 towards the Truck
6.53	The Departure of Drone 1 and 2 from the Truck towards Customer 1
11.86	The Arrival of Drone 1 and 2 to Customer 1
15.07	The Arrival of Drone 4 to the Truck
28.30	The Departure of Drone 5 from Customer 2 towards the Truck
33.45	The Departure of Drone 4 from the Truck towards Customer 2
39.75	The Arrival of Drone 4 to Customer 2
43.12	The Arrival of Drone 5 to the Truck
60.00	The End of the Tour

The explanation of the critical events in chronological order is as follows. At time 0.0, the truck starts its tour and departs from the depot. Interaction range with the first customer starts at time 0.01 and approximately one minute after the range is entered, at time 1.06, the fourth drone departs from the first customer towards the truck. The flight time of this drone is found to be 14.01 by the function $f_1^{Est,Arr}(1.06)$. Next, the truck enters loaded drone range at time 4.83 and at time 6.53, both of the drones in the truck depart through the first customer so that demand of the first customer is fully satisfied. The flight time of these two drones at 6.53 is found to be 5.33 by the function $f_1^{Est,Dep}(6.53)$ and the arrival of these two drones to first customer is at $6.53 + 5.33 = 11.86$. At time 15.07 (found by 1.06+14.01=15.07), the fourth drone arrives at the truck.

Because of the fact that the two customers in the example are located at the opposite quarters, no critical event occurs between $t = 14.86$ and $t = 28.30$. At $t = 28.30$, which is just after the starting time of the second customer's interaction region, the events related to the second customer start, and the first event is the departure of the fifth drone from the second customer towards the truck. The flight time of this drone is found to be 14.82 by the function $f_2^{Est,Arr}(28.30)$ and the arrival of the fifth drone to the truck is $28.30 + 14.82 = 43.12$. At time 33.45, the fourth drone departs from the truck towards the second customer and the corresponding flight time is found to be $f_2^{Est,Dep}(33.45) = 6.30$. Accordingly, the arrival of the fourth drone to the second customer is at time $33.45 + 6.30 = 39.75$. Finally, at $t = 60.0$ the truck arrives at the depot and the tour ends.

Next, the explained events will be given step by step with corresponding snapshots. Figure 36 gives the snapshot of the example's feasible solution at t=0.00. This is the time when the truck departs from the depot.

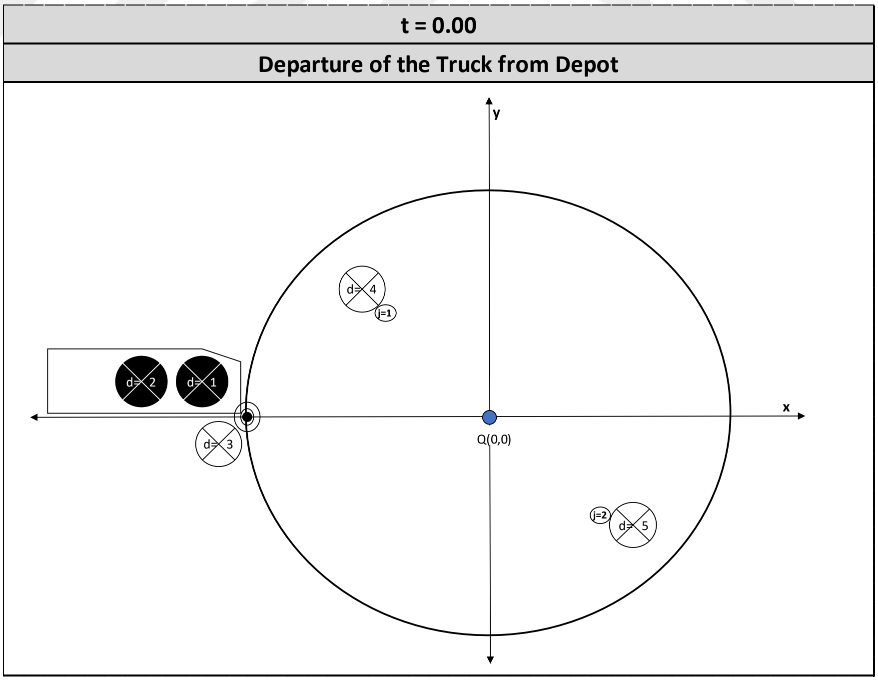


Figure 36. Snapshot at t = 0.0

Figure 37 gives the snapshot of the example's feasible solution at t=1.06. This is the time when the fourth drone departs from the first customer to the truck.

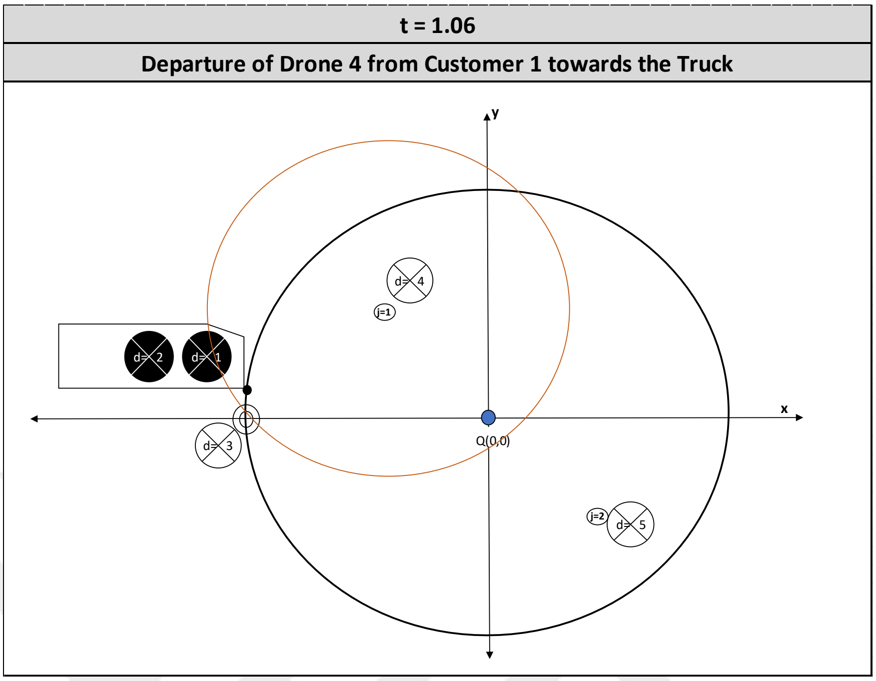


Figure 37. Snapshot at t = 1.06

Figure 38 gives the snapshot of the example's feasible solution at t=6.53. This is the time when both drones in the truck departs from the truck to the first customer.

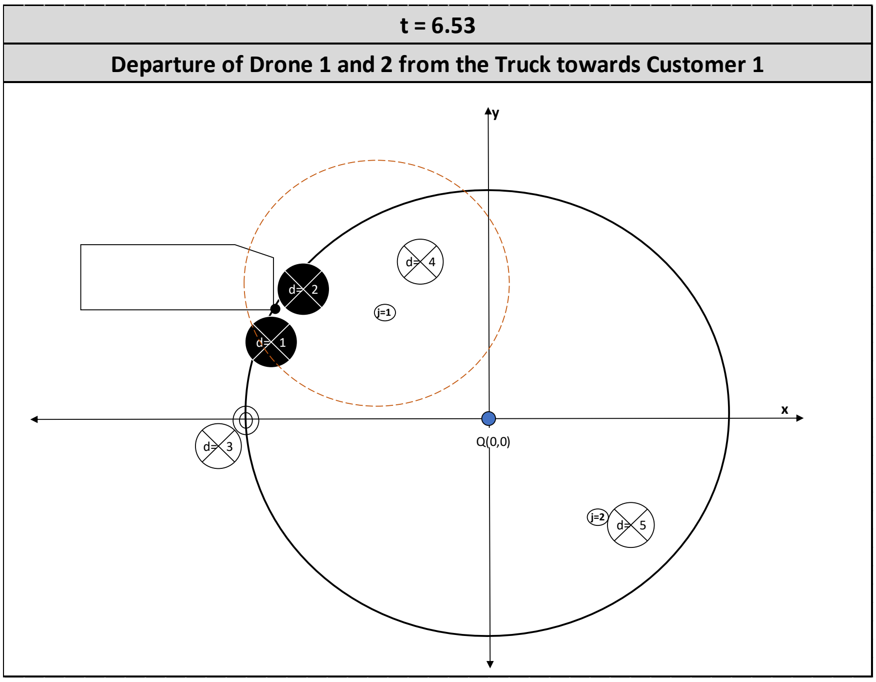


Figure 38. Snapshot at t = 6.53

Figure 39 gives the snapshot of the example's feasible solution at t=11.86. This is the time when the first two drones arrive at the first customer.

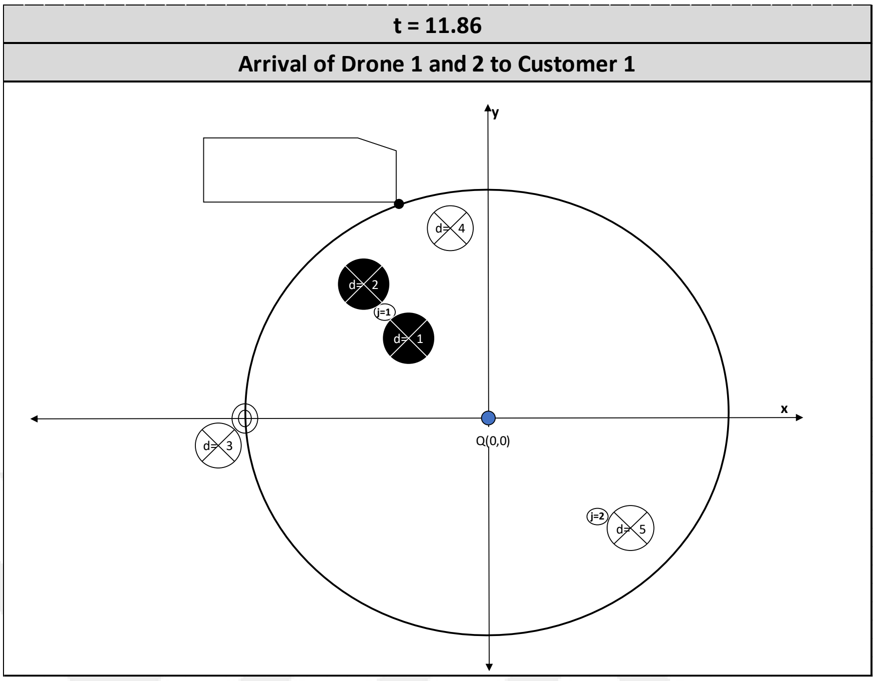


Figure 39. Snapshot at t = 11.86

Figure 40 gives the snapshot of the example's feasible solution at t=15.07. This is the time when the fourth drone arrives at the truck.

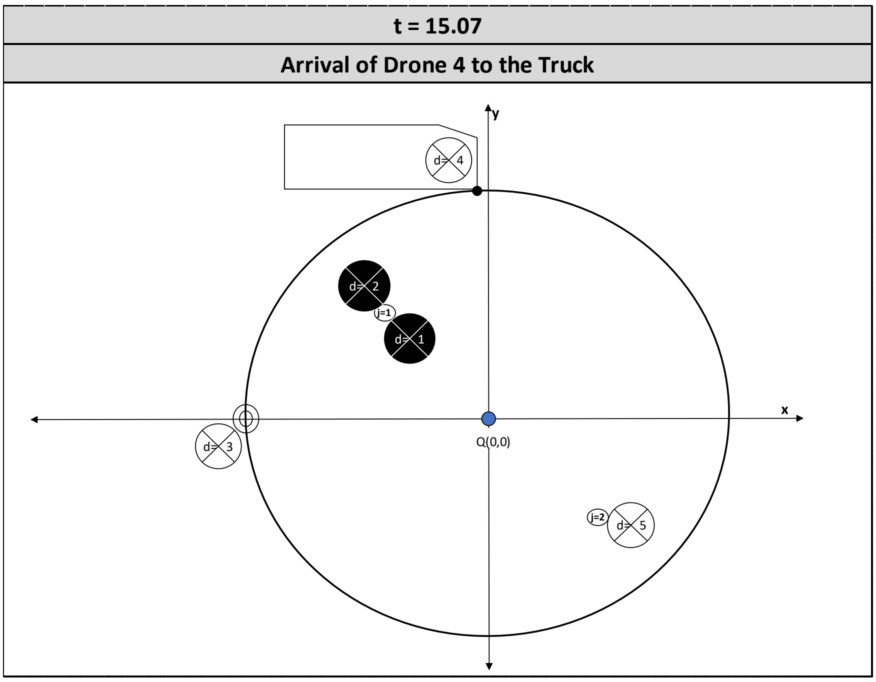


Figure 40. Snapshot at t = 15.07

Figure 41 gives the snapshot of the example's feasible solution at t=28.30. This is the time when the fifth drone departs from the second customer towards the truck.

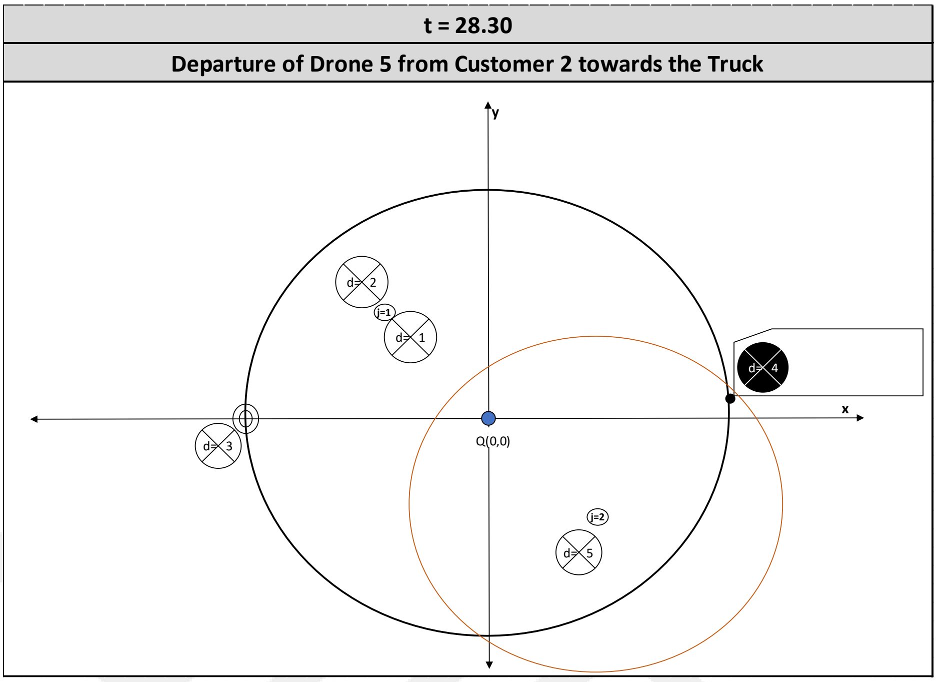


Figure 41. Snapshot at t = 28.30

Figure 42 gives the snapshot of the example's feasible solution at t=33.45. This is the time when the fourth drone departs from the truck towards the second customer.

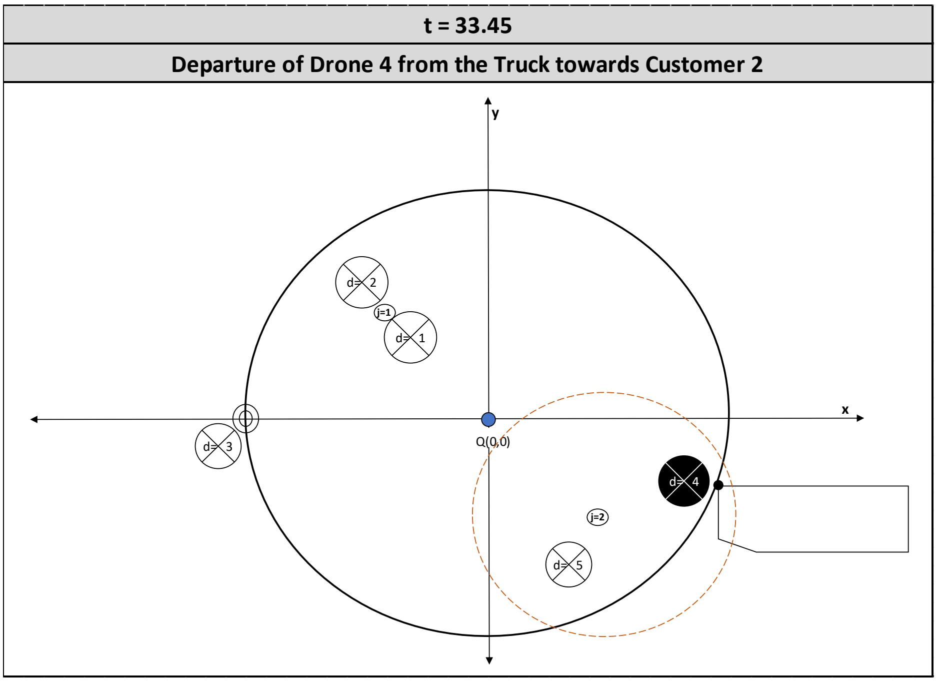


Figure 42. Snapshot at t = 33.45

Figure 43 gives the snapshot of the example's feasible solution at t=39.75. This is the time when the fourth drone arrives at the second customer.

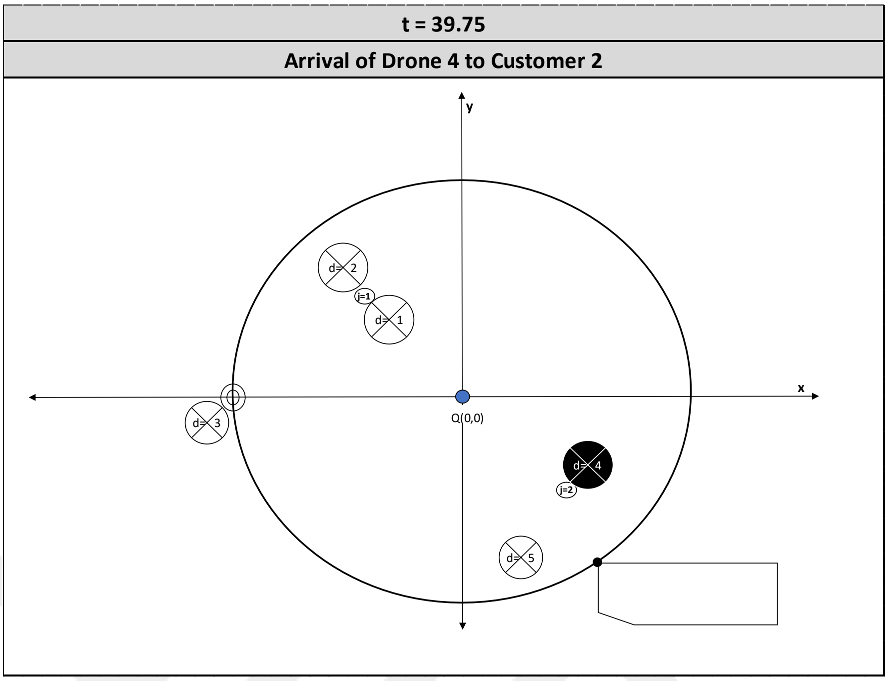


Figure 43. Snapshot at t = 39.75

Figure 44 gives the snapshot of the example's feasible solution at t=43.12. This is the time when the fifth drone arrives at the truck.

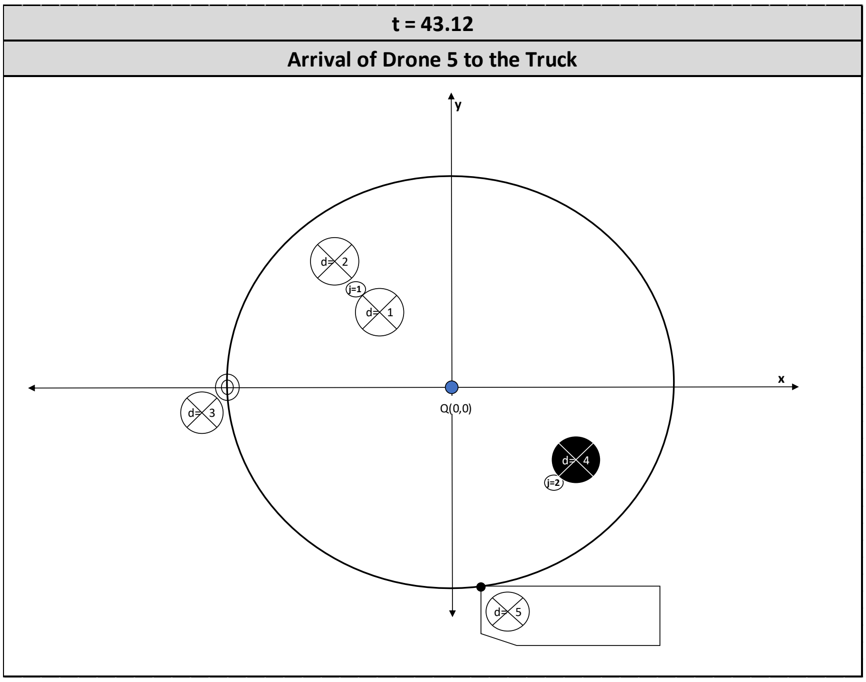


Figure 44. Snapshot at t = 43.12

Figure 45 gives the snapshot of the example's feasible solution at t=60. This is the time when the tour ends and the truck returns back to the depot.

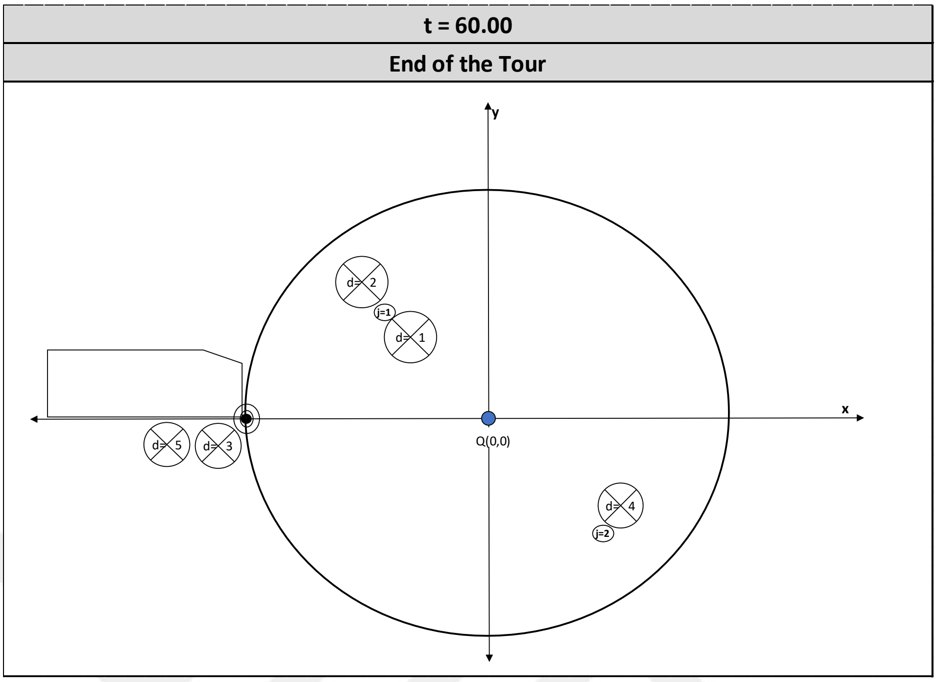


Figure 45. Snapshot at t = 60.0

4.5.3. Optimal Solution of the Sample Problem

We will introduce in this section the optimal solution of the sample problem. The problem is modelled and run in GAMS. The GAMS models for the CMR and DMR Models are given in Appendix A and Appendix B, respectively. First, we will explain the input parameters of the sample problem, and next, we will present the corresponding output.

The input parameters can be explained in three parts. The first part is the input of cost parameters, which are given in Table 16.

Table 16. Cost Input Parameters of the Sample Problem.

Source of the Cost	**Unit Cost (in Cents)**
Operating and fuel cost of the truck on the highway	6.51
Operating and battery usage cost of a drone	0.44
Operating and fuel cost of the truck in the city	8.39
Opportunity cost	14.13

The second part of the input parameters consists of the demands of the customers, given in Table 17.

Table 17. Demand Input Parameters of the Sample Problem

Customer: j	Demand: q(j)
1	2
2	1

Finally, the third part is related to the flight time coefficients of the customers. The earliest and latest interaction times for departure from the truck and arrival to the truck are given in the bottom part of Table 18 by tE and tL respectively. The regression coefficients of the quadratic regression function: $f_j^{Est}(t) = \mathrm{a} + \mathrm{b}t + \mathrm{c}t^2$ are given in a, b, and c column of the same table in the top part.

Table 18. Flight Time Input Parameters of the Sample Problem.

	CUSTOMER 1			CUSTOMER 2	
	Departure Coefficients	Arrival Coefficients		Departure Coefficients	Arrival Coefficients
a =	24.7315	16.7090	a =	365.2538	206.8815
b =	-4.7598	-2.6984	b =	-19.8942	-11.1193
c =	0.2739	0.1550	c =	0.2739	0.1531
tE =	4.83	0.01	tE =	32.45	27.41
tL =	12.55	17.57	tL =	40.17	45.22

Initial conditions of the sample problem are given in Figure 46 again so that the output is more easily followed.

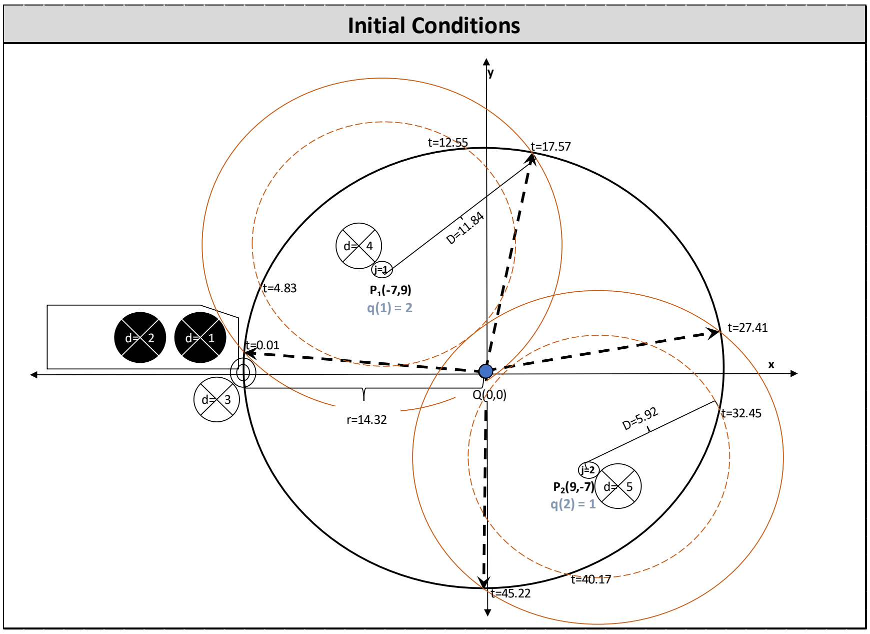


Figure 46. Initial Condition of the Sample Problem

Running the sample problem in GAMS, the objective function value and corresponding optimum solution values for the decision variables are given in Table 19. The minimum cost of the tour is found to be 607.30 cents. First two drones are taken from the depot, given by the decision variables $U_1^B = U_2^B = 1$. These drones are sent to the first customer, and the fourth drone is sent to the second customer, hence $X_{1,1}^B = X_{2,1}^B = X_{4,2}^B = 1$. The corresponding departure times of these drones are $X_{1,1} = X_{2,1} = 4.83\ and\ X_{4,2} = 40.17$ respectively. The fourth drone was taken from the first customer before it is sent to the second customer, and the fifth drone is taken from the second customer, which yields $Y_{4,1}^B = Y_{5,2}^B = 1$. The take-off times of these two drones are $Y_{4,1} = 0.01\ and\ Y_{5,2} = 32.24$ respectively.

Table 19. Optimum Solution Output of the Sample Problem

Optimal Solution	
Z* =	607.30

UB	
Drone	Solution
1	1
2	1
3	0
4	0
5	0

XB	Customer	
Drone	1	2
1	1	0
2	1	0
3	0	0
4	0	1
5	0	0

X	Customer	
Drone	1	2
1	4.83	0
2	4.83	0
3	0	0
4	0	40.17
5	0	0

YB	Customer	
Drone	1	2
1	0	0
2	0	0
3	0	0
4	1	0
5	0	1

Y	Customer	
Drone	1	2
1	0	0
2	0	0
3	0	0
4	0.01	0
5	0	32.24

Critical times and the summary of the events for the optimum solution of the sample problem are given in Table 20.

Table 20. Summary of the Critical Times and Events for the Optimum Solution

Time	Event
0.00	The Departure of the Truck from Depot
0.01	The Departure of Drone 4 from Customer 1 towards the Truck
4.83	The Departure of Drone 1 and 2 from the Truck towards Customer 1
12.96	The Arrival of Drone 1 and 2 to Customer 1
16.69	The Arrival of Drone 4 to the Truck
32.24	The Departure of Drone 5 from Customer 2 towards the Truck
39.77	The Arrival of Drone 5 to the Truck
40.17	The Departure of Drone 4 from the Truck towards Customer 2
48.30	The Arrival of Drone 4 to Customer 2
60.00	The End of the Tour

Finally, GAMS solution output with respect to the chronological order of the events is given again in Table 21. This table gives the same information as Table 20 but additionally, the activities of the drones can be separately followed in Table 21.

Table 21. Chronological Order of the Events and Corresponding Drone Activities

GAMS SOLUTION OUTPUT		DRONE			
TIME	EVENT	d1	d2	d4	d5
0.00	Departure of the Truck from Depot	In truck	In truck	At customer 1	At customer 2
0.01	Departure of Drone 4 from Customer 1 towards the Truck	-----	-----	Departure towards the truck	-----
4.83	Departure of Drone 1 and 2 from the Truck towards Customer 1	Departure towards customer 1	Departure towards customer 1	-----	-----
12.96	Arrival of Drone 1 and 2 to Customer 1	Arrival to customer 1	Arrival to customer 1	-----	-----
16.69	Arrival of Drone 4 to the Truck	-----	-----	Arrival to the truck	-----
32.24	Departure of Drone 5 from Customer 2 towards the Truck	-----	-----	-----	Departure towards the truck
39.77	Arrival of Drone 5 to the Truck	-----	-----	-----	Arrival to the truck
40.17	Departure of Drone 4 from the Truck towards Customer 2	-----	-----	Departure towards customer 2	-----
48.30	Arrival of Drone 4 to Customer 2	-----	-----	Arrival to customer 2	-----
60.00	End of the Tour	At customer 1	At customer 1	At customer 2	In truck

CHAPTER 5

NUMERICAL STUDY

5.1. Setup for Numerical Settings

The numerical settings are simply determinations of several problem parameters with respect to the database construction of the numerical study. The deriving parameters of the numerical study are the earliest times (t_j^{dep-e} and t_j^{arr-e}) and the latest times (t_j^{dep-l} and t_j^{arr-l}) that a drone keeps interaction with the truck in both directions: from the truck to a customer and vice versa, and the regression estimates of departure and arrival times ($f_{t,j}^{dep}$ and $f_{t,j}^{arr}$). The explanations of these parameters were given in Table 13 previously.

The calculation methodology of these parameters was built within this work so far. Here, these parameters are separately calculated for each of the customers located in the circular route. At first, the logic that customers are located will be explained. Next, two of the currently existing drones will be introduced with their necessary specifications to be used in the numerical settings. We will use these specifications to parametrize some measures so that they will build a bridge between drone specifications and model parameters. Finally, we will calculate the model parameters with the help of the findings.

5.1.1. Customer Locations

As stated earlier, this study will be set up on a numerical study on the service that a truck provides to customers placed in a route around which it travels with a speed of $V_{Tr} = 90km/h$. The radius of the route was calculated to be $r = 14.3239km$. In order to be able to place customers in convenient places in the circular route, we first determined the maximum distance from the truck's route at which a customer can be placed. We determined this distance to be 6km by evaluating the specifications of the drones which will form the basis for the numerical settings of this study. Evaluating these ideas, we draw another circle with the same center inside the milk-run tour circle. The customers will be placed between these two circles. The drawing of the two circles we mentioned is given in Figure 47.

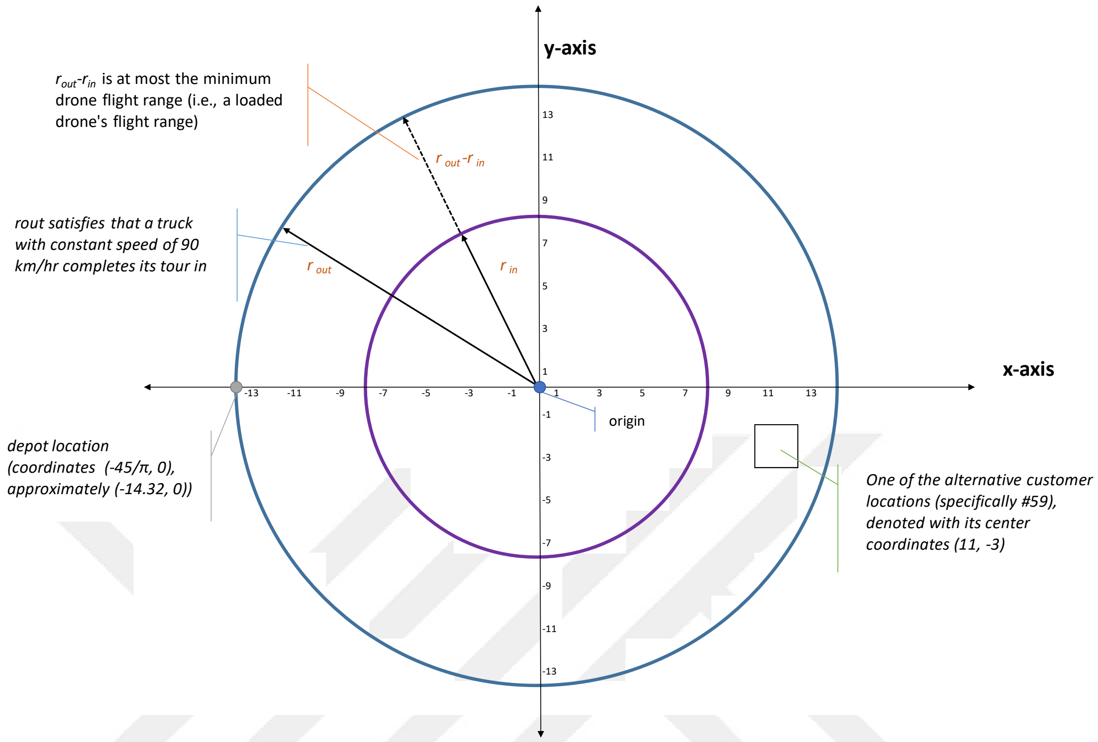


Figure 47. Customer Locations

In order to accommodate customers to suitable places, we draw pixel-like squares between these two circles. The outer circle, which is the milk-run tour, has a radius of 14.32km. In parallel to this setting, the coordinates are selected in a range of -14 and 14. We designate one side of these squares at 2km so that two or three customers can be located through the common radius of the inner circle and the outer circle. Customers will be at the center of gravity of these specified squares. 96 customer locations were created that could take place in this way. We have indexed the customers from 1 to 96 in order to be able to follow them in the runs of the model and to be able to select them randomly. The developed version by customer locations of the drawing Figure 47 in the explained manner is shown in Figure 48.

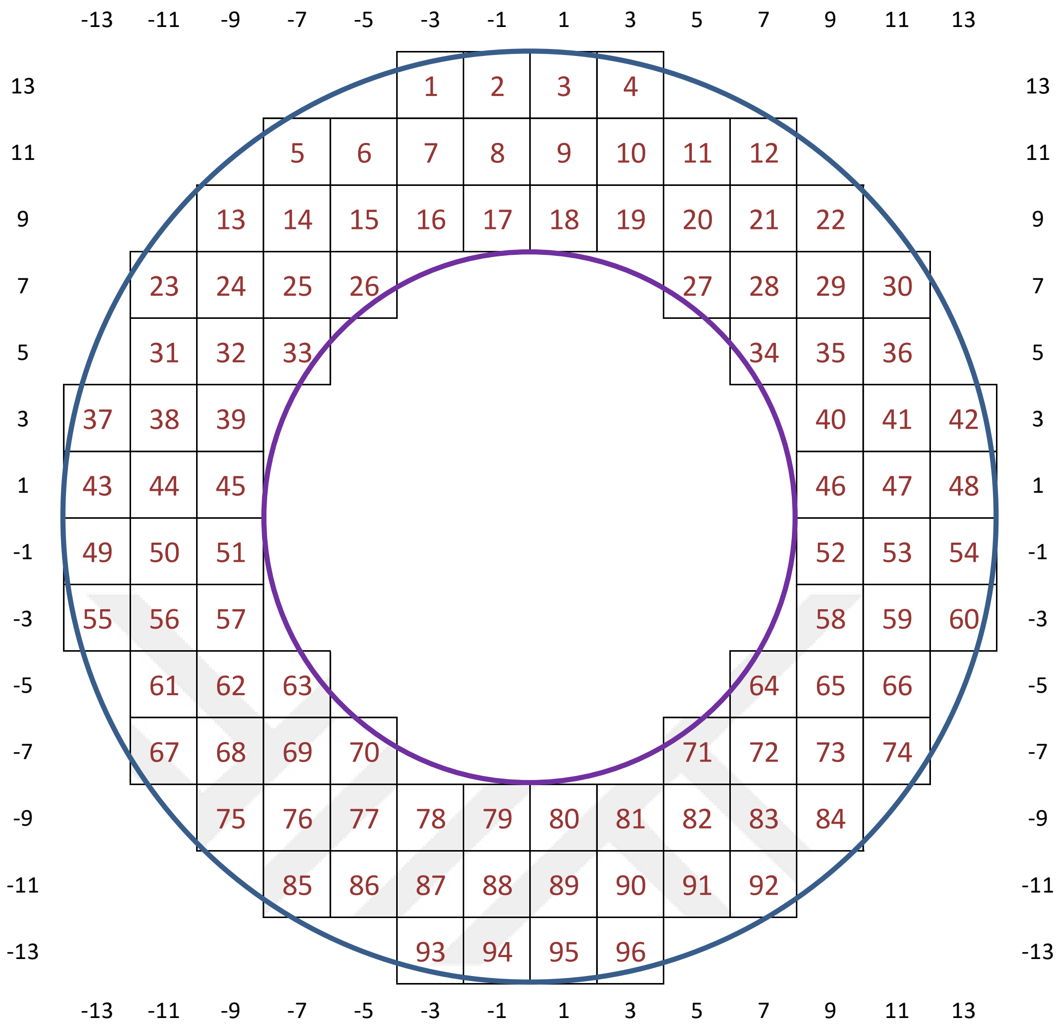


Figure 48. Exact Locations of the Customers

5.1.2. Drone and Truck Specifications

The settings for the truck speed of 90km/h and the milk-run time of one hour were selected so that this study can be carried out in accordance with real-life applications. Specifically, the truck we consider in this study is the Mercedes Sprinter Panelvan. This truck has the property of fixing its speed at 90km/h, and its fuel consumption is 9.4 liters per 100km in the city and 7.3 l/100km on the highway (Mercedes Benz, 2020). We use these parameters in determining the objective function and accordingly in the cost analysis.

Two drones produced by the DJI Company are suitable for this work due to their features. With similar concern, these two drones were chosen from the drones currently in use. The basic features of these two drones mentioned are as follows (The Drone Pro Shop, 2020)

1. DJI Matrice 210: maximum speed (without indicating empty or loaded) is given as 61.2 km/h in specifications.
2. DJI Matrice 210 V2: maximum speed (without indicating empty or loaded) is given as 61.2 km/h in specifications. We assume this speed for an empty drone.

The features of these two drones are given in Table 22.

Table 22. Features of the Drones

M210		
Max Payload		2.3 kg
Max Flight Time (with TB50 batteries)		13 min (full payload), 27 min (no payload)
Max Speed		61.2 km/h
M210 V2		
Max Payload		1.34 kg
Max Flight Time (with TB55 batteries)		24 min (full payload), 34 min (no payload)
Max Speed		61.2 km/h

5.1.3. Parametrization of Drone Specifications

Specific ratio parameters that will be used to determine the database to be used in the numerical study part are given in Table 23.

Table 23. Ratio Parameters

Ratio Parameter	**Explanation**	**Value Range**
$\boldsymbol{\beta}$	Range ratio of a loaded drone to an empty drone	$0 < \beta < 1$
$\boldsymbol{\delta}$	Speed ratio of the truck to an empty drone	$\delta > 1$ for DMR, $0 < \delta < 1$ for CMR
ϑ	Speed ratio of the truck on the highway to in the city	$\vartheta > 1$

To determine two setting values for the parameter β, the following facts are considered for the two drones. The flight times of the DJI Matrice 210 drone with TB50 batteries are 13 and 27 minutes for loaded and empty cases, respectively. Under real-life conditions, these times may not be attended due to battery performance, weather conditions, etc. Therefore, we use 15% less of these times and we considered the same speed for the empty and loaded drones. Consequently, the flight time's ratio defines the range ratio directly. We round the real ratio of 13/27= 0.48 to 0.5. Likewise, the flight times with TB55 batteries for the DJI Matrice 210 V2 are 24 and 34 minutes for loaded and empty cases, respectively. The corresponding rounded ratio of the real ratio of 24/34= 0.71 is 0.7. To sum up, two settings for the parameter value of β are 0.5 and 0.7.

As for β, we determined two settings for the parameter value for δ. The calculations for these values are as follows. The speed of the truck is 90km/h, which is the legal maximum speed limit for trucks on highways (General Directorate of Highways, 2020). The maximum speed for both of the drones DJI Matrice 210 and DJI Matrice 210 V2 when they are empty is 61.2 km/h. We rounded this value to 60km/h. We also assumed the case that under extreme conditions like high wind speed, rainy weather, etc., or with a less capacitated drone to be used, the speed of 60km/h may not be achieved. Accordingly, we have chosen a second setting value of 45 km/h. Using these two speeds which are 60km/h and 45km/h, the setting values for the parameter value of δ are $90/60 = 1.5$ and $90/45 = 2.0$.

To be used in runs of the model, we need to calculate the ranges of the drone when it is loaded and when it is empty. Our starting point for performing these calculations is the DJI Matrice 210 drone's maximum flight time when it is loaded. This flight time is 13 minutes. We have determined two maximum speed values when the DJI Matrice 210 drone is empty, which are 60km / h and 45km / h. Considering that the maximum time may not be reached in extreme conditions like bad weather conditions, we anticipated the case that the flight time may be 15% less. We used the values of $\beta = 0.7$ and $\delta = 2$ in the first step of our calculations. In the first step, we showed the loaded flight time, loaded range, empty drone range, and empty flight time values with variables $x_1, x_2, x_3, and\ x_4$, respectively in Table 24.

Table 24. Parameters for the Drones

DELTA	2
Empty Drone Range	x_3
BETA	0.7
Loaded Drone Range	x_2
Max empty speed	45
Loaded flight time	x_1
Empty flight time	x_4
Loaded range	x_2
Time/Max Time	0.85
Max Loaded Flight Time	13

As previously mentioned, x_1 value will be 85% of the max loaded flight time. It follows that $x_1 = 13 \cdot 0.85$. Based on the equation $Distance\ = Velocity \cdot Time$, we calculated the value of x_2 in minutes as $x_2 = \frac{45x_1}{60}$. Although the 45km / h drone speed given here is the empty drone speed, this calculation is still consistent due to the fact that 15% of loss in time is included in the calculation. In addition, we need to make such an approximation because we could not access the speed information when the drone was loaded. The parameter value β was the ratio of loaded drone range to empty drone range. Hence, an empty drone range can be found by dividing loaded drone range to β. The corresponding formula is$x_3 = \frac{x_2}{\beta}$. Finally, empty flight time is found by appealing to the formula $Time\ = Distance\ / Velocity$, which gives x_4 value as $x_4 = \frac{x_3}{45} \cdot 60$.

After using the formulas that we mentioned in the previous paragraph, Table 24 is updated and Table 25 is obtained.

Table 25. Updated Parameters for the Drones

DELTA	2
Empty Drone Range	11.84
BETA	0.7
Loaded Drone Range	8.29
Max empty speed	45
Loaded flight time	11.05
Empty flight time	15.79
Loaded range	8.29
Time/Max Time	0.85
Max Loaded Flight Time	13

Since the speed value corresponding to $\delta = 1.5$ value is 60, we calculated the empty range for this value by taking the drone speed 60km / h. Like Loaded flight time, we have accepted the empty flight time value constant for different speeds. If we designate empty drone range at a speed of 60km/h with the variable x_5, the variable x_5 and the other previously values required for the calculation of x_5 are given in Table 26.

Table 26. Parameters for Empty Drones

Max empty speed	45	60
Empty flight time	15.79	15.79
Empty range	11.84	x_5

In order to obtain the formula for x_5, we considered the formula for x_4 which was the formula that $x_4 = \frac{x_3}{45} \cdot 60$, and replaced the speed value 45 with 60. Accordingly, the formula for x_5 is given by: $x_5 = \frac{x_3}{60} \cdot 60$. Using this formula, we updated Table 26 as given in Table 27.

Table 27. Updated Parameters for Empty Drones

Max empty speed	45	60
Empty flight time	15.79	15.79
Empty range	11.84	15.79

We have created Table 28 by gathering all our calculations so far. This last table is the numerical setting table that we will use in model runs.

Table 28. Numerical Setting

DELTA	2	1.5	2	1.5
D: Empty Drone Range	11.84	15.79	11.84	15.79
BETA	0.7	0.7	0.5	0.5
Loaded Drone Range	8.29	11.05	5.92	7.89

5.1.4. *Determination of Model Parameters*

To start with, we will summarize the leading findings, illustrating the 29th customer, which can be found in Figure 48. As can be followed from Table 29, the 29. customer's coordinates are $(x, y) = (9,7)$. For the parameter values of $\delta = 2$ and $\beta = 0.5$, a loaded drone's range is found as 5.92km which is shown in Table 28. The center, intersection points, and the earliest and latest times for the 29th customer is given in Table 29. All these calculations are also performed for empty drones and are used where necessary, but not given here. The parametrization for empty drones is used for arrival flight times from the customer to the truck, whereas the parametrization for loaded drones is used for departure flight times from the truck to the customer in the mixed integer nonlinear programming model of this work.

Table 29. Intersection Points

			Intersection Points					
Index	x_C	y_C	x_E	y_E	x_L	y_L	t_E	t_L
29	9	7	6.93	12.53	13.85	3.63	19.82	27.54

Within the range of (tE, tL), the estimated flight times regression function for the 29th customer is found as in equation (71) using the methodology of chapter 4.3.1.

$$f_{\mathrm{j}}^{(E)}(t) = 157.8335 - 12.9844t + 0.2741t^2 \tag{71}$$

The general form of the estimated flight times regression model is:

$$f_{\mathrm{j}}^{(E)}(t) = \mathrm{a} + \mathrm{b}t + \mathrm{c}t^2 \tag{72}$$

We performed the same steps for each of the 96 customers that we applied to the 29th customer and obtained in following table to be used in model runs:

Table 30. Parameters of the Customers and Coefficients of Drone Flight Times

Cust. j	x_C	y_C	x_E	y_E	x_L	y_L	t_E	t_L	a	b	c
1	-3.00	13.00	-8.69	11.39	2.82	14.04	8.78	16.89	68.0518	-10.2899	0.4009
2	-1.00	13.00	-6.89	12.55	4.89	13.46	10.21	18.33	79.4558	-10.8184	0.3791
3	1.00	13.00	-4.89	13.46	6.89	12.55	11.67	19.79	96.1308	-11.9300	0.3791
4	3.00	13.00	-2.82	14.04	8.69	11.39	13.11	21.22	120.1469	-13.7629	0.4009
5	-7.00	11.00	-11.99	7.83	-2.02	14.18	5.53	13.65	37.1325	-7.2692	0.3791
…	…	…	…	…	…	…	…	…	…	…	…
29	9.00	7.00	6.93	12.53	13.85	3.63	19.82	27.54	157.8335	-12.9844	0.2741
…	…	…	…	…	…	…	…	…	…	…	…
94	-1.00	-13.00	4.89	-13.46	-6.89	-12.55	41.67	49.80	793.8375	-34.6159	0.3785
95	1.00	-13.00	6.89	-12.55	-4.89	-13.46	40.20	48.33	743.8781	-33.5057	0.3785
96	3.00	-13.00	8.69	-11.39	-2.82	-14.04	38.78	46.90	736.8902	-34.3112	0.4005

5.1.5. Settings for Classical Milk Run

The classical milk-run model will be the base model to be compared to the milk-run model integrated with drones, as is previously mentioned. Hence, in the last part of the section, we will introduce the settings for classical milk-run model in this study. In order to make the comparison of the classical milk-run model outputs and the outputs of the proposed model, we will run the problem instances that we will generate using the classical milk-run model and compare our findings. The classical milk-run application will be based on the standard traveling salesperson problem (TSP). The centers of the 96 possible customers are given in part in Table 31.

Table 31. Center Locations of Possible Customers

Index	Xc	Yc
1	-3	13
2	-1	13
3	1	13
4	3	13
5	-7	11
…	…	…
95	1	-13
96	3	-13

To be used in the classical milk-run solution, corresponding TSP optimal solutions will be found. For this purpose, paired distances between each of the two customers, and their distance to the depot are calculated. A part of the distance matrix is given in Table 32.

Table 32. Distance Matrix

Euclidean Distances		**Index**	**0**	**1**	**2**	**3**	**4**	**5**	**…**	**95**	**96**
		Xc	**14.32394**	-3	-1	1	3	-7	…	1	3
Index	**Xc**	**Yc**	**0**	13	13	13	13	11	…	-13	-13
0	**14.32394**	**0**	0.00	21.66	20.10	18.62	17.24	23.99	…	18.62	17.24
1	-3	13	21.66	0.00	2.00	4.00	6.00	4.47	…	26.31	26.68
2	-1	13	20.10	2.00	0.00	2.00	4.00	6.32	…	26.08	26.31
3	1	13	18.62	4.00	2.00	0.00	2.00	8.25	…	26.00	26.08
4	3	13	17.24	6.00	4.00	2.00	0.00	10.20	…	26.08	26.00
5	-7	11	23.99	4.47	6.32	8.25	10.20	0.00	…	25.30	26.00
…	…	…	…	…	…	…	…	…		…	…
95	1	-13	18.62	26.31	26.08	26.00	26.08	25.30	…	0.00	2.00
96	3	-13	17.24	26.68	26.31	26.08	26.00	26.00	…	2.00	0.00

5.2. Numerical Settings for the Model Runs

In this section, we will explain the methodology that we take runs of the numerical study. First, we mention the setting of the DMR model. As previously mentioned, we will use two distinct settings of δ and β values each. δ is the speed ratio of the truck to empty drone with two setting values of 2 and 1.5; and β is the range ratio of a loaded drone to empty drone with two setting values of 0.7 and 0.5. Two settings for the number of customers are 10 customers and 20 customers, which are fixed for each run, and their total number of demands are 30 and 60 respectively. The customers and their demands are randomized for different instances. Finally, we want to see the effect of the case whether the customers initially have drones or not. For this parameter, we will use 2 setting values, which are 0xNC and 3xNC. In other words, for instance, when the number of customers is 20, the number of empty drones at the customers will be 0 and 60. As a result, the total number of cases is $2 \cdot 2 \cdot 2 \cdot 2 = 16$. To create a variability for the possible effects of the parameters, we will randomly select 20 instances for each case. Accordingly, the overall number of instances makes $16 \cdot 20 = 320$. On the other hand, for the CMR model, the only valid setting is the number of customers. Delta, Beta, and empty drone parameters are not applicable for the CMR model. As a result, the CMR model has 2 cases. We also take runs for the same random instances for comparison purposes, and the overall number of instances makes $2 \cdot 20 = 40$. The information that is given in this paragraph and corresponding calculations are summarized in Table 33.

Table 33. Parameters and Their Settings in the Model Runs

Parameter	DMR MODEL		CMR MODEL	
	Values Assumed	**Number of Settings**	**Values Assumed**	**Number of Settings**
Delta	2.0, 1.5	2	N.A	N.A
Beta	0.7, 0.5	2	N.A	N.A
Number of customers	10, 20	2	10, 20	2
Empty drone	0NC, 3NC	2	N.A	N.A
Total number of cases	16		2	
Random instance per case	20		20	
Total number of instances	320		40	

At this step, the random instance selection mechanism needs an explanation. The pixelling mechanism of customer locations was explained before and demonstrated in Figure 48. As can be followed from this figure, a total of 96 possible customer locations exist. If the case of interest is, to illustrate, $\delta = 2.0$, $\beta = 0.7$, $NC = 20$, and the number of empty drones is 3xNC, this specific case must be randomly represented by 20 instances. In each of these 20 instances, 20 of 96 customers are selected randomly (i.e. each of $C(96{,}20)$ has an equal probability of selection), and the total demand of 60 items are distributed to 20 customers. The randomization process of customer selection and item distribution is performed as follows. There are 96 possible customers to be selected. First, 20 random numbers from a discrete uniform distribution from 1 to 96 are selected. Repeated numbers are reselected so that distinct 20 numbers, which are indices of the customers are obtained. Next, 60 continuous uniform random numbers between 0 and 1 are generated for the 60 items to be distributed among the customers. The cumulative distributions of 20 selected customers are formed by $F(j) = j/20$. Finally, 60 items are distributed to customers using the inverse transformation technique. Distribution of drones randomly to the customers is performed exactly in the same manner. The number of empty drones has 2 cases, which are 0 and 60 total number empty drones, and these drones are distributed to customers randomly. This specific setting is given in Table 34.

Table 34. An Instance of a Setting for the Model Runs

	Generator			Fixed			Drone Cases Generator	
i	**Rand(0,1)**	**Customer**	**Demand**	**i**	**Customer**	**Ranked**	**Case 0**	**Case 1**
1	0.775588	75	2	1	20	8	0	3
2	0.837577	81	2	2	66	13	0	3
3	0.781123	75	2	3	57	15	0	3
4	0.714461	69	3	4	27	16	0	4
5	0.853752	82	3	5	84	20	0	3
6	0.179044	18	3	6	53	27	0	4
7	0.924222	89	1	7	32	32	0	0
8	0.626325	61	4	8	85	33	0	3
9	0.820328	79	3	9	69	40	0	2
10	0.290711	28	4	10	51	49	0	3
11	0.099574	10	4	11	15	51	0	2
12	0.741614	72	4	12	33	53	0	3
13	0.921622	89	3	13	73	57	0	5
14	0.932781	90	1	14	40	66	0	5
15	0.430074	42	3	15	8	69	0	3
16	0.645275	62	2	16	49	71	0	1
17	0.949233	92	4	17	13	73	0	6
18	0.437992	43	7	18	95	84	0	2
19	0.13056	13	2	19	16	85	0	1
20	0.729477	71	3	20	71	95	0	4
		TOTAL	**60**			**TOTAL**	**0**	**60**

We will explain the numerical setting of the classical milk-run model formulation using the same 20 customers randomly selected. The corresponding distance matrix is given in Table 35.

Table 35. An Illustrative Distance Matrix

EUCLIDEAN DISTANCES		Index	0	8	13	15	16	20	27	32	33	40	49	51	53	57	66	69	71	73	84	85	95
		Xc	14.32394	-1	-9	-5	-3	5	5	-9	-7	9	-13	-9	11	-9	11	-7	5	9	9	-7	1
Index	Xc	Yc	0	11	9	9	9	9	7	5	5	3	-1	-1	-1	-3	-5	-7	-7	-7	-9	-11	-13
0	14.32394	0	0.00	18.86	25.00	21.32	19.52	12.96	11.66	23.85	21.90	6.11	27.34	23.35	3.47	23.52	6.00	22.44	11.66	8.79	10.46	23.99	18.62
8	-1	11	18.86	0.00	8.25	4.47	2.83	6.32	7.21	10.00	8.49	12.81	16.97	14.42	16.97	16.12	20.00	18.97	18.97	20.59	22.36	22.80	24.08
13	-9	9	25.00	8.25	0.00	4.00	6.00	14.00	14.14	4.00	4.47	18.97	10.77	10.00	22.36	12.00	24.41	16.12	21.26	24.08	25.46	20.10	24.17
15	-5	9	21.32	4.47	4.00	0.00	2.00	10.00	10.20	5.66	4.47	15.23	12.81	10.77	18.87	12.65	21.26	16.12	18.87	21.26	22.80	20.10	22.80
16	-3	9	19.52	2.83	6.00	2.00	0.00	8.00	8.25	7.21	5.66	13.42	14.14	11.66	17.20	13.42	19.80	16.49	17.89	20.00	21.63	20.40	22.36
20	5	9	12.96	6.32	14.00	10.00	8.00	0.00	2.00	14.56	12.65	7.21	20.59	17.20	11.66	18.44	15.23	20.00	16.00	16.49	18.44	23.32	22.36
27	5	7	11.66	7.21	14.14	10.20	8.25	2.00	0.00	14.14	12.17	5.66	19.70	16.12	10.00	17.20	13.42	18.44	14.00	14.56	16.49	21.63	20.40
32	-9	5	23.85	10.00	4.00	5.66	7.21	14.56	14.14	0.00	2.00	18.11	7.21	6.00	20.88	8.00	22.36	12.17	18.44	21.63	22.80	16.12	20.59
33	-7	5	21.90	8.49	4.47	4.47	5.66	12.65	12.17	2.00	0.00	16.12	8.49	6.32	18.97	8.25	20.59	12.00	16.97	20.00	21.26	16.00	19.70
40	9	3	6.11	12.81	18.97	15.23	13.42	7.21	5.66	18.11	16.12	0.00	22.36	18.44	4.47	18.97	8.25	18.87	10.77	10.00	12.00	21.26	17.89
49	-13	-1	27.34	16.97	10.77	12.81	14.14	20.59	19.70	7.21	8.49	22.36	0.00	4.00	24.00	4.47	24.33	8.49	18.97	22.80	23.41	11.66	18.44
51	-9	-1	23.35	14.42	10.00	10.77	11.66	17.20	16.12	6.00	6.32	18.44	4.00	0.00	20.00	2.00	20.40	6.32	15.23	18.97	19.70	10.20	15.62
53	11	-1	3.47	16.97	22.36	18.87	17.20	11.66	10.00	20.88	18.97	4.47	24.00	20.00	0.00	20.10	4.00	18.97	8.49	6.32	8.25	20.59	15.62
57	-9	-3	23.52	16.12	12.00	12.65	13.42	18.44	17.20	8.00	8.25	18.97	4.47	2.00	20.10	0.00	20.10	4.47	14.56	18.44	18.97	8.25	14.14
66	11	-5	6.00	20.00	24.41	21.26	19.80	15.23	13.42	22.36	20.59	8.25	24.33	20.40	4.00	20.10	0.00	18.11	6.32	2.83	4.47	18.97	12.81
69	-7	-7	22.44	18.97	16.12	16.12	16.49	20.00	18.44	12.17	12.00	18.87	8.49	6.32	18.97	4.47	18.11	0.00	12.00	16.00	16.12	4.00	10.00
71	5	-7	11.66	18.97	21.26	18.87	17.89	16.00	14.00	18.44	16.97	10.77	18.97	15.23	8.49	14.56	6.32	12.00	0.00	4.00	4.47	12.65	7.21
73	9	-7	8.79	20.59	24.08	21.26	20.00	16.49	14.56	21.63	20.00	10.00	22.80	18.97	6.32	18.44	2.83	16.00	4.00	0.00	2.00	16.49	10.00
84	9	-9	10.46	22.36	25.46	22.80	21.63	18.44	16.49	22.80	21.26	12.00	23.41	19.70	8.25	18.97	4.47	16.12	4.47	2.00	0.00	16.12	8.94
85	-7	-11	23.99	22.80	20.10	20.10	20.40	23.32	21.63	16.12	16.00	21.26	11.66	10.20	20.59	8.25	18.97	4.00	12.65	16.49	16.12	0.00	8.25
95	1	-13	18.62	24.08	24.17	22.80	22.36	22.36	20.40	20.59	19.70	17.89	18.44	15.62	15.62	14.14	12.81	10.00	7.21	10.00	8.94	8.25	0.00

5.3. Results

Two models were introduced in chapter 4, the first of which was the classical milk-run model (CMR) and the second one was the proposed model: A Milk-Run Distribution System Design for Integrating Drones (DMR). These two models are run using the settings introduced in this chapter so far. In this section, we will present the results we have obtained.

The results for the test instances for DMR and CMR Models with two different settings for the number of drones at the customers: $3 \times \mathrm{NC}$ and $0 \times \mathrm{NC}$ are given in Appendix C and D, respectively. These results are given for the 20 customers case. The results for 10 customers case are similarly found.

5.4. Comparison of Milk-Run Distribution with and without Drones

The findings are classified with respect to number of drones at the customers and the setting for the number of customers. In section 5.4.1, we present our findings for the 3xNC case, and in section 5.4.2, we present our findings for the 0xNC case. In both sections, at first, findings for 20 customers are presented and secondly, findings for 10 customers are presented.

5.4.1. Comparison of DMR and CMR Models with 3xNC Drones at Customers

The test instances are generated by a random selection of 20 customers. Each test instance is run for four pairs of delta and beta parameters. In each pair, 20 test instances are generated. The mean and standard deviations of the objective function values (total costs) of CMR and DMR models are calculated and presented in the Table 36.

Table 36. Mean and Standard Deviation of Models with $3 \times NC$ Drones at Customers for the Setting of 20 Customers

Delta	Beta	Zcmr			Zdmr		
		Mean	Stdev	CoV (Stdev/ Mean)	Mean	Stdev	CoV (Stdev/ Mean)
1.5	0.5	8101.3	114.6	0.0141	871.9	14.8	0.0169
1.5	0.7				873.4	13.2	0.0151
2	0.5				877.9	14.4	0.0164
2	0.7				885.7	13.0	0.0147

The average cost of the CMR model is 8101.3 with a standard deviation of 114.6. The coefficient of variation for the CMR costs is found to be 0.0141. In other words, the standard deviation is only 1.41% of the mean. The variation of cost is low, which supports the reliability of our findings. DMR model has slightly different means and standard deviations depending on its delta and beta parameters and the overall average cost is 877.2. Coefficient of variation values are also low as in the CMR case. However, the obtained objective function values are all lower compared to the CMR model. When the speed ratio of the truck on the highway and empty drone (delta) is held constant, and the range ratio of a loaded drone to an empty drone (beta) is increased, there is a slight increase in the total cost. On the other hand, when the range ratio of a loaded drone to an empty drone (beta) is held constant and the speed ratio of the truck on the highway and empty drone (delta) is increased, then total cost also increases.

Next, we calculated the cost gaps between these two models, whose results for the runs are given in Appendix C and D. The cost gap can be defined as a cost benefit in percentage in case the proposed model is used instead of the classical model. The gap percentages are calculated using equation (73).

$$\% \, GAP = \frac{z_{cmr} - z_{dmr}}{z_{cmr}} x100 \tag{73}$$

GAP percentages are calculated for each of the $(Beta, Delta)$ setting pairs for the setting of 20 customers, and the results are summarized in Figure 49:

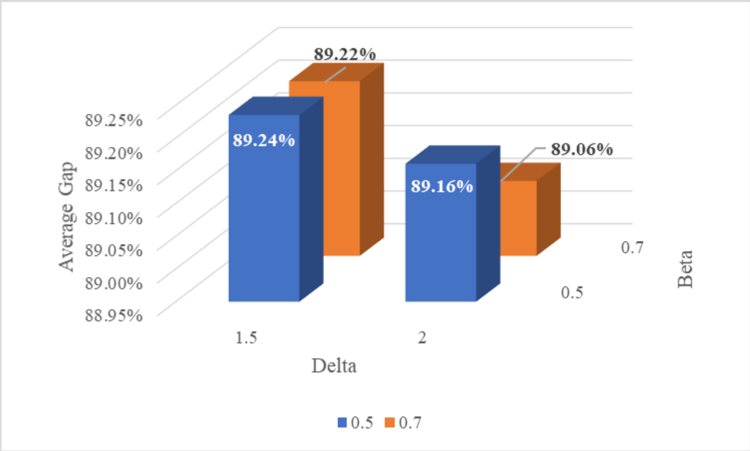


Figure 49. Average Gaps of DMR Models with $3 \times NC$ Drones at Customers with respect to the CMR Model for the Setting of 20 Customers

3xNC – DMR model resulted in lower costs compared to the CMR model and the average gap is 89.17 % for the setting of 20 customers. Slight differences are observed between $(Beta, Delta)$ setting pairs. These differences can be explained as follows. In case customers are allowed to hold 3 drones initially on the average, faster drones yield slightly lower costs. In return, faster drones give higher benefits. However, the drones with higher interaction ranges yield slightly higher costs and in return give lower benefits.

We repeat the same procedure for the setting of 10 customers. Accordingly, the test instances are now generated by a random selection of 10 customers. The mean and standard deviations of the objective function values (total costs) of CMR and DMR models are calculated and presented in Table 37.

Table 37. Mean and Standard Deviation of Models with 3xNC Drones at Customers for the Setting of 10 Customers

delta	beta	Zcmr			Zdmr		
		Mean	Stdev	CoV (Stdev/ Mean)	Mean	Stdev	CoV (Stdev/ Mean)
1.5	0.5	4703.8	113.1	0.0240	723.4	10.4	0.0144
1.5	0.7				732.2	13.2	0.0181
2	0.5				734.0	9.8	0.0134
2	0.7				737.7	10.5	0.0142

The average cost of the CMR model is 4703.8 with a standard deviation of 113.1. This average value is 58% of the finding for the 20 customers case. Considering the fact that the ratio of the average values is more than a half, we can interpret that serving 20 customers is more economical, in per customer sense. The obtained standard deviation for the 10 customers case is very close to the 20 customers case. Hence, the coefficient of variation for CMR costs increases from 0.0141 to 0.0240. In other words, the standard deviation is now 2.40% of the mean. DMR model has slightly different means and standard deviations depending on its delta and beta parameters and the overall average cost is 731.8. Like in the previous case of 20 customers, the obtained objective function values are all lower compared to the CMR model. However, the average value of DMR costs for the 10 customers case is 83% of the average for the 20 customers case. Comparing the differences for CMR and DMR findings, although CMR average cost for 10 customers is 42% less than that of 20 customers', DMR average cost for 10 customers is only 17% less than that of 20 customers'. The interpretation of this observation is that the saving for 10 customers case is less than the saving for 20 customers. Interpretation of the findings in terms of delta and beta parameters is as follows. When the speed ratio of the truck on the highway and empty drone (delta) is held constant, and the range ratio of a loaded drone to an empty drone (beta) is increased, there is a slight increase in the total cost. On the other hand, when the range ratio of a loaded drone to an empty drone (beta) is held constant and the speed ratio of the truck on the highway and empty drone (delta) is increased, then total cost also increases. Next, we will present our findings regarding the GAP measures. The GAP measures of 10 customers are calculated and presented in Figure 50.

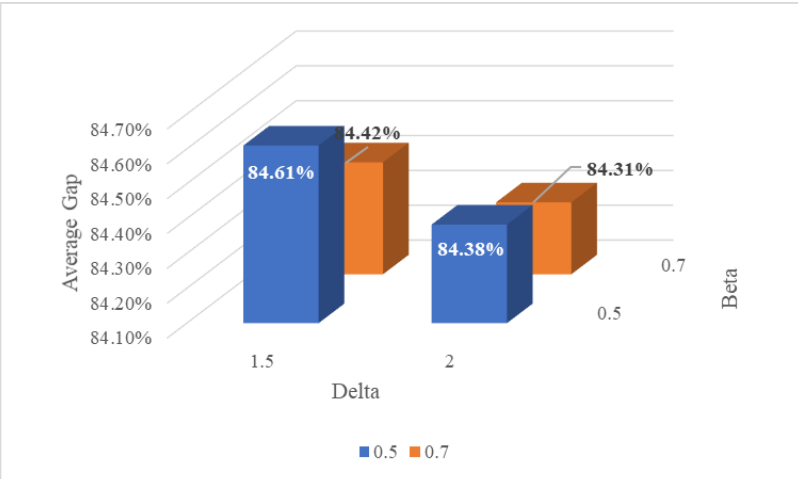


Figure 50. Average Gaps of DMR Models with 3 × NC Drones at Customers with respect to the CMR Model for the Setting of 10 Customers

3xNC – DMR model resulted in lower costs compared to the CMR model and the average GAP is 84.43% for the setting of 10 customers. This average is 4.74% less than the GAP obtained for the setting of 20 customers. As stated previously, the saving of DMR for 20 customers is higher than that of 10 customers'. Slight differences for average GAP values are observed between $(Beta, Delta)$ setting pairs. These differences can be explained as follows. In case customers are allowed to hold 3 drones initially on the average, faster drones yield slightly lower costs. In return, faster drones give higher benefits. However, the drones with higher interaction ranges yield slightly higher costs and in return give lower benefits.

5.4.2. Comparison of DMR and CMR Models with 0xNC Drones at Customers

Next, we will compare the setting pairs of delta and beta parameters in case no drones are allowed initially at the customers. The same 20 test instances with the previous case (3xNC drones at customers) are used for comparison purposes. The mean and standard deviations of the objective function values (total costs) of CMR and DMR models are calculated and presented in Table 38.

Table 38. Mean and Standard Deviation of Models with $0 \times NC$ Drones at Customers for the Setting of 20 Customers

Delta	Beta	Zcmr			Zdmr		
		Mean	**Stdev**	**CoV (Stdev/ Mean)**	**Mean**	**Stdev**	**CoV (Stdev/ Mean)**
1.5	0.5	8101.3	114.6	0.0141	795.8	9.5	0.0119
1.5	0.7				878.6	16.0	0.0182
2	0.5				795.1	7.3	0.0092
2	0.7				879.4	13.7	0.0156

The average cost of the CMR model is 8101.3 with a standard deviation of 114.6 and the coefficient of variation for CMR costs is 0.0141. DMR model has different means and standard deviations depending on parameters delta and beta, and again with low coefficient of variation values. When the speed ratio of the truck on the highway and empty drone (delta) is held constant, and the range ratio of a loaded drone to an empty drone (beta) is increased, total cost increases. This increase for 0xNC drones is relatively higher than the previous case with 3xNC drones. On the other hand, when the range ratio of a loaded drone to an empty drone (beta) is held constant and the speed ratio of the truck on the highway and empty drone (delta) is increased, then the total cost remains almost the same. This result is different from the previous case. For the 3xNC model, there was a slight increase in cost as a response to increasing delta when beta is held constant.

The comparison of GAP values of 0xNC-DMR and CMR Models for the setting of 20 customers is given in Figure 51:

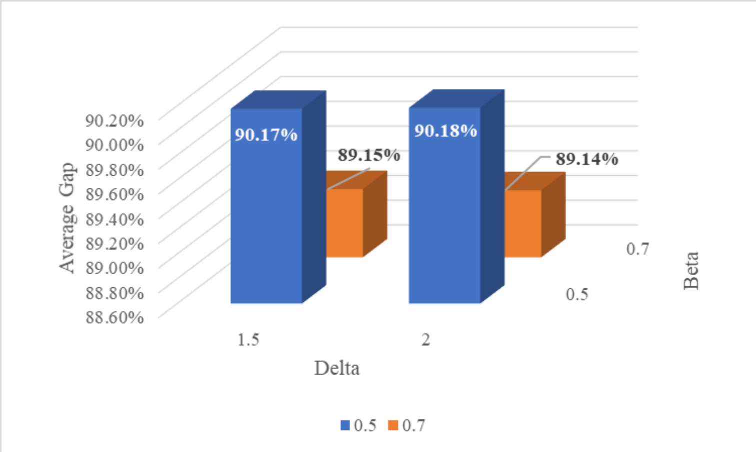


Figure 51. Average Gaps of DMR Models with $0 \times NC$ Drones at Customers with respect to the CMR Model for the Setting of 20 Customers

0xNC – DMR model resulted in lower costs compared to the CMR model and the average gap is 89.67 %. As the speed ratio of the truck on the highway to empty drone is increased there is almost no change in GAP measures. As the ratio of the range of a loaded drone to an empty drone is increased, the GAP between DMR and CMR models decreases.

We will repeat the same procedure for the setting of 10 customers, with 0xNC drones initially at the customers. The mean and standard deviation of objective function values are given in Table 39.

Table 39. Mean and Standard Deviation of Models with $0 \times NC$ Drones at Customers for the Setting of 10 Customers

delta	beta	Zcmr			Zdmr		
		Mean	Stdev	CoV (Stdev/ Mean)	Mean	Stdev	CoV (Stdev/ Mean)
1.5	0.5	4703.8	113.1	0.0240	692.0	4.2	0.0061
1.5	0.7				734.5	7.5	0.0102
2	0.5				691.2	3.3	0.0048
2	0.7				734.3	6.1	0.0083

The average cost of the CMR model is 4703.8 with a standard deviation of 113.1. DMR model has slightly different means and standard deviations depending on its delta and beta parameters and the overall average cost is 713.0. Like in the previous case of 20 customers, the obtained objective function values are all lower compared to the CMR model. Interpretation of the findings in terms of delta and beta parameters is as follows. When the speed ratio of the truck on the highway and empty drone (delta) is held constant, and the range ratio of a loaded drone to an empty drone (beta) is increased, total cost increases. This increase is relatively higher than the previous case with 3xNC drones. On the other hand, when the range ratio of a loaded drone to an empty drone (beta) is held constant and the speed ratio of the truck on the highway and empty drone (delta) is increased, then the total cost remains almost the same. This result for 0xNC is different from the previous case. For the 3xNC model, there was a slight increase in cost as a response to increasing delta when beta is held constant. Within the 0xNC case, these findings for 10 customers case has the same pattern for the 20 customers' case.

Next, we will present our findings regarding the GAP measures. The GAP measures of 10 customers are calculated and presented in Figure 52.

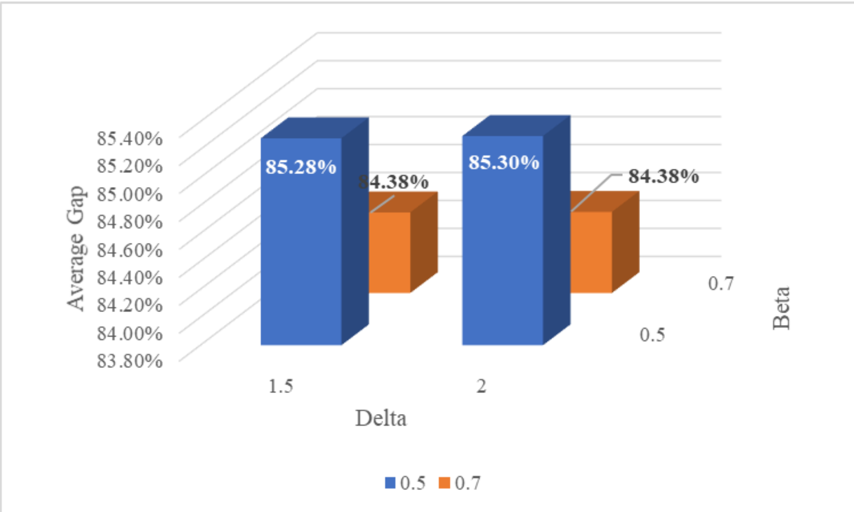


Figure 52. Average Gaps of DMR Models with $0 \times NC$ Drones at Customers with respect to the CMR Model for the Setting of 10 Customers

0xNC – DMR model resulted in lower costs compared to the CMR model and the average GAP is 84.83% for the setting of 10 customers. This average is 4.84% less than the GAP obtained for the setting of 20 customers. This result is very similar to the result for the case of 3xNC. Slight differences for average GAP values are observed between $(Beta, Delta)$ setting pairs. The differences between GAP values are as follows. As the speed ratio of the truck on the highway to empty drone is increased there is almost no change in GAP measures. As the ratio of the range of a loaded drone to an empty drone is increased, the GAP between DMR and CMR models decreases.

We think it will be useful to mention some of the arrangements we made in running the relevant models while making the comparisons in this chapter. Since the proposed DMR model contains too many variables, we could not get the optimum solution when we started the model without an initial solution. We think the probable reason for this situation is that GAMS is stuck at the local optimum in its solution algorithm and reports misleadingly the infeasibility of the problem. To overcome this problem, we thought of adding an initial solution to the model and when we added a suitable initial solution, GAMS reached the optimum solutions of the model. As an initial solution, we equated the $X_{d,j}^{B}$ values to 1 to find a feasible solution so that all the demands of the customers can be met, and we equated the $X_{d,j}$ values, which are the drone flight times, to the midpoints of the relevant range.

Another problem we had with model runs was that the objective function initially gave negative values. In order to solve this problem, we made the following changes to the objective function. We originally designed the objective function as the sum of three cost figures, which were the highway cost, the drone flight cost, and the holding cost of the drones at the customers. After determining that GAMS could not minimize the total of these costs, we eliminated the drone flight cost from the objective function and run the model again. In this case, GAMS got an optimum solution and we checked the consistency of this solution with the initial values. As a result, we run the model in a way that minimizes the sum of the two cost figures, and separately calculated the third cost figure of the drone flight costs, which created problems in the operation of the model.

Since holding cost, which is one of the cost figures in the model, can be considered as a type of opportunity cost, we eliminated this cost from the total cost used for comparison of DMR and CMR results. Namely, in the comparisons in this chapter, the cost of a DMR tour is designed to be as the sum of two cost figures, which are the drone flight costs and truck highway costs. In the next section, we will discuss the findings of this chapter in terms of their current usability and possible future usability. In this discussion, we will consider the depreciation cost of the drone by calculating the breakeven point of the proposed model.

5.4.3. Practical Comparison of DMR and CMR Models in the Long Run

As we mentioned previously, in this section, we will perform a cost analysis that the proposed model can be useful in the long run. We make the cost analysis for the case which has a higher cost benefit. We found previously that the cost benefit is higher for the setting of 20 customers. Average costs per milk-run tour for CMR and DMR models are calculated in the previous sections and presented in equations (74) and (75). The difference between these two costs is given in equation (76).

$$AverageCost(CMR) = 8101.3 \tag{74}$$

$$AverageCost(DMR) = 857.2 \tag{75}$$

$$Cost\ Difference = 8101.3 - 857.2 = 7244.1 \tag{76}$$

If two milk-run tours are considered to be made in a week, the resulting cost benefit is found as $2 \cdot 7244.1 = ¢14488.20$ (in cents), or equivalently, \$144.88. This corresponds to $52 \cdot 144.88 = \$7533.86$ per year. If the lifetime of a drone is considered to be 5 years, the total cost benefit during the lifetime of a drone is found to be $5 \cdot 7533.86 = \$37669.32$.

When the drones which are used to base the parameters in this study are evaluated, the approximate cost of a drone is approximately \$6500. Using this cost, the breakeven points below have been calculated.

The break-even point for the number of drones corresponding to the total cost benefit is as in equation (77).

$$BreakEven(Number\ of\ Drones) = \frac{37669.32}{6500} = 5.8 \tag{77}$$

Since the maximum number of drones (which is the total demand) used in the model runs is 60, the currently proposed model can be considered as very costly. In a situation where all demands can be met with 5 or fewer drones, the proposed model will be profitable.

The breakeven point corresponding to the cost of a drone is given in equation (78):

$$BreakEven(Cost\ of\ a\ Drone) = \frac{37669.32}{60} = 627.82 \tag{78}$$

The proposed model will be profitable if the cost of a unit drone is less than $627.82.

The breakeven point corresponding to the lifetime of a drone is given in equation (79):

$$BreakEven(Lifetime\ of\ a\ Drone) = \frac{60 \cdot 6500}{7533.864} = 51.77 \tag{79}$$

The proposed model will be profitable if the lifetime of a drone is more than 51.77 years.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

In this study, we have proposed a special type of milk-run distribution model and analyzed its possible benefits. The proposed model consists of a milk-run tour in which the truck never stops during the tour and all the delivery is performed by the drones. The abbreviation for the proposed model is DMR, which stands for Drone Milk-Run. The control model, namely the model to which we will compare our proposed model, is the Classical Milk-Run (abbr. CMR) model. In the classical milk-run model, a truck visits the customers through a predetermined route and satisfies their demands.

Having introduced both models, we take runs for both of them on separate numerical settings suitably designed for each of the models. For the CMR model, the only required setting is customer locations, which have two settings of 10 and 20 customers. Accordingly, 10 and 20 of 96 customers are randomly selected at each of the 20 instances run. Using the same customer locations, the DMR model makes use of the additional settings of those parameters which include two drone speeds, two flight ranges, and two cases which consists of no drones at the customers before the start of the milk-run and an average of 3 drones present at the customers before the start of the milk-run.

The findings are analyzed in two steps. At the first step, a cost gap measure, which is defined as “cost benefit in percentage in case the proposed model is used instead of classical model” is calculated and interpreted. For the setting of 20 customers, the cost savings for different $(Beta, Delta)$ setting pairs are found to be similar and the average gap is found to be 89.67% for no drones case and 89.17% for an average of 3 drones case. On the other hand, for the setting of 10 customers, the cost savings for different $(Beta, Delta)$ setting pairs are found to be similar and the average gap is found to be 84.83% for no drones case and 84.43% for an average of 3 drones case.

We observed that the prosed model has a higher cost benefit for the 20 customers setting compared to the 10 customers setting.

It seems from the findings of the GAP measures that a sufficiently high saving is achieved but further analysis is required to reach a more concrete result. For this purpose, we performed a cost analysis using the higher benefit case, which is the setting for 20 customers. Our purpose here is to point out the extent that the proposed model can be useful in the long run. Average costs per milk-run tour are calculated as ¢8101.3 and ¢857.2 for CMR and DMR models respectively, with a cost difference of ¢7244.1. Considering an average number of milk-runs as 2 per week and the average lifetime of a drone to be 5 years, this cost benefit corresponds to $37669.32. Taking this benefit as our reference point, we calculated the following breakeven points and determined the range that the proposed model is cost efficient.

The break-even point for the number of drones corresponding to the total cost benefit was found to be 5.8 drones. Since the maximum number of drones (which is the total demand) used in the model runs is 60, the currently proposed model can be considered as very costly. In a situation where all demands can be met with 5 or fewer drones, the proposed model will be profitable.

The breakeven point corresponding to the cost of a drone is found to be $627.82. However, the current cost of a drone on which we base our settings is approximately $6500. Accordingly, the proposed model is not currently cost efficient and will be profitable if the cost of a unit drone becomes less than $627.82 in the future.

The breakeven point corresponding to the lifetime of a drone is found to be 51.77 years. This lifetime was assumed to be 5 years for these calculations. Hence, the proposed model will be profitable if the lifetime of a drone will increase and become more than 51.77 years.

To sum up, the main finding of this study is that integrating drones in a milk-run distribution is beneficial compared to the classical milk-run under any problem setting for a single milk-run tour. However, the cost difference between those two milk-run types does not seem to be sufficient to invest in integrating drones as we propose under current cost figures. This result is also confirmed by several practitioners, some of whose opinions are summarized as follows.

MediaMarkt Performance and Talent Management Manager Oğuz Dokur stated that their branch deliveries are not periodic and that goods are usually delivered to the branch warehouse once and collectively. Therefore, he stated that the proposed model would not be suitable for MediaMarkt currently (2019). EKOL Logistic Research Development Center department's director Erdem Özsalih stated that the delivery by drones for Ekol Logistics' B2B transports would not be suitable at first, as the volume and weight of the delivered products will generally be more than the carrying capacity of drones. (2019). Cem Oğuz, Planning and Business Development Director of KolayGelsin, stated that it is possible to experience problems in terms of delivery location in drone transport since the typical target customer is an individual customer rather than a corporate customer. On the other hand, if there are separate delivery points for this process in workplaces, buildings, or sites, and if the drones can leave their delivery products in these areas, there may be an increase in delivery in drones at certain points (2019). DHL Supply Chain Turkey R&D Project Engineer Gizem CİDAL mentioned other points in drone delivery. Stating that she has doubts about the use of a sufficiently large number of drones in delivery to increase efficiency, Cidal said that she thought this scenario would be possible in the future and that such a study would be a good academic and visionary work (2019).

As a result of the cost analysis made, we evaluated that the proposed model will not be commercially profitable, at least in the near future. However, we have also shown that milk-run distribution integrating drones have a significant time advantage compared to the classical milk-run. Therefore, we can interpret that the proposed model can have applications in non-profit areas where time gain has great importance. Such applications can be found in the healthcare and military sectors.

An application that can be considered in the health sector is the use of drones in the process of organ transplantation. In April 2019, such a transportation is succeeded by the University of Maryland. A donor's kidney is first time transported by a drone to the surgeons at the University of Maryland Medical Center (UMMC) in Baltimore. This usage of drones supports the fact that drone transportation can present a faster and safer delivery in the health-care services sector, compared to the classical delivery methods (Clough, 2019).

Ergene and others state that drone usage for military logistics has an exponential increase in the last decade. They stress the three kinds of missions especially suitable for drone transportation: dull, dirty, and dangerous. To illustrate, the transportation of items that have biological, nuclear, or chemical characters are better transported by drones rather than by military personal because of the potential risks of delivering such items (Ergene et al., 2021).

This study is the first part of the BAP project of Çankaya University. In the latter parts of this project or for the investigators that work in this area, the following points and ideas can be recommended. First, the factor levels of the settings constructed in the work can be enlarged and new parameters such as truck's speed can be added. Secondly, the model constraints can be designed to capture more detail about the drone usage such that the maximum number of drones that can be simultaneously used or the minimum time required between corresponding deliveries. Moreover, instead of a single period optimization, a more complex model including a wider time horizon and the response to changing daily demands of customers can be studied. Finally, the proposed model in this study can be developed by considering hybrid milk-run delivery by using both the truck and drones.

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APPENDIX A. GAMS CMR MODEL

```
Set j/
$call                =xls2gms                r=SETS!b5:b25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=set.inc
$include set.inc
/;

alias(i,j);

parameter tau/
$call=xls2gms                                r=scalars!b11
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter nu/
$call=xls2gms                                 r=scalars!b5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter q(j)/
$call=xls2gms                                r=q(j)!b5:c25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter sigma/
$call=xls2gms                                 r=scalars!a5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter ccty/
$call=xls2gms                                 r=scalars!d5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter copc/
$call=xls2gms                                 r=scalars!h5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;


parameter table lambda(i,j)
$call=xls2gms                        r=lambda(ij)!c4:cx103
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
```

```
$include par.inc
;

display lambda;

variable
Z
Z_cty
Z_opc
;

binary  variable
V(i,j) ONE if truck goes from location i to j ZERO o.w.
;

positive variable
upsilon(i)
;

equations
obj
objcty
objopc
cin
cout
csub
;

obj..                Z =E= Z_cty + Z_opc;
objcty..                         Z_cty   =E=   ccty   *
(sum((i,j),lambda(i,j)*V(i,j) ))  ;
objopc..                       Z_opc   =E=   copc*   (60*
(sum((i,j),lambda(i,j)*V(i,j) )) / (sigma/nu) + (card(j)-
1)*20  - tau );
cin(j)..                              sum(i$(ord(i)   ne
ord(j)),V(i,j))=e=1;
cout(i)..                             sum(j$(ord(i)   ne
ord(j)),V(i,j))=e=1;
csub(i,j)$((ord(i)   ne   ord(j))and   (ord(i)   ge   2)   and
(ord(j)  ge  2))..           upsilon(i)  -  upsilon(j)  +
(card(j)-1)* V(i,j) =l= (card(j)-2);


Model CMR_210129 /all/;
option MIP = Cplex;
$onEcho > Cplex.opt
tilim 86400
itlim 1000000
epgap 0.00001
$offEcho
CMR_210129.optfile=1
```

```
Solve CMR_210129 using MIP minimizing z;

Z.L$(Z.L=0)=eps;
V.L(i,j)$(V.L(i,j)=0)=eps;

Display Z.L, Z_cty.L, Z_opc.L, V.L;

$onEcho >  ToWriteZ.txt
epsout=0
var=Z.L              rng=Z!D4:D4
$offEcho

$onEcho >  ToWriteZ_cty.txt
var=Z_cty.L              rng=Z!D6:D6
$offEcho

$onEcho >  ToWriteZ_opc.txt
var=Z_opc.L              rng=Z!D7:D7
$offEcho

$onEcho >  ToWriteV.txt
epsout=0
var=V.L              rng=V!D4:Z1000
$offEcho

execute_unload
'output_CMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx'
execute                                            'gdxxrw
output_CMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ.txt '
execute                                            'gdxxrw
output_CMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ_cty.txt '
execute                                            'gdxxrw
output_CMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ_opc.txt '
execute                                            'gdxxrw
output_CMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteV.txt '
;
```

APPENDIX B. GAMS DMR MODEL

```
Set j/
$call             =xls2gms              r=SETS!b5:b1000
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=set.inc
$include set.inc
/;

alias(j,jj);

Set d/
$call=xls2gms                           r=SETS!d5:d1000
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=set.inc
$include set.inc
/;

alias(d,dd);

Set SJD(d)/
$call=xls2gms                           r=SETS!e5:e1000
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=set.inc
$include set.inc
/;

Set S0D(d)/
$call=xls2gms                           r=SETS!f5:f1000
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=set.inc
$include set.inc
/;

parameter tau/
$call=xls2gms                             r=scalars!b11
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;
parameter bigM/
$call=xls2gms                              r=scalars!k5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;
parameter tsetup/
$call=xls2gms                              r=scalars!j5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter chld/
$call=xls2gms                              r=scalars!m5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
```

```
/;

parameter NC/
$call=xls2gms                                    r=scalars!o5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter q(j)/
$call=xls2gms                                   r=q(j)!b5:c25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter sigma/
$call=xls2gms                                    r=scalars!a5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter tDepE(j)/
$call=xls2gms                                  r=tD(j)!b5:c25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter tDepL(j)/
$call=xls2gms                                  r=tD(j)!g5:h25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter tArrE(j)/
$call=xls2gms                                  r=tA(j)!b5:c25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter tArrL(j)/
$call=xls2gms                                  r=tA(j)!g5:h25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter aArr(j)/
$call=xls2gms                                r=regA(j)!b5:c25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;
```

```
parameter bArr(j)/
$call=xls2gms                              r=regA(j)!g5:h25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter cArr(j)/
$call=xls2gms                              r=regA(j)!j5:k25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter aDep(j)/
$call=xls2gms                              r=regD(j)!b5:c25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter bDep(j)/
$call=xls2gms                              r=regD(j)!g5:h25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter cDep(j)/
$call=xls2gms                              r=regD(j)!j5:k25
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;
parameter chwy/
$call=xls2gms                                  r=scalars!c5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

parameter cdrn/
$call=xls2gms                                  r=scalars!f5
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
/;

set table dSDJ(d,j)
$call=xls2gms                               r=j-SDj!b4:w124
i=input_D1.5B0.5_Cust20_3NC_inst01.xlsx o=par.inc
$include par.inc
;

display
j,d,SJD,S0D,q,sigma,tdepe,tdepl,tArrE,tArrL,aArr,bArr,cAr
r,aDep,bDep,cDep,chwy,cdrn,chld,dSDJ;
```

```
variable
Z
Z_hwy
Z_drn
Z_hld
;

positive variable
X(d,j) The take off time of d from the truck to customer
j
Y(d,j) The take off time of d from customer j to the
truck

binary  variable
XB(d,j) ONE if d serves to customer j and ZERO o.w.
YB(d,j) ONE if d takes off from customer j and ZERO o.w.
UB(d)    ONE if d is taken off from the depot and ZERO
o.w.
;

equations
obj
objhwy
objdrn
objhld
c01
c02
c03
c04
c05
c06
c07
c08
c09
c10
c11
c12
c13
c14
c15
c16
c17
;

obj..                  Z =E= Z_hwy + Z_hld;
objhwy..              Z_hwy =E= chwy * sigma * tau/60;
objdrn..               Z_drn =E= cdrn *( (sum (d, (sum(j,
aArr(j)*YB(d,j)+  bArr(j)*Y(d,j)+  cArr(j)*sqr(Y(d,j))))
)) +  (sum (d, (sum(j, aDep(j)*XB(d,j)+ bDep(j)*X(d,j)+
cDep(j)*sqr(X(d,j))))    )) );
```

```
objhld..               Z_hld =E= chld *(3*NC + sum((d,j),
XB(d,j))- sum((d,j), YB(d,j)));

c01(d)$(not S0D(d))..         UB(d) =E= 0;
c02(d,j)$(not dSDJ(d,j))..    YB(d,j) =E= 0;

c03(d,j)$dSDJ(d,j)..          Y(d,j) =L= bigM*YB(d,j);

c04(d,j,jj)$(dSDJ(d,j) AND ord(jj)<ord(j))..      X(d,jj)
=E= 0;

c05(d)..                      sum(j,XB(d,j)) =L= 1;
c06(d,j)..                    X(d,j) =L= bigM*XB(d,j);
c07(j)..                      sum(d,XB(d,j)) =E= q(j);
c08(d)$S0D(d)..               sum(j,XB(d,j)) =L= UB(d);

c09(d,j)$dSDJ(d,j)..          sum(jj$(ord(jj) GE ord(j)),
XB(d,jj)) =L= YB(d,j);

c10(d,j,jj)$(dSDJ(d,j)   AND   (ord(jj)   GE   ord(j)))..
X(d,jj) =L= bigM*YB(d,j);

c11(d,j)$((not S0D(d)) and dSDJ(d,j))..        Y(d,j) +
(aArr(j) + bArr(j)*Y(d,j) + cArr(j)*sqr(Y(d,j))) + tSetup
=L=    sum(jj$(ord(jj)    GE    ord(j)),X(d,jj))+bigM*(1-
sum(jj$(ord(jj) GE ord(j)),XB(d,jj)));

c12(d,j)$dSDJ(d,j)..                Y(d,j)  +  (aArr(j)  +
bArr(j)*Y(d,j)      +      cArr(j)*sqr(Y(d,j)))       =L=
tArrL(j)+bigM*(1-YB(d,j));

c13(d,j)$dSDJ(d,j)..          tArrE(j) - bigM*(1-YB(d,j))
=L=       Y(d,j)    +    (aArr(j)    +    bArr(j)*Y(d,j)    +
cArr(j)*sqr(Y(d,j)));

c14(d,j)..               X(d,j) =L= tDepL(j)*XB(d,j);
c15(d,j)..               tDepE(j)*XB(d,j) =L= X(d,j);
c16(d,j)..               X(d,j) =L= tau;

c17(d,j)$dSDJ(d,j)..                 Y(d,j) + (aArr(j) +
bArr(j)*Y(d,j) + cArr(j)*sqr(Y(d,j))) =L= tau + bigM*(1-
YB(d,j));


Model DMR210129 /all/;
*option MINLP = DICOPT;

$include DMR_210128_1.inc


Solve DMR210129 using MINLP minimizing z;
```

```
Z.L$(Z.L=0)=eps;
X.L(d,j)$(X.L(d,j)=0)=eps;
Y.L(d,j)$(Y.L(d,j)=0)=eps;
XB.L(d,j)$(XB.L(d,j)=0)=eps;
YB.L(d,j)$(YB.L(d,j)=0)=eps;
UB.L(d)$(UB.L(d)=0)=eps;

Display Z.L, Z_hwy.L, Z_drn.L, Z_hld.L, UB.L, Y.L, YB.L,
X.L, XB.L;

parameter Arrflighttime(d,j);
Arrflighttime(d,j)=(aArr(j)*YB.l(d,j)  +  bArr(j)*Y.l(d,j)
+ cArr(j)*sqr(Y.l(d,j)));

parameter Depflighttime(d,j);
Depflighttime(d,j)=(aDep(j)*XB.l(d,j)  +  bDep(j)*X.l(d,j)
+ cDep(j)*sqr(X.l(d,j)));

$onEcho >  ToWriteZ.txt
epsout=0
var=Z.L              rng=Z!D4:Z1000
$offEcho

$onEcho >  ToWriteZ_hwy.txt
epsout=0
var=Z_hwy.L              rng=Z!D6:Z1000
$offEcho

$onEcho >  ToWriteZ_drn.txt
epsout=0
var=Z_drn.L              rng=Z!D7:Z1000
$offEcho

$onEcho >  ToWriteZ_hld.txt
epsout=0
var=Z_hld.L              rng=Z!D8:Z1000
$offEcho

$onEcho >  ToWriteUB.txt
epsout=0
var=UB.L              rng=UB!D4:DZ1000
$offEcho

$onEcho >  ToWriteYB.txt
epsout=0
var=YB.L              rng=YB!D4:Z1000
$offEcho

$onEcho >  ToWriteY.txt
epsout=0
```

```
var=Y.L              rng=Y!D4:Z1000'
$offEcho

$onEcho >  ToWriteXB.txt
epsout=0
var=XB.L              rng=XB!D4:Z1000
$offEcho

$onEcho >  ToWriteX.txt
epsout=0
var=X.L              rng=X!D4:Z1000
$offEcho


execute_unload
'output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx'
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
par=Arrflighttime             rng=Arrflighttime!D4:Z1000'
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
par=Depflighttime             rng=Depflighttime!D4:Z1000'
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ_hwy.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ_drn.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteZ_hld.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteUB.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteYB.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteY.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteXB.txt '
execute                                           'gdxxrw
output_DMR_210129_D1.5B0.5_Cust20_3NC_inst01.gdx
@ToWriteX.txt                                         ';
```

APPENDIX C. RESULTS FOR DMR AND CMR MODELS

Table 40. Results for 20 Customers 3NC DMR and CMR Models

No	Inst	delta	beta	#cust	NC	Zcty	Zopc	Zcmr	Zhwy	Zdrn	Zdmr	Gap
1	1	1.5	0.5	20	60	749.3	7328.0	8077.2	585.9	296.0	881.9	89.08%
2	2	1.5	0.5	20	60	766.4	7385.8	8152.3	585.9	289.6	875.5	89.26%
3	3	1.5	0.5	20	60	685.0	7111.5	7796.5	585.9	280.1	866.0	88.89%
4	4	1.5	0.5	20	60	747.7	7322.7	8070.3	585.9	289.8	875.7	89.15%
5	5	1.5	0.5	20	60	720.0	7229.4	7949.4	585.9	307.7	893.6	88.76%
6	6	1.5	0.5	20	60	791.4	7469.8	8261.2	585.9	299.7	885.6	89.28%
7	7	1.5	0.5	20	60	771.0	7401.1	8172.0	585.9	311.2	897.1	89.02%
8	8	1.5	0.5	20	60	792.1	7472.3	8264.5	585.9	273.5	859.4	89.60%
9	9	1.5	0.5	20	60	732.7	7272.2	8004.9	585.9	278.1	864.0	89.21%
10	10	1.5	0.5	20	60	741.7	7302.4	8044.0	585.9	285.7	871.6	89.16%
11	11	1.5	0.5	20	60	752.1	7337.6	8089.7	585.9	254.5	840.4	89.61%
12	12	1.5	0.5	20	60	736.7	7285.6	8022.3	585.9	293.0	878.9	89.04%
13	13	1.5	0.5	20	60	780.9	7434.6	8215.5	585.9	288.0	873.9	89.36%
14	14	1.5	0.5	20	60	792.0	7472.0	8264.1	585.9	293.7	879.6	89.36%
15	15	1.5	0.5	20	60	759.9	7363.8	8123.8	585.9	258.0	843.9	89.61%
16	16	1.5	0.5	20	60	730.0	7262.9	7992.9	585.9	302.6	888.5	88.88%
17	17	1.5	0.5	20	60	769.4	7395.8	8165.2	585.9	281.3	867.2	89.38%
18	18	1.5	0.5	20	60	741.8	7302.9	8044.7	585.9	282.1	868.0	89.21%
19	19	1.5	0.5	20	60	771.9	7404.2	8176.1	585.9	291.1	877.0	89.27%
20	20	1.5	0.5	20	60	763.7	7376.4	8140.1	585.9	264.3	850.2	89.56%
21	1	1.5	0.7	20	60	749.3	7328.0	8077.2	585.9	303.5	889.4	88.99%
22	2	1.5	0.7	20	60	766.4	7385.8	8152.3	585.9	282.7	868.6	89.35%
23	3	1.5	0.7	20	60	685.0	7111.5	7796.5	585.9	316.2	902.1	88.43%
24	4	1.5	0.7	20	60	747.7	7322.7	8070.3	585.9	289.3	875.2	89.16%
25	5	1.5	0.7	20	60	720.0	7229.4	7949.4	585.9	279.8	865.7	89.11%
26	6	1.5	0.7	20	60	791.4	7469.8	8261.2	585.9	294.3	880.2	89.35%
27	7	1.5	0.7	20	60	771.0	7401.1	8172.0	585.9	277.6	863.5	89.43%
28	8	1.5	0.7	20	60	792.1	7472.3	8264.5	585.9	284.3	870.2	89.47%
29	9	1.5	0.7	20	60	732.7	7272.2	8004.9	585.9	278.6	864.5	89.20%
30	10	1.5	0.7	20	60	741.7	7302.4	8044.0	585.9	281.5	867.4	89.22%
31	11	1.5	0.7	20	60	752.1	7337.6	8089.7	585.9	259.1	845.0	89.55%
32	12	1.5	0.7	20	60	736.7	7285.6	8022.3	585.9	293.5	879.4	89.04%
33	13	1.5	0.7	20	60	780.9	7434.6	8215.5	585.9	292.3	878.2	89.31%
34	14	1.5	0.7	20	60	792.0	7472.0	8264.1	585.9	288.7	874.6	89.42%
35	15	1.5	0.7	20	60	759.9	7363.8	8123.8	585.9	276.9	862.8	89.38%
36	16	1.5	0.7	20	60	730.0	7262.9	7992.9	585.9	284.5	870.4	89.11%
37	17	1.5	0.7	20	60	769.4	7395.8	8165.2	585.9	299.1	885.0	89.16%
38	18	1.5	0.7	20	60	741.8	7302.9	8044.7	585.9	278.4	864.3	89.26%
39	19	1.5	0.7	20	60	771.9	7404.2	8176.1	585.9	275.2	861.1	89.47%
40	20	1.5	0.7	20	60	763.7	7376.4	8140.1	585.9	313.6	899.5	88.95%
41	1	2	0.5	20	60	749.3	7328.0	8077.2	585.9	313.6	899.5	88.86%
42	2	2	0.5	20	60	766.4	7385.8	8152.3	585.9	283.1	869.0	89.34%
43	3	2	0.5	20	60	685.0	7111.5	7796.5	585.9	319.3	905.2	88.39%

Table 40 (Continued)

No	Inst	delta	beta	#cust	NC	Zcty	Zopc	Zcmr	Zhwy	Zdrn	Zdmr	Gap
44	4	2	0.5	20	60	747.7	7322.7	8070.3	585.9	286.1	872.0	89.20%
45	5	2	0.5	20	60	720.0	7229.4	7949.4	585.9	293.2	879.1	88.94%
46	6	2	0.5	20	60	791.4	7469.8	8261.2	585.9	318.5	904.4	89.05%
47	7	2	0.5	20	60	771.0	7401.1	8172.0	585.9	289.6	875.5	89.29%
48	8	2	0.5	20	60	792.1	7472.3	8264.5	585.9	293.0	878.9	89.37%
49	9	2	0.5	20	60	732.7	7272.2	8004.9	585.9	300.6	886.5	88.93%
50	10	2	0.5	20	60	741.7	7302.4	8044.0	585.9	284.0	869.9	89.19%
51	11	2	0.5	20	60	752.1	7337.6	8089.7	585.9	252.2	838.1	89.64%
52	12	2	0.5	20	60	736.7	7285.6	8022.3	585.9	289.6	875.5	89.09%
53	13	2	0.5	20	60	780.9	7434.6	8215.5	585.9	291.9	877.8	89.32%
54	14	2	0.5	20	60	792.0	7472.0	8264.1	585.9	293.0	878.9	89.36%
55	15	2	0.5	20	60	759.9	7363.8	8123.8	585.9	281.4	867.3	89.32%
56	16	2	0.5	20	60	730.0	7262.9	7992.9	585.9	298.6	884.5	88.93%
57	17	2	0.5	20	60	769.4	7395.8	8165.2	585.9	297.7	883.6	89.18%
58	18	2	0.5	20	60	741.8	7302.9	8044.7	585.9	282.4	868.3	89.21%
59	19	2	0.5	20	60	771.9	7404.2	8176.1	585.9	290.0	875.9	89.29%
60	20	2	0.5	20	60	763.7	7376.4	8140.1	585.9	282.2	868.1	89.34%
61	1	2	0.7	20	60	749.3	7328.0	8077.2	585.9	324.0	909.9	88.74%
62	2	2	0.7	20	60	766.4	7385.8	8152.3	585.9	293.8	879.7	89.21%
63	3	2	0.7	20	60	685.0	7111.5	7796.5	585.9	314.3	900.2	88.45%
64	4	2	0.7	20	60	747.7	7322.7	8070.3	585.9	296.7	882.6	89.06%
65	5	2	0.7	20	60	720.0	7229.4	7949.4	585.9	300.1	886.0	88.85%
66	6	2	0.7	20	60	791.4	7469.8	8261.2	585.9	329.4	915.3	88.92%
67	7	2	0.7	20	60	771.0	7401.1	8172.0	585.9	286.5	872.4	89.32%
68	8	2	0.7	20	60	792.1	7472.3	8264.5	585.9	300.9	886.8	89.27%
69	9	2	0.7	20	60	732.7	7272.2	8004.9	585.9	287.0	872.9	89.10%
70	10	2	0.7	20	60	741.7	7302.4	8044.0	585.9	292.4	878.3	89.08%
71	11	2	0.7	20	60	752.1	7337.6	8089.7	585.9	270.0	855.9	89.42%
72	12	2	0.7	20	60	736.7	7285.6	8022.3	585.9	298.9	884.8	88.97%
73	13	2	0.7	20	60	780.9	7434.6	8215.5	585.9	304.4	890.3	89.16%
74	14	2	0.7	20	60	792.0	7472.0	8264.1	585.9	294.9	880.8	89.34%
75	15	2	0.7	20	60	759.9	7363.8	8123.8	585.9	297.2	883.1	89.13%
76	16	2	0.7	20	60	730.0	7262.9	7992.9	585.9	314.7	900.6	88.73%
77	17	2	0.7	20	60	769.4	7395.8	8165.2	585.9	297.9	883.8	89.18%
78	18	2	0.7	20	60	741.8	7302.9	8044.7	585.9	299.5	885.4	88.99%
79	19	2	0.7	20	60	771.9	7404.2	8176.1	585.9	304.3	890.2	89.11%
80	20	2	0.7	20	60	763.7	7376.4	8140.1	585.9	290.0	875.9	89.24%

APPENDIX D. RESULTS FOR DMR AND CMR MODELS

Table 41. Results for 20 Customers 0NC DMR and CMR Models

No	Inst	delta	beta	#cust	NC	Zcty	Zopc	Zcmr	Zhwy	Zdrn	Zdmr	Gap
1	1	1.5	0.5	20	0	749.3	7328.0	8077.2	585.9	214.8	800.7	90.09%
2	2	1.5	0.5	20	0	766.4	7385.8	8152.3	585.9	212.0	797.9	90.21%
3	3	1.5	0.5	20	0	685.0	7111.5	7796.5	585.9	212.1	798.0	89.77%
4	4	1.5	0.5	20	0	747.7	7322.7	8070.3	585.9	181.9	767.8	90.49%
5	5	1.5	0.5	20	0	720.0	7229.4	7949.4	585.9	209.8	795.7	89.99%
6	6	1.5	0.5	20	0	791.4	7469.8	8261.2	585.9	217.0	802.9	90.28%
7	7	1.5	0.5	20	0	771.0	7401.1	8172.0	585.9	207.2	793.1	90.30%
8	8	1.5	0.5	20	0	792.1	7472.3	8264.5	585.9	213.2	799.1	90.33%
9	9	1.5	0.5	20	0	732.7	7272.2	8004.9	585.9	214.6	800.5	90.00%
10	10	1.5	0.5	20	0	741.7	7302.4	8044.0	585.9	220.3	806.2	89.98%
11	11	1.5	0.5	20	0	752.1	7337.6	8089.7	585.9	213.4	799.3	90.12%
12	12	1.5	0.5	20	0	736.7	7285.6	8022.3	585.9	218.1	804.0	89.98%
13	13	1.5	0.5	20	0	780.9	7434.6	8215.5	585.9	206.6	792.5	90.35%
14	14	1.5	0.5	20	0	792.0	7472.0	8264.1	585.9	205.2	791.1	90.43%
15	15	1.5	0.5	20	0	759.9	7363.8	8123.8	585.9	216.1	802.0	90.13%
16	16	1.5	0.5	20	0	730.0	7262.9	7992.9	585.9	216.5	802.4	89.96%
17	17	1.5	0.5	20	0	769.4	7395.8	8165.2	585.9	202.2	788.1	90.35%
18	18	1.5	0.5	20	0	741.8	7302.9	8044.7	585.9	209.4	795.3	90.11%
19	19	1.5	0.5	20	0	771.9	7404.2	8176.1	585.9	218.8	804.7	90.16%
20	20	1.5	0.5	20	0	763.7	7376.4	8140.1	585.9	188.6	774.5	90.48%
21	1	1.5	0.7	20	0	749.3	7328.0	8077.2	585.9	294.1	880.0	89.10%
22	2	1.5	0.7	20	0	766.4	7385.8	8152.3	585.9	290.0	875.9	89.26%
23	3	1.5	0.7	20	0	685.0	7111.5	7796.5	585.9	300.2	886.1	88.63%
24	4	1.5	0.7	20	0	747.7	7322.7	8070.3	585.9	244.2	830.1	89.71%
25	5	1.5	0.7	20	0	720.0	7229.4	7949.4	585.9	287.0	872.9	89.02%
26	6	1.5	0.7	20	0	791.4	7469.8	8261.2	585.9	305.7	891.6	89.21%
27	7	1.5	0.7	20	0	771.0	7401.1	8172.0	585.9	288.6	874.5	89.30%
28	8	1.5	0.7	20	0	792.1	7472.3	8264.5	585.9	303.1	889.0	89.24%
29	9	1.5	0.7	20	0	732.7	7272.2	8004.9	585.9	300.4	886.3	88.93%
30	10	1.5	0.7	20	0	741.7	7302.4	8044.0	585.9	314.9	900.8	88.80%
31	11	1.5	0.7	20	0	752.1	7337.6	8089.7	585.9	302.4	888.3	89.02%
32	12	1.5	0.7	20	0	736.7	7285.6	8022.3	585.9	309.2	895.1	88.84%
33	13	1.5	0.7	20	0	780.9	7434.6	8215.5	585.9	289.4	875.3	89.35%
34	14	1.5	0.7	20	0	792.0	7472.0	8264.1	585.9	284.5	870.4	89.47%
35	15	1.5	0.7	20	0	759.9	7363.8	8123.8	585.9	300.0	885.9	89.09%
36	16	1.5	0.7	20	0	730.0	7262.9	7992.9	585.9	302.1	888.0	88.89%
37	17	1.5	0.7	20	0	769.4	7395.8	8165.2	585.9	280.7	866.6	89.39%
38	18	1.5	0.7	20	0	741.8	7302.9	8044.7	585.9	293.2	879.1	89.07%
39	19	1.5	0.7	20	0	771.9	7404.2	8176.1	585.9	303.3	889.2	89.12%
40	20	1.5	0.7	20	0	763.7	7376.4	8140.1	585.9	261.7	847.6	89.59%
41	1	2	0.5	20	0	749.3	7328.0	8077.2	585.9	212.1	798.0	90.12%
42	2	2	0.5	20	0	766.4	7385.8	8152.3	585.9	214.0	799.9	90.19%
43	3	2	0.5	20	0	685.0	7111.5	7796.5	585.9	210.5	796.4	89.79%

Table 41 (Continued)

No	Inst	delta	beta	#cust	NC	Zcty	Zopc	Zcmr	Zhwy	Zdrn	Zdmr	Gap
44	4	2	0.5	20	0	747.7	7322.7	8070.3	585.9	188.5	774.4	90.40%
45	5	2	0.5	20	0	720.0	7229.4	7949.4	585.9	210.8	796.7	89.98%
46	6	2	0.5	20	0	791.4	7469.8	8261.2	585.9	213.7	799.6	90.32%
47	7	2	0.5	20	0	771.0	7401.1	8172.0	585.9	204.4	790.3	90.33%
48	8	2	0.5	20	0	792.1	7472.3	8264.5	585.9	210.2	796.1	90.37%
49	9	2	0.5	20	0	732.7	7272.2	8004.9	585.9	213.9	799.8	90.01%
50	10	2	0.5	20	0	741.7	7302.4	8044.0	585.9	216.4	802.3	90.03%
51	11	2	0.5	20	0	752.1	7337.6	8089.7	585.9	211.6	797.5	90.14%
52	12	2	0.5	20	0	736.7	7285.6	8022.3	585.9	214.6	800.5	90.02%
53	13	2	0.5	20	0	780.9	7434.6	8215.5	585.9	206.9	792.8	90.35%
54	14	2	0.5	20	0	792.0	7472.0	8264.1	585.9	206.8	792.7	90.41%
55	15	2	0.5	20	0	759.9	7363.8	8123.8	585.9	216.5	802.4	90.12%
56	16	2	0.5	20	0	730.0	7262.9	7992.9	585.9	213.1	799.0	90.00%
57	17	2	0.5	20	0	769.4	7395.8	8165.2	585.9	204.9	790.8	90.32%
58	18	2	0.5	20	0	741.8	7302.9	8044.7	585.9	208.1	794.0	90.13%
59	19	2	0.5	20	0	771.9	7404.2	8176.1	585.9	215.7	801.6	90.20%
60	20	2	0.5	20	0	763.7	7376.4	8140.1	585.9	191.4	777.3	90.45%
61	1	2	0.7	20	0	749.3	7328.0	8077.2	585.9	300.2	886.1	89.03%
62	2	2	0.7	20	0	766.4	7385.8	8152.3	585.9	295.7	881.6	89.19%
63	3	2	0.7	20	0	685.0	7111.5	7796.5	585.9	297.1	883.0	88.67%
64	4	2	0.7	20	0	747.7	7322.7	8070.3	585.9	252.6	838.5	89.61%
65	5	2	0.7	20	0	720.0	7229.4	7949.4	585.9	292.7	878.6	88.95%
66	6	2	0.7	20	0	791.4	7469.8	8261.2	585.9	304.5	890.4	89.22%
67	7	2	0.7	20	0	771.0	7401.1	8172.0	585.9	290.0	875.9	89.28%
68	8	2	0.7	20	0	792.1	7472.3	8264.5	585.9	298.8	884.7	89.29%
69	9	2	0.7	20	0	732.7	7272.2	8004.9	585.9	300.3	886.2	88.93%
70	10	2	0.7	20	0	741.7	7302.4	8044.0	585.9	309.0	894.9	88.87%
71	11	2	0.7	20	0	752.1	7337.6	8089.7	585.9	298.8	884.7	89.06%
72	12	2	0.7	20	0	736.7	7285.6	8022.3	585.9	305.3	891.2	88.89%
73	13	2	0.7	20	0	780.9	7434.6	8215.5	585.9	289.1	875.0	89.35%
74	14	2	0.7	20	0	792.0	7472.0	8264.1	585.9	286.3	872.2	89.45%
75	15	2	0.7	20	0	759.9	7363.8	8123.8	585.9	301.7	887.6	89.07%
76	16	2	0.7	20	0	730.0	7262.9	7992.9	585.9	303.8	889.7	88.87%
77	17	2	0.7	20	0	769.4	7395.8	8165.2	585.9	282.5	868.4	89.37%
78	18	2	0.7	20	0	741.8	7302.9	8044.7	585.9	292.9	878.8	89.08%
79	19	2	0.7	20	0	771.9	7404.2	8176.1	585.9	305.5	891.4	89.10%
80	20	2	0.7	20	0	763.7	7376.4	8140.1	585.9	263.4	849.3	89.57%