MULTIVARIATE CURVE FITTING FOR THE ANALYSIS OF SHORT MEMBERS AS THE BASIS FOR STRUCTURAL GRADING

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MULTIVARIATE CURVE FITTING FOR THE ANALYSIS OF SHORT MEMBERS AS THE BASIS FOR STRUCTURAL GRADING

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Submitted By

Approval of the Graduate School of Natural and Applied Sciences, Çankaya University.

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

MULTIVARIATE CURVE FITTING FOR THE ANALYSIS OF SHORT MEMBERS AS THE BASIS FOR STRUCTURAL GRADING

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In this study, multiple linear regression and multiple nonlinear regression methods were investigated and the most suitable multiple curve fitting method was selected according to the detailed analysis of short elements such as bamboo. The relationship between the physical properties of bamboo, compressive strength and ultimate tensile load was observed and multiple regression methods were applied on them. As a result, multiple linear regression method was determined as the most suitable method. In addition, the number of bamboos used for testing purposes has been reduced and economic savings have been achieved with the protection policy.

Keywords: Bamboo, Curve fitting, Multiple regression, least squares method, Compressive strength, Ultimate load

ÖZET

MULTIVARIATE CURVE FITTING FOR THE ANALYSIS OF SHORT MEMBERS AS THE BASIS FOR STRUCTURAL GRADING

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Bu çalışmada çoklu doğrusal regresyon ve çoklu doğrusal olmayan regresyon yöntemleri araştırılmış ve bambu gibi kısa elemanların detaylı analizine göre en uygun çoklu eğri uydurma yöntemi seçilmiştir. Bambunun fiziksel özellikleri ile basınç dayanımı ve nihai çekme yükü arasındaki ilişki gözlemlenmiş ve bunlar üzerinde çoklu regresyon yöntemleri uygulanmıştır. Sonucunda en uygun yöntem olarak çoklu doğrusal regresyon yöntemi belirlenmiştir. Ayrıca test amaçlı kullanılan bambuların sayısı azaltılarak koruma politikası ile ekonomik açıdan da tasarruf sağlanmıştır.

Anahtar Kelimeler: Bambu, Eğri uydurma, Çoklu Regresyon, En küçük kareler metodu, basınç dayanımı, Nihai çekme yükü

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LIST OF ABBREVIATIONS

kN	: Kilonewton
MPa	: Megapascal
mm	: Millimeter
gr	: Grams

CHAPTER 1

INTRODUCTION

Multivariate curve fitting is one in all the foremost elementary branches of statistics. Multiple linear regression and multiple nonlinear regression analyses are used frequently and in various ways in the most fundamental areas such as social sciences, health science, and other branches of applied sciences. Besides of these branches, various studies have been made on the factors affecting the dependent variable.

Investigating the relationship between variables has been one of the endeavors of science. Often, a researcher or an experimenter wants to find out if there is a relationship between two or more variables and show how this relationship is expressed by an equation. Because most of the problems we encounter in our daily life and in scientific research are related to determine whether there is a relationship between two (or more) variables. Data are often given for different values over a continuum. However, to guess at points between different values may be necessary, and to obtain intermediate estimates curve fitting techniques must be applied. A simplified version of a complex function may also be needed. One way to do this is to compute the values of the function at a certain number of different points along the range of interest (Karaoğlu, 2020).

The cause-and-effect relationship between the variables is investigated by regression analysis methods. In regression analysis, different regression analyses are used according to the values of the dependent variable. In addition, different regression analysis methods are used according to the existence of one or more independent variables. To set up models based on data sets in the literature; It is seen that regression techniques are frequently used in the solution of curve fitting problems (Kolb, 1984).

Regression analysis includes the following steps:

1- Statement of the problem

2- Selection of potentially relevant variables

3- Data collection
4- Model specification
5- Choice of fitting method
6- Model fitting
7- Model validation and criticism

using the chosen model(s) for the solution of the posed problem (Douglas & Montgomery, 2012).

In this thesis, it was aimed to examine the measurable properties of bamboo, which was chosen as a building material, and to determine the relationship between their strength against compressive strength and ultimate load (tensile) forces. In this direction, multiple linear regression, and multiple nonlinear regression methods will be investigated and the most suitable method that can be associated with bamboo will be determined.

1.1. BAMBOO

Bamboo has been used as a building material for a long time. Most traditional houses in Indonesia and Asia use bamboo as a building material. After the issues of sustainability and global warming arose, bamboo as a building material began to be widely discussed and considered. Some architects and civil engineers today tend to choose bamboo as a building material. Today, high-quality construction timber is rare due to deforestation. Wood also takes a long time to grow and be ready to be used as a building material. Meanwhile, bamboo can be harvested in a short time (3-5 years). When planted, bamboo also releases oxygen into the air, which industrial materials such as steel, plastic and concrete cannot do. For these reasons, bamboo has become widely recognized as an environmentally friendly building material (Nurdiah, 2016).

Bamboo, as a building material, has high flexibility and durability. With these properties, when used correctly, bamboo becomes a suitable building material for the main parts of a building (i.e., foundation, flooring, walls, and roofs). It has different uses in buildings thanks to its advantages such as easy availability, low cost, and high strength (Auwalu & Dickson, 2019).

The amount of energy required per 1 m3 per unit of stress for materials commonly used in the construction industry has been compared with that of bamboo.

It was found that steel requires 50 times more energy than bamboo to produce. Two tons of CO2 are generated during the production of one ton steel. In contrast, bamboo absorbs CO2 in addition to producing oxygen. The tensile strength of bamboo is relatively high and can reach 370 MPa, which makes it attractive to use bamboo as an alternative to steel, especially when considering the relationship between tensile strength and the specific weight of bamboo, which is six times greater than that for steel (Xiao, Inoue & Paudel, 2008).

1.2. LITERATURE REVIEW

In his master thesis Mustafa Doğmaz (2018), from Pamukkale University Institute of Science and Engineering, aimed to find linear relationships to calculate the long-term average flow and maximum instantaneous flow values for Western Mediterranean streams. This study includes linear equations belonging to 30 basins selected from the Western Mediterranean Region, basin physiographic parameters and land use rates obtained from the Digital Elevation Model. Doğmaz investigated the relationships between flow characteristics, physiographic parameters, and land use rates with multiple linear regression analysis. As a result of this study, he determined the best multiple linear regression model.

Zain & Abd (2009) has developed a mathematical model using statistical analysis in line with the concrete data he obtained from his experimental studies on the prediction of compressive strength of high-performance concrete. Concrete yielded an excellent correlation coefficient of 99.99% with the multiple nonlinear regression models (at all ages) because of compressive strength estimation at different ages (3, 7, 14, 28 and 91 days).

In this study, Çelik (2005) investigated and examined the level of alienation of secondary school students in terms of their methods of expressing anger and some socio-demographic variables. In the research, depending on the gender, the socio-economic level in the school and the style of understanding anger multivariate analysis of variance was used to compare alienation results.

Çerçi (2010) in this study, the aim is to explain the postpaid and prepaid revenues of three GSM operators in Turkey by using the Multivariate Regression Analysis method through the number of subscribers, communication traffic density, tariff unit price average and the number of base stations, and thus to perform an application with Multivariate Regression Analysis.

In this study, Hamzaoğlu (2013) performed a sample application respectively for all the regression analysis applications that are used frequently and intensively, such as simple linear, multiple linear regression, poisson regression, and binary logistic regression, and calculated power analysis.

As a result of detailed research and literature review, it has been determined that the multiple linear regression method was not used on bamboo in the construction department, specific building material.

1.3. PURPOSE OF STUDY

We will investigate whether there is a relationship between material properties and the final ultimate load and compressive strength values of bamboo to have an idea about the ultimate load and compressive strength of bamboo with different material properties without the need of any experiment results. To do this, we will form new equations by use of multivariate curve fitting techniques with the best correlation coefficients. We will use data of Guadua angustifolia type bamboo to generate these equations (Bahtiar, Trujillo & Nugroho, 2020).

With the created equations, we want to reduce the need for experimentation to find the ultimate load and compressive strength and relations between material properties of bamboo. By this way, maintaining the natural ecological balance without cutting and damaging less bamboo are the results we expect.

CHAPTER 2

MULTIVARIATE CURVE FITTING

Multivariate curve fitting is one of the procedures in data analysis and is helpful for prediction analysis showing graphically how the data points are related to one another whether it is in linear or non-linear model. Usually, the curve fit will find the concentrates along the curve, or it will just use to smooth the data and upgrade the presence of the plot. Curve fitting checks the relationship between independent variables and dependent variables with the objective of characterizing a good fit model. Curve fitting finds mathematical equation that best fits given information. (Vidyullatha & Rao, 2016).

2.1. MULTIPLE LINEAR REGRESSION

Multiple regression analysis method expresses the mathematical relationship between dependent variable y_i and multiple independent variables x_i.

A useful extension of linear regression is the case where y is a linear function of two or more independent variables. For example, y might be a linear function of x_1 and x_2, as in.

$$y_i = a_0 + a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + e \quad (2.1)$$

y_i= $dependent\ variable$

$a_0 = intercept$

$a_i = slope\ coefficients$

$x_i = independent\ variable\ (known\ constant)$

$e = residuals\ error$ (The e is the difference between the predicted value and the observed value.)

2.2. MULTIPLE NONLINEAR REGRESSION

In statistics, nonlinear regression could be a type of multivariate analysis within which observational data are modeled by a function which is a nonlinear combination of the model parameters and depends on one or many independent variables. The data are fitted by a method of successive approximations.

Nonlinear regression formula is same as linear regression, but equations are not linear and must be done linearization process.

$$y_i = a_0 {x_1}^{a_1} {x_2}^{a_2} {x_3}^{a_3} \dots . {x_n}^{a_n} + e \quad (2.2)$$

$$y_i = f(x_i; a_0, a_1, \dots, a_n) + e \quad (2.3)$$

$y_i = dependent\ variable$

$y_i = a_0 {x_1}^{a_1} {x_2}^{a_2} {x_3}^{a_3} \dots . {x_n}^{a_n} + e$

$e = residuals\ error$

$f(x_i; a_0, a_1, \dots, a_n)\ function\ of\ nonlinear\ regression$

2.3. ERROR ANALYSIS

2.3.1. Error Analysis of Multiple Linear Regression

2.3.1.1. Least Squares Method for Multiple Linear Regression

As with the previous cases, the “best” values of the coefficients are determined by setup the sum of the squares of the residuals

$$S_t = \sum (y_i - \bar{y})^2 \quad (2.4)$$

$$S_r = \sum_{i=1}^{n} e^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} \dots - a_n x_{ni})^2 \quad (2.5)$$

and differentiating with respect to each of the unknown coefficients,

$$\frac{\partial S_r}{\partial a_0} = -2 \sum_{i=1}^{n} (y_1 - a_0 - a_1 x_{1i} - a_2 x_{2i} \dots - a_n x_{ni}) \quad (2.6)$$

$$\frac{\partial S_r}{\partial a_1} = -2 \sum_{i=1}^{n} X_{1i} (y_1 - a_0 - a_1 x_{1i} - a_2 x_{2i} \dots - a_n x_{ni}) \quad (2.7)$$

$$\frac{\partial S_r}{\partial a_2} = -2 \sum_{i=1}^{n} X_{2i} (y_1 - a_0 - a_1 x_{1i} - a_2 x_{2i} \dots - a_n x_{ni}) \quad (2.8)$$

$$\frac{\partial S_r}{\partial a_n} = -2\sum_{i=1}^{n} X_{ni}(y_1 - a_0 - a_1x_{1i} - a_2x_{2i} \dots - a_nx_{ni}) \quad (2.9)$$

The coefficients yielding the minimum sum of the squares of the residuals are obtained by setting the partial derivatives equal to zero and expressing the result in matrix form as:

$$\begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} & . & . & \sum x_{ni} \\ \sum x_{1i} & \sum x_{1i}^2 & \sum x_{1i}x_{2i} & . & . & \sum x_{1i}x_{ni} \\ \sum x_{2i} & \sum x_{1i}x_{2i} & \sum x_{2i}^2 & . & . & \sum x_{2i}x_{ni} \\ . & . & . & & & . \\ . & . & . & & & . \\ \sum x_{ni} & \sum x_{ni}X_{1i} & \sum x_{ni}x_{2i} & . & . & \sum x_{ni}{}^2 \end{bmatrix} \begin{vmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_n \end{vmatrix} = \begin{bmatrix} \sum y_{1i} \\ \sum x_{1i}y_i \\ \sum x_{2i}y_i \\ . \\ . \\ \sum x_{ni}y_i \end{bmatrix} \quad (2.10)$$

Note that we have simplified the summation symbols; unless otherwise indicated, all summations are from *i* = 1 to *n.*

After solving equation (2.10) by Gauss elimination, a_0, a_1, a_2, a_n can be obtained (Chapra & Canale, 2010).

$$y_i = a_0 + a_1x_1 + a_2x_2 + \cdots + a_nx_n \quad (2.11)$$

$S_r = sum\ of\ the\ squares\ of\ the\ residual\ around\ the\ regression\ line$

$S_t = total\ sum\ of\ the\ squares\ around\ the\ mean$

2.3.2. Error Analysis of Nonlinear Multiple Regression

2.3.2.1. Linearization of Nonlinear Multiple Regression

Some nonlinear regression problems can be transformed to a linear regression model.

For example:

***linearization of power equation*:**

$$y_i = a_0{x_1}^{a_1}{x_2}^{a_2}{x_3}^{a_3} \dots .{x_n}^{a_n} \quad (2.12)$$

$$\log y_i = \log a_0 + a_1 \log x_1 + a_2 \log x_2 + a_3 \log x_3 + \cdots + a_n \log x_n \quad (2.13)$$

***linearization of exponential*:**

$$y_i = a_0e^{a_1x1+a_2x2+a_3x3}, \dots .e^{anxn} \quad (2.14)$$

$$\ln y_i = \ln a_0 + a_1 \ln x_1 + a_2 \ln x_2 + a_3 \ln x_3 \dots + a_n \ln x_n \quad (2.15)$$

In this way powered and exponential nonlinear equations are converted to linear (Chapra & Canale, 2010).

2.3.2.2. Least Squares Methods for Multiple Nonlinear Regression

The sum of the square of the residuals are:

$$S_r = \sum_{i=1}^{n} [(y_i - \hat{y_i})]^2 \tag{2.16}$$

$$\hat{y_i} = f(x_i; a_0, a_1, \dots, a_n) + e \tag{2.17}$$

$$S_r = \sum_{i=1}^{n} \left[\left(y_i - f(x_i; a_0, a_1, \dots, a_n)\right)\right]^2 \tag{2.18}$$

For example:

$$y_i = a_0 {x_1}^{a_1} {x_2}^{a_2} {x_3}^{a_3} \dots . {x_n}^{a_n} \tag{2.19}$$

$$\log y = \log a_0 + a_1 \log x_1 + a_2 \log x_2 + a_3 \log x_3 + \cdots + a_n \log x_n \tag{2.20}$$

To find the constants a_0, a_1.. a_n of the powered model, we minimize S_r by differentiating with respect to a_n equating the resulting equations to zero.

$$\sum_{i=1}^{n} [(\log y_i - \log a_0 - a_1 \log x_1 - a_2 \log x_2 - a_3 \log x_3 - \cdots - a_n \log x_n)]^2 \tag{2.21}$$

$$\frac{\partial S_r}{\partial a_0} = -2 \sum_{i=1}^{n} (logy_i - loga_0 - a_1 logx_{1i} - a_2 logx_{2i} \dots - a_n logx_{ni}) \tag{2.22}$$

$$\frac{\partial S_r}{\partial a_1} = -2 \sum_{i=1}^{n} logX_{1i}(logy_i - loga_0 - a_1 logx_{1i} - a_2 logx_{2i} \dots - a_n logx_{ni}) \tag{2.23}$$

$$\frac{\partial S_r}{\partial a_2} = -2 \sum_{i=1}^{n} logX_{2i}(logy_i - loga_0 - a_1 logx_{1i} - a_2 logx_{2i} \dots - a_n logx_{ni}) \tag{2.24}$$

$$\frac{\partial S_r}{\partial a_n} = -2 \sum_{i=1}^{n} logx_{ni}(logy_i - loga_0 - a_1 logx_{1i} - a_2 logx_{2i} \dots - a_n logx_{ni}) \tag{2.25}$$

$$\begin{bmatrix} n & \sum logx_{1i} & \sum logx_{2i} & . & . & \sum logx_{ni} \\ \sum logx_{1i} & \sum logx_{1i}^2 & \sum logx_{1i}logx_{2i} & . & . & \sum logx_{1i}logx_{ni} \\ \sum logx_{2i} & \sum logx_{2i}logx_{1i} & \sum logx_{2i}^2 & . & . & \sum logx_{2i}logx_{ni} \\ . & . & . & & & . \\ . & . & . & & & . \\ \sum logx_{ni} & \sum logx_{ni}logx_{1i} & \sum logx_{ni}logx_{2i} & . & . & \sum logx_{ni}^2 \end{bmatrix} \begin{vmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_n \end{vmatrix} = \begin{bmatrix} \sum logy_i \\ \sum logx_{1i}logy_i \\ \sum logx_{2i}logy_i \\ . \\ . \\ \sum logx_{ni}logy_i \end{bmatrix} \quad (2.26)$$

After solving equation (2.26) by Gauss elimination, a_0, a_1, a_2, a_n can be obtained (Chapra & Canale, 2010).

$$\log y_i = \log a_0 + a_1 \log x_1 + a_2 \log x_2 + a_3 \log x_3 + \cdots + a_n \log x_n \quad (2.27)$$

2.3.3. Correlation Coefficient for Multiple Linear and Nonlinear Regression

The Multiple Correlation Coefficient is expressed with the letter r, and this expression shows us the degree of relationship between independent variables and dependent variables. r always takes a positive value and is expressed by ($0 \leq r \leq 1$). By squaring r, the multiple coefficient determination r^2 is obtained. The multiple coefficient determination r^2 shows the rate at which the independent variables together explain the changes in the dependent variable.

$$r^2 = \frac{St - Sr}{St} \quad (2.28)$$

$r^2 = coefficient\ of\ determination$

$r = correlation\ coefficient$

$0 \leq r \leq 1$

$r = r^2 = 1 \quad determination\ of\ perfect\ fit$

$r = r^2 = 0 \quad there\ are\ not\ any\ relationship\ between\ variable$

2.3.4. Standard Error of The Estimate for Multiple Linear and Nonlinear Regression

The standard error of estimate is the measure of variation of an observation made around the calculated regression line; It is simply used to verify the precision of the predictions made with the regression line. Also, a standard deviation that measures the variation in the data set from its mean, the standard error of estimation also measures the variation in the real values of y from the calculated values of y (predicted) on the regression. It is calculated as a standard deviation, and here the deviations are the vertical distance from each point on the average relationship line (Business Jargons, 2020).

$$s_{y/x} = \sqrt{\frac{s_r}{n-(m+1)}} \quad (2.29)$$

$s_{y/x} = standard\ error\ of\ estimate$

$m = order\ of\ polynomial$

$n = number\ of\ data$

$S_r = sum\ of\ the\ squares\ of\ the\ residual\ around\ the\ regression\ line$

CHAPTER 3

APPLICATION OF MULTIPLE LINEAR REGRESSION IN MATLAB

At first, it is aimed to curve fit 113 bamboo samples by printing the regression code in MATLAB. For this analysis, we need to use two parameters to estimate the ultimate load. After that one by one the amount of parameters are added. In first part the dependent variable is ultimate load and in the second part the dependent variables changed to compressive strength load.

Data and MATLAB code have shown in the appendix part.

3.1. PREDICTION OF ULTIMATE LOAD BY USE OF AVERAGE LENGTH (mm) AND AVERAGE THICKNESS (mm)

At this stage, two parameters were used (independent variables), average length, and average thickness, to estimate the ultimate load value. As a result, it is reached to the high value of correlation coefficient, then it's shown that these parameters have a direct relationship with the ultimate load value.

$$y_i = a_0 + a_1 x_1 + a_2 x_2 \quad (3.1)$$

$$y_i(kN) = -50.420 + 0.826 \ * average\ length\ (mm) + 19.413 * average\ thickness\ (mm)$$

$y_i = ultimate\ load(kN)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$a_0 = -50.420$

$a_1 = 0.826$

$a_2 = 19.413$

$r^2 = 0.820$

$r = 0.9055$

Standard Error of The Estimate: 42.285 kN

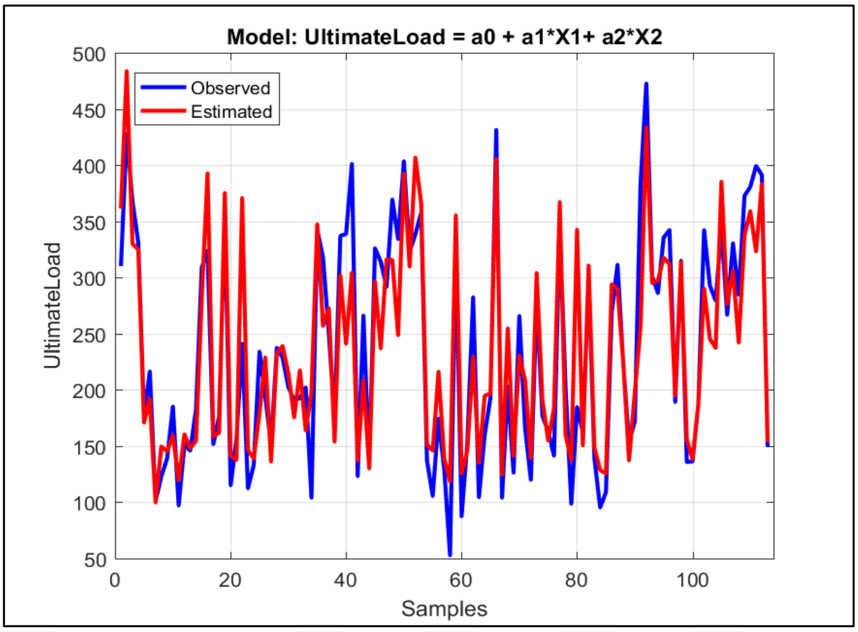

Figure 1. Curve fitting of ultimate load according to average length and average thickness

3.2. PREDICTION OF ULTIMATE LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm) AND AVERAGE EXTERNAL DIAMETER (mm)

At this stage, the ultimate load values are estimated by increasing the number of parameters to three. In addition to the average length and thickness values, the average diameter is also included in the calculation. As a result, it is observed that by increasing the number of parameters, a higher correlation coefficient value and reduction in the error value can be obtained.

$$y_i = a_0 + a_1x_1 + a_2x_2 + a_3x_3 \quad (3.2)$$

$$y_i(kN) = -124.935 - 0.258 * \ average\ length\ (mm) + 12.490 * average\ thickness\ (mm) + 2.466 * average\ external\ diameter\ (mm)$$

$y_i = ultimate\ load(kN)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$a_0 = -124.935$

$a_1 = -0.258$

$a_2 = 12.490$

$a_3 = 2.466$

$r^2 = 0.895$

$r = 0.946$

Standard Error of The Estimate: 32.263 kN

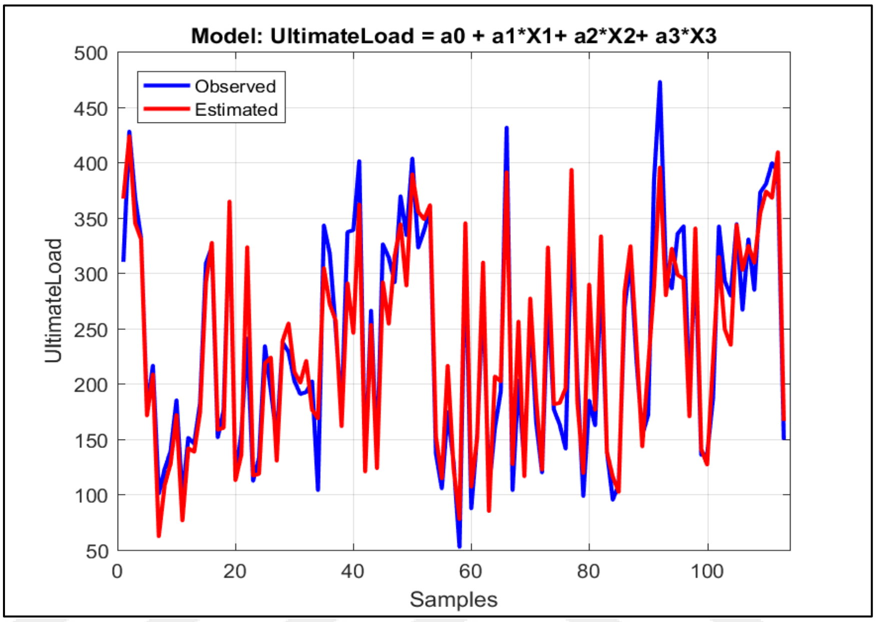

Figure 2. Curve fitting of ultimate load according to average length, average thickness, and average external diameter

3.3. PREDICTION OF ULTIMATE LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm), AVERAGE EXTERNAL DIAMETER (mm) AND WEIGHT BEFORE COMPRESSION TEST (gr)

In this section, the numerical value of weight before compression test were added as fourth parameter to this calculation. Thus, it is observed that a higher correlation coefficient according to these three parameters value and reduced the error value to a lower level.

$$y_i = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \quad (3.3)$$

$$y_i ultimate\ load\ (kN) = 19.550 - 0.998 * \ average\ length\ (mm) + 8.612 * \ average\ thickness\ (mm) + 1.303 * average\ external\ diameter\ (mm) + 0.344 * weight\ before\ compression\ test\ (gr)$$

$y_i = ultimate\ load\ (kN)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$x_4 = weight\ before\ compression\ test\ (gr)$

$a_0 = 19.550$

$a_1 = -0.998$

$a_2 = 8.612$

$a_3 = 1.303$

$a_4 = 0.344$

$r^2 = 0.911$

$r = 0.954$

Standard Error of The Estimate: 29.726 kN

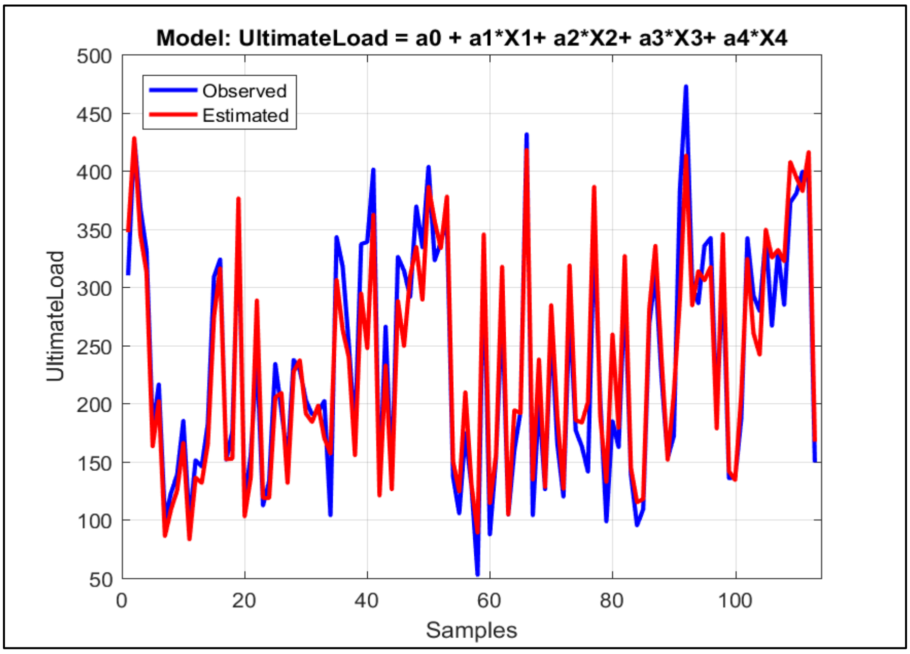

Figure 3. Curve fitting of ultimate load according to average length, average thickness, average external diameter, and weight before compression test

3.4. PREDICTION OF COMPRESSIVE STRENGTH BY USE OF AVERAGE LENGTH (mm) AND AVERAGE THICKNESS (mm)

In this part of the research, the aim is the estimation of the compressive strength value. Again, average thickness and average length values were used as predictor parameters, but the dependent variable changed as compressive strength. As a result, it is found out that these parameters were also related to the compressive strength, but when calculations compared with the ultimate load value, it is observed that this relationship is weaker.

$$y_i = a_0 + a_1 x_1 + a_2 x_2 \qquad (3.4)$$

$$y_i\ (MPa) = 117.183\ -\ 0.206\ average\ length\ (mm) - 1.826 * average\ thickness\ (mm)$$

$y_i = compressive\ strength (MPa)$

$x_1 = average\ length (mm)$

$x_2 = average\ thickness (mm)$

$a_0 = 117.183$

$a_1 = -0.206$

$a_2 = -1.826$

r^2 =0.423

$r = 0.650$

Standard Error of The Estimate: 11.016 MPa

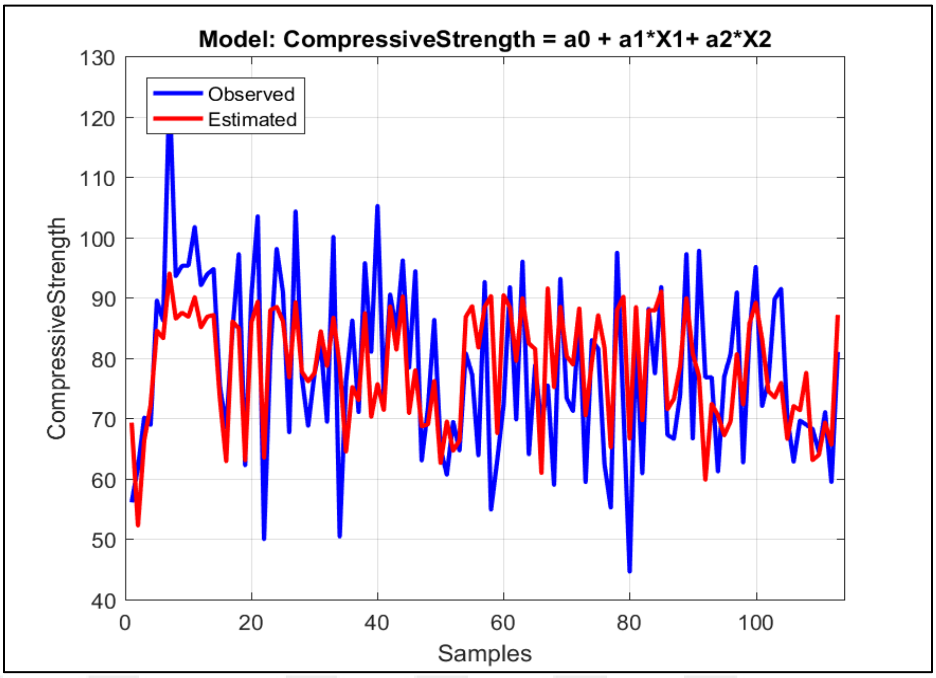

Figure 4. Curve fitting of compressive strength according to average length, and average thickness

3.5. PREDICTION OF COMPRESSIVE STRENGTH LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm) AND AVERAGE EXTERNAL DIAMETER (mm)

In this section, the average external diameter value was added in addition to these, thickness and length parameters which were used to estimate the compressive strength value. As a result, it is noticed that there is a slight change in correlation coefficient value.

$$y_i = a_0 + a_1x_1 + a_2x_2 + a_3x_3 \qquad (3.5)$$

$$y_i\ (MPa) = 120.274\ -\ 0.161\ average\ length\ (mm) - 1.538 * average\ thickness\ (mm) - 0.102 * average\ external\ diameter(mm)$$

$y_i = compression\ strength(MPa)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$a_0 = 120.274$

$a_1 = -0.161$

$a_2 = -1.538$

$a_3 = -0.102$

r^2 =0.429

$r = 0.655$

Standard Error of The Estimate: 10.957 MPa

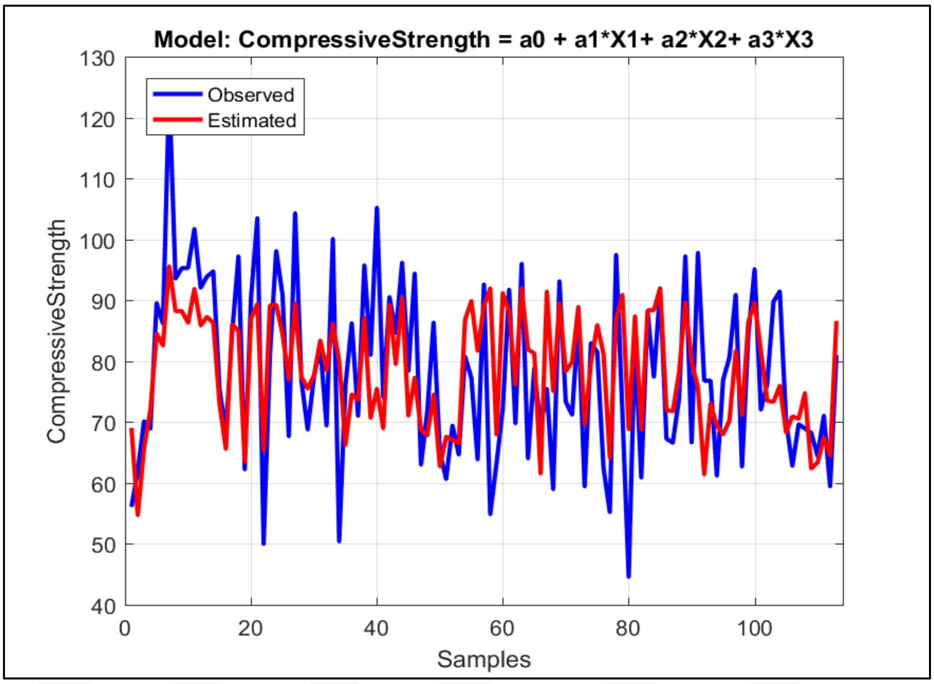

Figure 5. Curve fitting of compressive strength according to average length, average thickness, and average external diameter

3.6. PREDICTION OF COMPRESSIVE STRENGTH LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm), AVERAGE EXTERNAL DIAMETER (mm) AND WEIGHT BEFORE COMPRESSION TEST (gr)

In this section, the weight before compression test numerical value was added in addition to the three parameters which were used to estimate the compressive strength value earlier. As a result, obtained a slightly higher correlation coefficient value with the fourth parameter used.

$$y_i = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \quad (3.6)$$

$$y_i\ (MPa) = 150.824 - 0.317\ \ average\ length\ (mm) -$$
$$2.358\ \ average\ thickness\ (mm) -$$

$$0.348\ average\ external\ diameter\ (mm) + 0.073 * weight\ before\ compression\ test\ (gr)$$

$y_i = compressive\ strength(MPa)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$x_4 = weight\ before\ compression\ test\ (gr)$

$a_0 = 150.824$

$a_1 = -0.317$

$a_2 = -2.358$

$a_3 = -0.348$

$a_4 = 0.073$

$r^2 = 0.462$

$r = 0.680$

Standard Error of The Estimate: 10.632 MPa

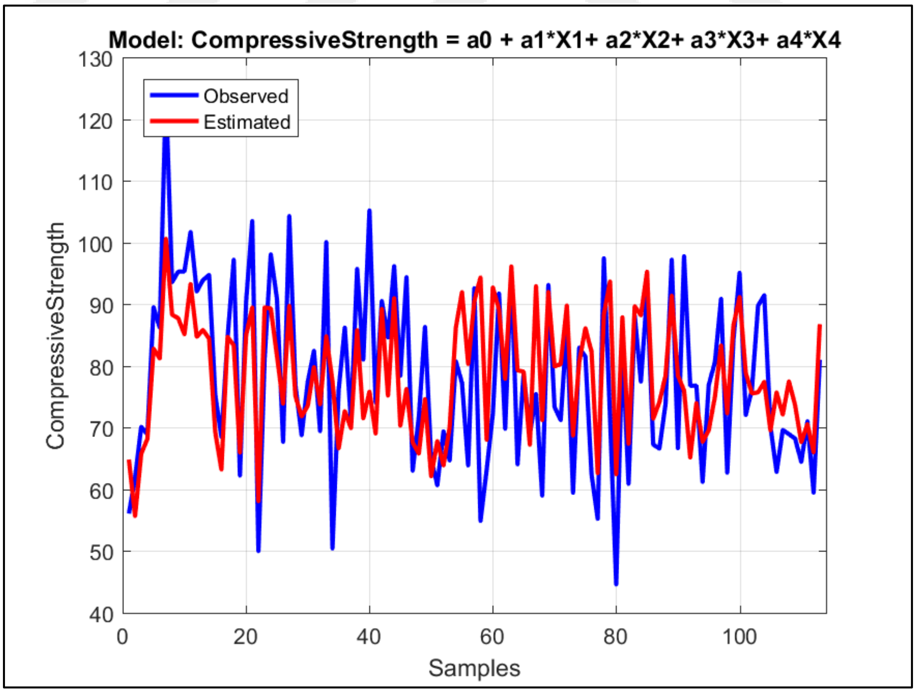

Figure 6. Curve fitting of compressive strength according to average length, average thickness, average external diameter, and weight before compression test

CHAPTER 4

APPLICATION OF MULTIPLE NONLINEAR REGRESSION IN MATLAB

Until this part of the study, whether there is a linear relationship between the ultimate load, compressive strength, and physical properties were analysed, but at this stage, whether there is a nonlinear relationship between these parameters will be investigated. When the multiple nonlinear regression methods were examined, the saturation growth model was not suitable for the bamboo data at hand, the power model and the exponential model gave similar results, so the power (logarithmic) model was chosen as the method.

Data and MATLAB code shown in the appendix part.

4.1. PREDICTION OF ULTIMATE LOAD BY USE OF AVERAGE LENGTH (mm) AND AVERAGE THICKNESS (mm)

In this part of the research, the aim is the estimation of the ultimate load value with multiple nonlinear regression. In this calculation the average thickness and average length values was used as predictor parameters, but the equation was nonlinear (powered or logarithmic). As a result, it was determined that the thickness and length parameters also have a non-linear relationship with the ultimate load value.

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) \quad (4.1)$$

$$\log(y_i) = \log(8.3343) + 0.2078 * \log(average\ length) + 1.005 * \log(average\ thickness)$$

$y_i = ultimate\ load(kN)$

$x_1 = average\ length\ (mm)$

$x_2 = average\ thickness(mm)$

$a_0 = 8.3343$

$a_1 = 0.2078$

$a_2 = 1.0054$

$r^2 = 0.838$

$r = 0.915$

Standard Error of The Estimate: 0.081 kN

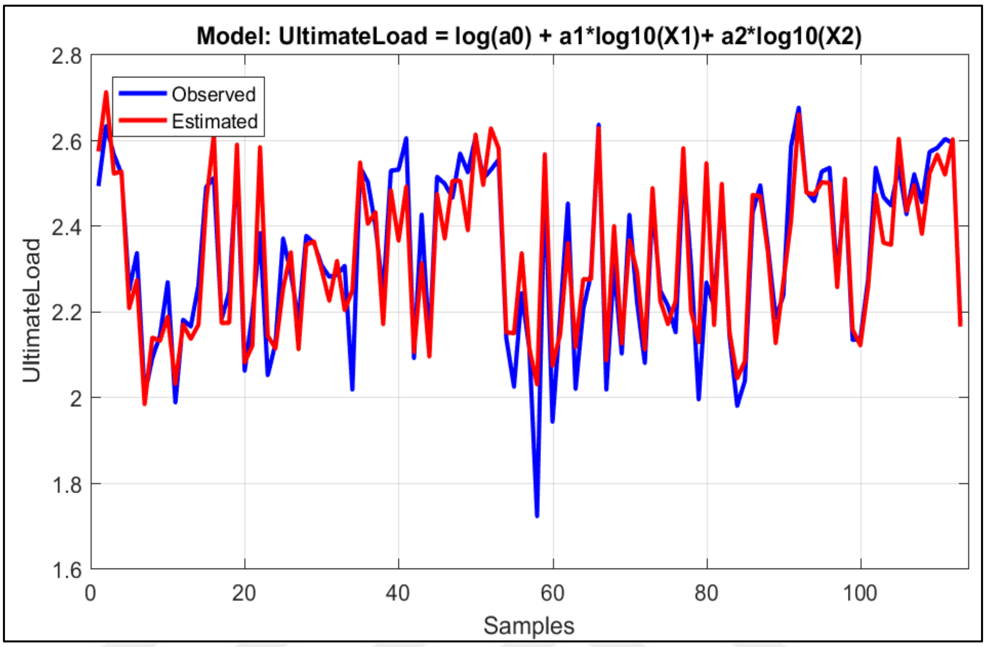

Figure 7. Nonlinear curve fitting of ultimate load according to average length, and average thickness

4.2. PREDICTION OF ULTIMATE LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm) AND AVERAGE EXTERNAL DIAMETER (mm)

At this stage the non-linear relationship of the ultimate load value observed, which is tried to estimate at this stage, with the parameters, the average external diameter value was added in addition to the length and thickness values. As a result, it is reached to higher correlation coefficient value than the calculation that made by using of two parameters.

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) + a_3 \log(x_3) \qquad (4.2)$$

$$\log(y_i) = \log(0.582) - 0.199 * \log(average\ length) + 0.589 * \log(average\ thickness) + 1.186 * \log(average\ external\ diameter)$$

$y_i = ultimate\ load(kN)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$a_0 = 0.5819$

$a_1 = -0.1993$

$a_2 = 0.5892$

$a_3 = 1.1859$

$r^2 = 0.903$

$r = 0.950$

Standard Error of The Estimate: 0.063 kN

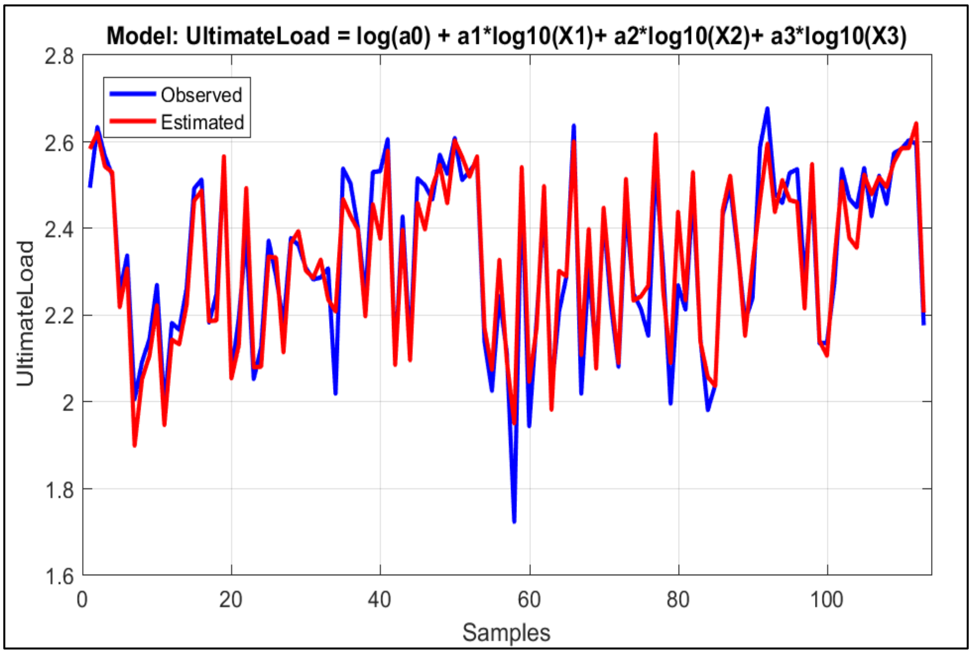

Figure 8. Nonlinear curve fitting of ultimate load according to average length, average thickness, and average external diameter

4.3. PREDICTION OF ULTIMATE LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm), AVERAGE EXTERNAL DIAMETER (mm) AND WEIGHT BEFORE COMPRESSION (gr)

In this part, while trying to estimate the nonlinear relationship of the ultimate load value with the selected parameters, the average external diameter value was added as a fourth parameter. As a conclusion, the calculations which are made with using four parameters indicated that the correlation coefficient value was slightly higher.

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) + a_3 \log(x_3) + a_4 \log(x_4) \quad (4.3)$$

$$\log(y_i) = \log(2.6213) - 0.3806 * \log(average\ length) + 0.4552 * \log(average\ thickness) + 0.8231 * \log(average\ external\ diameter) + 0.2416 * \log(weight\ before\ compression)$$

$y_i = ultimate\ load(kN)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$x_4 = weight\ before\ compression(gr)$

$\mathrm{a}_0 = 2.6213$

$a_1 = -0.3806$

$a_2 = 0.4552$

$a_3 = 0.8231$

$a_4 = 0.2416$

$r^2 = 0.906$

$r = 0.952$

Standard Error of The Estimate: 0.062 kN

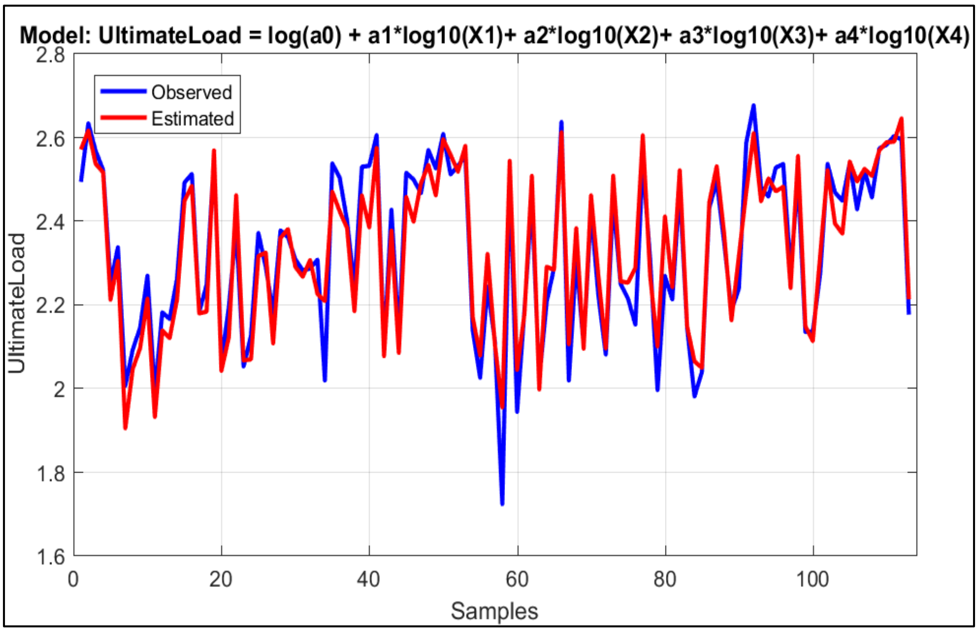

Figure 9. Nonlinear curve fitting of ultimate load according to average length, average thickness, average external diameter, and weight before compression test

4.4. PREDICTION OF COMPRESSIVE STRENGTH LOAD BY USE OF AVERAGE LENGTH (mm) AND AVERAGE THICKNESS (mm)

In this part of the research, the aim is the estimation of the compressive strength value, and the average thickness and average length values were used as parameters. As a result, it is observed that these parameters also have a nonlinear relationship with the compressive strength. When compared to them with the calculations which were made in the relations with the ultimate load value, it is concluded that this relationship is also weaker.

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) \tag{4.4}$$

$$\log(y_i) = \log(336.8349) - 0.1933 * \log(average\ length) - 0.2622 * \log(average\ thickness)$$

$y_i = compressive\ strength(MPa)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$a_0 = 336.8349$

$a_1 = -0.1933$

$a_2 = -0.2622$

$r^2 = 0.415$

$r = 0.644$

Standard Error of The Estimate: 0.063 MPa

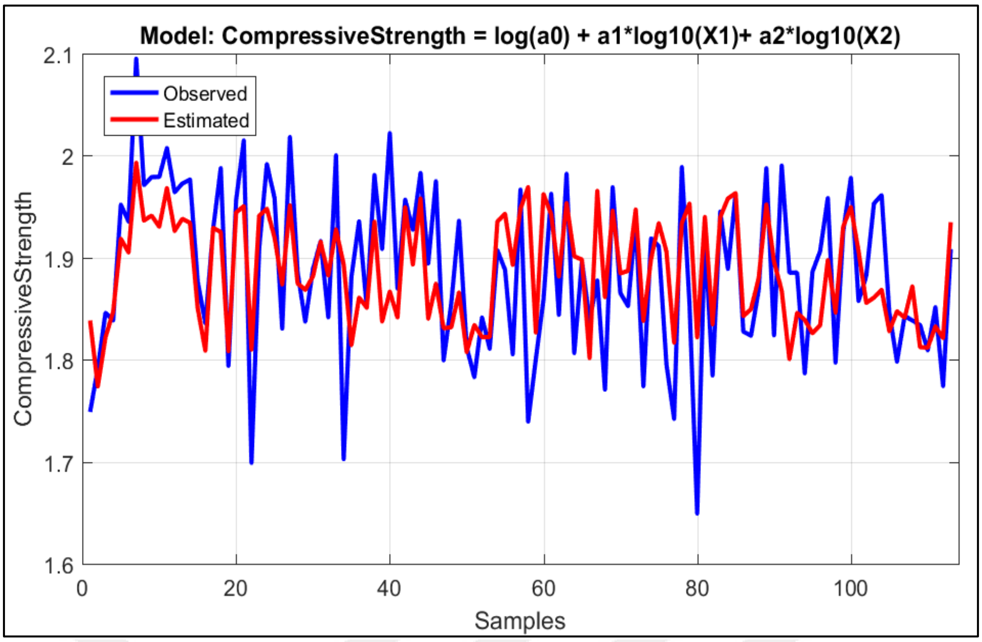

Figure 10. Nonlinear curve fitting of compressive strength according to average length, and average thickness

4.5. PREDICTION OF COMPRESSIVE STRENGTH LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm) AND AVERAGE EXTERNAL DIAMETER (mm)

At this stage, while estimating the compressive strength value, the average external diameter value was added to the parameters and nonlinear relationship between them was observed. As a result of this parameter's effect, correlation coefficient value demonstrated a slight increase, and error value a little reduced.

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) + a_3 \log(x_3) \quad (4.5)$$

$$\log(y_i) = \log(308.693) - 0.207 * \log(average\ length) - 0.276 * \log(average\ thickness) + 0.039 * \log(average\ external\ diameter)$$

$y_i = compressive\ strength(MPa)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$a_0 = 308.693$

$a_1 = -0.207$

$a_2 = -0.276$

$a_3 = 0.039$

$r^2 = 0.415$

$r = 0.644$

Standard Error of The Estimate: 0.063 MPa

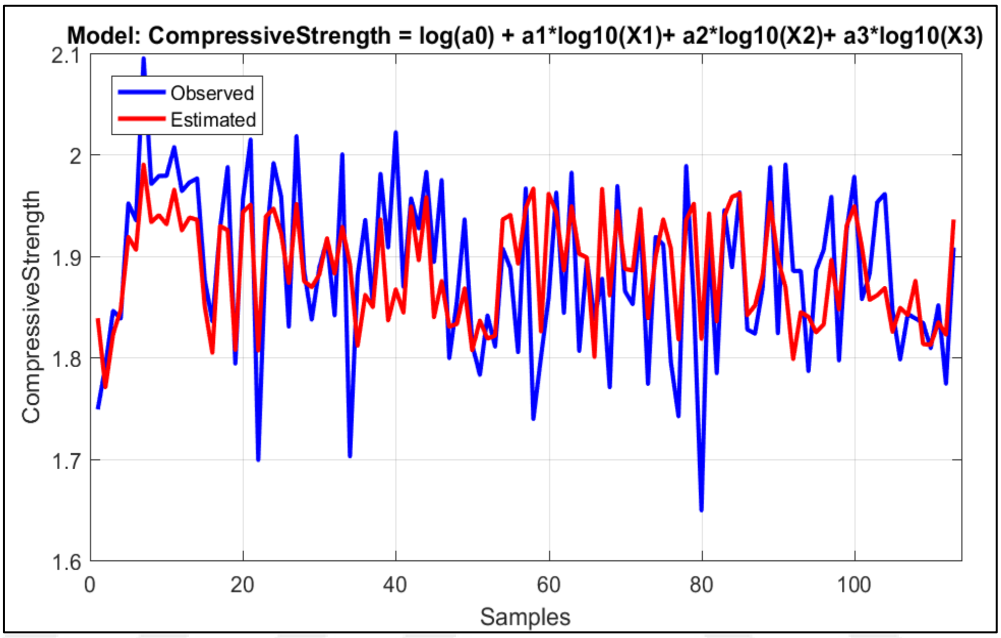

Figure 11. Nonlinear curve fitting of compressive strength according to average length, average thickness, and average external diameter

4.6. PREDICTION OF COMPRESSIVE STRENGTH LOAD BY USE OF AVERAGE LENGTH (mm), AVERAGE THICKNESS (mm), AVERAGE EXTERNAL DIAMETER (mm) AND WEIGHT BEFORE COMPRESSION TEST (gr)

In this part of the research, in addition to the average thickness, length and diameter parameters, whose nonlinear relationship was found out while estimating the compressive strength value, also it is included that the weight before compression value as a fourth parameter in our calculations. Again, a similar result was obtained and an increase in the correlation coefficient value in direct proportion to the number of parameters was observed, albeit slightly.

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) + a_3 \log(x_3) + a_4 \log(x_4) \quad (4.6)$$

$$\log(y_i) = \log(1392.143) - 0.388 * \log(average\ length) - 0.410 * \log(average\ thickness) - 0.324 * \log(average\ external\ diameter) + 0.242 * \log(weight\ before\ compression)$$

$y_i = compressive\ strength(MPa)$

$x_1 = average\ length(mm)$

$x_2 = average\ thickness(mm)$

$x_3 = average\ external\ diameter(mm)$

$x_4 = weight\ before\ compression(gr)$

$\mathrm{a}_0 = 1392.143$

$a_1 = -0.388$

$a_2 = -0.410$

$a_3 = -0.324$

$a_4 = 0.242$

$r^2 = 0.435$

$r = 0.659$

Standard Error of The Estimate: 0.061 MPa

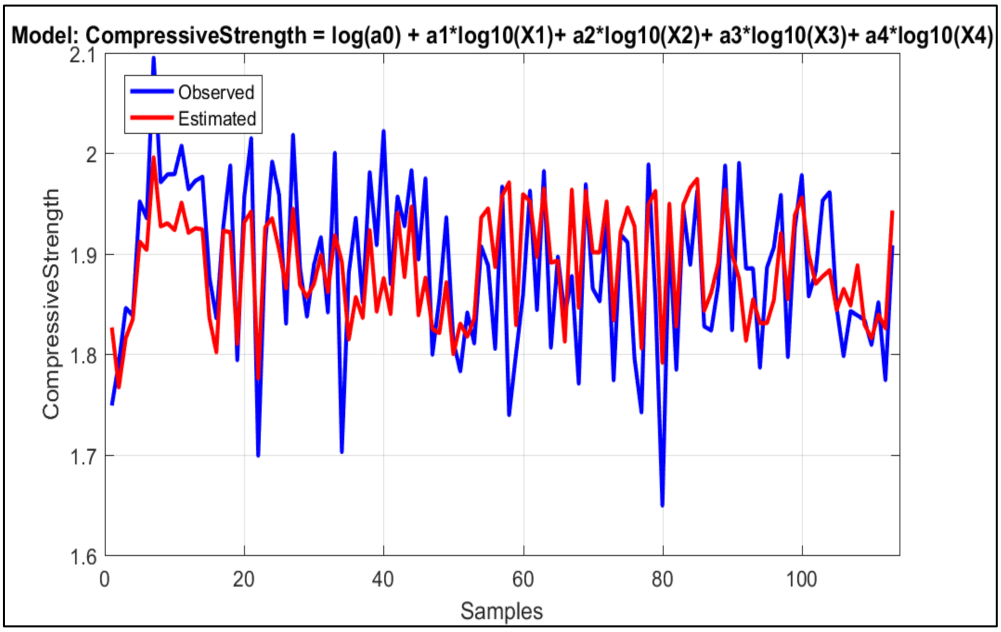

Figure 12. Nonlinear curve fitting of ultimate load according to average length, average thickness, average external diameter, and weight before compression test

CHAPTER 5

CONCLUSION

As a result of research on multiple regression methods and literature reviews, comparisons were made with the physical properties of bamboo. The structural properties of the material, together with the currently measured numerical data, and the relationships between their compressive strength and ultimate (tensile) load were transferred to the graphic language, and a linear and nonlinear relationship was observed between them. To understand the relationship between compressive strength and ultimate (tensile) load and the parameters of bamboo, the numerical value of (r) was taken as an indicator. This correlation coefficient value (r) is used to measure how strong the relationship between any two variables. According to table 1, it can be understood that when the number of variables is increased, the (r) value is higher; that is, more predictable results are obtained. When the results of the multiple linear regression method were examined, the highest correlation coefficient value (r) was 0.954 for ultimate load and 0.680 for compressive strength. On the other hand, when the multiple nonlinear regression was examined, the correlation coefficient value (r) was the highest 0.952 for ultimate load and 0.659 for compressive strength. Therefore, the multiple linear regression method was determined as the most appropriate method.

The compressive strength and ultimate load values were tried to be estimated using the multiple linear regression method, and at this point, the available data of bamboo were used as variables. And when the number of these variables increased, more predictable results were obtained.

The ultimate load value range is (52.7 kN -- 472.7 kN) and the compressive strength value range is (44.6 MPa -- 124.5MPa). Although the error value measured for the ultimate load was higher than the error value of compressive strength, it gave more predictable results because the range of ultimate load values was wider.

When linear and nonlinear relationships between compressive strength, ultimate load and used parameters on calculations were compared, it was observed that the error value was lower for nonlinear relationships. At this point as a reason for that, on nonlinear equations logarithmic calculations were taken from all values.

Table 1. r, r model and equations

r	MULTIPLE LINEAR REGRESSION
	Ultimate load
$\boldsymbol{r = 0.905}$	$y_i = -50.42 + 0.826x_1 + 19.41x_2$
$\boldsymbol{r = 0.946}$	$y_i = -124.93 - 0.258\, x_1 + 12.49\, x_2 + 2.46\, x_3$
$\boldsymbol{r = 0.954}$	$y_i = 19.55 - 0.998\, x_1 + 8.612\, x_2 + 1.303\, x_3 + 0.344\, x_4$
	Compressive Strength
$\boldsymbol{r = 0.650}$	$y_i = 117.18 - 0.206\, x_1 - 1.826\, x_2$
$\boldsymbol{r = 0.655}$	$y_i = 120.27 - 0.161\, x_1 - 1.528\, x_2 - 0.102\, x_3$
$\boldsymbol{r = 0.680}$	$y_i = 150.82 - 0.317\, x_1 - 2.358x_2 - 0.348\, x_3 + 0.073\, x_4$
	MULTIPLE NONLINEAR REGRESSION
	Ultimate Load
$\boldsymbol{r = 0.915}$	$\log(y_i) = \log(8.33) + 0.2078 \log(x_1) + 1.005 \log(x_2)$
$\boldsymbol{r = 0.950}$	$\log(y_i) = \log(0.582) - 0.199 \log(x_1) + 0.589 \log(x_2) + 1.186 \log(x_3)$
$\boldsymbol{r = 0.952}$	$\log(y_i) = \log(2.621) - 0.381 \log(x_1) + 0.455 \log(x_2) + 0.823 \log(x_3) + 0.242 \log(x_4)$
	Compressive Strength
$\boldsymbol{r = 0.644}$	$\log(y_i) = \log(336.835) - 0.193 \log(x_1) - 0.262 \log(x_2)$
$\boldsymbol{r = 0.644}$	$\log(y_i) = \log(308.693) - 0.207 \log(x_1) - 0.276 \log(x_2) + 0.039 \log(x_3)$
$\boldsymbol{r = 0.659}$	$\log(y_i) = \log(1392) - 0.388 \log(x_1) - 0.410 \log(x_2) - 0.324 \log(x_3) + 0.242 \log(x_4)$

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APPENDIX

CODE OF MULTIPLE LINEAR REGRESSION IN MATLAB

```
_% multipleLinearRegression
%% CONSTANTS
clc;
variablesY={'CompressiveStrength','UltimateLoad'};
variablesX={'AverageLength','AverageThickness','AverageEx
ternalDiameter','WeightBeforeCompression'};

%% CONSTANTS
FileData='book2_new.xlsx';
OutputFolder='LinearRegOutput';
if ~isdir(OutputFolder), mkdir(OutputFolder); end

%% READING THE DATA
[Data] = xlsread(FileData);
for ii=1:length(variablesY)

    fprintf(1,'Analysis for:
%s\n',upper(variablesY{ii}));
    if strcmpi(variablesY(ii),'CompressiveStrength')
        Y = Data(:,4);
    else
        Y = Data(:,5);
    end

    % LINEAR REGRESSION MODEL
    for jj=1:4 % Estimator Number
        X=Data(:,6:6+jj-1);

        mdl = fitlm(X,Y);

        % Printing parameters
        fprintf(1,'\n');
        fprintf(1,'%d.) Model: y = a0 ',jj);
        for mm=1:jj
            fprintf(1,'+ a%d*X%d ',mm,mm);
        end
        fprintf(1,'\n');

        fprintf(1,'%s = a0 ',variablesY{ii});
```

```
        for mm=1:jj
            fprintf(1,'+ a%d*%s ',mm,variablesX{jj});
        end
        fprintf(1,'\n\n');

        fprintf(1,'Constant    Value    p-Value
Result \n');
        fprintf(1,'--------    -----    -------      ----
-- \n');
        fprintf(1,'%5s %11.3f %11.3e %40s
\n','a0',table2array(mdl.Coefficients(1,1)),table2array(m
dl.Coefficients(1,end)),'Accepted (Statistically
significant)');

        pValueCritical=0.05;
        pValueMatrix=nan(1,length(mdl.CoefficientNames)-
1);
        for k=2:length(mdl.CoefficientNames)
            pValueMatrix(k-
1)=table2array(mdl.Coefficients(k,end));
            if pValueMatrix(k-1) < pValueCritical
                Result='Accepted (Statistically
significant)';
            else
                Result='Not Accepted ((Statistically not
significant)';
            end
            fprintf(1,'%5s %11.3f %11.3e %40s
\n',['a',num2str(k-
1)],table2array(mdl.Coefficients(k,1)),table2array(mdl.Co
efficients(k,end)),Result);
        end
        clear Result

        fprintf(1,'\n');

        fprintf(1,'The R^2 of the regression model:
%.3f\n',mdl.Rsquared.Ordinary);
        fprintf(1,'  The R of the regression model:
%.3f\n\n',sqrt(mdl.Rsquared.Ordinary));

        yy = feval(mdl,X);
        fprintf(1,'Mean absulate error of the regression
model: %.3f\n\n',mean(abs(Y-yy)));

        % PLOTTING THE RESULTS

        % a.) Observed versus Model
        h1= plot(Y,'-b','LineWidth',2); hold on;
        h2= plot(yy,'-r','LineWidth',2); hold off;
```

```
        grid on
        xlim([0 length(Y)+1]);
        legend([h1 h2
],'Observed','Estimated','Location','northwest')

        xlabel('Samples')
        ylabel(variablesY{ii})
        ModStr=getModelEquation(variablesY{ii},jj);
        title(['Model: ',ModStr])

FileName=[variablesY{ii},'_Model',num2str(jj),'_1_Observe
dAndForecasted.png'];
        saveas(gca,fullfile(OutputFolder,FileName));
        clear h1 h2 FileName

        % b.) Error time series
        stem(Y-yy,'-
.b','MarkerFaceColor','red','MarkerEdgeColor','green','Ma
rkerSize',5);
        xlabel('Samples')
        ylabel('Error (Observed - Estimated)')
        title(['Model: ',ModStr])
        grid on

FileName=[variablesY{ii},'_Model',num2str(jj),'_2_ErrorVa
riation.png'];
        saveas(gca,fullfile(OutputFolder,FileName));
        clear FileName

        % c.) Error histogram
        histogram(Y-yy,15);
        grid on
        xlabel('Error (Observed - Estimated)')
        ylabel('Occurance Amount')
        title(['Model: ',ModStr])

FileName=[variablesY{ii},'_Model',num2str(jj),'_3_ErrorHi
stogram.png'];
        saveas(gca,fullfile(OutputFolder,FileName));
        clear FileName

        % Printing the ouptput

FileName=[variablesY{ii},'_Model',num2str(jj),'_Output.tx
t'];
        fid=fopen(fullfile(OutputFolder,FileName),'wt');
        fprintf(fid,'Observation   Estimated \n');
        fprintf(fid,'-----------   --------- \n');
        fprintf(fid,'%8.3f %12.3f \n',[Y yy]');
        fclose(fid);
```

```
        clear ans fid mdl X yy

    end % Estimator
    clear Y

end % ParameterY

% >>>>>>>>>>>>>>>>>> END OF MAIN PROGRAM
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

function ModelEquation = getModelEquation(varY,EstNumber)
ModelEquation = [varY,' = a0 '];
for mm=1:EstNumber
    ModelEquation=[ModelEquation '+
a',num2str(mm),'*X',num2str(mm)];
end
```
(MathWorks, 2021).

CODE OF MULTIPLE NONLINEAR REGRESSION IN MATLAB

```
multipleLog10Regression
%% CONSTANTS
clc;
variablesY={'CompressiveStrength','UltimateLoad'};
variablesX={'AverageLength','AverageThickness','AverageEx
ternalDiameter','WeightBeforeCompression'};

%% CONSTANTS
FileData='book2_new.xlsx';
OutputFolder='LinearRegLog10Output';
if ~isdir(OutputFolder), mkdir(OutputFolder); end

%% READING THE DATA
[Data] = xlsread(FileData);
for ii=1:length(variablesY)

    fprintf(1,'Analysis for:
%s\n',upper(variablesY{ii}));
    if strcmpi(variablesY(ii),'CompressiveStrength')
        Y = Data(:,4);
    else
        Y = Data(:,5);
    end
    Y = log10(Y);

    % LINEAR REGRESSION MODEL
    for jj=1:4 % Estimator Number
        X=Data(:,6:6+jj-1);
        X = log10(X);

        mdl = fitlm(X,Y);

        % Printing parameters
        fprintf(1,'\n');
        fprintf(1,'%d.) Model: log10(y) = log10(a0)
',jj);
        for mm=1:jj
            fprintf(1,'+ a%1d*log10(X%1d)  ',mm,mm);
        end
        fprintf(1,'\n');

        fprintf(1,'log10(%s) = log10(a0)
',variablesY{ii});
        for mm=1:jj
```

```
            fprintf(1,'+ a%1d*log10(X%s)
',mm,variablesX{mm});
        end
        fprintf(1,'\n\n');

        a0=table2array(mdl.Coefficients(1,1));
        fprintf(1,'log10(%s) =
log10(%.4f)',variablesY{ii},10.^a0);
        for k=1:jj
            fprintf(1,'+ %.4f*log10(%s)
',table2array(mdl.Coefficients(k+1,1)),variablesX{k});
        end
        fprintf(1,'\n\n');

        fprintf(1,'Constant    Value    p-Value
Result \n');
        fprintf(1,'--------    -----    -------      ----
-- \n');
        fprintf(1,'%5s %11.3f %11.3e      %-60s
\n','a0',table2array(mdl.Coefficients(1,1)),table2array(m
dl.Coefficients(1,end)),'Accepted (Statistically
significant)');

        pValueCritical=0.05;
        pValueMatrix=nan(1,length(mdl.CoefficientNames)-
1);
        for k=2:length(mdl.CoefficientNames)
            pValueMatrix(k-
1)=table2array(mdl.Coefficients(k,end));
            if pValueMatrix(k-1) < pValueCritical
                Result='Accepted (Statistically
significant)';
            else
                Result='Not Accepted ((Statistically not
significant)';
            end
            fprintf(1,'%5s %11.3f %11.3e      %-60s
\n',['a',num2str(k-
1)],table2array(mdl.Coefficients(k,1)),table2array(mdl.Co
efficients(k,end)),Result);
        end
        clear Result

        fprintf(1,'\n');

        fprintf(1,'The R^2 of the regression model:
%.3f\n',mdl.Rsquared.Ordinary);
        fprintf(1,'  The R of the regression model:
%.3f\n\n',sqrt(mdl.Rsquared.Ordinary));
```

```
        yy = feval(mdl,X);
        fprintf(1,'Mean absulate error of the regression
model: %.4f\n\n',mean(abs(Y-yy)));

        % PLOTTING THE RESULTS

        % a.) Observed versus Model
        h1= plot(Y,'-b','LineWidth',2); hold on;
        h2= plot(yy,'-r','LineWidth',2); hold off;
        grid on
        xlim([0 length(Y)+1]);
        legend([h1 h2
],'Observed','Estimated','Location','northwest')

        xlabel('Samples')
        ylabel(variablesY{ii})
        ModStr=getModelEquation(variablesY{ii},jj);
        title(['Model: ',ModStr])

FileName=[variablesY{ii},'_Log10_Model',num2str(jj),'_1_O
bservedAndForecasted.png'];
        saveas(gca,fullfile(OutputFolder,FileName));
        clear h1 h2 FileName

        % b.) Error time series
        stem(Y-yy,'-
.b','MarkerFaceColor','red','MarkerEdgeColor','green','Ma
rkerSize',5);
        xlabel('Samples')
        ylabel('Error (Observed - Estimated)')
        title(['Model: ',ModStr])
        grid on

FileName=[variablesY{ii},'_Log10_Model',num2str(jj),'_2_E
rrorVariation.png'];
        saveas(gca,fullfile(OutputFolder,FileName));
        clear FileName

        % c.) Error histogram
        histogram(Y-yy,15);
        grid on
        xlabel('Error (Observed - Estimated)')
        ylabel('Occurance Amount')
        title(['Model: ',ModStr])

FileName=[variablesY{ii},'_Log10_Model',num2str(jj),'_3_E
rrorHistogram.png'];
        saveas(gca,fullfile(OutputFolder,FileName));
        clear FileName
```

```
        % Printing the ouptput

FileName=[variablesY{ii},'_Log10_Model',num2str(jj),'_Out
put.txt'];
        fid=fopen(fullfile(OutputFolder,FileName),'wt');
        fprintf(fid,'Observation   Estimated \n');
        fprintf(fid,'-----------   ---------- \n');
        fprintf(fid,'%8.3f %12.3f \n',[Y yy]');
        fclose(fid);
        clear ans fid mdl X yy

    end % Estimator
    clear Y

end % ParameterY

% >>>>>>>>>>>>>>>>>>> END OF MAIN PROGRAM
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

function ModelEquation = getModelEquation(varY,EstNumber)
ModelEquation = [varY,' = a0 '];
for mm=1:EstNumber
    ModelEquation=[ModelEquation '+
a',num2str(mm),'*log10(X',num2str(mm),')'];
End
```

(MathWorks, 2021).

DATA OF BAMBOO (Bahtiar, Trujillo & Nugroho, 2020).

Number	No	Sample Code	Compressive strength (assuming hollow) (s_{cw}, MPa)	Ultimate tensile load (Fu, kN)	Average length (mm) x1	Average Thickness (mm)x2	Average External Diameter (mm)x3	Weight Before compression Test (gr)(Wa) x4
1	81	48b	56.1	310.1	71.7	18.2	115.1	270.0
2	47	I74(16)	61.9	427.9	115.0	22.6	120.0	500.7
3	101	I80(14t)	70.1	367.7	117.5	14.6	129.0	433.1
4	83	9b	68.9	331.2	76.2	16.1	111.1	249.5
5	49	S37(39t)	89.5	177.5	92.5	7.5	92.1	150.3
6	96	67t	86.2	216.5	87.0	8.8	99.9	184.8
7	52	S42(31t)	124.5	100.9	70.8	4.7	59.4	55.6
8	31	S48(32b)	93.6	123.2	92.6	6.4	72.2	95.9
9	38	S48(34t)	95.3	139.0	88.1	6.3	79.7	103.0
10	28	S37(39t)	95.3	185.1	82.9	7.3	92.1	135.2
11	85	S43t	101.7	97.0	87.0	5.0	65.3	63.9
12	44	S48(34b)	92.1	151.3	95.8	6.8	83.9	129.8
13	39	S25(45b)	93.9	145.9	91.1	6.4	84.2	112.4
14	88	40b	94.7	182.4	84.3	7.0	94.9	134.8
15	53	I26(08b)	75.3	308.8	98.4	13.0	113.7	271.7
16	26	I4	68.5	323.9	98.5	18.6	99.4	305.0
17	87	NC03b	84.6	151.6	89.9	6.9	89.2	134.6
18	74	3	97.2	176.1	96.1	6.8	91.1	148.7
19	25	S71(13b)	62.2	355.8	110.1	17.2	122.8	460.0
20	76	S43b	90.4	115.0	103.9	5.5	79.7	105.5
21	35	S29(46b)	103.5	155.6	79.5	6.3	82.3	101.2
22	46	I19(05b)	50.0	241.0	110.1	17.0	107.2	269.6
23	84	3t	81.1	112.4	83.9	6.6	73.7	88.0
24	32	S53(33t)	98.1	133.1	85.3	6.1	76.8	92.1
25	6	I70(20b)	90.9	234.1	74.0	8.7	103.0	146.7
26	48	S92(35b)	67.7	194.1	110.4	9.7	103.9	235.2
27	95	40t	104.3	150.8	81.1	6.2	80.9	100.9
28	56	I27(12T)	76.9	237.5	100.6	10.3	105.9	237.5
29	36	I26(08t)	68.8	228.8	108.0	10.3	112.9	259.3
30	2	I19(05t)	77.4	202.5	115.2	8.8	103.8	220.6
31	89	NC04	82.5	190.6	89.9	7.8	102.0	157.1
32	1	M76(19b)	69.4	192.6	104.0	9.4	103.6	193.8
33	33	S37(39)	100.1	202.1	80.9	7.6	92.3	133.1
34	41	S35(43b)	50.4	103.9	117.3	7.8	91.8	196.3
35	29	I65(26)	76.0	343.1	119.3	15.4	108.6	382.4
36	82	48t	86.2	317.4	102.3	11.5	113.6	287.2
37	20	I42(10t)	71.0	250.5	107.6	12.1	105.1	251.1
38	73	85b	95.7	173.6	82.4	7.0	89.3	120.2
39	45	I29(49b)	81.0	337.0	107.8	13.6	111.2	351.5
40	27	I29(49T)	105.2	338.8	110.2	10.3	109.6	308.5
41	93	I84(15t)	74.1	401.2	97.2	14.1	136.1	408.9
42	86	85t	90.5	123.2	86.0	6.0	78.3	97.5
43	80	9t	84.6	266.1	89.3	9.6	114.4	206.4
44	90	85t	96.2	131.3	78.2	6.0	78.8	88.9
45	100	I86	78.4	326.0	106.9	13.3	112.7	330.6
46	102	I67(11b)	94.4	314.1	95.1	10.8	109.2	261.0
47	104	I79	63.0	291.6	109.5	14.2	117.8	357.9
48	108	I36(03t)	71.7	369.4	107.5	14.3	129.1	380.6

49	109	I57(36t)	86.3	334.2	100.7	11.1	122.0	335.5
50	111	I36(18t)	65.0	403.5	100.9	18.5	125.3	420.6
51	112	I84(15t)	60.7	323.1	108.1	13.9	135.5	430.5
52	115	67b	69.4	338.7	74.4	20.4	96.6	252.4
53	91	48b	64.7	357.6	91.1	17.6	117.7	420.1
54	51	S25(31t)	80.8	137.2	89.0	6.6	88.4	136.8
55	50	S48(32b)	77.3	105.6	79.3	6.7	71.3	96.0
56	60	S37(39t)	63.9	174.4	81.2	10.3	94.9	171.6
57	67	S53(33t)	92.6	129.1	85.5	6.1	78.8	118.1
58	75	S43t	54.9	52.7	86.2	5.0	65.7	76.8
59	78	9b	63.2	327.0	89.8	17.1	113.5	350.0
60	77	S13(47t)	72.3	87.5	80.0	5.7	73.8	86.7
61	55	S48(34b)	91.8	153.7	80.9	6.7	86.6	135.3
62	68	M88(37b)	69.8	282.5	88.5	10.7	131.5	358.2
63	18	S42(31t)	96.0	104.4	76.7	6.3	61.3	79.9
64	58	I3(09)	64.0	160.1	91.5	8.7	99.8	176.2
65	62	I31(09t)	78.9	193.3	97.1	8.6	99.2	190.7
66	57	S7(13b)	68.9	431.5	104.3	19.1	123.7	515.0
67	40	S13(47b)	75.4	103.9	71.9	5.9	79.8	92.0
68	4	I19(05t)	59.0	202.9	104.1	11.3	108.3	243.7
69	63	S60(44b)	93.1	126.2	83.5	6.3	74.7	117.4
70	66	M95(48b)	73.3	265.8	81.7	11.0	115.9	293.0
71	61	S35(43b)	71.2	165.6	108.5	8.7	93.5	233.8
72	23	S25(45t)	86.7	119.9	87.0	6.1	78.7	113.6
73	65	I26(08b)	59.4	280.9	104.5	13.8	122.7	361.9
74	34	S74(29t)	83.0	176.9	118.4	7.4	99.4	265.4
75	98	40b	81.5	163.2	84.4	7.0	98.2	174.5
76	24	S79(29b)	62.5	141.4	105.3	7.6	102.9	254.6
77	92	I41(02b)	55.2	347.1	99.5	17.3	133.1	418.0
78	19	S37(39t)	97.5	202.7	73.7	7.7	93.6	160.1
79	16	S48(32b)	71.6	98.6	73.5	6.5	73.7	99.3
80	12	I31(09b)	44.6	184.8	106.1	15.7	99.6	233.7
81	94	3b	83.6	162.5	76.9	7.1	94.4	151.4
82	37	I28(50b)	60.9	300.5	105.7	14.1	125.5	371.5
83	30	S48(34t)	88.1	139.1	82.5	6.7	81.3	126.4
84	71	S43b	77.4	95.2	97.4	5.1	82.2	121.7
85	10	S42(31)	91.8	109.1	75.1	5.9	70.3	90.4
86	21	I42(10b)	67.2	268.8	103.7	13.3	108.8	313.5
87	64	I89(04b)	66.6	311.4	93.5	13.6	123.4	383.3
88	70	S92(35b)	73.8	222.9	99.5	9.9	106.9	271.4
89	43	S29(46b)	97.2	153.0	75.2	6.5	84.0	122.2
90	79	NC04	66.7	172.3	102.8	8.4	106.3	234.0
91	97	I67(11b)	97.8	384.2	89.0	12.0	116.2	304.9
92	15	I66(41b)	76.8	472.7	92.3	21.0	114.3	452.8
93	17	S27(12b)	76.8	301.3	96.1	13.7	104.8	308.2
94	8	I28(50T)	61.2	286.1	109.4	13.2	125.7	365.0
95	69	S29(49b)	76.9	335.6	120.1	13.9	114.1	401.8
96	13	I65(26)	80.6	342.2	106.3	14.1	109.7	404.2
97	5	S35(43t)	90.9	189.2	105.6	8.1	89.7	225.0
98	14	I3(42)	62.7	315.1	83.0	15.2	120.2	351.8
99	99	40t	84.2	135.8	94.1	6.6	84.1	143.9
100	72	3t	95.1	136.2	80.3	6.3	78.7	110.5
101	42	I70(20b)	72.0	187.2	91.9	8.3	107.5	196.3
102	3	I86(40m)	76.2	342.2	83.0	14.0	116.1	336.3
103	7	I29(49T)	89.8	292.7	124.2	10.0	114.2	378.3
104	11	I27(12T)	91.5	279.6	111.5	10.1	106.7	313.6
105	103	67b	70.2	344.3	75.0	19.3	100.3	314.3
106	105	I57(36t)	62.8	266.8	112.6	12.0	124.3	443.4
107	106	I79(07t)	69.6	330.4	96.3	14.3	120.1	375.3
108	107	M84(06)	68.9	284.7	94.7	11.0	130.1	385.6
109	110	I84(15t)	68.2	372.7	137.7	14.1	137.2	653.7
110	113	I80(14t)	64.5	380.7	114.9	16.2	132.2	515.9
111	114	I36(03t)	71.0	399.3	99.7	15.0	134.3	460.2
112	116	I36(18t)	59.4	391.3	84.2	18.7	130.5	433.5
113	117	85b	81.0	149.4	84.9	6.9	92.2	154.0

$$\boldsymbol{y_i = a_0 + a_1x_1 + a_2x_2}$$

$$y_i ultimate\ load\ (kN) = -50.420 + 0.826\ average\ length\ (m) + 19.413\ average\ thickness\ (mm)$$

Observation	Estimated
-----------	---------
310.14	361.35
427.87	483.741
367.68	329.739
331.15	325.225
177.47	170.834
216.46	191.716
100.91	99.698
123.16	149.695
138.95	145.253
185.13	159.519
97.03	119.341
151.25	160.44
145.86	148.108
182.39	154.453
308.82	282.442
323.86	392.773
151.63	158.42
176.05	161.817
355.76	375.329
115.01	141.265
155.61	137.528
240.97	370.888
112.36	146.462
133.08	138.75
234.06	179.442
194.12	228.958
150.78	136.121
237.46	232.313
228.76	239.099
202.54	214.792
190.62	175.488
192.61	217.469
202.14	163.797
103.94	198.134
343.07	347.445
317.43	256.901
250.52	272.765
173.59	153.74
337.02	301.923

338.82	241.022
401.17	304.297
123.15	136.91
266.14	208.816
131.29	129.978
326.04	296.48
314.07	236.977
291.64	316.135
369.43	315.708
334.22	248.577
403.5	392.617
323.06	309.652
338.67	406.828
357.6	366.039
137.15	151.445
105.55	145.843
174.44	216.173
129.11	138.724
52.65	118.547
327.03	355.387
87.47	125.404
153.71	145.976
282.51	229.693
104.37	135.129
160.05	194.682
193.27	196.937
431.46	405.763
103.94	124.299
202.92	254.797
126.18	141.056
265.76	230.544
165.64	208.698
119.93	139.083
280.9	304.088
176.9	190.703
163.18	154.797
141.41	183.348
347.05	367.16
202.73	160.064
98.6	137.047
184.75	342.636
162.47	150.541
300.49	310.732
139.05	148.424
95.23	128.561
109.05	125.564
268.79	294.121

311.37	289.9
222.89	224.092
152.96	137.166
172.26	197.686
384.21	256.07
472.72	433.909
301.25	295.105
286.11	296.984
335.6	317.659
342.22	312.031
189.2	194.499
315.06	313.939
135.83	156.027
136.21	138.196
187.21	187.49
342.22	290.073
292.73	245.492
279.58	237.267
344.26	385.391
266.75	276.307
330.44	306.105
284.72	242.132
372.74	337.647
380.69	359.093
399.33	323.099
391.29	383.145
149.41	153.384

$$\boldsymbol{y_i = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3}$$

$y_i ultimate\ load\ (kN) = -124.935 - 0.258\ average\ length\ (m) + 12.490\ average\ thickness\ (mm) + 2.466\ average\ external\ diameter\ (mm)$

Observation	Estimated
-----------	---------
310.14	367.16
427.87	423.679
367.68	344.984
331.15	330.397
177.47	171.351
216.46	208.503
100.91	62.205
123.16	108.628
138.95	127.801
185.13	171.859
97.03	76.512

151.25	141.83
145.86	138.524
182.39	174.387
308.82	291.822
323.86	327.45
151.63	158.464
176.05	160.306
355.76	364.736
115.01	112.899
155.61	136.122
240.97	323.473
112.36	117.144
133.08	118.671
234.06	218.486
194.12	223.775
150.78	130.514
237.46	238.715
228.76	254.529
202.54	210.735
190.62	200.988
192.61	220.71
202.14	176.555
103.94	168.784
343.07	304.523
317.43	272.06
250.52	257.176
173.59	161.636
337.02	290.812
338.82	245.846
401.17	362.064
123.15	120.712
266.14	253.298
131.29	123.706
326.04	291.736
314.07	254.172
291.64	314.926
369.43	344.083
334.22	288.653
403.5	389.26
323.06	355.407
338.67	348.604
357.6	361.296
137.15	152.588
105.55	114.506
174.44	216.382
129.11	123.521
52.65	77.682

327.03	345.154
87.47	107.01
153.71	151.003
282.51	309.589
104.37	84.985
160.05	206.659
193.27	202.13
431.46	391.134
103.94	127.371
202.92	256.228
126.18	116.428
265.76	277.123
165.64	186.685
119.93	122.404
280.9	323.248
176.9	181.632
163.18	182.707
141.41	196.013
347.05	393.326
202.73	183.147
98.6	119.322
184.75	289.689
162.47	176.485
300.49	333.337
139.05	138.388
95.23	115.992
109.05	102.363
268.79	282.997
311.37	324.343
222.89	236.671
152.96	143.405
172.26	215.571
384.21	288.522
472.72	395.494
301.25	279.834
286.11	322.014
335.6	298.414
342.22	294.746
189.2	170.455
315.06	340.406
135.83	140.915
136.21	127.009
187.21	220.566
342.22	314.695
292.73	248.981
279.58	235.104
344.26	343.568

266.75	302.891
330.44	324.641
284.72	309.365
372.74	354.264
380.69	373.812
399.33	367.817
391.29	409.245
149.41	166.421

$$\boldsymbol{y_i = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4}$$

$y_i ultimate\ load\ (kN) = 19.550 - 0.998\ \ average\ length\ (m) + 8.612\ \ average\ thickness\ (mm) + 1.303\ average\ external\ diameter\ (mm) + 0.344\ weight\ before\ compression\ test\ (gr)$

Observation	Estimated
-----------	---------
310.14	347.236
427.87	428.166
367.68	345.034
331.15	312.802
177.47	163.164
216.46	202.046
100.91	86.097
123.16	109.029
138.95	125.402
185.13	166.196
97.03	83.21
151.25	136.326
145.86	131.771
182.39	165.501
308.82	274.565
323.86	316.153
151.63	152.157
176.05	152.437
355.76	376.384
115.01	103.018
155.61	136.511
240.97	288.642
112.36	118.689
133.08	118.767
234.06	205.232
194.12	209.099
150.78	131.74
237.46	227.473
228.76	237.034

202.54	191.261
190.62	184.102
192.61	198.15
202.14	170.227
103.94	156.958
343.07	306.271
317.43	263.093
250.52	239.47
173.59	155.484
337.02	294.58
338.82	247.477
401.17	362.303
123.15	120.84
266.14	232.735
131.29	126.19
326.04	288.157
314.07	249.433
291.64	309.45
369.43	334.505
334.22	289.204
403.5	386.305
323.06	356.461
338.67	333.524
357.6	377.924
137.15	149.905
105.55	124.363
174.44	209.715
129.11	130.135
52.65	88.928
327.03	345.424
87.47	114.369
153.71	155.699
282.51	317.586
104.37	104.546
160.05	194.186
193.27	191.649
431.46	417.97
103.94	134.558
202.92	237.838
126.18	128.22
265.76	284.564
165.64	188.777
119.93	126.566
280.9	318.717
176.9	185.699
163.18	183.53
141.41	201.258

347.05	386.286
202.73	189.451
98.6	132.597
184.75	259.355
162.47	178.958
300.49	326.823
139.05	144.642
95.23	115.07
109.05	117.915
268.79	280.504
311.37	335.554
222.89	238.226
152.96	151.577
172.26	208.361
384.21	290.432
472.72	413.143
301.25	284.328
286.11	313.688
335.6	305.949
342.22	317.323
189.2	178.457
315.06	345.637
135.83	141.789
136.21	134.253
187.21	207.327
342.22	324.23
292.73	260.364
279.58	241.962
344.26	349.364
266.75	325.408
330.44	332.029
284.72	322.325
372.74	407.461
380.69	394.198
399.33	382.582
391.29	416.131
149.41	167.175

$\boldsymbol{y_i = a_0 + a_1 x_1 + a_2 x_2}$

y_i *Compressive load* $(kN) = 117.183 - 0.206$ *average length* $(m) -$ 1.826*average thickness* (mm)

Observation	Estimated
-----------	---------
56.078	69.28

61.855	52.232
70.136	66.399
68.926	72.108
89.509	84.538
86.198	83.279
124.455	94.007
93.571	86.515
95.285	87.508
95.309	86.836
101.707	90.087
92.074	85.096
93.895	86.858
94.748	87.128
75.312	73.286
68.499	62.894
84.598	86.042
97.217	84.926
62.227	63.049
90.411	85.862
103.488	89.333
49.986	63.47
81.11	87.929
98.081	88.473
90.898	86.094
67.683	76.775
104.296	89.255
76.854	77.721
68.772	76.137
77.421	77.505
82.452	84.432
69.425	78.68
100.075	86.681
50.411	78.795
76.035	64.495
86.225	75.187
71.022	73.022
95.741	87.44
81.012	70.249
105.214	75.674
74.08	71.387
90.531	88.555
84.616	81.375
96.177	90.213
78.393	70.875
94.388	77.989
63.011	68.703
71.691	68.997

86.331	76.184
64.951	62.602
60.668	69.49
69.4	64.657
64.679	66.361
80.76	86.809
77.269	88.579
63.871	81.724
92.615	88.457
54.871	90.262
63.189	67.532
72.278	90.404
91.751	88.366
69.81	79.509
95.989	89.915
64.045	82.428
78.881	81.491
68.88	60.938
75.447	91.551
58.975	75.154
93.126	88.485
73.327	80.304
71.229	78.932
86.707	88.225
59.427	70.476
82.961	79.352
81.537	87.087
62.453	81.724
55.211	65.18
97.465	87.956
71.585	90.145
44.586	66.647
83.623	88.447
60.887	69.687
88.141	87.922
77.445	87.89
91.766	91.026
67.244	71.514
66.608	73.212
73.848	78.633
97.22	89.912
66.65	80.695
97.793	76.972
76.779	59.825
76.809	72.397
61.191	70.508
76.93	67.204

80.607	69.499
90.878	80.639
62.686	72.294
84.202	85.721
95.096	89.175
72.039	83.053
76.222	74.537
89.76	73.464
91.458	75.864
70.195	66.596
62.818	72.047
69.632	71.335
68.938	77.547
68.236	63.063
64.455	63.963
71.036	69.3
59.45	65.629
80.997	87.146

$\boldsymbol{y_i = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3}$

$y_i compressive\ load\ (kN) = 120.274\ - 0.161\ average\ length\ (m) - 1.538\ \ average\ thickness\ (mm) - 0.102 average\ external\ diameter$

Observation	Estimated
-----------	---------
56.078	69.039
61.855	54.724
70.136	65.767
68.926	71.894
89.509	84.517
86.198	82.583
124.455	95.562
93.571	88.219
95.285	88.232
95.309	86.324
101.707	91.864
92.074	85.868
93.895	87.255
94.748	86.301
75.312	72.897
68.499	65.604
84.598	86.04
97.217	84.989
62.227	63.488
90.411	87.039

103.488	89.392
49.986	65.437
81.11	89.145
98.081	89.306
90.898	84.474
67.683	76.99
104.296	89.488
76.854	77.456
68.772	75.497
77.421	77.673
82.452	83.374
69.425	78.546
100.075	86.152
50.411	80.013
76.035	66.275
86.225	74.558
71.022	73.668
95.741	87.113
81.012	70.71
105.214	75.474
74.08	68.99
90.531	89.227
84.616	79.53
96.177	90.473
78.393	71.071
94.388	77.275
63.011	68.753
71.691	67.82
86.331	74.521
64.951	62.741
60.668	67.592
69.4	67.073
64.679	66.558
80.76	86.761
77.269	89.879
63.871	81.715
92.615	89.088
54.871	91.957
63.189	67.956
72.278	91.168
91.751	88.157
69.81	76.194
95.989	91.996
64.045	81.931
78.881	81.276
68.88	61.545
75.447	91.424

58.975	75.095
93.126	89.507
73.327	78.372
71.229	79.845
86.707	88.917
59.427	69.682
82.961	79.728
81.537	85.929
62.453	81.198
55.211	64.095
97.465	86.998
71.585	90.88
44.586	68.843
83.623	87.371
60.887	68.749
88.141	88.338
77.445	88.411
91.766	91.989
67.244	71.976
66.608	71.783
73.848	78.111
97.22	89.653
66.65	79.954
97.793	75.626
76.779	61.419
76.809	73.03
61.191	69.47
76.93	68.002
80.607	70.216
90.878	81.637
62.686	71.196
84.202	86.348
95.096	89.639
72.039	81.681
76.222	73.515
89.76	73.319
91.458	75.954
70.195	68.331
62.818	70.944
69.632	70.566
68.938	74.758
68.236	62.373
64.455	63.353
71.036	67.445
59.45	64.546
80.997	86.605

$$\boldsymbol{y_i = a_0 + a_1x_1 + a_2x_2 + a_3x_3a_4x_4}$$

$$y_i compressive\ load\ (kN) = 150.824 - 0.317\ \ average\ length\ (m) - 2.358\ \ average\ thickness\ (mm) - 0.348 average\ external\ diameter\ (mm) + 0.073\ \ weight\ before\ compression\ test\ (gr)$$

Observation	Estimated
-----------	---------
56.078	64.826
61.855	55.672
70.136	65.777
68.926	68.173
89.509	82.786
86.198	81.217
124.455	100.614
93.571	88.304
95.285	87.725
95.309	85.127
101.707	93.28
92.074	84.704
93.895	85.828
94.748	84.422
75.312	69.248
68.499	63.215
84.598	84.707
97.217	83.325
62.227	65.951
90.411	84.95
103.488	89.474
49.986	58.073
81.11	89.472
98.081	89.326
90.898	81.672
67.683	73.887
104.296	89.747
76.854	75.079
68.772	71.798
77.421	73.556
82.452	79.804
69.425	73.776
100.075	84.814
50.411	77.512
76.035	66.645
86.225	72.662
71.022	69.924

95.741	85.812
81.012	71.506
105.214	75.818
74.08	69.041
90.531	89.254
84.616	75.182
96.177	90.999
78.393	70.315
94.388	76.273
63.011	67.595
71.691	65.795
86.331	74.637
64.951	62.116
60.668	67.815
69.4	63.884
64.679	70.074
80.76	86.194
77.269	91.963
63.871	80.306
92.615	90.486
54.871	94.335
63.189	68.014
72.278	92.724
91.751	89.15
69.81	77.885
95.989	96.132
64.045	79.294
78.881	79.059
68.88	67.219
75.447	92.944
58.975	71.206
93.126	92
73.327	79.945
71.229	80.287
86.707	89.797
59.427	68.724
82.961	80.588
81.537	86.103
62.453	82.307
55.211	62.606
97.465	88.331
71.585	93.687
44.586	62.429
83.623	87.893
60.887	67.372

88.141	89.66
77.445	88.216
91.766	95.277
67.244	71.449
66.608	74.153
73.848	78.44
97.22	91.381
66.65	78.429
97.793	76.029
76.779	65.151
76.809	73.981
61.191	67.71
76.93	69.595
80.607	74.99
90.878	83.329
62.686	72.302
84.202	86.533
95.096	91.17
72.039	78.882
76.222	75.531
89.76	75.726
91.458	77.404
70.195	69.556
62.818	75.705
69.632	72.128
68.938	77.498
68.236	73.621
64.455	67.663
71.036	70.567
59.45	66.002
80.997	86.764

$$\mathbf{log}(\boldsymbol{y_i}) = \mathbf{log}(\boldsymbol{a_0}) + \boldsymbol{a_1}\,\mathbf{log}(\boldsymbol{x_1}) + \boldsymbol{a_2}\,\mathbf{log}(\boldsymbol{x_2})$$

$\log(y_i)\ Ultimate = \log(8.3343) + 0.2078 * \log(average\ length) + 1.005 * \log(average\ thickness)$

Observation	Estimated
-----------	---------
2.492	2.572
2.631	2.711
2.565	2.521
2.52	2.525

2.249	2.207
2.335	2.272
2.004	1.983
2.09	2.138
2.143	2.131
2.267	2.187
1.987	2.03
2.18	2.169
2.164	2.135
2.261	2.169
2.49	2.454
2.51	2.612
2.181	2.172
2.246	2.173
2.551	2.588
2.061	2.08
2.192	2.119
2.382	2.583
2.051	2.143
2.124	2.113
2.369	2.253
2.288	2.337
2.178	2.111
2.376	2.355
2.359	2.362
2.307	2.297
2.28	2.224
2.285	2.317
2.306	2.202
2.017	2.248
2.535	2.547
2.502	2.404
2.399	2.431
2.24	2.169
2.528	2.482
2.53	2.364
2.603	2.49
2.09	2.104
2.425	2.312
2.118	2.094
2.513	2.473
2.497	2.369
2.465	2.504
2.568	2.504
2.524	2.389
2.606	2.612
2.509	2.494

2.53	2.626
2.553	2.58
2.137	2.151
2.023	2.148
2.242	2.335
2.111	2.112
1.721	2.029
2.515	2.566
1.942	2.072
2.187	2.146
2.451	2.359
2.019	2.116
2.204	2.275
2.286	2.274
2.635	2.627
2.017	2.085
2.307	2.398
2.101	2.124
2.424	2.365
2.219	2.29
2.079	2.11
2.449	2.487
2.248	2.224
2.213	2.17
2.15	2.224
2.54	2.58
2.307	2.2
1.994	2.128
2.267	2.545
2.211	2.167
2.478	2.497
2.143	2.152
1.979	2.043
2.038	2.083
2.429	2.471
2.493	2.468
2.348	2.337
2.185	2.125
2.236	2.268
2.585	2.411
2.675	2.659
2.479	2.476
2.457	2.472
2.526	2.501
2.534	2.499
2.277	2.256
2.498	2.509

2.133	2.157
2.134	2.12
2.272	2.255
2.534	2.472
2.466	2.36
2.447	2.355
2.537	2.602
2.426	2.434
2.519	2.494
2.454	2.38
2.571	2.522
2.581	2.565
2.601	2.519
2.592	2.601
2.174	2.164

$\mathbf{Log}(y_i) = \mathbf{log}(a_0) + a_1\,\mathbf{log}(x_1) + a_2\,\mathbf{log}(x_2) + a_3\,\mathbf{log}(x_3)$

$\log(y_i)Ultimate = \log(8.3343) + 0.2078 * \log(average\ length) + 1.0054 * \log(average\ thickness) + 1.1859 * \log(average\ external\ diameter)$

Observation	Estimated
-----------	---------
2.492	2.581
2.631	2.618
2.565	2.541
2.52	2.527
2.249	2.217
2.335	2.305
2.004	1.897
2.09	2.051
2.143	2.104
2.267	2.22
1.987	1.944
2.18	2.141
2.164	2.131
2.261	2.223
2.49	2.461
2.51	2.485
2.181	2.184
2.246	2.186
2.551	2.564
2.061	2.052
2.192	2.129
2.382	2.491
2.051	2.078
2.124	2.079

2.369	2.333
2.288	2.33
2.178	2.112
2.376	2.364
2.359	2.391
2.307	2.301
2.28	2.284
2.285	2.326
2.306	2.234
2.017	2.206
2.535	2.465
2.502	2.426
2.399	2.395
2.24	2.195
2.528	2.454
2.53	2.374
2.603	2.577
2.09	2.083
2.425	2.395
2.118	2.094
2.513	2.457
2.497	2.396
2.465	2.494
2.568	2.544
2.524	2.456
2.606	2.6
2.509	2.562
2.53	2.517
2.553	2.564
2.137	2.168
2.023	2.072
2.242	2.325
2.111	2.092
1.721	1.948
2.515	2.539
1.942	2.044
2.187	2.168
2.451	2.495
2.019	1.98
2.204	2.3
2.286	2.287
2.635	2.598
2.017	2.106
2.307	2.396
2.101	2.075
2.424	2.445
2.219	2.251

2.079	2.088
2.449	2.512
2.248	2.232
2.213	2.241
2.15	2.266
2.54	2.615
2.307	2.253
1.994	2.088
2.267	2.436
2.211	2.232
2.478	2.527
2.143	2.136
1.979	2.055
2.038	2.034
2.429	2.441
2.493	2.519
2.348	2.36
2.185	2.15
2.236	2.312
2.585	2.462
2.675	2.593
2.479	2.436
2.457	2.509
2.526	2.463
2.534	2.458
2.277	2.214
2.498	2.546
2.133	2.138
2.134	2.105
2.272	2.326
2.534	2.506
2.466	2.376
2.447	2.353
2.537	2.522
2.426	2.477
2.519	2.516
2.454	2.493
2.571	2.551
2.581	2.583
2.601	2.583
2.592	2.64
2.174	2.204

$$\log(y_i) = \log(a_0) + a_1 \log(x_1) + a_2 \log(x_2) + a_3 \log(x_3) + a_4 \log(x_4)$$

$\log(y_i)Ultimate = \log(2.6213) - 0.3806 * \log(average\ length) + 0.4552 * \log(average\ thickness) + 0.8231 * \log(average\ external\ diameter) + 0.2416 * \log(weight\ before\ compression)$

Observation	Estimated
-----------	---------
2.492	2.569
2.631	2.614
2.565	2.534
2.52	2.514
2.249	2.21
2.335	2.303
2.004	1.903
2.09	2.044
2.143	2.094
2.267	2.213
1.987	1.93
2.18	2.137
2.164	2.118
2.261	2.211
2.49	2.446
2.51	2.482
2.181	2.177
2.246	2.182
2.551	2.567
2.061	2.04
2.192	2.12
2.382	2.46
2.051	2.065
2.124	2.067
2.369	2.314
2.288	2.322
2.178	2.106
2.376	2.358
2.359	2.379
2.307	2.289
2.28	2.265
2.285	2.304
2.306	2.223
2.017	2.207
2.535	2.468
2.502	2.421
2.399	2.381
2.24	2.182
2.528	2.459
2.53	2.383
2.603	2.573
2.09	2.075

2.425	2.375
2.118	2.083
2.513	2.455
2.497	2.397
2.465	2.489
2.568	2.532
2.524	2.459
2.606	2.593
2.509	2.556
2.53	2.516
2.553	2.577
2.137	2.168
2.023	2.077
2.242	2.319
2.111	2.102
1.721	1.953
2.515	2.542
1.942	2.042
2.187	2.177
2.451	2.506
2.019	1.995
2.204	2.288
2.286	2.282
2.635	2.61
2.017	2.104
2.307	2.381
2.101	2.093
2.424	2.459
2.219	2.267
2.079	2.093
2.449	2.506
2.248	2.254
2.213	2.251
2.15	2.286
2.54	2.603
2.307	2.266
1.994	2.098
2.267	2.409
2.211	2.24
2.478	2.519
2.143	2.146
1.979	2.063
2.038	2.047
2.429	2.442
2.493	2.529
2.348	2.369
2.185	2.161

2.236	2.314
2.585	2.468
2.675	2.608
2.479	2.445
2.457	2.5
2.526	2.469
2.534	2.48
2.277	2.238
2.498	2.554
2.133	2.147
2.134	2.111
2.272	2.316
2.534	2.519
2.466	2.392
2.447	2.368
2.537	2.54
2.426	2.493
2.519	2.523
2.454	2.506
2.571	2.567
2.581	2.586
2.601	2.588
2.592	2.643
2.174	2.211

$$\mathbf{log}(\boldsymbol{y_i}) = \mathbf{log}(\boldsymbol{a_0}) + \boldsymbol{a_1}\,\mathbf{log}(\boldsymbol{x_1}) + \boldsymbol{a_2}\,\mathbf{log}(\boldsymbol{x_2})$$

$$\log(y_i)\ Compressive = \log(336.8349) - 0.1933 * \log(average\ length) - 0.2622 * \log(average\ thickness)$$

Observation	Estimated
-----------	---------
1.749	1.838
1.791	1.774
1.846	1.822
1.838	1.847
1.952	1.918
1.935	1.905
2.095	1.993
1.971	1.936
1.979	1.941

1.979	1.93
2.007	1.968
1.964	1.926
1.973	1.938
1.977	1.934
1.877	1.85
1.836	1.809
1.927	1.929
1.988	1.925
1.794	1.808
1.956	1.944
2.015	1.95
1.699	1.81
1.909	1.941
1.992	1.948
1.959	1.92
1.83	1.874
2.018	1.951
1.886	1.875
1.837	1.869
1.889	1.882
1.916	1.916
1.842	1.883
2	1.928
1.703	1.893
1.881	1.814
1.936	1.861
1.851	1.851
1.981	1.935
1.909	1.837
2.022	1.867
1.87	1.841
1.957	1.95
1.927	1.893
1.983	1.958
1.894	1.84
1.975	1.874
1.799	1.831
1.855	1.832
1.936	1.866
1.813	1.808
1.783	1.834
1.841	1.822
1.811	1.822
1.907	1.935
1.888	1.943
1.805	1.893

1.967	1.948
1.739	1.969
1.801	1.827
1.859	1.962
1.963	1.942
1.844	1.881
1.982	1.954
1.806	1.901
1.897	1.898
1.838	1.802
1.878	1.966
1.771	1.861
1.969	1.946
1.865	1.885
1.853	1.887
1.938	1.947
1.774	1.838
1.919	1.899
1.911	1.934
1.796	1.906
1.742	1.817
1.989	1.934
1.855	1.953
1.649	1.822
1.922	1.94
1.785	1.835
1.945	1.94
1.889	1.958
1.963	1.963
1.828	1.843
1.824	1.85
1.868	1.88
1.988	1.952
1.824	1.896
1.99	1.868
1.885	1.801
1.885	1.846
1.787	1.839
1.886	1.826
1.906	1.834
1.958	1.898
1.797	1.846
1.925	1.93
1.978	1.95
1.858	1.906
1.882	1.856
1.953	1.861

1.961	1.869
1.846	1.828
1.798	1.847
1.843	1.841
1.838	1.872
1.834	1.812
1.809	1.812
1.851	1.833
1.774	1.821
1.908	1.935

$$\mathbf{log}(\boldsymbol{y_i}) = \mathbf{log}(\boldsymbol{a_0}) + \boldsymbol{a_1}\,\mathbf{log}(\boldsymbol{x_1}) + \boldsymbol{a_2}\,\mathbf{log}(\boldsymbol{x_2}) + \boldsymbol{a_3}\,\mathbf{log}(\boldsymbol{x_3})$$

$$\log(y_i) Compressive = \log(308.693) - 0.207 * \log(average\ length) - 0.276 * \log(average\ thickness) + 0.039 * \log(average\ external\ diameter)$$

Observation	Estimated
-----------	---------
1.749	1.839
1.791	1.771
1.846	1.823
1.838	1.847
1.952	1.919
1.935	1.906
2.095	1.99
1.971	1.934
1.979	1.94
1.979	1.931
2.007	1.965
1.964	1.925
1.973	1.938
1.977	1.936
1.877	1.851
1.836	1.805
1.927	1.93
1.988	1.925
1.794	1.808
1.956	1.943
2.015	1.951
1.699	1.807
1.909	1.939
1.992	1.947
1.959	1.922
1.83	1.874
2.018	1.951

1.886	1.875
1.837	1.869
1.889	1.882
1.916	1.918
1.842	1.883
2	1.929
1.703	1.892
1.881	1.812
1.936	1.862
1.851	1.85
1.981	1.936
1.909	1.837
2.022	1.867
1.87	1.844
1.957	1.949
1.927	1.896
1.983	1.958
1.894	1.84
1.975	1.875
1.799	1.83
1.855	1.833
1.936	1.868
1.813	1.807
1.783	1.836
1.841	1.819
1.811	1.822
1.907	1.936
1.888	1.941
1.805	1.893
1.967	1.947
1.739	1.966
1.801	1.826
1.859	1.961
1.963	1.943
1.844	1.886
1.982	1.949
1.806	1.902
1.897	1.898
1.838	1.801
1.878	1.966
1.771	1.861
1.969	1.944
1.865	1.887
1.853	1.886
1.938	1.947
1.774	1.839
1.919	1.899

1.911	1.936
1.796	1.907
1.742	1.818
1.989	1.936
1.855	1.952
1.649	1.818
1.922	1.942
1.785	1.836
1.945	1.939
1.889	1.958
1.963	1.962
1.828	1.842
1.824	1.851
1.868	1.881
1.988	1.953
1.824	1.897
1.99	1.869
1.885	1.798
1.885	1.845
1.787	1.84
1.886	1.825
1.906	1.833
1.958	1.896
1.797	1.847
1.925	1.93
1.978	1.949
1.858	1.909
1.882	1.857
1.953	1.861
1.961	1.868
1.846	1.825
1.798	1.849
1.843	1.842
1.838	1.876
1.834	1.813
1.809	1.812
1.851	1.835
1.774	1.823
1.908	1.936

$$\mathbf{log}(\boldsymbol{y_i}) = \mathbf{log}(\boldsymbol{a_0}) + \boldsymbol{a_1}\,\mathbf{log}(\boldsymbol{x_1}) + \boldsymbol{a_2}\,\mathbf{log}(\boldsymbol{x_2}) + \boldsymbol{a_3}\,\mathbf{log}(\boldsymbol{x_3}) + \boldsymbol{a_4}\,\mathbf{log}(\boldsymbol{x_4})$$

$$\log(y_i)\ Compressive = \log(1392.143) - 0.388 \ * \log(average\ length) - 0.410 \ * \log(average\ thickness) - 0.324 \ * \log(average\ external\ diameter) + 0.242 \ * \log(weight\ before\ compression)$$

Observation	Estimated
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1.749	1.827
1.791	1.767
1.846	1.816
1.838	1.835
1.952	1.912
1.935	1.904
2.095	1.996
1.971	1.927
1.979	1.93
1.979	1.924
2.007	1.951
1.964	1.921
1.973	1.925
1.977	1.924
1.877	1.836
1.836	1.802
1.927	1.923
1.988	1.921
1.794	1.81
1.956	1.931
2.015	1.942
1.699	1.776
1.909	1.926
1.992	1.935
1.959	1.904
1.83	1.866
2.018	1.945
1.886	1.869
1.837	1.857
1.889	1.87
1.916	1.899
1.842	1.862
2	1.918
1.703	1.892
1.881	1.815
1.936	1.857
1.851	1.836
1.981	1.923
1.909	1.842
2.022	1.876
1.87	1.84
1.957	1.941
1.927	1.877
1.983	1.947
1.894	1.839

1.975	1.876
1.799	1.825
1.855	1.821
1.936	1.871
1.813	1.8
1.783	1.83
1.841	1.817
1.811	1.835
1.907	1.936
1.888	1.945
1.805	1.887
1.967	1.958
1.739	1.971
1.801	1.829
1.859	1.959
1.963	1.952
1.844	1.897
1.982	1.965
1.806	1.891
1.897	1.893
1.838	1.813
1.878	1.964
1.771	1.846
1.969	1.963
1.865	1.902
1.853	1.901
1.938	1.952
1.774	1.834
1.919	1.921
1.911	1.946
1.796	1.927
1.742	1.806
1.989	1.949
1.855	1.962
1.649	1.791
1.922	1.95
1.785	1.827
1.945	1.949
1.889	1.966
1.963	1.974
1.828	1.843
1.824	1.861
1.868	1.89
1.988	1.964
1.824	1.899
1.99	1.875
1.885	1.813

1.885	1.854
1.787	1.831
1.886	1.831
1.906	1.854
1.958	1.921
1.797	1.855
1.925	1.938
1.978	1.956
1.858	1.899
1.882	1.87
1.953	1.878
1.961	1.884
1.846	1.844
1.798	1.865
1.843	1.848
1.838	1.888
1.834	1.829
1.809	1.816
1.851	1.839
1.774	1.826
1.908	1.943