DESIGN OF AN OVERCONSTRAINED MANIPULATOR FOR REHABILITATION PURPOSES

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FEBRUARY 2016

DESIGN OF AN OVERCONSTRAINED MANIPULATOR FOR REHABILITATION PURPOSES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES OF
ÇANKAYA UNIVERSITY

BY
KUNTAY YILMAZ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
IN
THE DEPARTMENT OF
MECHANICAL ENGINEERING

FEBRUARY 2016

Title of the Thesis: **Design of an Overconstrained Manipulator for Rehabilitation Purposes**

Submitted by **Kuntay Yılmaz**

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ABSTRACT

DESIGN OF AN OVERCONSTRAINED MANIPULATOR FOR REHABILITATION PURPOSES

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February 2016, 43 pages

In this thesis, an overconstrained mechanism for upper extremity rehabilitation is introduced. The geometry of the selected manipulator was arranged that it fits the exact motion of the arm upper extremity and acts as an exoskeleton. Inverse kinematics calculations are shown for describing the motion of actuators for a desired arm motion. Lagrange Formulation is used for the inverse dynamic model of the system. Due to the geometry of the manipulator, kinematic and dynamic calculations are applied to the two spherical subspaces of the manipulator using imaginary joints. Workspace analysis has been made to verify the motion that the mechanism needs for the specified rehabilitation tasks.

Keywords: Rehabilitation Robotics, Overconstrained Manipulators, Kinematic Analysis, Dynamic Analysis, Exoskeleton, Workspace Analysis.

ÖZ

REHABİLİTASYON AMAÇLI AŞIRI KAPALI MANİPÜLATÖR TASARIMI

YILMAZ, Kuntay
Yüksek Lisans, Makine Mühendisliği Anabilim Dalı
Tez Yöneticisi: Asst. Prof. Dr. Özgün SELVİ
Şubat 2016, 43 sayfa

Bu tezde, üst ekstremitenin rehabilitasyonunda kullanılabilecek bir kısıtlı mekanizma tanıtılmıştır. Seçilen mekanızmanın geometrisi üst ekstremitenin hareket aralığına göre ayarlanmış olup dış iskelet görevini üstlenmektedir. İstenen hareketi tanımlayabilmek için Euler açıları yardımı ile ters kinematik hesaplamaları yapılmıştır. Sistemin dinamik modeli Lagrange denklemleri ile elde edilmiştir. Mekanizmanın geometrisinden ötürü oluşan iki küresel altuzay'ın kinematik ve dinamik hesabı imgesel bir mafsal yardımı ile yapılmıştır. Mekanizmanın çalışma aralığı belirlenmiş ve bazı rehabilitasyon egzersizleri için çözüm olarak kullanılabileceği gösterilmiştir.

Anahtar Kelimeler: Rehabilitasyon Robotları, Aşırı Kapalı Manipülatörler, Kinematik Analiz, Dinamik Analiz, Dış İskelet, Çalışma Alanı Analizi.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to Asst. Prof. Dr. Özgün SELVİ for his supervision, guidance, suggestions, and encouragement throughout the development of this thesis.

I thank my family for their valuable support and patience.

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LIST OF ABBREVIATIONS

DOF	Degrees Of Freedom
R	Revolute Joint
P	Prismatic Joint
S	Spherical Joint
ROM	Range Of Motion
GUI	Graphical User Interface

1. BACKGROUND & LITERATURE SURVEY

Robotics, the rising trend which has elevated to much higher grounds throughout this decade has had great influences on many medical and industrial applications. One of the fields where we can observe robotics more frequently nowadays is Physiotherapy & Rehabilitation. According to WHO, World Report on Disability, around one billion people in the world suffer from disabilities and 200 million of them face difficulty functioning. Due to the increase in life span and chronic health conditions it is expected that this number will increase in the following years [1]. Rehabilitation may refer to different things depending on the field but in this thesis it will strictly be referred to the aspect of physical therapy for the human limbs, within the medical field. Number of patients in need of rehabilitation increasing means more time and effort consumed by practitioners, which leads to need of increase in staff. Rehabilitation generally requires systematic and repetitive movements therefore controlled mechanisms such as robots can help with the assistance process of rehabilitation. Rehabilitation Robotics is a challenging field with a high potential to support people with severe disabilities in their daily life. The field of rehabilitation robotics makes use of the precision, repeatability and monitoring properties of robots to ensure a more controlled and labour force free environment. The objective is to support people to perform tasks in their daily lives, at home or at work [2]. The most important parts for the human body, which needs rehabilitation to perform daily activities are shoulder, arm, wrist, hip, leg, and foot. In this thesis, the emphasis will be on the upper extremity, the region from the forequarter to fingers, and specifically the motion between the shoulder and elbow will be examined.

1.1 Upper Extremity Rehabilitation

The need for rehabilitation of the upper extremity mostly occur from the orthopaedic deficiencies, injuries on bones, joints and muscles or neurological problems like nervous system injuries or paralysis. Orthopaedic injuries include damage taken by the bones, joints and/or muscle tissues, the causes may be accidents such as car crashes or faulty movements causing impacts. Neurologic injuries include strokes, spinal cord injuries neuron damages etc. The simple representation of the bone and muscle structure of the upper extremity can be seen in Figures (1-2).

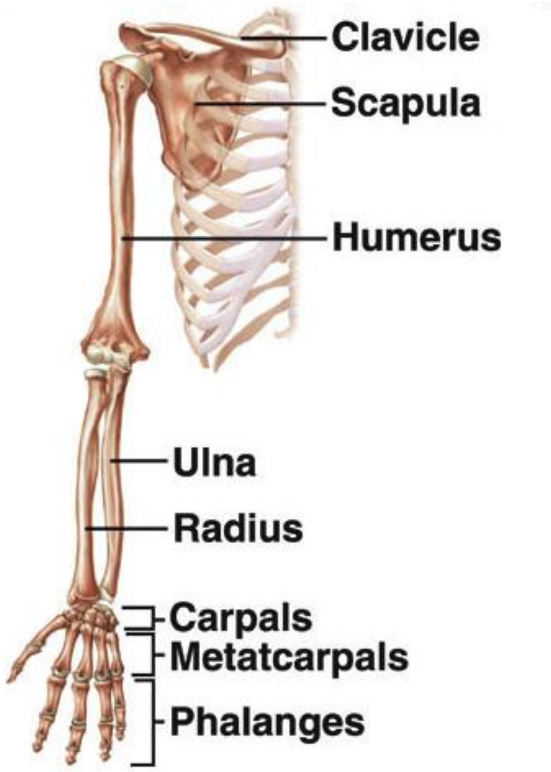


Figure 1. Upper Extremity Bone Structure, courtesy of Rutgers University Anatomy and Physiology Lecture Notes [27]

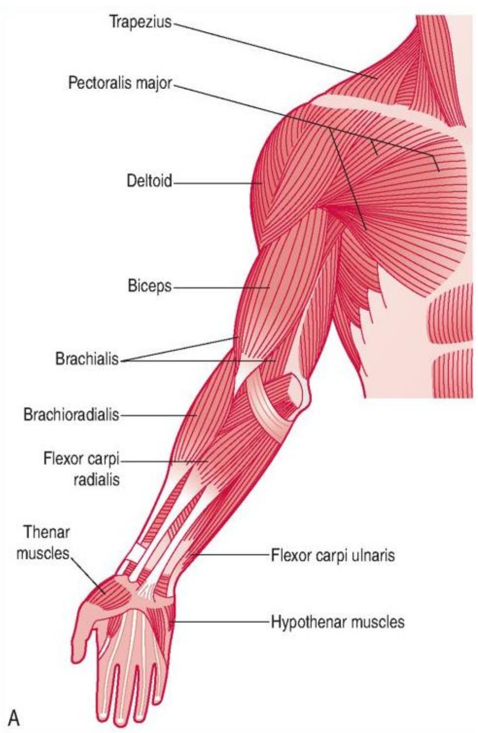


Figure 2. Upper Extremity Muscle Structure, courtesy of Medical Dictionary. [28]

In both of the injury cases, after the necessary medical surgery and comfortable protection conditions are met, several studies show that repetitive motion of the effected limb proves to be an improving activity to regain motor control [3,4,5]. The possible movements of the human arm and their limits are presented in the figure below:

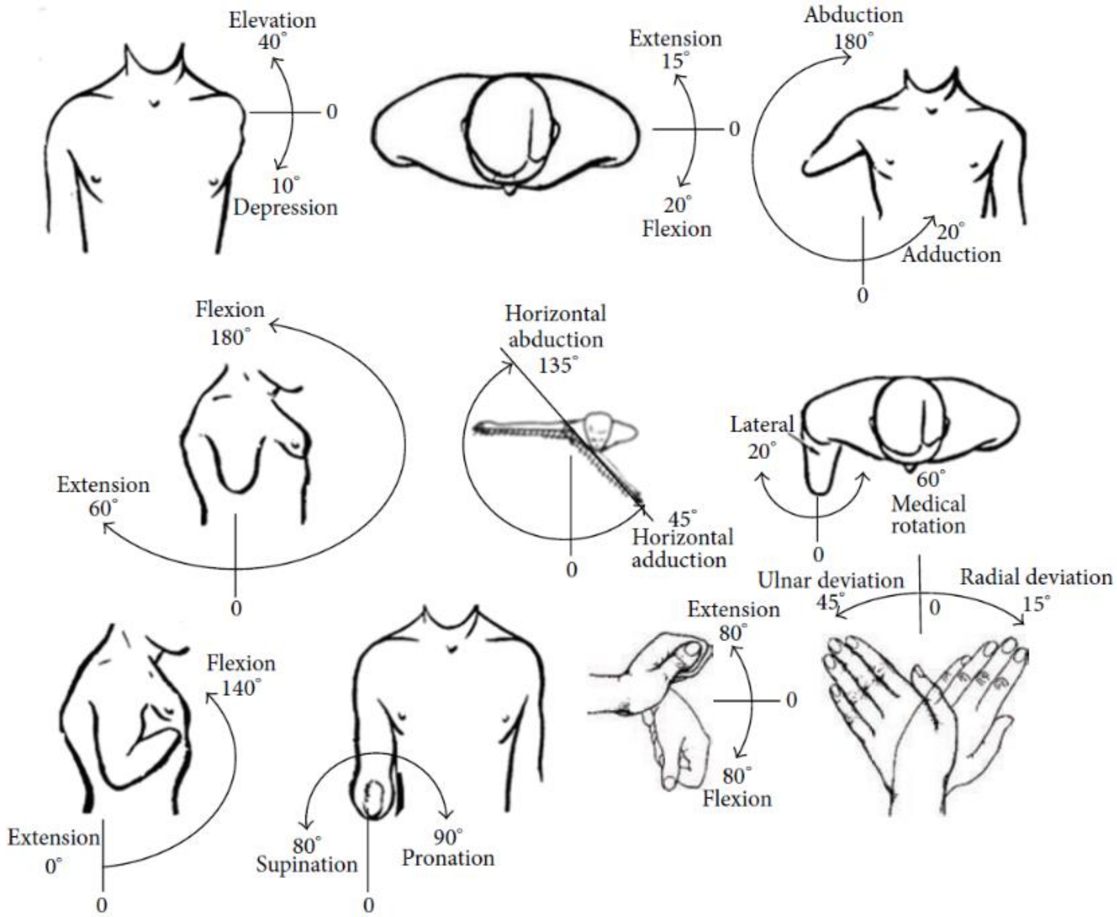
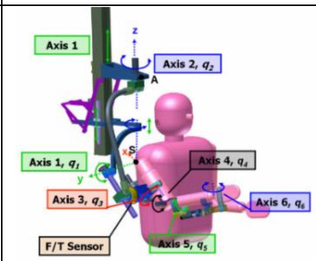
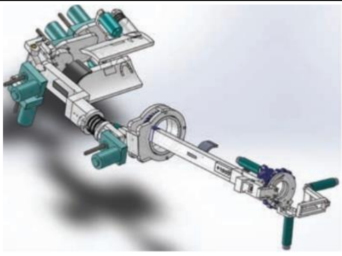
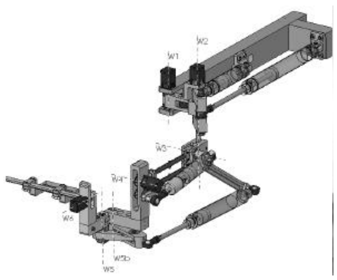


Figure 3. The Range of Motion (ROM) of Upper Extremity [10]

There are three main manual therapy methods to help patients recover motion on their injured limbs: 1) Passive exercising, where the patient has no movement of the limb and the motion must be done by the help of a physiotherapist. 2) Active Exercising is when the patient has control over the limb but needs to improve muscular endurance or joint flexibility. 3) Active-assisted exercising is used to assist patients with insufficient control and/or strength externally until they can do it by their selves [6]. These methods are used to help patients conduct their exercises, where the exercises are selected such that the range of motion (ROM) of the injured limb can return to acceptable values. Athletes need to regain full potential of their limb because of their professional needs but for a citizen, the ability to perform daily activities to maintain living is sufficient.

1.2 Serial Rehabilitation Robots

The classification of robotics in the field of rehabilitation is done by the limb that it's trying to rehabilitate and the kinematic structure of the mechanism used to actualize this objective. The possible kinematic structures for robots are known as serial robots, parallel robots and hybrid robots [7]. Serial manipulators (open-loop manipulators) consist of several links connected in series by various types of joints [8]. These manipulators have good operating characteristics (large workspace, high flexibility and manoeuvrability) but have disadvantages such as low precision, low stiffness and low power. Serial manipulators, having easier manufacturing and assembling properties are widely used in the rehabilitation field. Some of the serial manipulator robots for upper extremity rehabilitation can be seen in the table below [9-15]:

Rehabilitation Robots For Upper Extremity					
Name of Publication	***Authors - Year***	***Focus Area***	***Kinematics***	***Illustration***	***Image***
ARMin – Exoskeleton for Arm Therapy in Stroke Patients	T. Nef, M. Mihelj, G. Kiefer, C. Perndl, R. Müller - 2007	Human arm (fitting to its range of motion)	Serial Structure 6 DOF	Provide neurological treatment of the arm by task-oriented repetitive movements.	
Design of an Arm Exoskeleton with Scapula Motion for Shoulder Rehabilitation	S. K. Manna, S. Bhaumik - 2010	Human Arm (Shoulder griddle to wrist)	Serial Structure 10 DOF	Improving the Motion adaptability by adding the shoulder griddle movement.	
A Pneumatic Robot for Re-: Rationale and Mechanical Design	R. J. Sanchez, J, E. Wolbrecht, R. Smith, J. Liu, S. Rao, S. Cramer, T. Rahman, J. E. Bobrow, D. J. Reinkensmeyer - 2005	Arm movement with hand grip included	Serial Structure 5 DOF	Pneumatic robot for functional movement training of the arm and hand after stroke	

Wrist Rehabilitation in Chronic Stroke Patients by Means of Adaptive, Progressive Robot-Aided Therapy	V. Squeri, L. Masia, P. Giannoni, G. Sandini, and P. Morasso - 2014	Wrist	Serial Structure 3 DOF	A haptic three DoF robot quantifying motor impairment and assisting wrist and forearm articular movements.	
A new force-feedback arm exoskeleton for haptic interaction in Virtual Environments	A. Frisoli F. Rocchi S. Marcheschi A. Dettori F. Salsedo M. Bergamasco - 2005	Whole Arm	Serial Structure 5DOF	The exoskeleton is very effective for simulating the touch by hand of large objects or the manipulation within the whole workspace of the arm	
An End-effector Arm Rehabilitation Robot with VE	N. Angsupasirikul, R. Chancharoen - 2015	Horizontal arm movement	Serial Structure RP 2 DOF	End-effector arm rehabilitation robot with active, passive and assisted modes. Includes virtual game environment and uses EMG	
A universal haptic device for arm and wrist rehabilitation	J. Oblak, I. Cikajlo, Z. Matjačić - 2009	Two modes, Wrist + Arm mode	Serial Structure 2 DOF	Has 2 mechanical configurations which can switch between arm and wrist modes to provide specific treatment	

Table 1. Serial Rehabilitation Robots for Upper Extremity

1.3 Parallel & Hybrid Rehabilitation Robots

Parallel manipulators (closed-loop manipulators) usually consists of a moving platform that is connected to the fixed base by multiple legs [8]. The manipulators provide advantages like lower moving masses, higher rigidity, better accuracy and payload-to-weight ratio but these manipulators usually have limited workspace and non-isotropic input/output relations. These characteristics make them viable choices in industrial applications where strength and rigidity is prioritized over workspace. Hybrid manipulators use parallel and serial manipulators together to gather the advantageous characteristics of both of these mechanisms for certain objectives. A table is given below to show some of the parallel and hybrid rehabilitation robots made [16-19].

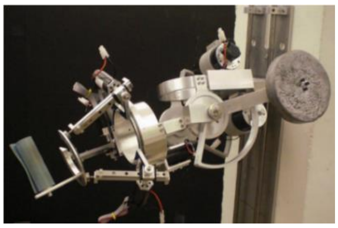
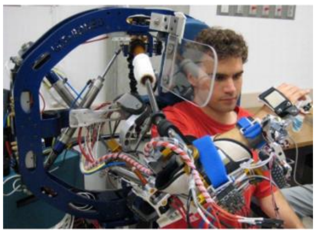
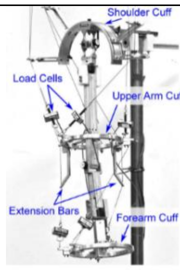
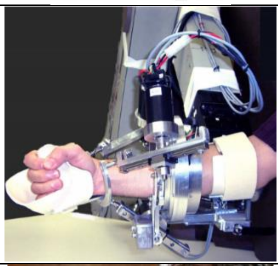
Rehabilitation Robots For Upper Extremity					
Name of Publication	***Authors - Year***	***Focus Area***	***Kinematics***	***Illustration***	***Image***
Mechanical Design of a Distal Arm Exoskeleton for Stroke and Spinal Cord Injury Rehabilitation	A. U. Pehlivan, Ö. Çelik, M. K. O'Malley - 2011	From forearm to wrist	Hybrid Structure 3 RPS 5 DOF	Rehabilitation for upper extremity after stroke, spinal cord injury neurological injuries.	
Optimization of a Parallel Shoulder Mechanism to Achieve a High Force low mass robotic arm exoskeleton	J. Klein, S. Spencer, J. Allington, J. E. Bobrw, D. J. Reikensmeyer - 2010	Human Arm (shoulder movements)	Parallel structure 2 RRPS 3 DOF	Mostly focuses on producing the ROM (range of motion) of the human arm.	
A Cable Driven Upper Arm Exoskeleton for Upper Extremity Rehabilitation	Y. Mao, S. K. Agrawal - 2011	Shoulder to forearm	Parallel structure 5 DOF	To achieve force control with cable manipulator	
The RiceWrist: A Distal Upper Extremity Rehabilitation Robot for Stroke Therapy	M. K. O'Malley, C. Burgar - 2006	From forearm to wrist	Hybrid structure 3 RPS 4 DOF	Provide therapy in the specified region via force-feedback	
Development of Wrist Rehabilitation Equipment Using Pneumatic Parallel Manipulator	M. Takaiwa, T. Noritsugu - 2005	Wrist	Parallel Structure 3 DOF	Stewart type platform with pneumatic actuators providing minute control property	

Table 2. Parallel & Hybrid Rehabilitation Robots for Upper Extremity

1.4 Overconstrained Manipulators

One type of manipulator not mentioned above is the overconstrained manipulator, which can be considered as an inner branch of the parallel manipulator. Overconstrained manipulators are mechanisms that have full cycle motion despite failing the Kutzbach criterion [20]. The formula given in Equation (1.1) can be used to define the mobility of a mechanism [21].

$$M = \sum_{i=1}^{N} f_i - \sum_{i=1}^{L} \lambda_i \tag{1.1}$$

In the above equation, $\sum\lambda_i$ represents the total subspace number of the independent loops, $\sum f_i$ is the total mobility of the joints and M is the mobility (DOF) of the system. Overconstrained manipulators have certain advantages over the other more known manipulators. They hold the strength and rigidity capabilities of parallel manipulators, they can be fitted on the subspace motion perfectly and they can achieve mobility with fewer links and joints. Most of the overconstrained manipulators suggested to be used throughout the years can be seen in the works of Baker, Waldron and Philips [22-24].

1.5 Motion between the Shoulder and the Elbow

To design a manipulator for a specified motion, firstly the surroundings then the types of motion needed has to be known. The primary goal of this thesis is to achieve the motion needed to perform some of the rehabilitative upper extremity exercises. The shoulder and elbow is taken into account and an overconstrained manipulator has been proposed for the 5 DOF subspace. The specified motion range for the mentioned are can be defined by rotations as shown as in Figure 4.

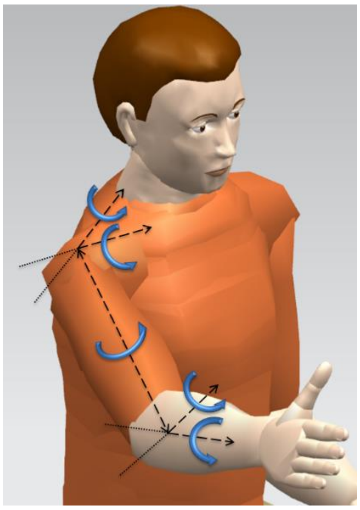


Figure 4. Arm Motion Defined by Rotations

1.6 Objective

The objective is to introduce overconstrained mechanisms into upper extremity rehabilitation by making use of their reliable characteristics and show how they can be modified and calculated for different purposes. In the light of this goal, kinematic and dynamic calculations of the mechanism is made and the workspace that the mechanism can reach is found.

2. THE MECHANISM AND IT'S KINEMATICS

The specified motion of the arm was shown in terms of rotations in Figure 4. It is seen that only rotational motions are required, therefore the first starting point in this project was to see if the specified motion can be achieved by the use of a spherical manipulator. Through research and trials, it was observed that the spherical 3RRRRR parallel mechanism could be an appropriate selection for the intended 5 DOF motion. The suggested mechanism for the 5 DOF workspace of the upper extremity can be found in the work of X. Kong and C. Gosselin [25]. According to research, the referred mechanism has not gone through any kinematic or dynamic calculations thus far and at first sight seems to be a complicated mechanism for these types of calculations. To solve the mechanism and to reduce the complexity of the system, we proposed to add an imaginary joint and separate the system so that it can be solved as a two-part system with less number of joints. The system, with the imaginary joint, becomes a 5 DoF double spherical manipulator with a configuration of (RRR)-(RR) and 3 limbs, the schematic drawing of the manipulator is given in Figure 5.a. The schematic drawing of the double spherical manipulator with an added imaginary joint is given in Figure 5.b. This procedure simplifies the kinematic and dynamic solving process and makes it easier to see the motions separately. After the separation, 3RRRR and 3RRR manipulators are produced and both of these systems can be solved using the inverse kinematic approach.

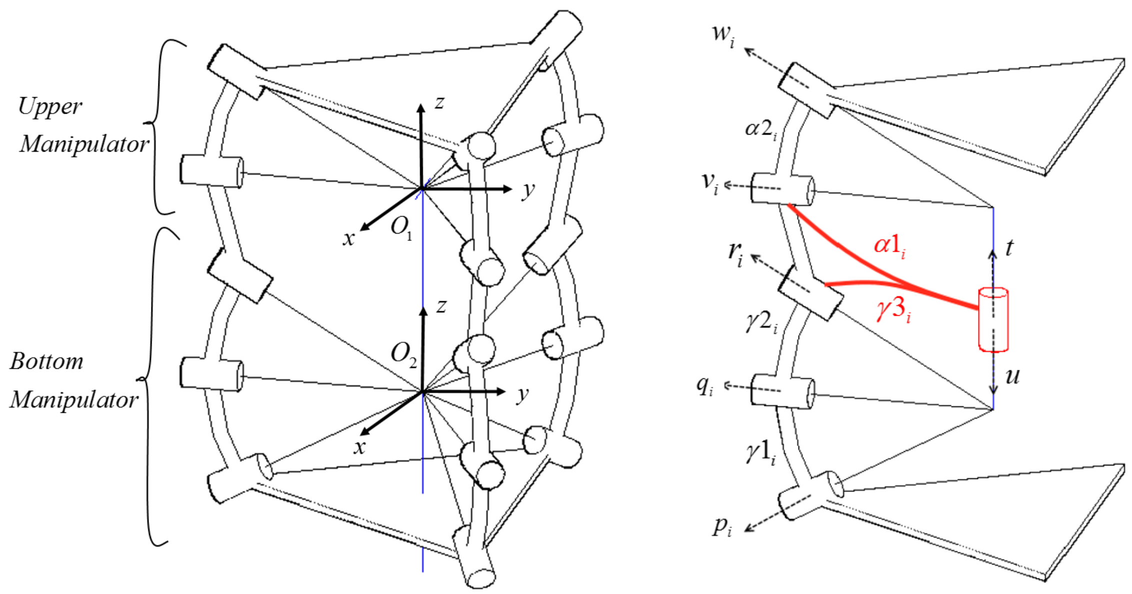


Figure 5. a. Schematic view of the 3RRRRR Manipulator, b. The Double Spherical Manipulator with Imaginary joints added

2.1 Upper Manipulator Kinematic Analysis (3-RRR)

The produced 3 leg system is a spherical 3 DOF manipulator where all the legs can be solved using the same kinematic equation. The drawing of the manipulator with the added imaginary joints is given in Figure 6.a. The red coloured imaginary joints constitute the active joints of the system. The drawing for a leg of the upper manipulator with the necessary vector indications are given in Figure 6.b.

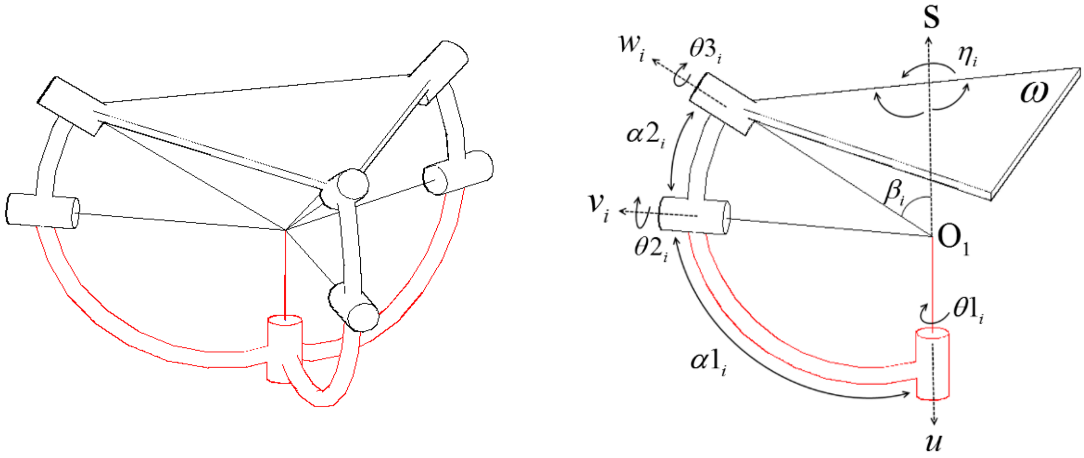


Figure 6. a. Drawing of Upper Manipulator, b. An Upper Manipulator Leg with Vector Assignments

To find the orientation of rigid bodies and vectors, rotation matrices Rot_c are used, where subscripts denote the axis that is used for the rotation. Rotation matrices for x,y,z axes with a random ϖ angle are given in Equation (2.1) below.

$$Rot_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varpi & -\sin\varpi \\ 0 & \sin\varpi & \cos\varpi \end{pmatrix}, Rot_y = \begin{pmatrix} \cos\varpi & 0 & \sin\varpi \\ 0 & 1 & 0 \\ -\sin\varpi & 0 & \cos\varpi \end{pmatrix}, Rot_z = \begin{pmatrix} \cos\varpi & -\sin\varpi & 0 \\ \sin\varpi & \cos\varpi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.1)$$

u, $\mathbf{v_i}$ and $\mathbf{w_i}$ are unit vectors of the upper manipulator which are parallel to the revolute joint axes. The three legs are identically structured and are defined by the link and orientation angles $\alpha 1_i$, $\alpha 2_i$, β_i, η_i and joint angles $\theta 1_i$, $\theta 2_i$, $\theta 3_i$ where **i = 1, 2, 3**. β_i and η_i define the geometry of the platform with respect to the middle point O_1 and the end effector $\mathbf{O_r}$. The first links for each leg (imaginary connections), which are between the vectors **u** and $\mathbf{v_i}$, constitute the inputs of the system. The unit vector **u** is given in Equation (2.2).

$$\mathbf{u} = [0 \quad 0 \quad -1]^T \quad (2.2)$$

The upper manipulator will be used for the elbow pronation/supination, flexion/extension and lateral/medical rotation motions, the ZYZ rotation sequence defines this motion and can be used to define the platform/end point position. The platform position is constructed by the rotation sequence of ZYZ using the inverse kinematics approach. For the rotations of ζ, ψ and

ξ, the rotation matrix R_o and the end effector $\mathbf{O_r}$, which is defined by rotating the vector $\mathbf{u}$, the following equations are obtained:

$$
\begin{gathered}
R_o = Rot_z(\zeta) \cdot Rot_y(\psi) \cdot Rot_z(\xi) \\
\mathbf{O_r} = R_o \cdot \mathbf{u}
\end{gathered}
\tag{2.3}
$$

The platform joints vectors $\mathbf{w_i}$ can be derived by using the rotation matrix found in Equation (2.3). $\mathbf{w0_i}$ can be interpreted as the platform shape and the necessary calculations, using the platform orientation angles β_i and η_i, are given in Equation (2.4):

$$
\begin{aligned}
\mathbf{w0_i} &= Rot_z(\eta_i) \cdot Rot_z(\beta_i) \\
\mathbf{w_i} &= R_o \cdot \mathbf{w0_i}
\end{aligned}
\tag{2.4}
$$

The $\mathbf{w_i}$ vector obtained from the above equation can be used as given in Equation (2.5):

$$
\mathbf{w_i} = [wx_i \quad wy_i \quad wz_i]^T \tag{2.5}
$$

To solve the system by using the platform orientation as inputs a closure equation definition is needed. By calculating the $\mathbf{w_i}$ vectors by forward kinematics from $\mathbf{u}$ vector we obtain $\mathbf{wf_i}$, which is given in Equation (2.6).

$$
\mathbf{wf_i} = Rot_z(\theta 1_i) \cdot Rot_x(\alpha_1) \cdot Rot_z(\theta 2_i) \cdot Rot_x(\alpha_2) \cdot \mathbf{u} \tag{2.6}
$$

The open form of Equation (2.6) is given in Equation (2.7).

$$
\mathbf{wf_i} = \begin{pmatrix}
-\cos\alpha 2_i \cdot \sin\alpha 1_i \cdot \sin\theta 1_i - \sin\alpha 2_i \cdot \left(\cos\alpha 1_i \cdot \cos\theta 2_i \cdot \sin\theta 1_i + \cos\theta 1_i \cdot \sin\theta 2_i\right) \\
\cos\alpha 2_i \cdot \cos\theta 1_i \cdot \sin\alpha 1_i + \sin\alpha 2_i \cdot \left(\cos\alpha 1_i \cdot \cos\theta 1_i \cdot \cos\theta 2_i - \sin\theta 1_i \cdot \sin\theta 2_i\right) \\
-\cos\alpha 1_i \cdot \cos\alpha 2_i + \cos\theta 2_i \cdot \sin\alpha 1_i \cdot \sin\alpha 2_i
\end{pmatrix} \tag{2.7}
$$

The closure equation is obtained by forming the relationship $\mathbf{w_i} = \mathbf{wf_i}$. Using the first two rows of the matrix in Equation (2.7), which gives two equations, $\cos\theta 2_i$, and $\sin\theta 2_i$ can be solved as shown in Equation (2.8).

$$
\begin{aligned}
\cos\theta 2_i &= -\cot\alpha 2_i \cdot \tan\alpha 1_i + \csc\alpha 2_2 \cdot \sec\alpha 1_i \cdot \left(-\sin\theta 1_i \cdot wx_i + \cos\theta 1_i \cdot wy_i\right) \\
\sin\theta 2_i &= -\cos\theta 1_i \cdot \csc\alpha 2_i \left(wx_i + \tan\theta 1_i \cdot wy_i\right)
\end{aligned}
\tag{2.8}
$$

To obtain the values for $\theta 2_i$, $Atan2\left(cos\ \theta 2, sin\ \theta 2\right)$[8], the four-quadrant inverse tangent, should be used. Resultant solution of the mentioned operations is given in Equation (2.9).

$$
\theta 2_i = \mathrm{Atan}\,2\begin{pmatrix}
-\cos\theta 1_i \cdot \csc\alpha 2_i \left(wx_i + \tan\theta 1_i \cdot wy_i\right), \\
-\cot\alpha 2_i \cdot \tan\alpha 1_i + \csc\alpha 2_i \cdot \sec\alpha 1_i \cdot \left(-\sin\theta 1_i \cdot wx_i + \cos\theta 1_i \cdot wy_i\right)
\end{pmatrix} \tag{2.9}
$$

The last row of the matrix given in Equation (2.7) is shown in Equation (2.10). The obtained $\theta 2$ values will be inputs for this equation.

$$wz_i - (-\cos\alpha 1_i \cdot \cos\alpha 2_i + \cos\theta 2_i \cdot \sin\alpha 1_i \cdot \sin\alpha 2_i) = 0 \qquad (2.10)$$

To solve for $\theta 1$, Tangent half-angle formulas are used as given below and the kinematic solution of the system is completed.

$$\sin\theta 1_i = \frac{2\tan\theta 1_i / 2}{1+\tan^2\theta 1_i / 2} \quad , \quad \cos\theta 1_i = \frac{1-\tan^2\theta 1_i / 2}{1+\tan^2\theta 1_i / 2}$$

2.2 Bottom Manipulator Kinematic Analysis (3-RRRR)

The produced 3 leg system is a spherical 5 DOF redundant parallel spherical manipulator. The actuation of the system (upper and bottom manipulator) will be maintained in this part of the system. Drawing of the manipulator with the added imaginary joints is given in Figure 7.a where the blue arrows denote the active joints. The drawing for a leg of the upper manipulator with the necessary vector indications are given in Figure 7.b.

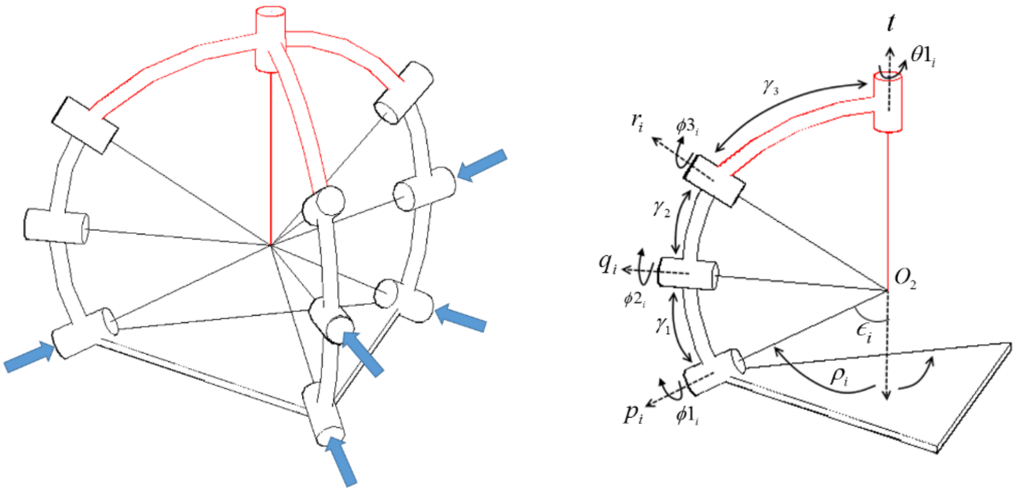


Figure 7. a. Drawing of Bottom Manipulator, b. A Bottom Manipulator Leg with Vector Assignments

t, $\mathbf{p_i}$, $\mathbf{q_i}$, $\mathbf{r_i}$ are unit vectors of the bottom manipulator which are parallel to the axes of the revolute joints. The three legs are defined by the angles $\gamma 1_i$, $\gamma 2_i$, $\gamma 3_i$, ϵ_i, κ_i, joint angles $\phi 1_i$, $\phi 2_i$, $\phi 3_i$ and platform angles of $\theta 1_i$, $\theta 2_i$, $\theta 3_i$ where i = 1, 2, 3. ϵ_i and κ_i define the geometry of the base platform with respect to $\mathbf{p_i}$ vectors and O_2 point respectively. The last link for each leg (imaginary connections), which are between the vectors $\mathbf{r_i}$ and **t**, constitute the moving platform therefore becoming the outputs of the system. Note that, the $\theta 1_i$, $\theta 2_i$, $\theta 3_i$ values found in the upper manipulator are used to specify the moving platform angles for the bottom manipulator. Firstly we describe our initial unit vector **t** by Equation (2.11).

$$\mathbf{t} = [0 \quad 0 \quad 1]^T \tag{2.11}$$

The inverse kinematic solution of this manipulator closely resembles to the solution of the upper manipulator. Even though this manipulator is 5 DOF, the 3 DOF motion is used to actuate the upper manipulator, and we only need to define the shoulder flexion/extension and horizontal abduction/adduction motions to this manipulator. Moving Platform/End effector position of this manipulator can be obtained by XY rotations, the rotation matrix Rb_o and the end effector vector $\mathbf{Ob_r}$ formed by rotating the $\mathbf{t}$ vector is given below:

$$\begin{aligned} Rb_o &= Rot_x(\varphi) \cdot Rot_y(\sigma) \\ \mathbf{Ob_r} &= Rb_o \cdot \mathbf{t} \end{aligned} \tag{2.12}$$

The vectors of the last link joints connecting to the end effector, $\mathbf{r_i}$ are formed by using the end effector rotation given in Equation (2.12) and taking inputs from the upper manipulator, which is shown in Equation (2.13).

$$\mathbf{r_i} = Rb_o \cdot Rot_z(\theta 1_i) \cdot Rot_x(\gamma 3_i) \cdot \mathbf{t} \tag{2.13}$$

A closure equation has to be formed as done in the upper manipulator, the unit vectors $\mathbf{p_i}$ and $\mathbf{r_i}$ can be defined as shown in Equation (2.14).

$$\mathbf{p_i} = \begin{bmatrix} px_i & py_i & pz_i \end{bmatrix}^T, \ \mathbf{r_i} = \begin{bmatrix} rx_i & ry_i & rz_i \end{bmatrix}^T \tag{2.14}$$

Then we can obtain the position of the unit vectors $\mathbf{p_i}$ and $\mathbf{rf_i}$ (the forward approach of $\mathbf{r_i}$) by forward kinematics as given in Equation (2.15).

$$\begin{aligned} \mathbf{p_i} &= Rot_z(\rho_i) \cdot Rot_x(\epsilon_i) \cdot \mathbf{t} \\ \mathbf{rf_i} &= Rot_z(\rho_i) \cdot Rot_x(\epsilon_i) \cdot Rot_z(\phi 1_i) \cdot Rot_x(\varphi_1) \cdot Rot_z(\phi 2_i) \cdot Rot_x(\gamma_2) \cdot \mathbf{t} \end{aligned} \tag{2.15}$$

Applying the relation $\mathbf{rf_i} = \mathbf{r_i}$, expanding the values of Equation (2.15) and using Equation (2.14) together with it provides the values given in Equation (2.16).

$$\begin{aligned} rx_i &= \left(\cos\phi 1_i \cos\phi 2_i - \cos\gamma 1_i \sin\phi 1_i \sin\phi 2_i\right) px_i \\ &+(\sin\gamma 1_i \sin\gamma 2_i \sin\phi 1_i + \cos\gamma 2_i \left(-\cos\gamma 1_i \cos\phi 2_i \sin\phi 1_i - \cos\phi 1_i \sin\phi 2_i\right)) py_i \\ &+(\cos\gamma 2_i \sin\gamma 1_i \sin\phi 1_i - \sin\gamma 2_i \left(-\cos\gamma 1_i \cos\phi 2_i \sin\phi 1_i - \cos\phi 1_i \sin\phi 2_i\right)) pz_i \\ ry_i &= \left(cos\phi 2_i \sin\phi 1_i + \cos\gamma 1_i \cos\phi 1_i \sin\phi 2_i\right) px_i \\ &+(-\cos\phi 1_i \sin\gamma 1_i \sin\gamma 2_i + \cos\gamma 2_i (\cos\gamma 1_i \cos\phi 1_i \cos\phi 2_i - \sin\phi 1_i \sin\phi 2_i)) py_i \\ &+(\cos\gamma 2_i \sin\gamma 1_i \sin\phi 1_i - \sin\gamma 2_i \left(-\cos\gamma 1_i \cos\phi 2_i \sin\phi 1_i - \cos\phi 1_i \sin\phi 2_i\right)) pz_i \\ rz_i &= \sin\gamma 1_i \sin\phi 2_i \, px_i + \left(\cos\gamma 2_i \cos\phi 2_i \sin\gamma 1_i + \cos\gamma 1_i \sin\gamma 2_i\right) py_i \\ &+\left(\cos\gamma 1_i \cos\gamma 2_i - \cos\phi 2_i \sin\gamma 1_i \sin\gamma 2_i\right) pz_i \end{aligned} \tag{2.16}$$

rx_i, ry_i are used to form equations in terms of $\cos\phi 2_i$ and $\sin\phi 2_i$. Using the four-quadrant inverse tangent, $\text{Atan2}\left(\cos\phi 2, \sin\phi 2\right)$ [8], the values of $\phi 2_i$ for i = 1, 2, 3 are obtained. Using the $\cos\phi 2_i$ and $\sin\phi 2_i$ equations in rz_i and solving it accordingly, the values of $\phi 1_i$ are obtained in terms of upper manipulator joints, bottom manipulator joints and end effector rotation angles.

3. DYNAMIC ANALYSIS OF THE MECHANISM

Dynamic analysis plays an important role to this manipulator because of its need for precise control. After achieving the kinematic analysis and obtaining the joint rates as functions, the obtained values can be used to determine the external forces and moments of the system through dynamic analysis. The method used will be the Lagrange Formulation, by taking the external forces and moments of the system as a couple acting on joint O_1 and moving the couple on the same axis and directing it to O_2 both systems can be solved independently. The system in the presence of external forces is shown in Figure 8.

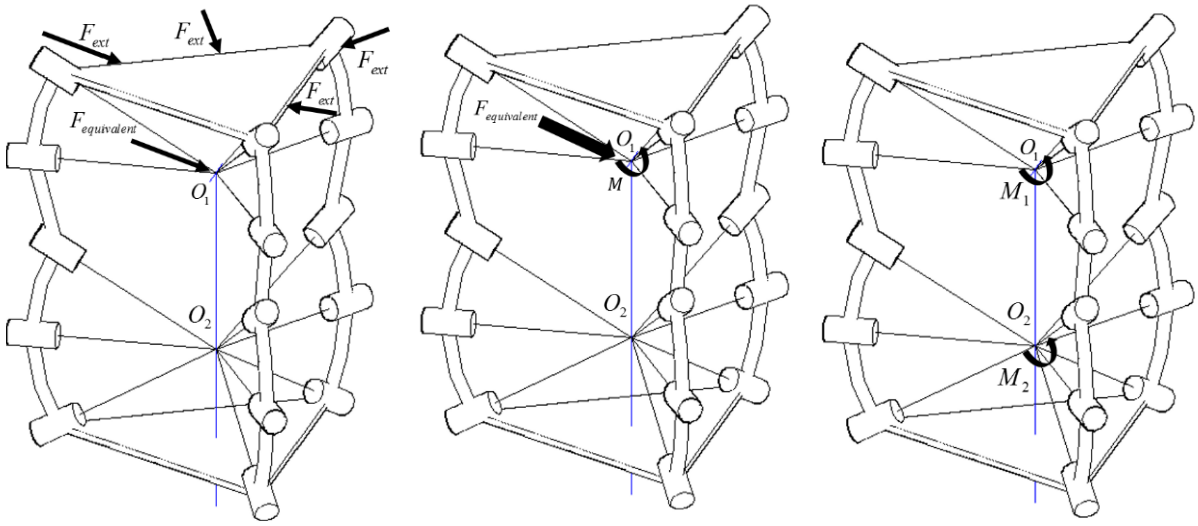


Figure 8. a. Manipulator while external forces are acting, b. External forces composed as force-moment couple on O_1, c. External forces composed as moments on O_1 and O_2

The Lagrange Formulation for parallel manipulators is given in Equation (3.1). [8]

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_j}\right) - \frac{\delta L}{\delta q_j} = Q_j + \sum_{i=1}^{k} \lambda_i \left(\frac{\delta \Gamma_i}{\delta q_j}\right) \quad for\ j = 1\, to\ n \tag{3.1}$$

Where L denotes the Lagrangian function, q_j the j th generalized coordinate, Q_j as j th generalized force, Γ_i as i th constraint function, k as DOF number, n as number of coordinates and λ_i as the Lagrangian Multiplier.

3.1 Upper Manipulator Dynamic Analysis (3-RRR)

The angular velocity of the end effector is found from Equation (3.2) [8]

$$\omega_n = \sum_{i=1}^{n} \dot{\theta 1}_i \, z_{i-1} \tag{3.2}$$

Where ω_n denotes the angular velocity of the end effector, $\dot{\theta 1}_i$ being the angular velocity of the i th link and z_{i-1} being the respective axis. This equation implies that the angular velocity of the links are additive, thus the resulting end effector velocity equation for the upper manipulator of the mechanism is given in Equation (3.3).

$$\omega_u = \mathbf{u} \cdot \dot{\theta}1_i + \mathbf{v_i} \cdot \dot{\theta}2_i + \mathbf{w_i} \cdot \dot{\theta}3_i \tag{3.3}$$

The rotation of the upper manipulator platform, R_o was found in Equation (2.3) and is going to be used to define the upper manipulator moving platform velocity. The rotations occurring in the bottom manipulator effects the upper manipulator, thus the total rotation R_u becomes as given in Equation (3.4).

$$R_u = Rot_x(\varphi) \cdot Rot_y(\sigma) \cdot R_o \tag{3.4}$$

Accordingly, we can find the velocity of the end effector by Equation (3.5).

$$\omega_u = \frac{\delta R_u}{\delta \varphi} \cdot R_u{}^T \cdot \dot{\varphi} + \frac{\delta R_u}{\delta \sigma} \cdot R_u{}^T \cdot \dot{\sigma} + \frac{\delta R_u}{\delta \xi} \cdot R_u{}^T \cdot \dot{\xi} + \frac{\delta R_u}{\delta \psi} \cdot R_u{}^T \cdot \dot{\psi} + \frac{\delta R_u}{\delta \zeta} \cdot R_u{}^T \cdot \dot{\zeta} \tag{3.5}$$

The Jacobian matrix is an important calculation which can reveal the dexterity and singularity characteristics of a mechanism. Jacobian matrix of parallel manipulators usually are in the form as given in Equation (3.6).

$$J_q \cdot \dot{\mathbf{q}} = J_x \cdot \dot{\mathbf{x}} \tag{3.6}$$

The Jacobian matrix relation for the upper manipulator is given in Equation (3.7).

$$J_{q,u} \cdot \dot{\mathbf{q}}_\mathbf{u} = J_{x,u} \cdot \dot{\mathbf{x}}_\mathbf{u} \tag{3.7}$$

Where vector $\mathbf{q_u}$ contains the input angles [$\theta1_1$ $\theta1_2$ $\theta1_3$] and vector $\mathbf{x_u}$ contains the output angles [ζ ψ ξ]. Using Equation (3.3) and multiplying it by ($\mathbf{v_i} \times \mathbf{w}_i$), we can eliminate the passive joints $\theta2_i$, $\theta3_i$ and obtain the relation between the inputs and outputs of the system as given in Equation (3.8).

$$\mathbf{u} \cdot (\mathbf{v_i} \times \mathbf{w_i}) \cdot \dot{\theta}1_i = \omega_u \cdot (\mathbf{v_i} \times \mathbf{w_i}) \ , \qquad \text{for } i = 1, 2, 3 \tag{3.8}$$

For simplicity in showing, we define the vectors as;

$$\mathbf{u_i} = \begin{pmatrix} ux_i \\ uy_i \\ uz_i \end{pmatrix}, \mathbf{v_i} = \begin{pmatrix} vx_i \\ vy_i \\ vz_i \end{pmatrix}, \mathbf{w_i} = \begin{pmatrix} wx_i \\ wy_i \\ wz_i \end{pmatrix}, \qquad \text{for } i = 1, 2, 3$$

The results obtained from Equation (3.8) and Equation (3.5) can be written in a relation such as in Equation (3.7) as given in Equation (3.9).

Where:

$$J_{q,\mathrm{u}}=\begin{pmatrix} Jqu_{1,1} & 0 & 0 \\ 0 & Jqu_{2,2} & 0 \\ 0 & 0 & Jqu_{3,3} \end{pmatrix}, \dot{\mathbf{q}}_{\mathbf{u}}=\begin{pmatrix} \dot{\theta 1}_1 \\ \dot{\theta 1}_2 \\ \dot{\theta 1}_3 \end{pmatrix}, J_{x,\mathrm{u}}=\begin{pmatrix} Jxu_{1,1} & Jxu_{1,2} & Jxu_{1,3} \\ Jxu_{2,1} & Jxu_{2,2} & Jxu_{2,3} \\ Jxu_{3,1} & Jxu_{3,2} & Jxu_{3,3} \end{pmatrix}, \dot{\mathbf{x}}_{\mathbf{u}}=\begin{pmatrix} \dot{\xi} \\ \dot{\psi} \\ \dot{\zeta} \end{pmatrix} \quad (3.9)$$

$$Jqu_{1,1}=uz_1\left(-vy_1wx_1+vx_1wy_1\right)+uy_1\left(vz_1wx_1-vx_1wz_1\right)+ux_1\left(-vz_1wy_1+vy_1wz_1\right)$$
$$Jqu_{2,2}=uz_2\left(-vy_2wx_2+vx_2wy_2\right)+uy_2\left(vz_2wx_2-vx_2wz_2\right)+ux_2\left(-vz_2wy_2+vy_2wz_2\right),$$
$$Jqu_{3,3}=uz_3\left(-vy_3wx_3+vx_3wy_3\right)+uy_3\left(vz_3wx_3-vx_3wz_3\right)+ux_3\left(-vz_3wy_3+vy_3wz_3\right),$$

$$Jxu_{1,1}=Jxu_{2,1}=Jxu_{3,1}=$$
$$\cos\varphi\sin\sigma\cdot\left(-vy_1wx_1+vx_1wy_1\right)-\sin\sigma\sin\varphi\cdot\left(vz_1wx_1-vx_1wz_1\right)-\cos\sigma\cdot\left(-vz_1wy_1+vy_1wz_1\right)$$
$$Jxu_{1,2}=Jxu_{2,2}=Jxu_{3,2}=-\left(\cos\sigma\cos\varphi\sin\xi+\cos\xi\sin\varphi\right)\left(-vy_1wx_1+vx_1wy_1\right)-\left(\cos\xi\cos\varphi-\cos\sigma\sin\xi\sin\varphi\right)$$
$$\cdot\left(vz_1wx_1-vx_1wz_1\right)-\sin\xi\sin\sigma\cdot\left(-vz_1wy_1+vy_1wz_1\right)$$
$$Jxu_{1,3}=Jxu_{2,3}=Jxu_{3,3}=-\left(\cos\xi\cos\sigma\cos\varphi\cos\psi-\cos\psi\sin\xi\sin\varphi-\cos\varphi\sin\sigma\sin\psi\right)$$
$$\cdot\left(-vy_1wx_1+vx_1wy_1\right)-\left(-\cos\varphi\cos\psi\sin\xi-\sin\varphi\left(\cos\xi\cos\sigma\cos\psi-\sin\sigma\sin\psi\right)\right)\cdot\left(vz_1wx_1-vx_1wz_1\right)$$
$$-\left(\cos\xi\cos\psi\sin\sigma+\cos\sigma\sin\psi\right)\cdot\left(-vz_1wy_1+vy_1wz_1\right)$$

The Lagrangian Function L is given in Equation (3.10).

$$L=T-V \quad (3.10)$$

Where T is the total Kinetic Energy and V is the total Potential Energy of the system. Kinetic and potential energy of system components are formulated as given in Equation (3.11).

$$\begin{aligned} &\sum_{j=1}^{3}T_{\mathrm{u},linki}=\frac{1}{2}\omega_i^T\cdot I_{u,linki}\cdot\omega_i, \sum_{j=1}^{3}V_{\mathrm{u},linki}=m\cdot g\cdot h_i \\ &T_{\mathrm{u},platform}=\frac{1}{2}\omega_i^T\cdot I_{u,\mathrm{platform}}\cdot\omega_i, V_{\mathrm{u,platform}}=m\cdot g\cdot h_p \end{aligned}\quad , \quad for \quad i=1,2 \quad (3.11)$$

All the parts are selected as a part of hollow cylinder where the general formula for the inertia tensor I is given below in Equation (3.12).

$$I=\begin{pmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{pmatrix}$$
$$Ixx=\int_{\phi 1}^{\phi 2}\int_{r1}^{r2}\int_{h1}^{h2}\rho r\left(r^2(\sin\theta)^2+z^2\right)\mathrm{d}z\mathrm{d}r\mathrm{d}\theta, Iyy=\int_{\phi 1}^{\phi 2}\int_{r1}^{r2}\int_{h1}^{h2}\rho r\left(r^2(\cos\theta)^2+z^2\right)\mathrm{d}z\mathrm{d}r\mathrm{d}\theta, \quad (3.12)$$
$$Izz=\int_{\phi 1}^{\phi 2}\int_{r1}^{r2}\int_{h1}^{h2}\rho r\left(r^2\right)\mathrm{d}z\mathrm{d}r\mathrm{d}\theta, \quad \text{Off-Diagonal Terms}=0$$

Using the Lagrange equation, Equation (3.1), we find a list of λ_i in Equation (3.13) by using $J_{x,u}$ instead of Γ, moments as M_{upper} = [$M\,\zeta$, $M\,\psi$, $M\,\xi$] and vector **q** as $\mathbf{x_u}$ = [$\zeta\ \ \psi\ \ \xi$];

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}_{u,j}}\right)-\frac{\delta L}{\delta x_{u,j}}=M_{upper,j}+\sum_{i=1}^{3}\lambda_i\left(J_{x,u}[j,i]\right),\quad for\ j=1\ to\ 3 \tag{3.13}$$

Torques $\tau = [\tau_1\ \tau_2\ \tau_3]$ are found below in Equation (3.14) by using the found λ_i values in Equation (3.13) and replacing $\left(\frac{\delta\Gamma_i}{\delta q_j}\right)$ by $J_{q,u}$ and vector **q** as $\mathbf{q_u} = [\theta 1_1\ \theta 1_2\ \theta 1_3]$.

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_{u,j}}\right)-\frac{\delta L}{\delta q_{u,j}}=\tau_j+\sum_{i=1}^{3}\lambda_i\left(J_{q,u}[j,\mathrm{i}]\right),\quad for\ j=1\ to\ 3 \tag{3.14}$$

3.2 Bottom Manipulator Dynamic Analysis (3-RRRR)

The angular velocity of the bottom manipulator end effector is found by converting the vector and joint rates for the bottom manipulator by the usage of Equation (3.3), the resultant is given in Equation (3.15).

$$\omega_b=\mathbf{p_i}\cdot\dot{\phi 1}_i+\mathbf{q_i}\cdot\dot{\phi 2}_i+\mathbf{r_i}\cdot\dot{\phi 3}_i \tag{3.15}$$

The rotation of the bottom manipulator platform was found in Equation (2.12) and accordingly, we can find the velocity of the bottom manipulator end effector by Equation (3.16).

$$\omega_b=\frac{\delta R_b}{\delta\varphi}\cdot R_b^{\ T}\cdot\dot{\varphi}+\frac{\delta R_b}{\delta\sigma}\cdot R_b^{\ T}\cdot\dot{\sigma}+\frac{\delta R_b}{\delta\theta 1_i}\cdot R_b^{\ T}\cdot\dot{\theta 1}_i \tag{3.16}$$

To find the Jacobian matrix of this manipulator, we use the same method as the upper part for i = 1 but for i = 2, 3, $\phi 2_i$ will not be eliminated since it is not a passive joint, therefore the respective elimination equations are given in Equation (3.17, 3.18, 3.19).

$$\begin{aligned}&\omega_{b,1}=(\mathbf{p_1}\cdot\dot{\phi 1}_1+\mathbf{q_1}\cdot\dot{\phi 2}_1+\mathbf{r_1}\cdot\dot{\phi 3}_1)\cdot(\mathbf{q_1}\times\mathbf{r_1})\\&\dot{\phi 1}_1\ p_1.\left(q_1\times r_1\right)-\omega_{b,1}.\left(q_1\times r_1\right)=0\end{aligned} \tag{3.17}$$

$$\begin{aligned}&\omega_{b,2}=(\mathbf{p_2}\cdot\dot{\phi 1}_2+\mathbf{q_2}\cdot\dot{\phi 2}_2+\mathbf{r_2}\cdot\dot{\phi 3}_2)\cdot(\mathbf{r_2})\\&\dot{\phi 1}_2\cdot\left(p_2\times r_2\right)+\dot{\phi 2}_2\cdot\left(q_2\times r_2\right)-\omega_{b,2}\times r_2=0\end{aligned} \tag{3.18}$$

$$\begin{aligned}&\omega_{b,3}=(\mathbf{p_3}\cdot\dot{\phi 1}_3+\mathbf{q_3}\cdot\dot{\phi 2}_3+\mathbf{r_3}\cdot\dot{\phi 3}_3)\cdot(\mathbf{r_3})\\&\dot{\phi 1}_3\cdot\left(p_3\times r_3\right)+\dot{\phi 2}_3\cdot\left(q_3\times r_3\right)-\omega_{b,3}\times r_3=0\end{aligned} \tag{3.19}$$

Equation (3.17) gives one equation and Equations (3.18, 3.19) give six equations, we chose the one from Equation (3.17) and four of Equations (3.18, 3.19) to find the Jacobian matrix. The Jacobian matrix relation, with simple vector assignments are given in Equation (3.20).

$$J_{q,\mathrm{b}} \cdot \dot{\mathbf{q}}_{\mathbf{b}} = J_{x,\mathrm{b}} \cdot \dot{\mathbf{x}}_{\mathbf{b}} \tag{3.20}$$

For simplicity, we define the vectors as:

$$\mathbf{p_i} = \begin{pmatrix} px_i \\ py_i \\ pz_i \end{pmatrix}, \mathbf{q_i} = \begin{pmatrix} qx_i \\ qy_i \\ qz_i \end{pmatrix}, \mathbf{r_i} = \begin{pmatrix} rx_i \\ ry_i \\ rz_i \end{pmatrix}, \quad \text{for } i = 1,2,3$$

The open form of Jacobian matrix presentations are given in Equation (3.21).

$$J_{\mathrm{q,b}} = \begin{pmatrix} Jqb_{1,1} & 0 & 0 & 0 & 0 \\ 0 & Jqb_{2,2} & Jqb_{2,3} & 0 & 0 \\ 0 & Jqb_{3,2} & Jqb_{3,3} & 0 & 0 \\ 0 & 0 & 0 & Jqb_{4,4} & Jqb_{4,5} \\ 0 & 0 & 0 & Jqb_{5,4} & Jqb_{5,5} \end{pmatrix}, \dot{\mathbf{q}}_{\mathbf{b}} = \begin{pmatrix} \dot{\phi 1}_1 \\ \dot{\phi 1}_2 \\ \dot{\phi 2}_2 \\ \dot{\phi 1}_3 \\ \dot{\phi 2}_3 \end{pmatrix}$$

$$J_{x,\mathrm{b}} = \begin{pmatrix} Jxb_{1,1} & Jxb_{1,2} & Jxb_{1,3} & 0 & 0 \\ 0 & Jxb_{2,2} & Jxb_{2,3} & 0 & 0 \\ Jxb_{3,1} & Jxb_{3,2} & Jxb_{3,3} & 0 & 0 \\ 0 & Jxb_{4,2} & Jxb_{4,3} & 0 & 0 \\ Jxb_{5,1} & Jxb_{5,2} & Jxb_{5,3} & 0 & 0 \end{pmatrix}, \dot{\mathbf{x}}_{\mathbf{b}} = \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\sigma}_1 \\ \dot{\theta 1}_1 \\ \dot{\theta 1}_2 \\ \dot{\theta 1}_3 \end{pmatrix} \tag{3.21}$$

Where:

$$Jqb_{1,1} = pz_1\left(-qy_1rx_1 + qx_1ry_1\right) + py_1\left(qz_1rx_1 - qx_1rz_1\right) + px_1\left(-qz_1ry_1 + qy_1rz_1\right)$$
$$Jqb_{2,2} = -pz_2ry_2 + py_2rz_2,\ Jqb_{2,3} = -qz_2ry_2 + qy_2rz_2,\ Jqb_{3,2} = pz_2rx_2 - px_2rz_2,$$
$$Jqb_{3,3} = qz_2rx_2 - qx_2rz_2,\ Jqb_{4,4} = -pz_3ry_3 + py_3rz_3,\ Jqb_{4,5} = -qz_3ry_3 + qy_3rz_3,$$
$$Jqb_{5,4} = pz_3rx_3 - px_3rz_3,\ Jqb_{5,5} = qz_3rx_3 - qx_3rz_3$$

$$Jxb_{1,1} = qz_1ry_1 - qy_1rz_1,\ Jxb_{3,1} = rz_2,\ Jxb_{5,1} = rz_3,\ Jxb_{1,2} = -\sin\varphi\left(-qy_1rx_1 + qx_1ry_1\right) - \cos\varphi\left(qz_1rx_1 - qx_1rz_1\right),$$
$$Jxb_{2,2} = \sin\varphi ry_2 - \cos\varphi rz_2,\ Jxb_{3,2} = -\sin\varphi rx_2,\ Jxb_{4,2} = \sin\varphi ry_3 - \cos\varphi rz_3,\ Jxb_{5,2} = -\sin\varphi rx_3$$
$$Jxb_{1,3} = -\cos\sigma\cos\varphi\left(-qy_1rx_1 + qx_1ry_1\right) + \cos\sigma\sin\varphi\left(qz_1rx_1 - qx_1rz_1\right) - \sin\sigma\left(-qz_1ry_1 + qy_1rz_1\right)$$
$$Jxb_{2,3} = \cos\sigma\cos\varphi ry_2 + \cos\sigma\sin\varphi rz_2,\ Jxb_{3,3} = -\cos\sigma\cos\varphi rx_2 + \sin\sigma rz_2$$
$$Jxb_{4,3} = \cos\sigma\cos\varphi ry_3 + \cos\sigma\sin\varphi rz_3,\ Jxb_{5,3} = -\cos\sigma\cos\varphi rx_3 + \sin\sigma rz_3$$

The kinetic and potential energy of the links and the platform in the bottom manipulator are found by the equations given in Equation (3.22) where Equation (3.11) can be applied to this equation with the appropriate change of boundaries.

$$\sum_{j=1}^{3} T_{\mathrm{b},legj,linki} = \frac{1}{2}\omega b_i^T \cdot I_{b,\mathrm{link}i} \cdot \omega b_i, \sum_{j=1}^{3} V_{\mathrm{b,legj},linki} = m \cdot g \cdot h_i$$
$$\sum_{j=1}^{3} T_{\mathrm{b},platformi} = \frac{1}{2}\omega b_i^T \cdot I_{b,platformi} \cdot \omega b_i, \sum_{j=1}^{3} V_{\mathrm{b,platformi}} = m \cdot g \cdot h_i \quad , \quad for \quad i = 1,2 \qquad (3.22)$$

Afterwards, by the usage of Equation (3.1), we find list of λb_i in Equation (3.23) by using $J_{x,b}$ as Γ, moments as M_{bottom} = [$M\varphi$ $M\sigma$ $M\theta 1_1$ $M\theta 1_2$ $M\theta 1_3$] and by replacing vector **q** by **x_b** = [φ σ $\theta 1_1$ $\theta 1_2$ $\theta 1_3$].

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}_{\mathrm{b},j}}\right) - \frac{\delta L}{\delta x_{b,j}} = M_{bottom,j} + \sum_{i=1}^{5} \lambda b_i \left(J_{x,b}[j,\mathrm{i}]\right), \quad for\ j = 1\ to\ 5 \qquad (3.23)$$

Torques τb = [τb_1 τb_2 τb_3 τb_4 τb_5] are found below in Equation (3.24) by using the found λb_i values from Equation (3.23), replacing $J_{q,b}$ as Γ and vector **q** by **q_b** = [$\phi 1_1$ $\phi 1_2$ $\phi 2_2$ $\phi 1_3$ $\phi 2_3$].

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_{b,j}}\right) - \frac{\delta L}{\delta q_{b,j}} = \tau b_j + \sum_{i=1}^{5} \lambda b_i \left(J_{q,b}[j,\mathrm{i}]\right), \quad for\ j = 1\ to\ 5 \qquad (3.24)$$

4. DESIRED MOTIONS & MECHANISM WORKSPACE ANALYSIS

The mechanism, consisting of spherical manipulators, reveals two spherical surfaces where the manipulator can move in. The first space is centred at the shoulder joint and the second is centred at the elbow joint. Workspace should be defined with respect to the range of motion of the arm given in Figure 3 and for the motions required by the rehabilitation exercises. The main aim in designing this manipulator is to help patients with passive rehabilitation activities and help decrease the working load of physiotherapists. Some of the passive exercises to regain the range of motion of the upper extremity can be found in the University Of Miami Miller School Of Medicine Library RehabTeamSite [29]. To define the motions more clearly, the reference medical planes will be shown in Figure 9.

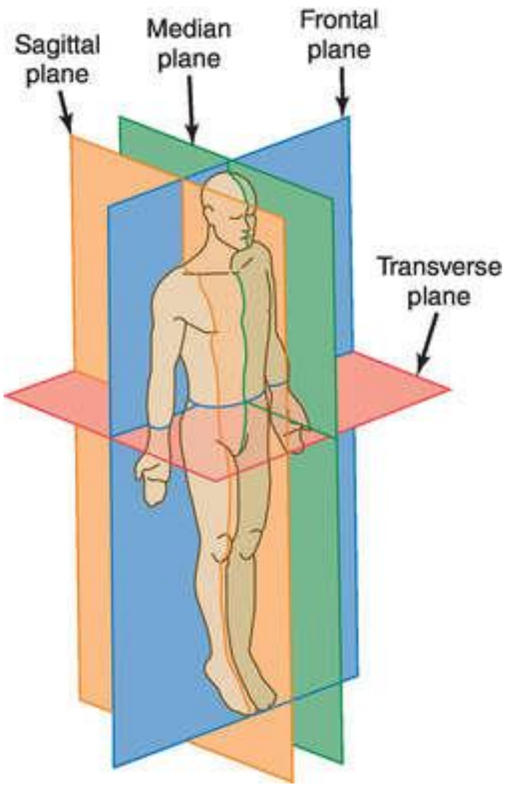


Figure 9. The Medical Reference Planes of Human Body, courtesy of Medical Dictionary the free dictionary. [30]

Shoulder flexion and extension exercise, in which the arm rests on the frontal plane and is moved from the side of the body over the head, is shown in Figure 10. Shoulder abduction and adduction exercise, focusing on pulling the arm away from the body as much as possible on a parallel to the transverse plane, is shown in Figure 11. Shoulder internal and external rotation, achieved by positioning the upper arm at shoulder position (on the frontal plane) and turning the arm towards the transverse plane, is shown in Figure 12. Note that in these exercises the shoulder joint must be stabilized and the actions should be done slowly to prevent injury.

Figure 10. Shoulder Flexion and Extension, courtesy of RehabTeamSite. [29]

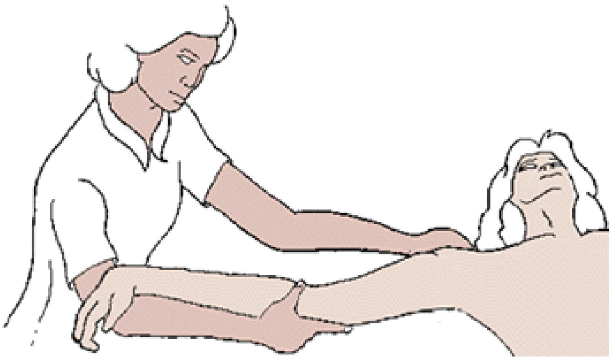


Figure 11. Shoulder Abduction and Adduction, courtesy of RehabTeamSite. [29]

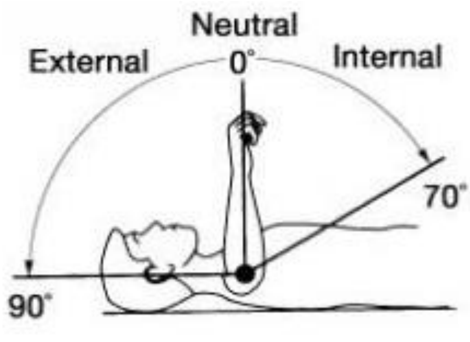


Figure 12. Shoulder Internal and External Rotation., courtesy of CSMI solutions. [31]

Elbow flexion and extension exercise, where the upper arm is positioned in the frontal plane and the elbow is bent so that the hand touches the shoulder, can be seen in Figure 13. Forearm supination and pronation, where the forearm is twisted so that the hand faces the ceiling then the ground, is shown in Figure 14.

Figure 13 Elbow Flexion and Extension, courtesy of RehabTeamSite. [29]

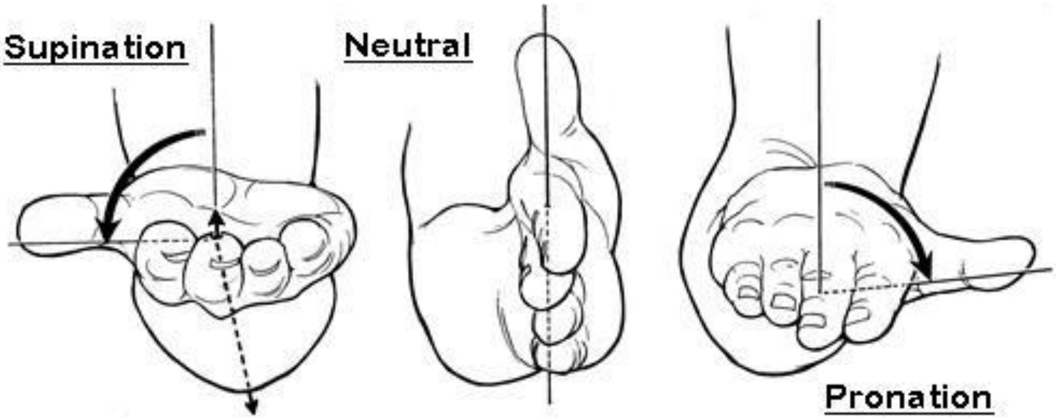


Figure 14 Forearm Supination and Pronation, courtesy of studyblue. [32]

These exercises require the usage of maximum limits of the upper extremity motions and are harder to accomplish with parallel mechanisms because of the possible singularity occurrences (workspace limitations).

The same methodology used for the kinematic – dynamic calculations is conducted while finding the workspace, the manipulator is again considered as two parts separated by imaginary joints. The main goal of the workspace analysis will not focus on the passive exercises directly, but will try to achieve the highest possible ROM for the given rotational motions.

4.1 Workspace of the Upper Manipulator

The upper manipulator is a 3 DOF manipulator where the motion of the elbow pronation and supination, flexion and extension and lateral/medial rotation (or shoulder internal and external rotation) are defined. The platform position of the mechanism was found in chapter 1 by the use of ZYZ rotation sequence but this definition hinders the workspace finding process because of its complicated positioning. A different approach in applying 3 rotations to the system can be done by taking the first two rotations as the vector position in spherical coordinates and the third rotation as the rotation around the found vector. The drawback of this method is that, because the conversion process contains trigonometric identities such as *arctan,* therefore singularities occur in specifying the position. A simpler way to define the platform position is by applying the Euler-Rodriguez formula [26]. The rotation matrix, to rotate a vector in the direction of **s** vector with an angle of Φ, *Rot(***s**, Φ*)* is given below in Equation (4.1):

$$\mathbf{s} = \begin{pmatrix} sx \\ sy \\ sz \end{pmatrix}, \qquad Rot(\mathbf{s},\Phi) = \cos\Phi I + \sin\Phi[\mathbf{s}]_\times + (1-\cos\Phi)\mathbf{s}\otimes\mathbf{s}$$

$$where \quad \mathbf{s}\otimes\mathbf{s} = \begin{pmatrix} sx^2 & sxsy & sxsz \\ sxsy & sy^2 & sysz \\ sxsz & sysz & sz^2 \end{pmatrix} \quad and \quad [\mathbf{s}]_\times = \begin{pmatrix} 0 & -sz & sy \\ sz & 0 & -sx \\ -sy & sx & 0 \end{pmatrix} \tag{4.1}$$

$$Rot(\mathbf{s},\Phi) = \begin{pmatrix} \cos\Phi + s_x^2(1-\cos\Phi) & s_x s_y(1-\cos\Phi) - s_z\sin\Phi & s_x s_z(1-\cos\Phi) + s_y\sin\Phi \\ s_x s_y(1-\cos\Phi) + s_z\sin\Phi & \cos\Phi + s_y^2(1-\cos\Phi) & s_y s_z(1-\cos\Phi) - s_x\sin\Phi \\ s_x s_z(1-\cos\Phi) - s_z\sin\Phi & s_y s_z(1-\cos\Phi) + s_x\sin\Phi & \cos\Phi + s_z^2(1-\cos\Phi) \end{pmatrix}$$

Firstly, by the use of the angles ζ and ψ, a rotation matrix is formed the same way it is done in Equation (4.1). The obtained equation can define a vector in a spherical space but the rotation around this vector is necessary for rehabilitation purposes. The final rotation matrix Rot_e and the resultant end point vector **Ori** is defined below in Equation (4.2).

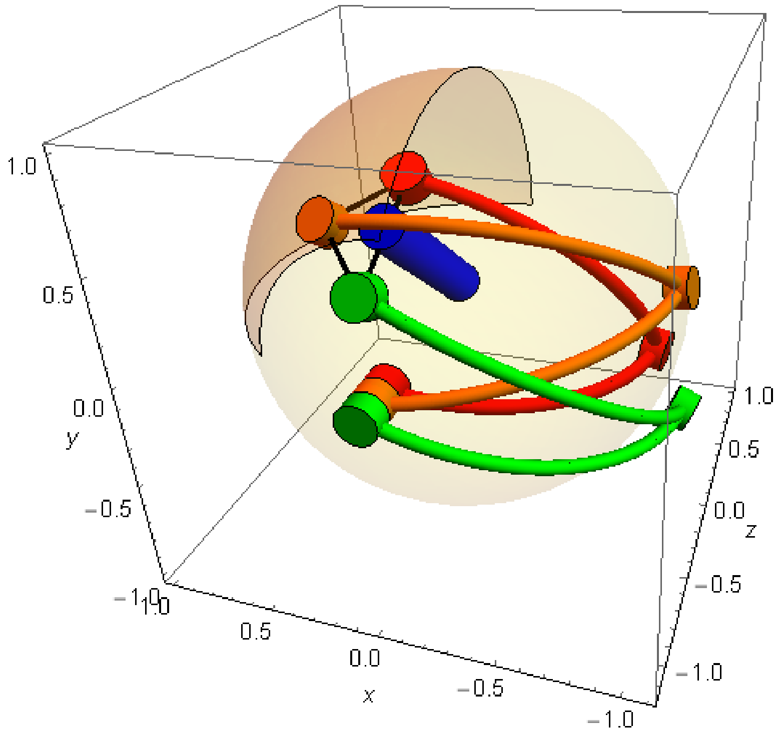
$$\begin{gathered} \mathbf{u} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \mathbf{vp} = \begin{pmatrix} -\sin\zeta \\ \cos\zeta \\ 0 \end{pmatrix} \quad \mathbf{zz} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ r1 = Rot(\mathbf{vp},\psi) \quad r2 = Rot(\mathbf{zz},\xi) \\ Rot_e = r1\cdot r2 \\ \mathbf{Ori} = Rot_e \cdot u \end{gathered} \tag{4.2}$$

The rest of the kinematic procedure continues to stay the same as shown in Chapter 1.

A simple way to show the motion done by the manipulator is to apply the kinematic equations to form a GUI (Graphical User Interface) and manipulate the mechanism. To determine whether the given positions are viable in terms of singularity and dexterity the determinant of the obtained Jacobian matrices will be used. Through experimenting, a workspace that is plausible enough to accomplish the upper manipulator motions are observed with the following angles:

$$\begin{array}{ll} \textit{Leg Angles}: & \textit{Platform Angles}: \\ \alpha 1_i = 85^\circ \quad \alpha 2_i = 95^\circ & \eta_1 = 0^\circ \ \eta_2 = 90^\circ \ \eta_3 = 0^\circ \\ \textit{for } i = 1,2,3 & \chi_1 = 15^\circ \ \chi_2 = 15^\circ \ \chi_3 = -15^\circ \end{array}$$

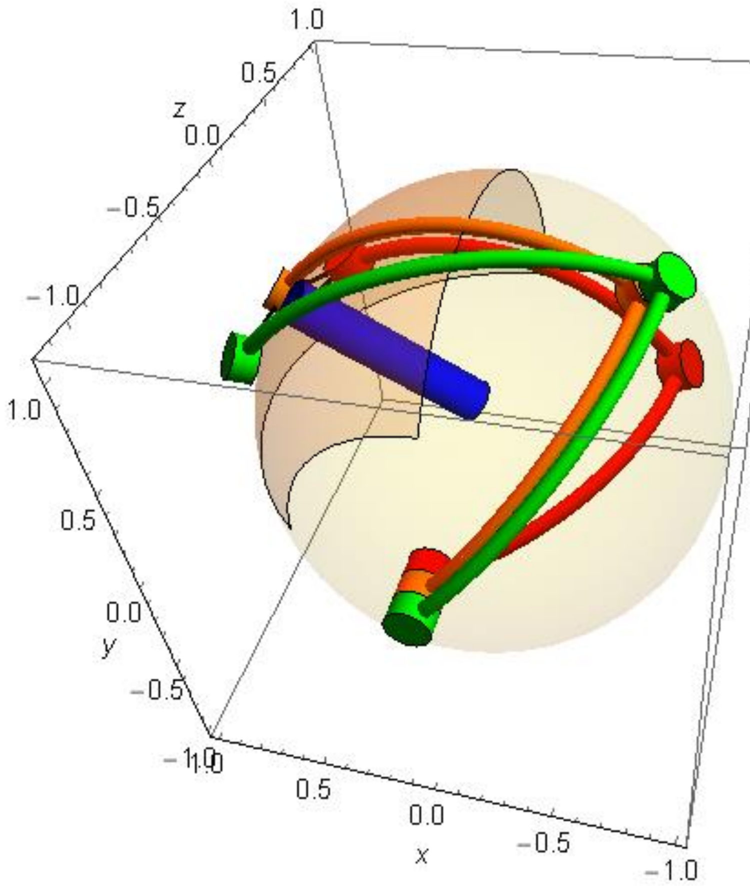
In the figures below, the blue arrow represents the vector from the elbow joint to forearm, a black triangle platform is formed by the usage of the platform angles given above and the desired workspace (ROM of the elbow) is specified by an orange surface on the sphere. Every figure will have a representation on a human drawing so that the position is easily understood.



$$\textit{Det}[\text{Jacobian}] = 0.4452$$

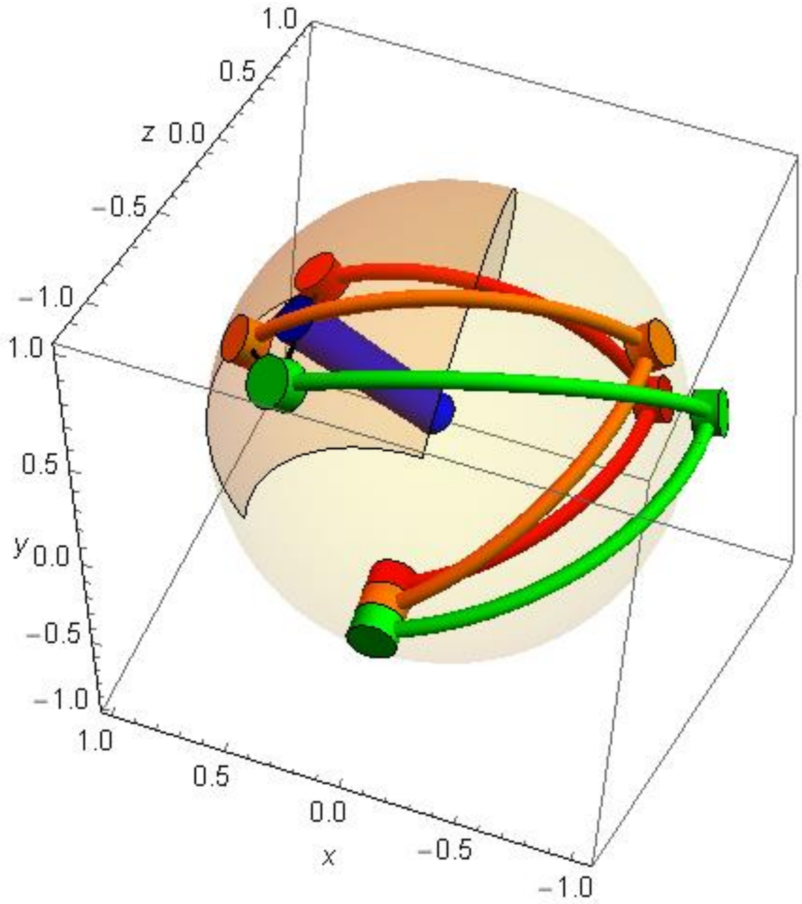
Figure 15. The Upper Manipulator Shown, 140° Flexion, 0° Elbow Pronation and medial rotation

Note that the Origin of the given sphere in the figure above represents the elbow joint, +Z axes refers to the forearm while it is on the Sagittal plane, +X is parallel to the Frontal plane and is towards the head and +Y is horizontal to the transverse plane. The determinant of the Jacobian Matrix is given to see whether the given motion is close to a singular position or not. From observation of the GUI, maximal limits of elbow flexion of 150° and shoulder internal/external rotation of 90° can be obtained. Pronation/supination examples are given below:



$$Det[\text{Jacobian}] = 0.3051$$

Figure 16. Upper Manipulator at 115° Elbow Flexion and 40° Shoulder Internal Rotation and 0° Forearm Pronation/supination



$$Det[\text{Jacobian}] = 0.0581$$

Figure 17. Upper Manipulator at 115° Elbow Flexion and 40° Shoulder Internal Rotation and 40° Forearm Pronation

4.2 Workspace of the Bottom Manipulator

The bottom manipulator holds two rotations and they relate to the flexion/extension and horizontal abduction/adduction of the shoulder. In Figure (18), the desired area and the undesired area are given with respect to the axes.

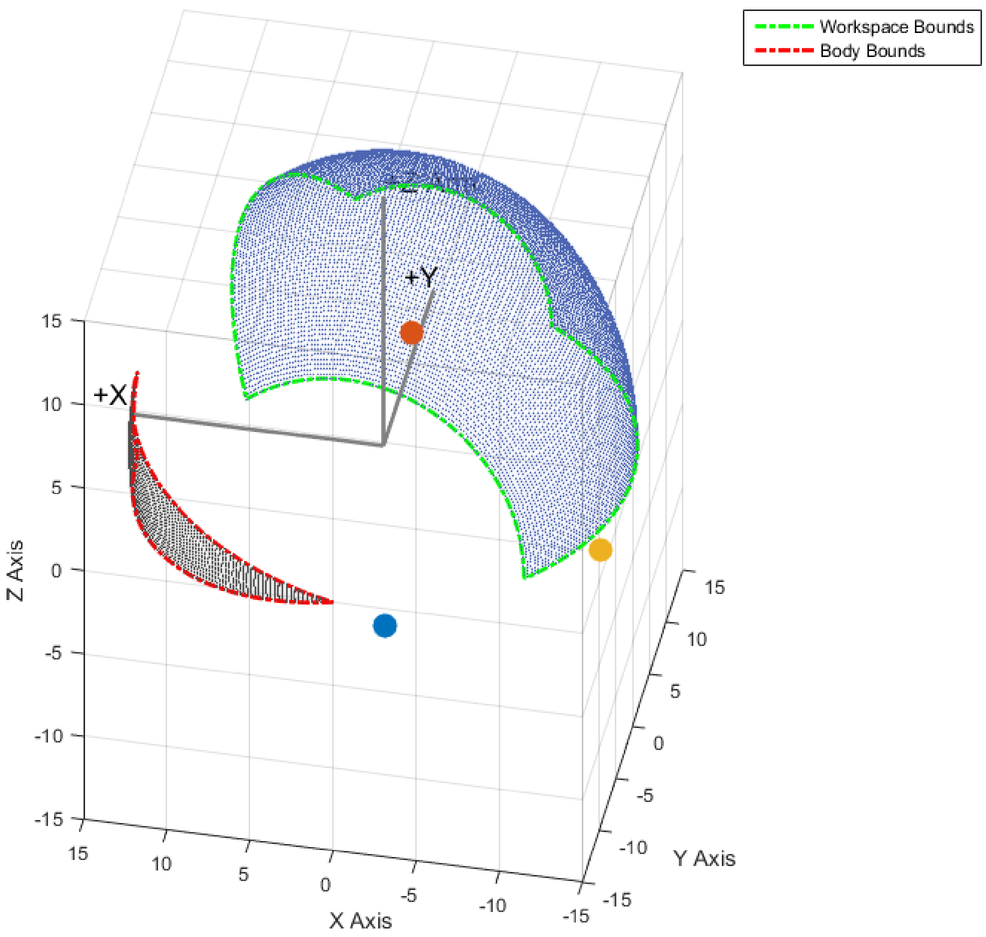
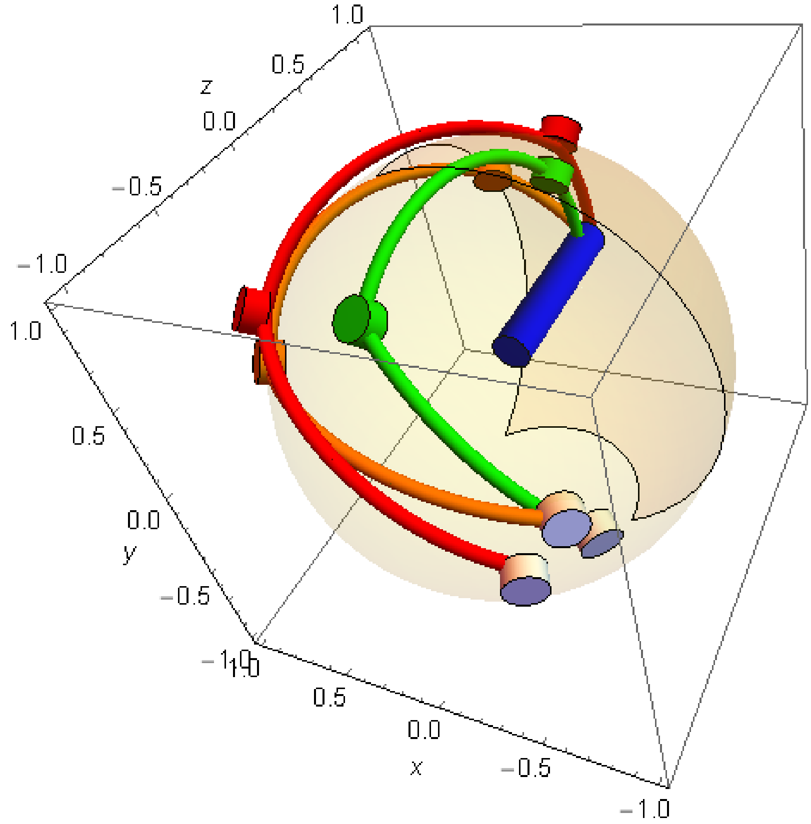


Figure 18. The Bottom Manipulator Workspace Definition

Note that the Origin of the given axes in the figure above represents the shoulder joint, +Z axes refers to the arm while it is on the Sagittal plane, +X is parallel to the Frontal plane and is towards the head and +Y is horizontal to the transverse plane. Finding values for the bottom manipulator to obtain the desired workspace is harder to achieve by the graphical representation mentioned. The reason for the complexity is that the bottom manipulator takes inputs from the upper manipulator and the area required for the shoulder ROM is bigger than the elbow. The same method used in the upper manipulator is used here and the main goal is to see whether it can achieve the workspace requirements of some passive rehabilitation exercises.

Experimenting shows that the mechanism can reach the intended positions by changing the ground points. Figures were given below to see whether the mechanism can be modified such that it can achieve the ROM of human arm or be capable of forming some of the basic exercise movements.



$Det[\text{Jacobian}] = -82.5281$

Figure 19. Upper arm on the Sagittal Plane,0° from the transverse plane

The grey cylinders represent the ground joints for the three legs. This is a sample configuration which aims to show a shoulder flexion exercise. The ground joint and link angles used for the mechanism above is given below, while the upper manipulator is also at the same configuration as the bottom manipulator:

Link Angles:

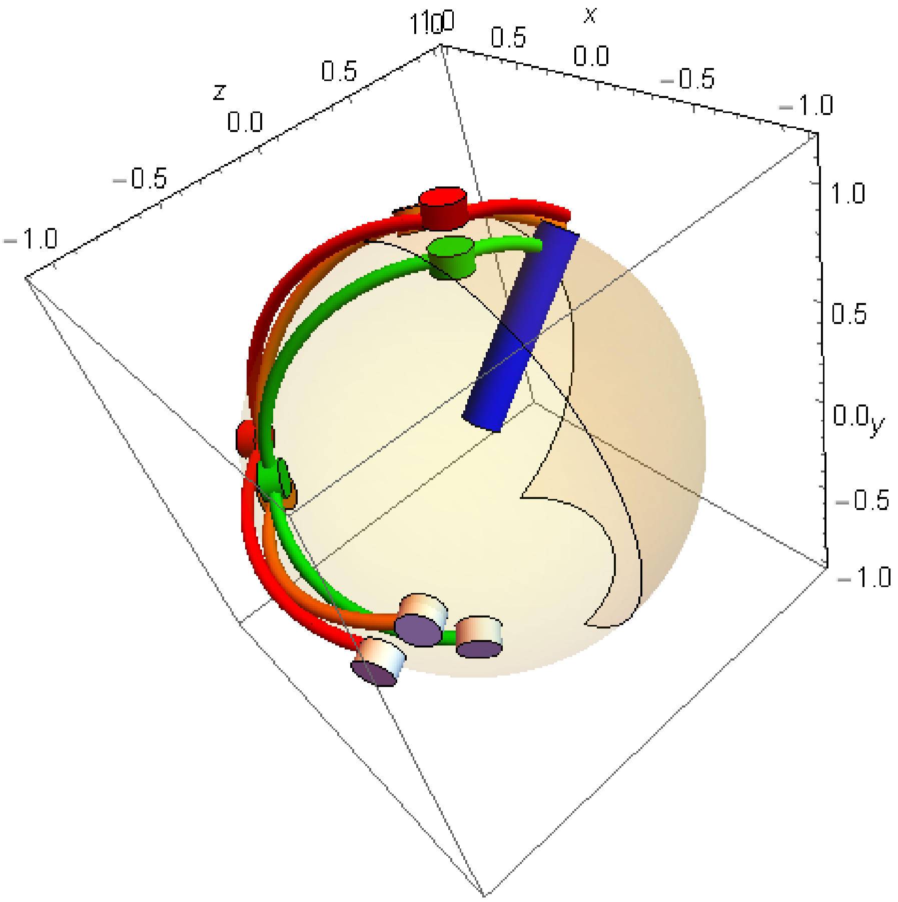
$\gamma 1_i = 85°$ $\gamma 2_i = 95°$ $\gamma 3_i = 35°$

for $i = 1, 2, 3$

Ground Joint Angles:

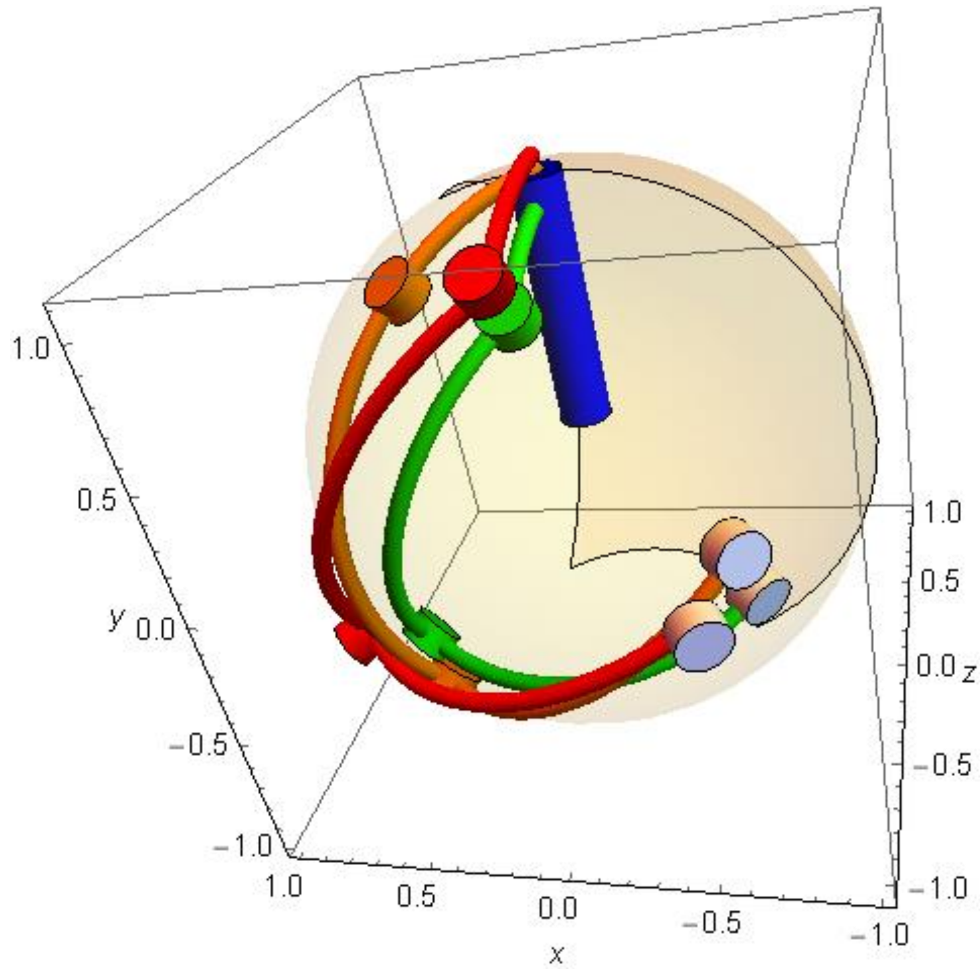
$\kappa_1 = 110°$ $\kappa_2 = 85°$ $\kappa_3 = 100°$

$\varepsilon_1 = -130°$ $\varepsilon_2 = -140°$ $\varepsilon_3 = -150°$



$Det[\text{Jacobian}] = -1.0843$

Figure 20. Upper arm on the Sagittal Plane, 45° from the Transverse Plane



$Det[\text{Jacobian}] = -0.1267$

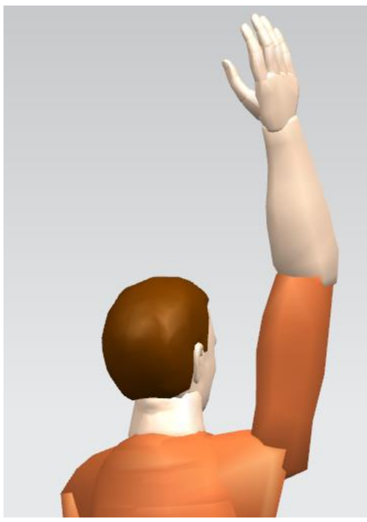
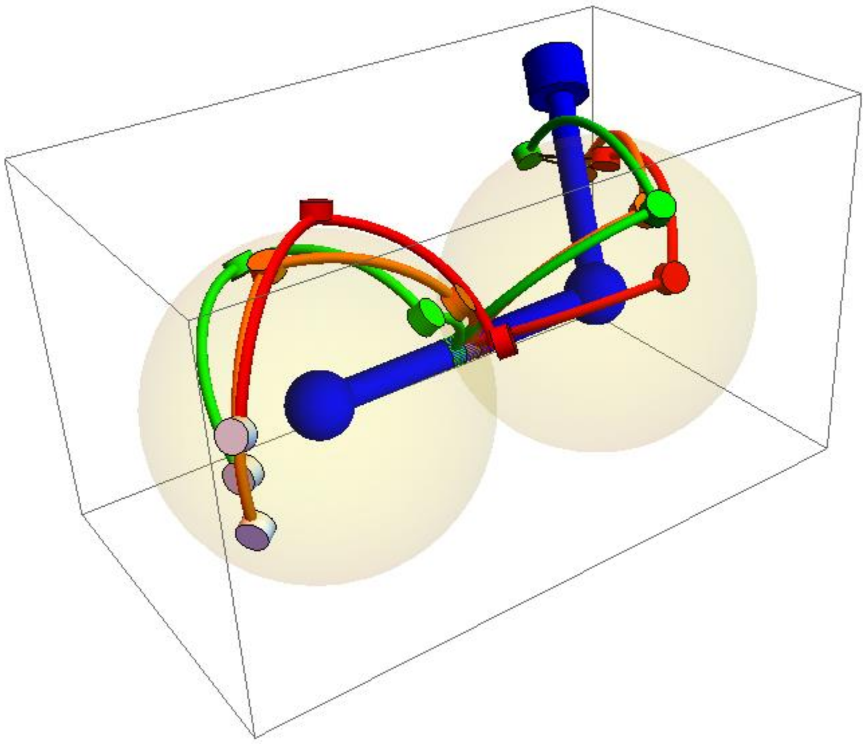


Figure 21. Upper arm on the Sagittal Plane, 90° from the Transverse Plane

A movement from parallel of the transverse plane to the perpendicular (upwards direction) to the transverse plane is achieved in Figures (19-21).

4.3 Results of Workspace Analysis

In this section, the same graphical method mentioned in the subchapters above (4.1 – 4.2) will be used for both the upper and bottom manipulator in the same space. Due to number of variables the calculation to find the biggest possible workspace has not been done, instead formations are found for some of the passive exercises used for the shoulder and elbow rehabilitation. The figures below (24-26) show the whole system while conducting the shoulder flexion/extension exercise.



Upper Manipulator	*Bottom Manipulator*
$Det[\text{Jacobian}] = 0.3644$	$Det[\text{Jacobian}] = -0.3052$

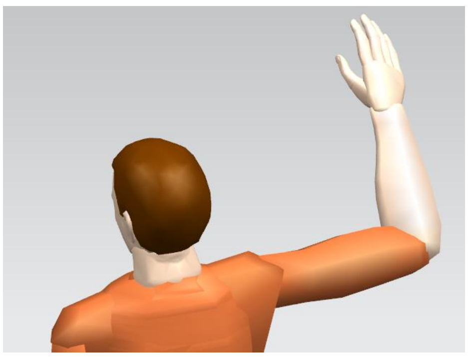


Figure 22. Mechanism Doing Shoulder Flexion/Extension, mode 1

The upper arm is on the frontal plane and the angles are represented as modes, which are 0°, 45°,90° from the transverse plane, the elbow joint is at 45° flexion, 70° abduction. 90° + $\theta 1_i$ is used instead of $\theta 1_i$ for the bottom manipulator so that a solid object can be formed between the joints of the upper and bottom manipulator when the imaginary joints are removed.

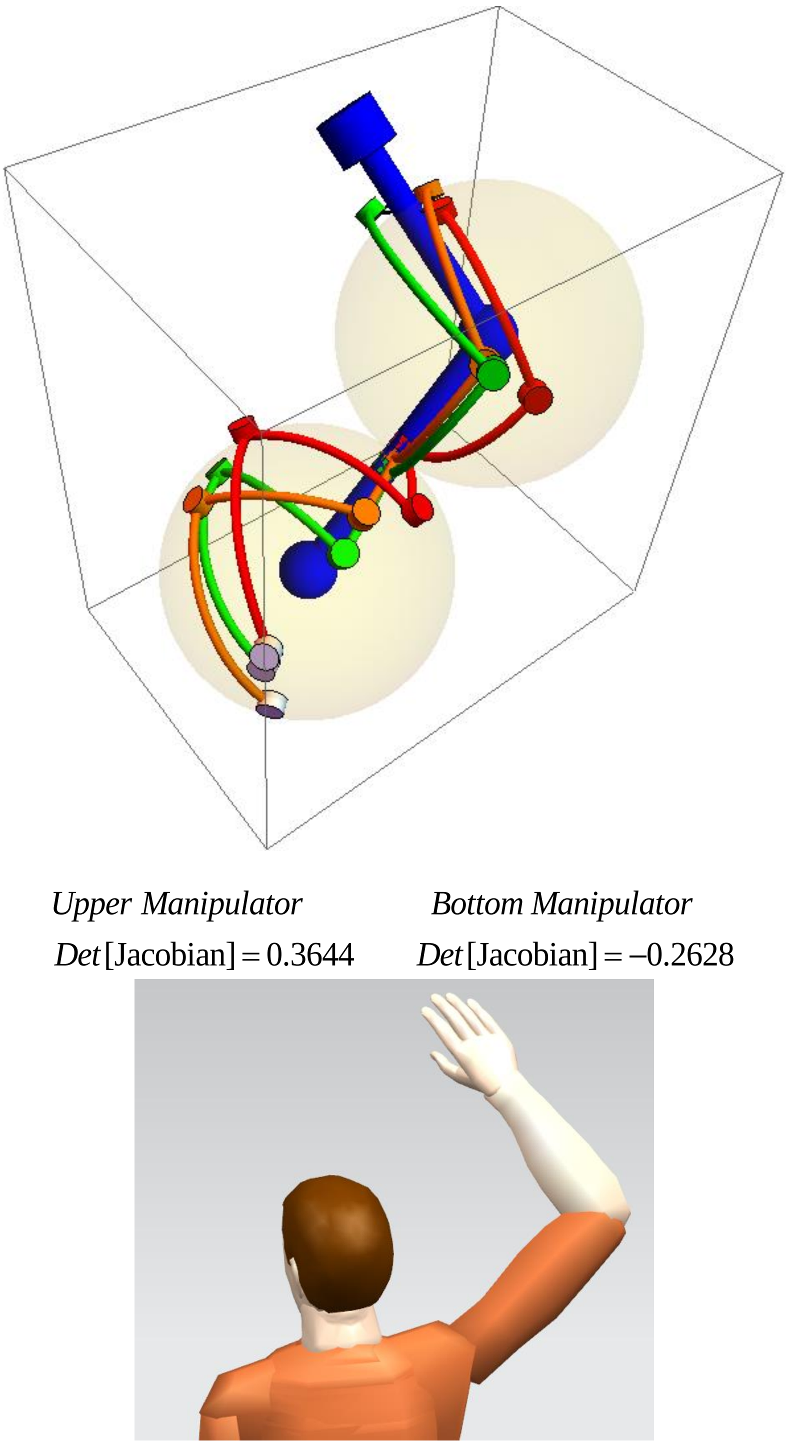
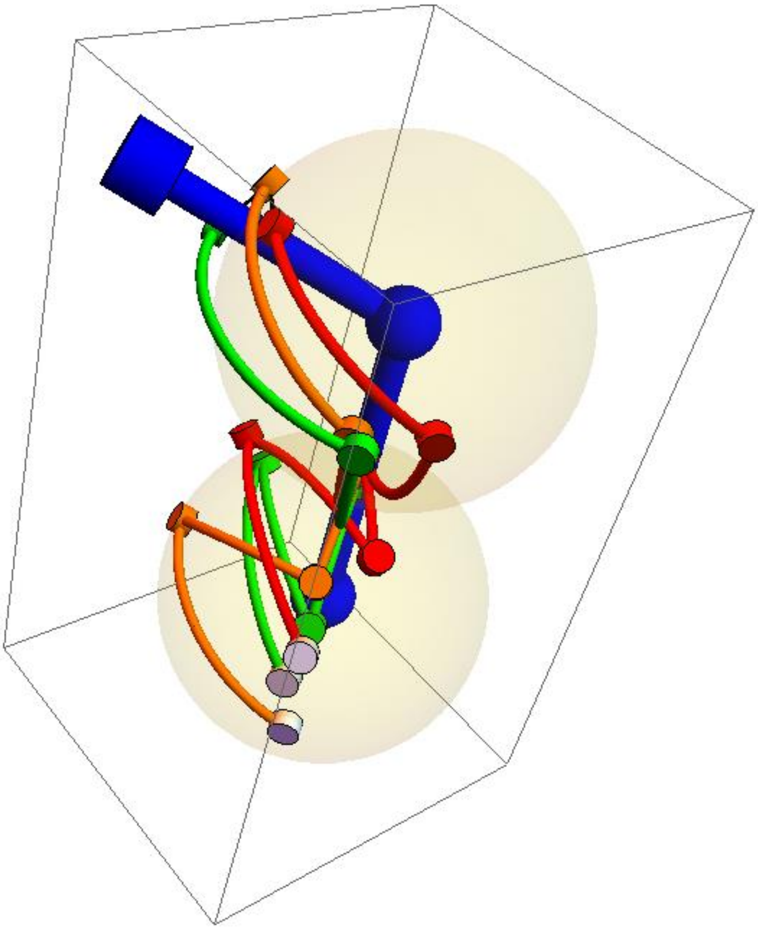


Figure 23. Mechanism Doing Shoulder Flexion/Extension, mode 2



Upper Manipulator *Bottom Manipulator*

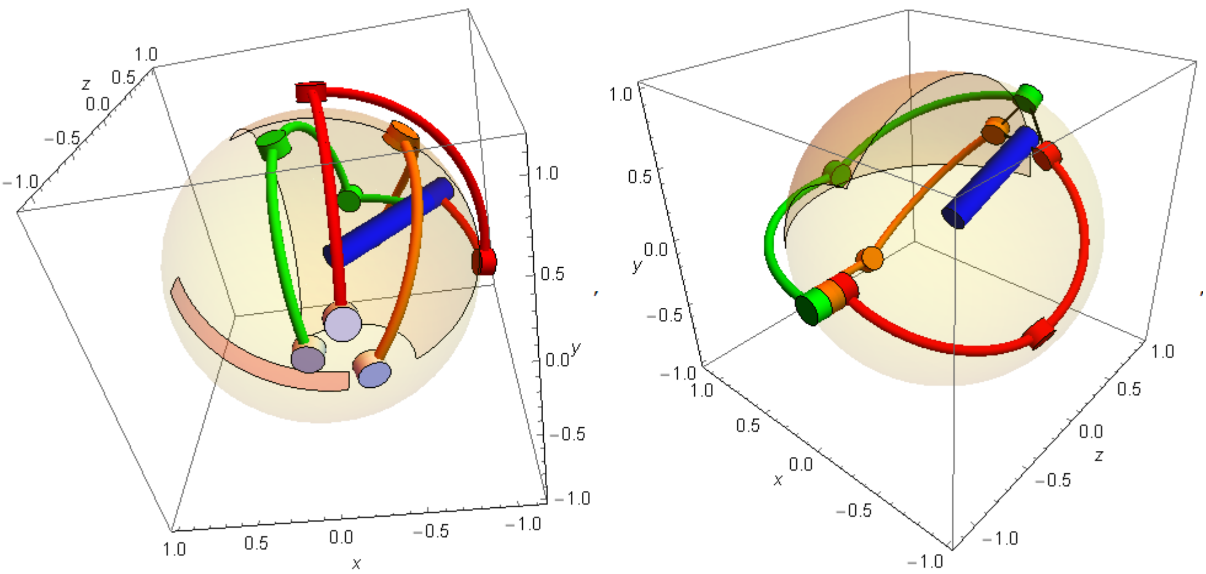
$Det[\text{Jacobian}] = 0.3644$ $Det[\text{Jacobian}] = -0.3052$

Figure 24 Mechanism Doing Shoulder Flexion/Extension, mode 3

Most of the exercises mentioned before can be used on the scapular plane, which is an important area for the upper extremity rehabilitation cases. To show that the mechanism as a whole is capable of doing the mentioned exercises without the need of adjusting, best values for the upper manipulator workspace were applied by trial and error. The selected inputs for the bottom manipulator are given below:

$$\begin{array}{ll} \textit{Leg Angles}: & \textit{Base Platform Angles}: \\ \gamma 1_i = 85^\circ \quad \gamma 2_i = 95^\circ \quad \gamma 3_i = 35^\circ & \kappa_1 = 120^\circ \; \kappa_2 = 110^\circ \; \kappa_3 = 220^\circ \\ \textit{for} \quad i = 1,2,3 & \varepsilon_1 = -180^\circ \; \varepsilon_2 = -160^\circ \; \varepsilon_3 = -200^\circ \end{array}$$

The exercises conducted by the manipulator with the above inputs are given below, the manipulator on the left side of the figure shows the bottom manipulator and the right side shows the upper manipulator.



Upper Manipulator $\quad$ *Bottom Manipulator*

$Det[\text{Jacobian}] = -21.41 \quad Det[\text{Jacobian}] = -0.6783$

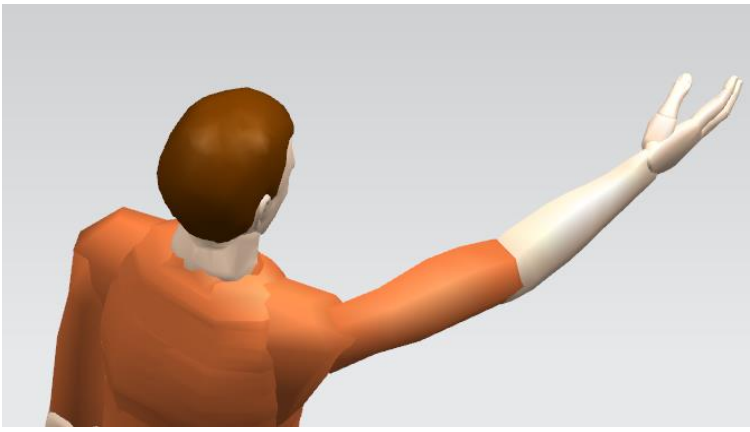
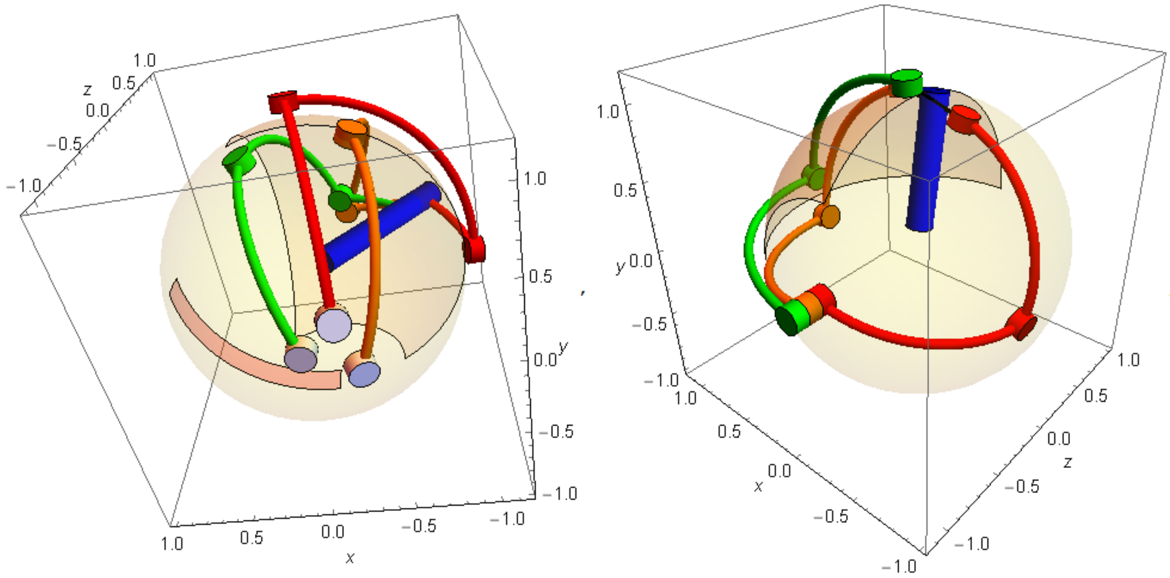


Figure 25. Mechanism Doing Elbow Flexion/Extension, 20° from the Transverse Plane



Upper Manipulator *Bottom Manipulator*

$Det[\text{Jacobian}] = -0.3736$ $Det[\text{Jacobian}] = -0.1778$

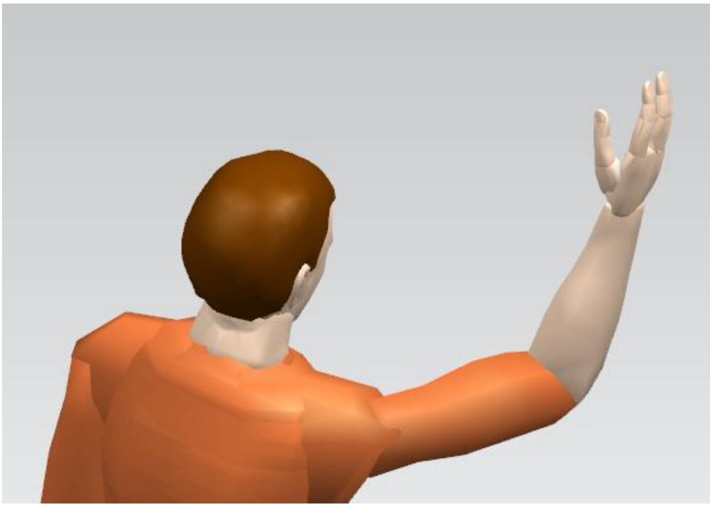
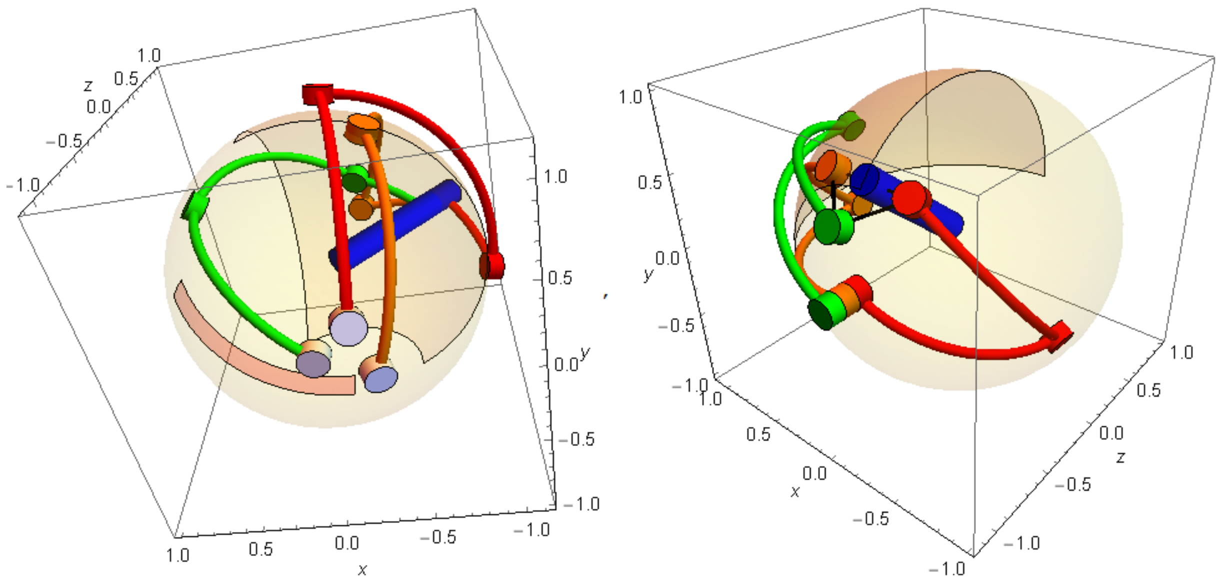


Figure 26. Mechanism Doing Elbow Flexion/Extension, 80° from the Transverse Plane



Upper Manipulator *Bottom Manipulator*

$Det[\text{Jacobian}] = 5.4828$ $Det[\text{Jacobian}] = -0.2570$

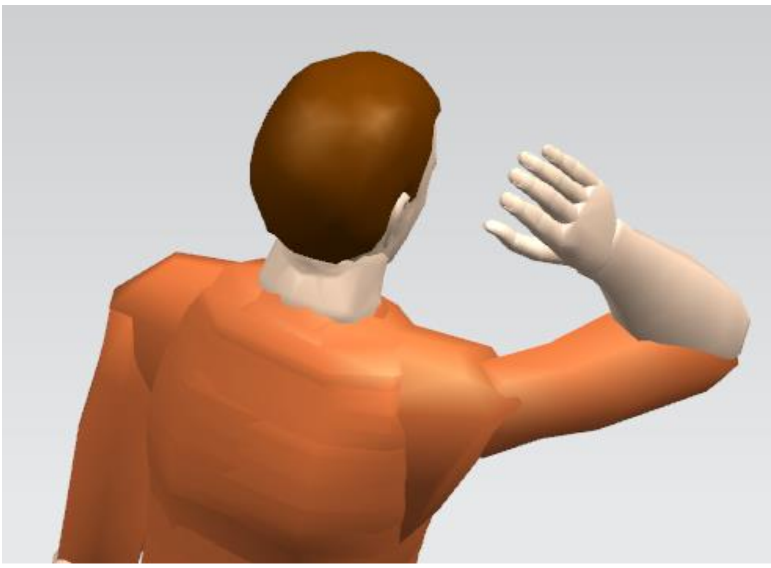
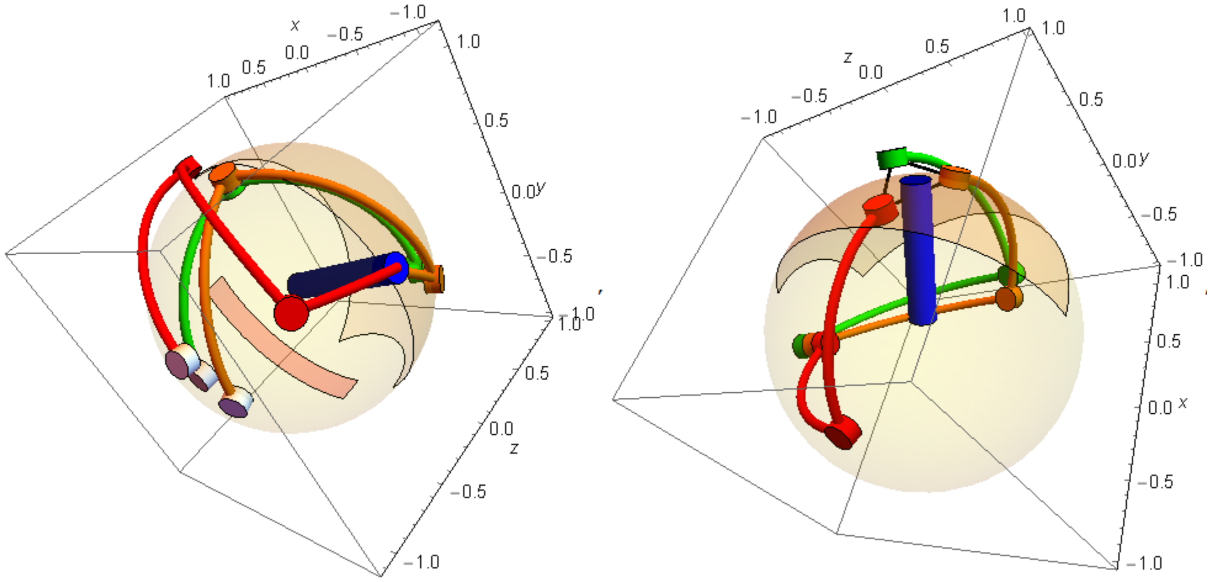


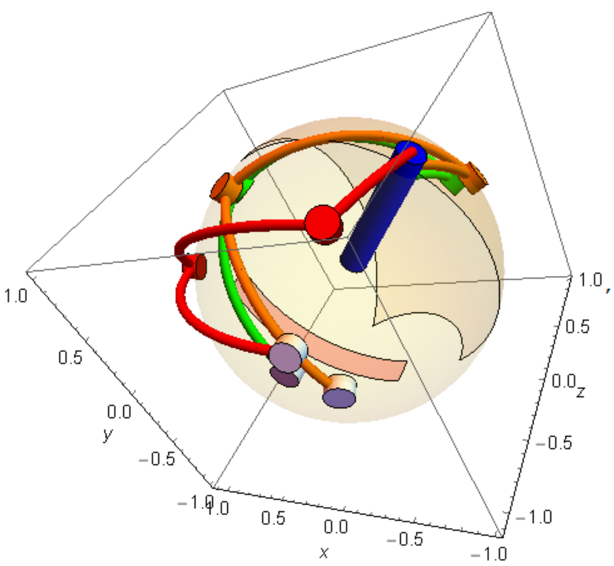
Figure 27. Mechanism Doing Elbow Flexion/Extension, 140° from the Transverse Plane

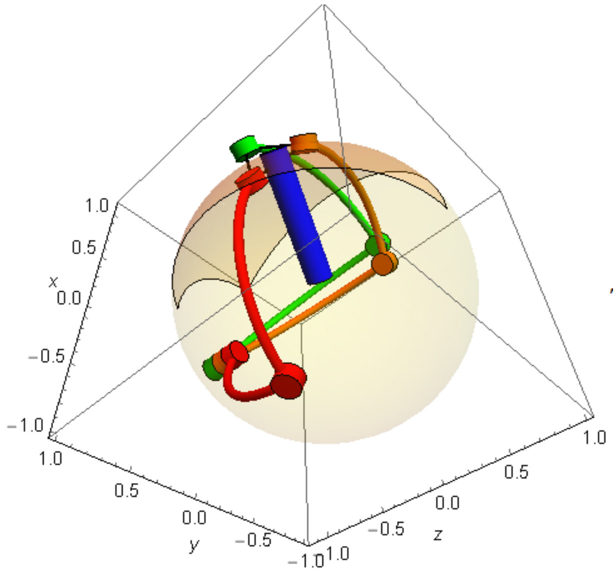


Upper Manipulator *Bottom Manipulator*

$Det[\text{Jacobian}] = 2.1313$ $Det[\text{Jacobian}] = -0.012$

Figure 28. Mechanism Doing Shoulder Flexion/Extension, 10° from the Transverse Plane

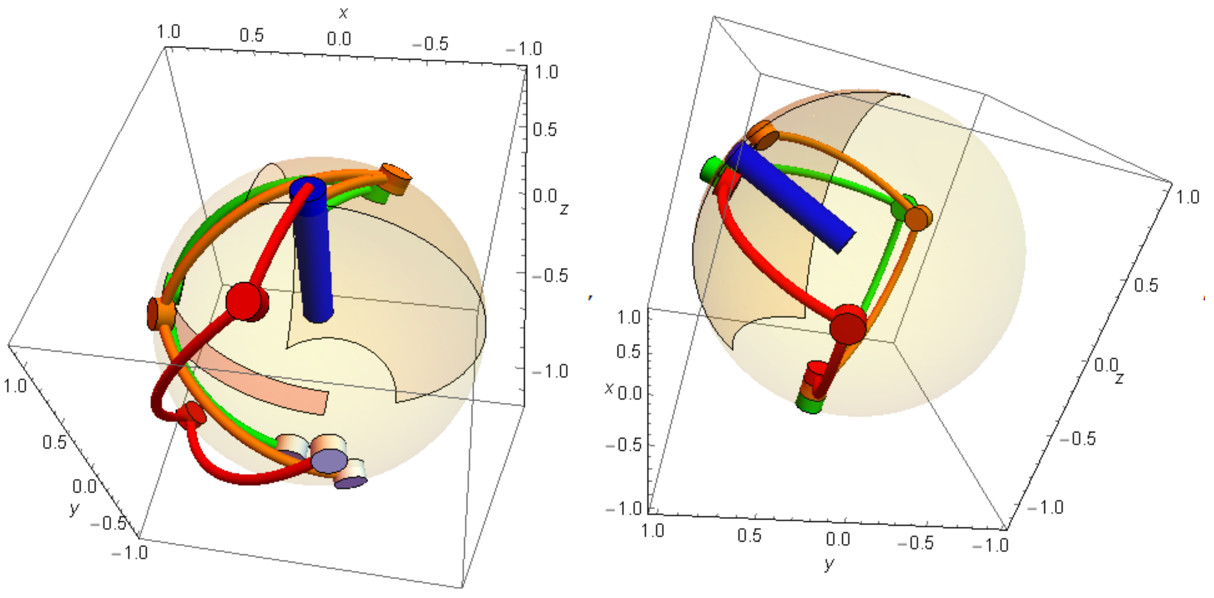




Upper Manipulator *Bottom Manipulator*

$Det[\text{Jacobian}] = 2.1313$ $Det[\text{Jacobian}] = -0.0439$

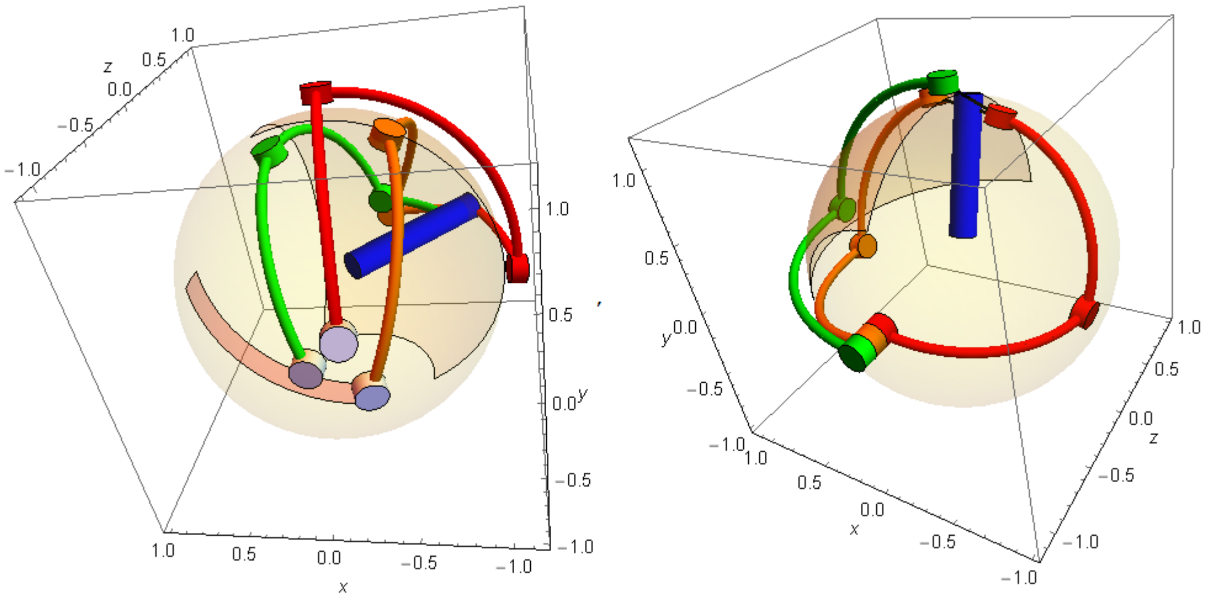
Figure 29. Mechanism Doing Shoulder Flexion/Extension, 45° from the Transverse Plane



Upper Manipulator *Bottom Manipulator*

Det[Jacobian] = 2.1313 *Det*[Jacobian] = −0.0546

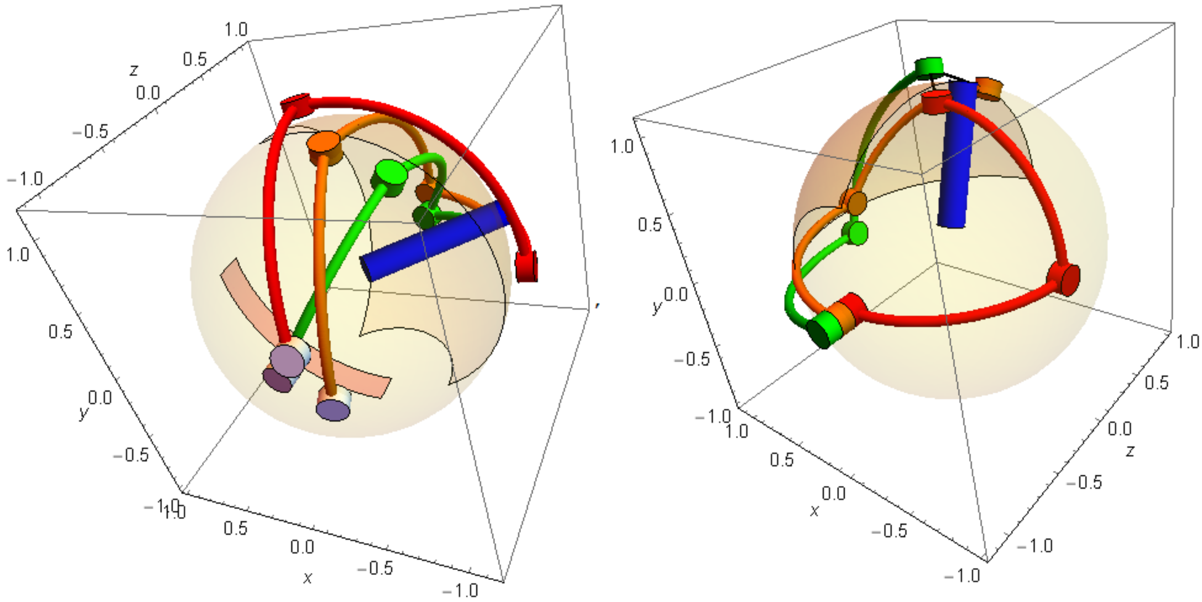
Figure 30. Mechanism Doing Shoulder Flexion/Extension, 90° from the Transverse Plane



Upper Manipulator *Bottom Manipulator*

$Det[\text{Jacobian}] = 0.484$ $Det[\text{Jacobian}] = -0.1889$

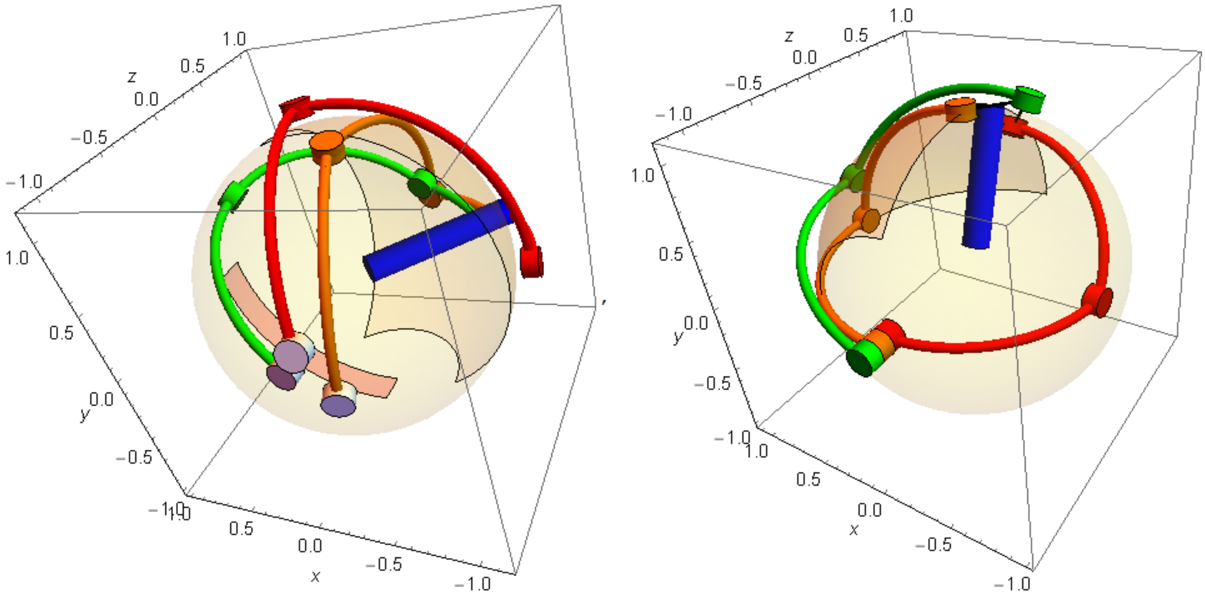
Figure 31. Mechanism Doing Forearm Pronation/Supination, Neutral



Upper Manipulator *Bottom Manipulator*

Det[Jacobian] = 30.4736 Det[Jacobian] = −0.0152

Figure 32. Mechanism Doing Forearm Pronation/Supination, 80° pronation



Upper Manipulator *Bottom Manipulator*

$Det[\text{Jacobian}] = -14.1386$ $Det[\text{Jacobian}] = 0.013$

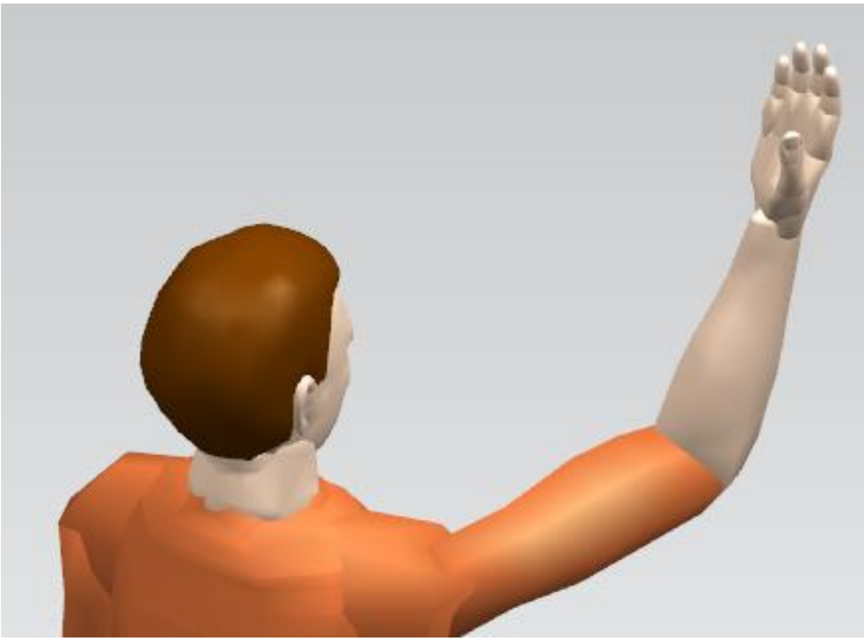


Figure 33. Mechanism Doing Forearm Pronation/Supination, 80° of inwards turn

It can be seen from the figures above that with the given initial inputs, the bottom manipulator can roam in between an area surrounded by the planes which are defined with respect to the medical axes. The planes defined by 45° - 90° from the sagittal plane towards the frontal plane and 0° - 90° from the transverse plane towards the frontal plane limits this motion. The defined motion is enough for most of the rehabilitation tasks.

5. CONCLUSIONS & FUTURE WORKS

An overconstrained mechanism has been defined for the 5 DOF motion between the shoulder and the elbow. The kinematic and dynamic calculations of the mechanism were made by adding imaginary joints and using rotation matrices. Graphical workspace analysis has been done to verify if the mechanism can do the intended motions. The results showed that adding imaginary joints to the system can simplify the kinematic and dynamic solving process of the overconstrained manipulator and is a viable solution technique for overconstrained mechanisms. The upper manipulator, including the motions from the elbow and shoulder internal/external rotation, has the capability of accomplishing the full range of motion of the original human joint. Finding the maximum workspace of the bottom manipulator is harder to realize due to the amount of inputs, nevertheless working modes for specific rehabilitation motions can be obtained as seen throughout this thesis.

For future work, an optimization algorithm may be developed for the workspace of the upper and bottom manipulator so that the optimal mechanism in terms of dexterity, dynamic capabilities and workspace ROM can be found. Control algorithm can be implemented so that the mechanism will be able to conduct the specific requirements for the passive and active exercise modes.

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