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Comprehending the Model of Omicron Variant Using Fractional Derivatives

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Abstract

The world is grappled with an unprecedented challenges due to Corona virus. We are all battling this epidemic together, but we have not been able to defeat this epidemic yet. A new variant of this virus, named "Omicron" is spreading these days. The fractional differential equations are providing us with better tools to study the mathematical model with memory effects. In this paper, we will consider an extended SER mathematical model with quarantined and vaccinated compartment to speculate the Omicron variant. This extended Susceptible Exposed Infected Recovered SER model involves equations that associate with the group of individuals those are susceptible (S), exposed (E): this class includes the individuals who are infected but not yet infectious, infectious (W): this class includes the individuals who are infected but not yet Quarantined, quarantined (Q): this class includes those group of people who are infectious, confirmed and quarantined, recovered (R) this class includes the group of individuals who have recovered, and vaccinated (V): this class includes the group of individuals who have been vaccinated. The non-negativity and of the extended SER model is analysed, the equilibrium points and the basic reproduction number are also calculated. The proposed model is then extended to the mathematical model using AB derivative operator. Proof for the existence and the uniqueness for the solution of fractional mathematical model in sense of AB fractional derivative is detailed and a numerical method is detailed to obtain the numerical solutions. Further we have discussed the efficiency of the vaccine against the Omicron variant via graphical representation.

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Key Words: Covid-19, Omicron Variant, SER Model, Fractional derivative; Existence and Uniqueness; Predictor-Corrector Method.

1 Introduction

As we all know how the Corona virus has wreaked havoc all over the world, how people have lost their lives, and how the world is struggling to get rid of this virus. It's going to be about two years that we are still in this disastrous situation. We all are very well aware about the destruction that was created by the second wave of Covid-19. Just a few months back we all took a sigh of relief from this virus but on November 26, 2021, WHO designated the new variant of Corona virus, named "Omicron". Although the transmissibility and the severity of this variant is not much clear but the data shows that the rate of hospitalization is increasing day by day. Also no new symptoms are yet known for the Omicron variant and by now the situation is not as dangerous as the Delta variant but it should be very clear to us that all variants of Covid-19 including the Omicron variant can cause severe disease or death. After the second wave of Covid-19 many of us got vaccinated, although WHO is working on to study the effectiveness of the vaccines on the Omicron variant but the data till now shows that the vaccines proved to be effective against severe disease and death from this variant.
At the present time, the research is still going on to study the assessments of the transmissibility of the Omicron variant, severity of the infection, symptoms of the infection, performance of the vaccines and the effectiveness of the treatment for this new variant of Covid-19 but WHO advice every country to fasten its vaccination drive so that more and more people could be vaccinated before the variant could create a havoc like the Delta variant.

The mathematical models are capable of decision making, saving lives, assisting in policy and many more. These are helpful in understanding the conditions needed to sustain lives, and provide us ways to study and predict the behaviour of the spread. The concept of derivatives and integrals pay a lot in formulation of these mathematical models. Here, the extended SER mathematical model for the Omicron variant of Covid-19 is taken in account. This extended SER model involves equations that associate with the group of individuals those are susceptible susceptible (S), exposed (E): this class includes the individuals who are infected but not yet infectious, infectious (W): this class includes the individuals who are infected but not yet Quarantined, quarantined (Q): this class includes those group of people who are infectious, confirmed and quarantined, recovered (R) this class includes the group of individulas who have recovered, and vaccinated (V): this class includes the group of individuals who have been vaccinated.

As we know that the present is explained better by the past. A system of fractional differential equations are effective in studying the effect of memory on the epidemic rise and evolution. A type of memory effect that is commonly spotted in epidemiology and ecology is "Hysteresis". Hysteresis [1] is the dependence of the state of the system on its history. Hysteresis can be found in chemistry (liquid-solid phase transitions), physics (magnetic hysteresis, electrical hysteresis), engineering (control system, aerodynamics, electronic circuits), biology (cell biology and genetics, immunology, Respiratory physiology, Voice and speech physiology, Neuroscience, Ecology and epidemiology). Hysteresis is a common phenomenon in ecology and epidemiology, in which the observed equilibrium of system requires the knowledge of the past history of the system. A similar is the case with the corona virus infection, in which we see that the past experiences tell us that the hygienic habits like washing the hands regularly, avoid touching of eyes, nose and mouth and social distancing are the protective measures to prevent its spread. The vaccine can also have memory effect i.e., what is its lasting effect by which the body can defend itself through immune memory. Different fractional derivative operators give better insight into the dynamics of mathematical models. The applications of fractional derivatives [2, 3] can be widely seen in the fields of physics [4, 5, 6], the mathematical models with fractional derivatives to study the Covid-19 are given in [7, 8, 9, 10, 11, 12], the effect of Covid-19 on the digestive system is studied in [13], the case study of Covid cases in Argentina with real data based on daily cases is studied in [14], [15] studies the effect of facemasks against the Corona Virus, [16] gives the analysis of first and second wave of the Covid in Iran and Japan, [17] studies the Covid-19 model with the effect of isolation and quarantine, [18] gives the dynamics of Ebola virus using different fractional derivatives, [19], the mathematical model for Zika virus transmission can be seen in [20],[21] gives the mathematical modeling of human liver with Caputo-Fabrizio derivative operator, [22] gives the mathematical model of Spanish flu and many other applications in the field of biology can be seen in [23, 24, 25, 26, 27]. The different applications in the field of mathematics are listed in [28, 29, 30, 31]. The commonly used fractional derivative operators [2, 3] involves Riemann-Liouville, Caputo, Caputo-Fabrizio, Atangana-Baleanu, Yang-Abdel Cattani, and many more. The applications of fractional derivatives using Riemann-Liouville operator can be seen in [32, 33], the applications of the Caputo fractional derivatives can be seen in [34, 35, 36, 37, 38, 39, 40]. In this work, we have taken in account the AB fractional derivative whose kernel is non-singular, exponential and non-local. The advantage of the AB derivative is that it provides a tool to understand the memory effect. As most of the biological systems have after-effects or memory so the modeling of such biological systems using fractional order derivatives with non-singular and non-local kernel have many advantages in which the effects like memory are considered in order to provide a better

understanding.
The paper is presented as follows. In Section 2, we have proposed the extended SER mathematical model for the Omicron variant and the non-negativity of the extended SER model is analysed. In Section 3 the equilibrium points and the basic reproduction number are also calculated and the proposed model is then extended to the fractional mathematical model in sense of AB fractional derivative operator, the proof for the existence and the uniqueness for the solution of fractional mathematical model in sense of AB fractional derivative is briefed. In Section 4, the numerical approach is detailed to find the numerical solutions. In Section 5, the stability of the numerical solutions is discussed. In Section 6, we have discussed the efficiency of the vaccine against the Omicron variant via graphical representation. Lastly, Section 7 shows the conclusion of the paper.

2 Extended SER Mathematical Model of Omicron Variant

Classifying the total population $\mathcal{N}$(t) into different groups of individuals as susceptible (S), exposed (E), infectious (W), quarantined (Q), recovered (R), and vaccinated (V), we develop a model that involves equations that associate with the group of individuals those are susceptible (S), exposed (E), infectious (W), quarantined (Q), recovered (R), and vaccinated (V) The extended SER mathematical model for Omicron infection is given as

$$
\begin{aligned}
\frac{dS}{dt} &= A - \frac{\beta_1}{1+\alpha_1 W(t)} S(t)\, W(t) - c\, S(t) - v\, S(t), && S(0) = S_0,\\
\frac{dE}{dt} &= \frac{\beta_1}{1+\alpha_1 W(t)} S(t)\, W(t) - \lambda\, E(t) + \delta \frac{\beta_1}{1+\alpha_1 W(t)} V(t)\, W(t), && \\
& && E(0) = E_0,\\
\frac{dW}{dt} &= \lambda\, E(t) - q\, W(t), && W(0) = W_0,\\
\frac{dQ}{dt} &= q\, W(t) - (1-\chi)\, r\, Q(t) - \chi\, \sigma\, Q(t), && Q(0) = Q_0,\\
\frac{dR}{dt} &= (1-\chi)\, r\, Q(t), && R(0) = R_0,\\
\frac{dV}{dt} &= c\, S(t) - \delta \frac{\beta_1}{1+\alpha_1 W(t)} V(t)\, W(t), && V(0) = V_0.
\end{aligned}
\tag{2.1}
$$

Where, A, represents new births and new residents per unit time, β_1 represents transmission rate, c is the vaccination rate (rate of people who are vaccinated), v is the natural death rate, λ is the average latent time, δ is the vaccine inefficacy, q is the infection time, χ is the case fatality rate, r is the time of recovery, σ is the time until death, α_1 is the saturation effect. The parameter $\delta, 0 \leq \delta \leq 1$ represents the vaccine inefficacy, this means that the vaccine is 100% effective when the value of δ is zero.

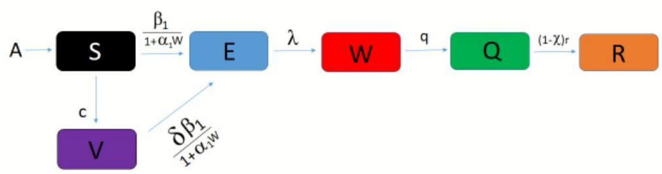


Figure 1: Disease transmission flow of the extended SER Model.

The figure-1 shows the transmission flow of the Omicron. This figure gives a general representation about how the total population $\mathcal{N}$ moves to the susceptible class of individuals, the individuals from susceptible class move to the exposed class of group with the transmission rate β_1 then λ is the time taken by the exposed group of individuals to become infected and the infected group of individuals are moved to the class of quarantined class, the quarantined individuals are then moved to the recovered class. Further in this figure, we have also seen the effect of vaccine. The susceptible individuals are vaccinated by the rate 'c', if the vaccine in inefficient then the vaccinated group of individuals will be moved to the exposed class and then to the infected class, the infected individuals are quarantined and the quarantined individuals are recovered.

Theorem 1. *[41] The solution $S(t), E(t), W(t), Q(t), R(t), V(t)$ of system (2.1) will be positive $\forall\ t \geq 0$, whenever $S_0, E_0, W_0, Q_0, R_0, V_0 \geq 0$.*

Proof. Consider

$$\frac{dS}{dt} = A - \frac{\beta_1}{1 + \alpha_1 W(t)} S(t)\, W(t) - c\, S(t) - v\, S(t). \tag{2.2}$$

Clearly

$$A - \frac{\beta_1}{1 + \alpha_1 W(t)} S(t)\, W(t) - c\, S(t) - v\, S(t) \geq -vS(t).$$

So,

$$\frac{dS}{dt} \geq -v\, S(t).$$

On solving we get

$$S(t) = S_0\, e^{-v\, t} \geq 0.$$

In a similar way, we can show $E(t), W(t), Q(t), R(t), V(t) \geq 0$. This proves the positivity of the solutions. $\square$

3 Fractional Extended SER Mathematical Model of Omicron Variant In Frame of AB Derivative Operator

In this section, we will first find the equilibrium points and the basic reproduction number. Then we will extended the proposed mathematical model to the fractional mathematical model using AB derivative.

3.1 Equilibrium Points and Basic Reproduction Number

We will find the equilibrium points and the basic reproduction number R_0 for system (2.1)

3.1.1 Equilibrium Points

For a dynamical system, the equilibrium points is the value of the variables where the variables do not change with time. As derivatives represent the change in the variables, so derivatives in the mathematical model should be zero to find the equilibrium points. Therefore solving the following system of equations will give the equilibrium points.

$$
\begin{aligned}
A - \frac{\beta_1}{1+\alpha_1 W^*(t)} S^*(t)\, W^*(t) - c\, S^*(t) - v\, S^*(t) &= 0, \\
\frac{\beta_1}{1+\alpha_1 W^*(t)} S^*(t)\, W^*(t) - \lambda\, E^*(t) + \delta \frac{\beta_1}{1+\alpha_1 W^*(t)} V^*(t)\, W^*(t) &= 0, \\
\lambda\, E^*(t) - q\, W^*(t) &= 0, \\
q\, W^*(t) - (1-\chi)\, r\, Q^*(t) - \chi\, \sigma\, Q^*(t) &= 0, \\
(1-\chi)\, r\, Q^*(t) &= 0, \\
c\, S^*(t) - \delta \frac{\beta_1}{1+\alpha_1 W^*(t)} V^*(t)\, W^*(t) &= 0.
\end{aligned}
\tag{3.1}
$$

We get

$$\begin{aligned} S^* &= \frac{A}{c+v}, \\ E^* &= 0, \\ I^* &= 0, \\ Q^* &= 0, \\ R^* &= 0, \\ V^* &= \frac{A\,c}{c\,v+\,v^2}. \end{aligned} \tag{3.2}$$

3.1.2 Basic Reproduction Number

The basic reproduction number of an infection is the expected number of cases directly generated by one case in a population where all the individuals are susceptible to the infection. We will now compute the basic reproduction number R_0 using the next generation matrix.
The basic reproduction number R_0 is given by $R_0 = \rho(\mathcal{F}\,\mathcal{V}^{-1})$, where $\mathcal{F}\,\mathcal{V}^{-1}$ is the Next generation matrix.
From system (2.1) we conclude

$$\mathcal{F} = \begin{bmatrix} 0 & \frac{\beta_1\,A\,c\,\delta}{c\,v+v^2} + \frac{\beta_1\,A}{c+v} \\ 0 & 0 \end{bmatrix}$$

$$\mathcal{V} = \begin{bmatrix} -\lambda - v & 0 \\ \lambda & -q - v \end{bmatrix}$$

The next generation matrix is given by

$$\mathcal{F}\,\mathcal{V}^{-1} = \begin{bmatrix} -\frac{\left(\frac{\beta_1 A c\delta}{c\,v+v^2} + \frac{\beta_1 A}{c+v}\right)\lambda}{(\lambda+v)\,(q+v)} & -\frac{\left(\frac{\beta_1 A c\delta}{c\,v+v^2} + \frac{\beta_1 A}{c+v}\right)}{(q+v)} \\ 0 & 0 \end{bmatrix}$$

$$R_0 = -\frac{A\beta_1 c\delta + \lambda + A\beta_1\lambda + v}{c\lambda q v + (c+\lambda+q)v^3 + v^4 + (c\lambda + (c+\lambda)q)v^2}. \tag{3.3}$$

3.2 Fractional Extended SER Mathematical Model of Omicron Variant in Frame of AB Derivative Operator

We now extend the SER mathematical model to the fractional extended SER mathematical model of Omicron Variant in sense of AB derivative operator. Furthermore, in order of ensuring that the right- and left-hand sides of the consequent fractional model possess the same dimension $(time)^{-\nu}$, all the parameters having the dimension $(time)^{-1}$ are replaced by power of ν, while the other parameters remain unchanged.

$$
\begin{aligned}
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} S(t) &= A^{\nu} - \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} S(t)\, W(t) - c^{\nu}\, S(t) - v^{\nu}\, S(t), \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad S(0) = S_0, \\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} E(t) &= \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} S(t)\, W(t) - \lambda^{\nu}\, E(t) + \delta \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)}\, V(t)\, W(t), \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad E(0) = E_0, \\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} W(t) &= \lambda^{\nu}\, E(t) - q^{\nu}\, W(t), \qquad W(0) = W_0, \\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} Q(t) &= q^{\nu}\, W(t) - (1-\chi)\, r^{\nu}\, Q(t) - \chi\, \sigma^{\nu}\, Q(t), \qquad Q(0) = Q_0, \\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} R(t) &= (1-\chi)\, r^{\nu}\, Q(t), \qquad R(0) = R_0, \\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} V(t) &= c^{\nu}\, S(t) - \delta \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)}\, V(t)\, W(t), \qquad V(0) = V_0.
\end{aligned}
\tag{3.4}
$$

here, ${}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu}$ is the AB fractional derivative of order ν which is defined as follows.

Definition 1. *[42] Let $h \in H^1(a,b)$, $b > a$, let $0 < \nu < 1$, then the Atangana-Baleanu fractional derivative is given as*

$$
{}_{b}^{\mathcal{ABC}}\varsigma_t^{\nu}(h(t)) = \frac{K(\nu)}{1-\nu} \int_b^t h'(\rho) E_{\nu} \left[-\nu \frac{(t-\rho)^{\nu}}{1-\nu} \right] d\rho. \tag{3.5}
$$

Where, ${}_{b}^{\mathcal{ABC}}\varsigma_t^{\nu}$ is the AB fractional derivative of order ν in Caputo sense, E_{ν} is the Mittag-Leffler function and $K(\nu)$ is the normalization function such that $K(0) = K(1) = 1$.

Definition 2. *[42] The fractional integral of AB fractional derivative of order ν, $0 < \nu < 1$ is given as*

$$
\mathcal{I}^{\mathcal{AB}}{}_t^{\nu}(h(t)) = \frac{1-\nu}{K(\nu)} h(t) + \frac{\nu}{K(\nu)\Gamma(\nu)} \int_0^t h(\rho)(t-\rho)^{\nu-1} d\rho. \tag{3.6}
$$

Theorem 2. *[42] For $h \in H^1(a,b)$, $b > a$, y is continuous, bounded function then, the fractional differential equation*

$$
\mathcal{I}^{\mathcal{AB}}{}_t^{\nu}(h(t)) = y(t),
$$

gives a solution which is unique given as

$$
h(t) = \frac{1-\nu}{K(\nu)} y(t) + \frac{\nu}{K(\nu)\Gamma(\nu)} \int_0^t y(\rho)(t-\rho)^{\nu-1} d\rho.
$$

3.3 Existence and Uniqueness of the Solution

Theorem 3. *[43] Following functions*

$$\begin{aligned}
G_1(t,S) &= A^\nu - \frac{\beta_1^\nu}{1+\alpha_1\,W(t)} S(t)\,W(t) - c^\nu\,S(t) - v^\nu\,S(t),\\
G_2(t,E) &= \frac{\beta_1^\nu}{1+\alpha_1\,W(t)} S(t)\,W(t) - \lambda^\nu\,E(t) + \delta\frac{\beta_1^\nu}{1+\alpha_1\,W(t)}\,V(t)\,W(t),\\
G_3(t,W) &= \lambda^\nu\,E(t) - q^\nu\,W(t),\\
G_4(t,Q) &= q^\nu\,W(t) - (1-\chi)\,r^\nu\,Q(t) - \chi\,\sigma^\nu\,Q(t),\\
G_5(t,R) &= (1-\chi)\,r^\nu\,Q(t),\\
G_6(t,V) &= c^\nu\,S(t) - \delta\,\frac{\beta_1^\nu}{1+\alpha_1\,W(t)}\,V(t)\,W(t).
\end{aligned} \tag{3.7}$$

fulfil the Lipschitz condition, further, the contractions when:

(i) $0 < p_1 < 1$,

(ii) $0 < p_2 < 1$,

(iii) $0 < p_3 < 1$,

(iv) $0 < p_4 < 1$,

(v) $0 < p_5 < 1$,

(vi) $0 < p_6 < 1$,

Proof Consider

$$G_1(t,S) = A^\nu - \frac{\beta_1^\nu}{1+\alpha_1\,I(t)} S(t)\,I(t) - c^\nu\,S(t) - v^\nu\,S(t).$$

Let S_1 and S_2 be two functions, then

$$\begin{aligned}
\|G_1(t,S_1) - G_1(t,S_2)\| &= \left\| \left(\frac{\beta_1^\nu}{1+\alpha_1 I(t)} I(t) - c - v \right) (S_1(t) - S_2(t)) \right\| \\
&\leq c + v + \beta_1^\nu \left\| \frac{I(t)}{1+\alpha_1 I(t)} \right\| \|S_1(t) - S_2(t)\|
\end{aligned} \tag{3.8}$$

Let $l_1 = sup_t\|S(t)\|$, $l_2 = sup_t\|E(t)\|$, $l_3 = sup_t\|I(t)\|$, $l_4 = sup_t\|Q(t)\|$,$l_5 = sup_t\|R(t)\|$, $l_6 = sup_t\|D(t)\|$, $l_7 = sup_t\|V(t)\|$,
then

$$\|G_1(t,S_1) - G_1(t,S_2)\| \leq p_1\|S_1(t) - S_2(t)\|,$$

where

$$p_1 = c + v + \beta_1^\nu \frac{l_3}{1+\alpha_1\,l_3}.$$

Hence, Lipschitz's condition for $G_1(t,S)$ holds and if $0 < p_1 < 1$, then contraction holds for $G_1(t,S)$. We can similarly prove the result for $G_2(t,E)$, $G_3(t,I)$, $G_4(t,Q)$, $G_5(t,R)$, $G_6(t,D)$, $G_7(t,V)$.

Theorem 4. *[43] The fractional extended SER mathematical model for Omicron Variant*

$$\begin{aligned}
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} S(t) &= A^{\nu} - \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} S(t)\, W(t) - c^{\nu}\, S(t) - v^{\nu}\, S(t), & S(0) = S_0,\\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} E(t) &= \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} S(t)\, W(t) - \lambda^{\nu}\, E(t) + \delta \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} V(t)\, W(t), & \\
& & E(0) = E_0,\\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} W(t) &= \lambda^{\nu}\, E(t) - q^{\nu}\, W(t), & W(0) = W_0,\\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} Q(t) &= q^{\nu}\, W(t) - (1-\chi)\, r^{\nu}\, Q(t) - \chi\, \sigma^{\nu}\, Q(t), & Q(0) = Q_0,\\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} R(t) &= (1-\chi)\, r^{\nu}\, Q(t), & R(0) = R_0,\\
{}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} V(t) &= c^{\nu}\, S(t) - \delta \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} V(t)\, W(t), & V(0) = V_0.
\end{aligned} \tag{3.9}$$

gives a unique solution under the constraints, we can search for a t_{max} which satisfies

$$\frac{1-\nu}{K(\nu)} p_i + \frac{t_{max}^{\nu}}{K(\nu)\Gamma(\nu)} p_i < 1, \quad for\ i = 1, 2, 3, 4, 5, 6, 7. \tag{3.10}$$

Proof. □

Consider

$${}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} S(t) = A^{\nu} - \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} S(t)\, W(t) - c^{\nu}\, S(t) - v^{\nu}\, S(t), \qquad S(0) = S_0. \tag{3.11}$$

Let

$$G_1(t, S) = A^{\nu} - \frac{\beta_1^{\nu}}{1+\alpha_1 W(t)} S(t)\, W(t) - c^{\nu}\, S(t) - v^{\nu}\, S(t).$$

Then equation (3.11) can be written as

$${}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} S(t) = G_1(t, S). \tag{3.12}$$

Using theorem 2, we get

$$S(t) = S_0 + \frac{1-\nu}{K(\nu)} G_1(t, S(t)) + \frac{\nu}{K(\nu)\Gamma(\nu)} \int_0^t (t-\rho)^{\nu-1} G_1(\rho, S(\rho)) d\rho. \tag{3.13}$$

Let $X = (0, T)$ and define an operator $Y:\ \mathcal{C}(X, \mathbb{R}^6) \longrightarrow \mathcal{C}(X, \mathbb{R}^6)$ such that

$$Y[S(t)] = S_0 + \frac{1-\nu}{K(\nu)} G_1(t, S(t)) + \frac{\nu}{K(\nu)\Gamma(\nu)} \int_0^t (t-\rho)^{\nu-1} G_1(\rho, S(\rho)) d\rho. \tag{3.14}$$

So equation (3.13) can be seen as $Y[S(t)] = S(t)$. Define the supremum norm on X as $\|S\| = Sup_{t\in X}|S(t)|$. Then $\mathcal{C}(X, \mathbb{R}^6)$ and $\|.\|$ defines a Banach Space.

Consider

$$Y[S_1(t)] - Y[S_2(t)] = \frac{1-\nu}{K(\nu)} \left(G_1(t, S_1(t)) - G_2(t, S_2(t))\right) + \frac{\nu}{K(\nu)\Gamma(\nu)} \int_0^t (t-\rho)^{\nu-1} \left(L_1(\rho, S_1(\rho)) - G_1(\rho, S_1(\rho))\right) d\rho. \quad (3.15)$$

Taking modulus on equation (3.15) and then using the triangle inequality we get

$$|Y[S_1(t)] - Y[S_2(t)]| \le \frac{1-\nu}{K(\nu)} \left|(G_1(t, S_1(t)) - G_2(t, S_2(t)))\right| + \frac{\nu}{k(\nu)\Gamma(\nu)} \int_0^t \left|(t-\rho)^{\nu-1} \left(G_1(\rho, S_1(\rho)) - G_1(\rho, S_1(\rho))\right) d\rho\right|. \quad (3.16)$$

As the function $G_1(t, S(t))$ agrees with the Lipschitz condition, we have

$$|Y(S_1) - Y(S_2)| \le \left(\frac{1-\nu}{K(\nu)} p_1 + \frac{t_{max}^{\nu}}{K(\nu)\Gamma(\nu)} p_1\right) |S_1 - S_2|. \quad (3.17)$$

Also equation (3.17) will be a contraction only if

$$\frac{1-\nu}{K(\nu)} p_1 + \frac{t_{max}^{\nu}}{K(\nu)\Gamma(\nu)} p_1 < 1. \quad (3.18)$$

Using the Banach Fixed Point theorem, we finally govern the existence of a solution which is unique as well for the fractional mathematical extended SEIR model to speculate the Omicron variant in the sense of AB derivative operator.

4 Predictor Corrector Method.

A predictor corrector method is an efficient method to find the numerical solutions of the nonlinear systems. In this method, a point that is close to the converged solution of the ongoing step is predicted first and then the Newton's method starts iterations at the point predicted. The predictor recognizes the pattern of the last converged solutions to find and predict the starting point of the next step. Lastly, the corrector performs the iterations starting from the predicted point until convergence is achieved.

4.1 Numerical Solution Using Predictor-Corrector Method

Predictor–Corrector [45] is the most effective method that is derived for all most all the fractional order derivatives. We will now apply this method in AB sense to find the solution of the proposed model. We assume

$$Z(t) = S(t), E(t), W(t), Q(t), R(t), V(t).$$

and

$$Z_0(t) = S_0(t), E_0(t), W_0(t), Q_0(t), R_0(t), V_0(t).$$

To understand the method consider

$$ {}_{0}^{\mathcal{ABC}}\varsigma_t^{\nu} Z(t) = P(t, Z(t)), \quad t \geq 0, \quad Z(0) = Z_0. \tag{4.1}$$

The above equation reduces to the fractional voltera equation which is given as

$$Z(t) = Z_0(t) + (1-\nu)P(t_{j+1}, Z_{j+1}) + \frac{\nu}{\Gamma(\nu)} \int_0^{t_{j+1}} (t_{j+1} - \rho)^{\nu-1} P(\rho, Z(\rho)) d\rho. \tag{4.2}$$

Now proceeding as per the method given in [2] for $\nu \in [0,1], 0 \leq t \leq T$ and setting $h = \frac{T}{N}$ and $t_n = nh$ for $n = 0, 1, 2, \cdots, N$, where N is positive integer, the Predictor-Corrector formula [2] of the above problem is given as

$$Z_{j+1} = Z_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} P(t_{j+1}, Z_{j+1}^M) + \sum_{i=0}^{j} b_{j+1}, iP(t_i, Z_i) \right). \tag{4.3}$$

where

$$b_{j+1,i} = \begin{cases} j^{\nu+1} - (j-\nu)(j+1)^{\nu} & \text{if} i = 0 \\ (j-i+2)^{\nu+1} + (j-i)^{\nu+1} - 2(j-i+1)^{\nu+1} & \text{if } 1 \leq i \leq j \\ 1, & i = j+1 \end{cases}$$

and

$$b_{j+1,j+1} = 1 + \frac{(1-\nu)\Gamma(\nu+2)}{\nu h^{\nu}}$$

. The Predictor-Corrector formula is given as

$$Z_{j+1}^M = Z_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} P(t_i, Z_i), \tag{4.4}$$

where

$$a_{j+1,i} = \begin{cases} -(j-i)^{\nu} + (j-i+1)^{\nu} & \text{if} i = 0, \cdots, j-1 \\ 1 + \frac{(1-\nu)\Gamma(\nu)}{h^{\nu}} & \text{if } j = i \end{cases}$$

Using the above Predictor-Corrector method [2], the numerical solution for the considered model is given as

$$
\begin{aligned}
S_{j+1} &= S_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} G_1(t_{j+1}, S^M_{j+1}) + \sum_{i=0}^{j} b_{j+1}, i G_1(t_i, S_i) \right), \\
E_{j+1} &= E_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} G_2(t_{j+1}, E^M_{j+1}) + \sum_{i=0}^{j} b_{j+1}, i G_2(t_i, E_i) \right), \\
W_{j+1} &= W_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} G_3(t_{j+1}, W^M_{j+1}) + \sum_{i=0}^{j} b_{j+1}, i G_3(t_i, W_i) \right), \\
Q_{j+1} &= Q_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} G_4(t_{j+1}, Q^M_{j+1}) + \sum_{i=0}^{j} b_{j+1}, i G_4(t_i, W_i) \right), \\
R_{j+1} &= R_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} G_5(t_{j+1}, R^M_{j+1}) + \sum_{i=0}^{j} b_{j+1}, i G_5(t_i, R_i) \right), \\
V_{j+1} &= V_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \left(b_{j+1,j+1} G_6(t_{j+1}, V^M_{j+1}) + \sum_{i=0}^{j} b_{j+1}, i G_6(t_i, V_i) \right),
\end{aligned}
\tag{4.5}
$$

where,

$$
\begin{aligned}
S^M_{j+1} &= S_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} G_1(t_i, S_i), \\
E^M_{j+1} &= E_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} G_2(t_i, E_i), \\
W^M_{j+1} &= W0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} G_3(t_i, W_i), \\
Q^M_{j+1} &= Q_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} G_4(t_i, Q_i), \\
R^M_{j+1} &= R_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} G_5(t_i, R_i), \\
V^M_{j+1} &= V_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} G_6(t_i, V_i).
\end{aligned}
\tag{4.6}
$$

5 Stability of Numerical Solutions

Lemma 1. *[44] If* $0 < \nu < 1$ *and* k_1 *is a positive integer, then there exists positive constants* $D_{\nu,1}$ *and* $D_{\nu,2}$ *that depends only on* ν*, such that*

$$(k_1+1)^{\nu} - k_1^{\nu} \leq D_{\nu,1}(k_1+1)^{\nu-1}, \tag{5.1}$$

and

$$(k_1+2)^{\nu+1} - 2(k_1+1)^{\nu+1} + k_1^{\nu+1} \leq D_{\nu,2}(k_1+1)^{\nu-1}. \tag{5.2}$$

Lemma 2. *[44] Assume that* $x_{q,m} = (m-q)^{\nu-1}$*,* $q = 1, 2, 3, \cdots, m-1$ *and* $x_{q,m} = 0$ *for* $q \geq m$*, also* $\nu, K, h, T > 0$*,* $k_1 h \leq T$ *and* k_1 *is a positive integer. Let* $\sum_{q=k_1}^{q=m} x_{q,m}|e_q|$ *for* $q > m > 1$*. If*

$$|e_q| \leq Kh^{\nu} \sum_{q=1}^{m=1} x_{q,m}|e_q| + |p_0|, m = 1, 2, \cdots, k_1,$$

$$|e_{k_1}| \leq M|p_0|, k_1 = 1, 2, \cdots$$

where M *is a positive constant independent of* k_1 *and* h*.*

Theorem 5. *[44] The numerical solution* (4.5)-(4.6) *are conditionally stable.*

Proof. Let $\tilde{Z}_0, \tilde{Z}_i (i = 0, 1, \cdots j+1)$ and $\tilde{Z}_{j+1}^M$, $(j = 0, 1, \cdots N-1)$ be perturbations of Z_0, Z_i, Z_{j+1}^M respectively. Then the following perturbation equations are obtained by using equations (4.5), (4.6)

$$\tilde{Z}_{j+1}^M = \tilde{Z}_0 + \frac{h^{\nu}}{\Gamma(\nu)} \sum_{i=0}^{j} a_{j+1,i} \left(P(t_i, Z_j + \tilde{Z}_i) - P(t_j, Z_j) \right), \tag{5.3}$$

$$\begin{aligned} \tilde{Z}_{j+1} = \tilde{Z}_0 + \frac{\nu h^{\nu}}{\Gamma(\nu+2)} \Bigg(& b_{j+1,j+1} \left(P(t_{j+1}, Z_{j+1}^M + \tilde{Z}_{j+1}^M) - P\left(t_{j+1}, Z_{j+1}^M\right) \right) \\ & + \sum_{i=0}^{j} b_{j+1,i} \left(P(t_i, Z_i + \tilde{Z}_i) - P(t_i, Z_i) \right) \Bigg), \end{aligned} \tag{5.4}$$

using Lipschitz condition, we get

$$|\tilde{Z_{i+1}}| \leq \zeta_0 + \frac{\nu h^{\nu K}}{\Gamma(\nu+2)} \left(b_{j+1,j+1}|\tilde{Z}_{j+1}^M| + \sum_{i=1}^{j} b_{i,j+1}|\tilde{Z}_j| \right), \tag{5.5}$$

where, $\zeta_0 = max_{0 \leq j \leq N} \left\{ |\tilde{Z}_0| + \frac{\nu h^{\nu} K_{b_{j,0}}}{\Gamma(\nu+2)} |\tilde{Z}_0| \right\}$ From [33], we can write

$$|\tilde{Z}_{j+1}^M| \leq v_0 + \frac{h^{\nu} K}{\Gamma(\nu)} \sum_{i=1}^{j} a_{i,j+1}|\tilde{Z}_i|, \tag{5.6}$$

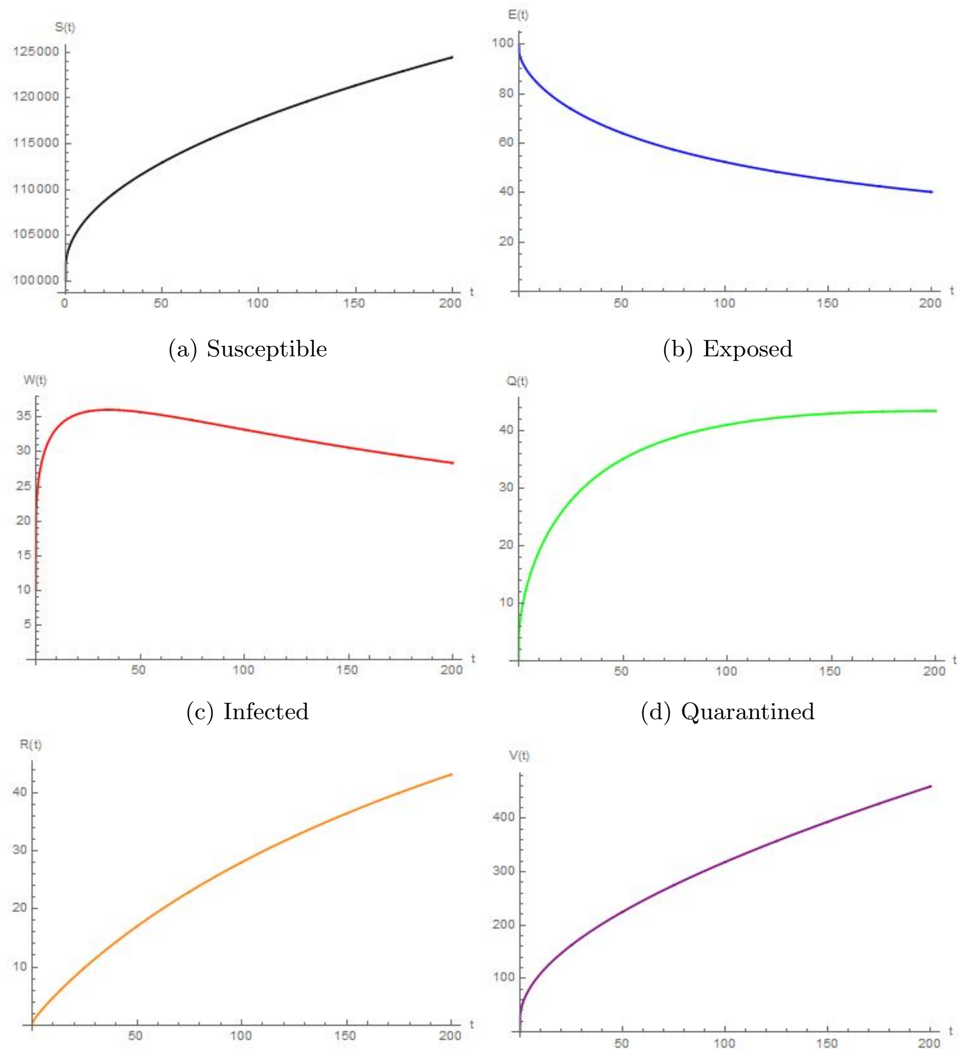
where $v_0 = max_{0\leq j\leq N}\left\{|\tilde{Z}_0|+\frac{h^{\nu}Ka_{n,0}}{\Gamma(\nu)}|\tilde{Z}_0|\right\}$. Substituting $|\tilde{Z}^p_{i+1}|$ from equation (5.6) into (5.5), we get

$$\begin{aligned}|\tilde{Z}_{j+1}| &\leq \gamma_0 + \frac{\nu h^{\nu}K}{\Gamma(\nu+2)}\sum_{i=1}^{j}\left(b_{j+1,i} + \frac{h^{\nu}K\, b_{j+1,j+1}a_{j+1,i}}{\Gamma(\nu)}\right)|\tilde{Z}_i| \\ &\leq \gamma_0 + \frac{\nu h^{\nu}K\, D_{\nu,2}}{\Gamma(\nu+2)}\sum_{i=1}^{j}(j+1-i)^{\nu-1}|\tilde{Z}_i|, \qquad (5.7)\end{aligned}$$

where, $\gamma_0 = max\left\{v_0 + \frac{\nu h^{\nu}Kb_{j+1,j+1}}{\Gamma(\nu+2)}\zeta_0\right\}$. Also from Lemma 1, $D_{\nu,2}$ is a positive constant that depends only on ν, and as h is assumed to be small enough. Finally from Lemma 2, we get $|\tilde{Z}_{j+1}|\leq D\gamma_0$. With this we are done with the proof. □

6 Analysis of Behaviour of The Solution Using Graphical Representations

In this section, we will present a graphical representation for the analysis of behaviour of the solutions of the above mentioned model for different fractional order derivatives. In figure 2-7, the graphical representations shows the behaviour of the solution at different fractional orders ie., $\nu = 0.5$, $\nu = 0.7$, $\nu = 0.81$, $\nu = 0.85$, $\nu = 0.899$, $\nu = 0.95$, when the inefficacy of the vaccine is very low $\delta = 0.0005$ that means that the vaccines are efficiently working against the Omicron variant. Figure 8-10 represents the behaviour of the solutions when the vaccine is inefficient against the Omicron variant.
The values of the parameters stated in the model are as follows
$A^{\nu} = 2300$ person per day, $\beta_1 = 8.58\times10^{-9}$, $c = 3.54\times10^{-4}$, $v = 3\times10^{-5}$, $\lambda^{-\nu} = 5.5$ days, $\delta = 0.0005$, $q^{-\nu} = 3.8$ days, $\chi = 0.014$, $r^{-\nu} = 10$ days, $\sigma^{-\nu} = 15$ days, $\alpha_1 = 0.001$.



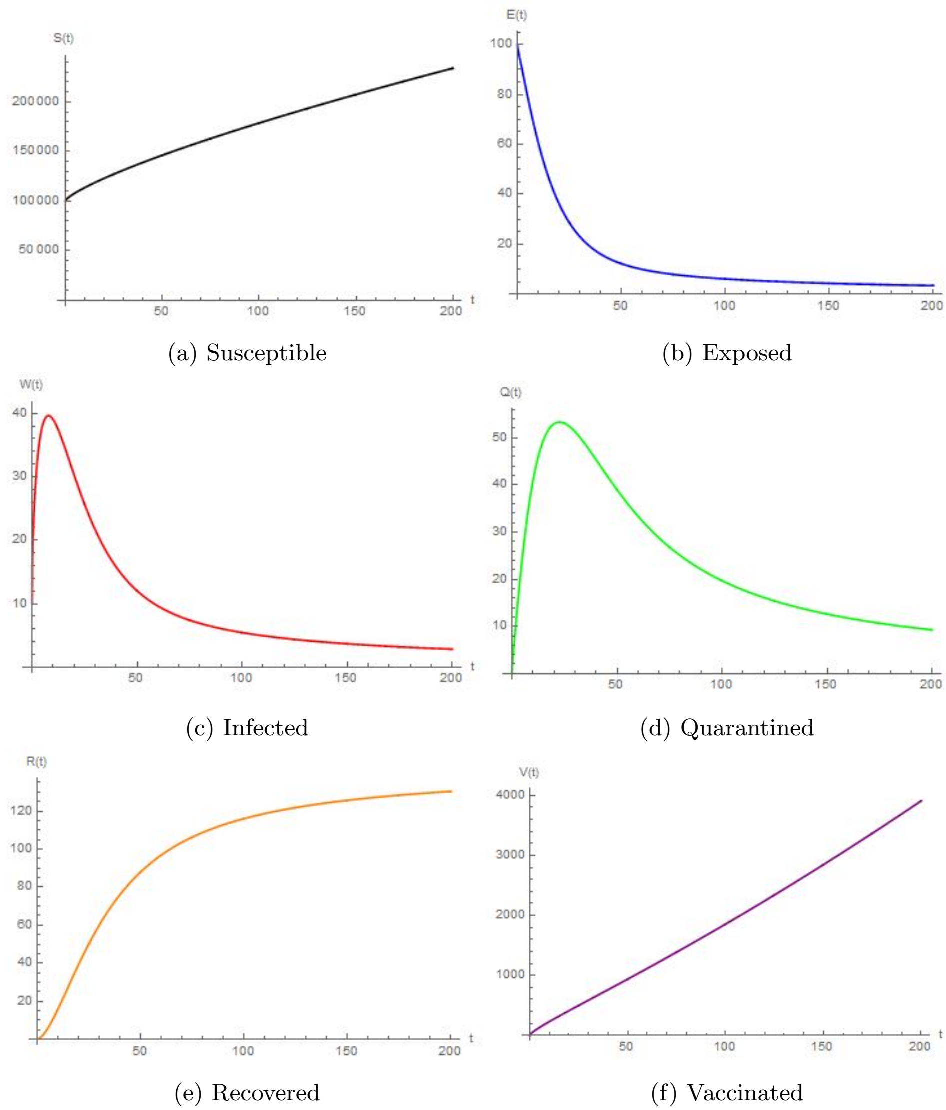
(a) Susceptible (b) Exposed

(c) Infected (d) Quarantined

(e) Recovered (f) Vaccinated

Figure 2: The behaviour of the solution when the vaccine's inefficacy is very low and the value of the fractional order is $\nu = 0.5$

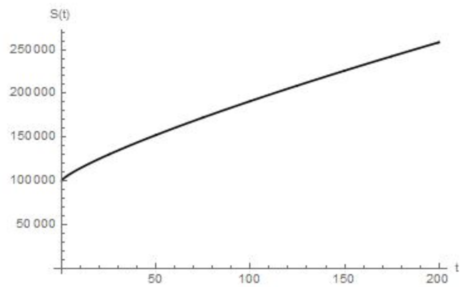
The above figure shows the behaviour of the susceptible, exposed, infected, quarantined, recovered, and the vaccinated group of individuals along with the time when the inefficacy of the vaccine is very low and the value of the fractional order derivative is $\nu = 0.5$. We can see that if the vaccine is working good against the omicron infection, then the exposed, infected, and the quarantined group of individuals decreases and the recovered group increases.



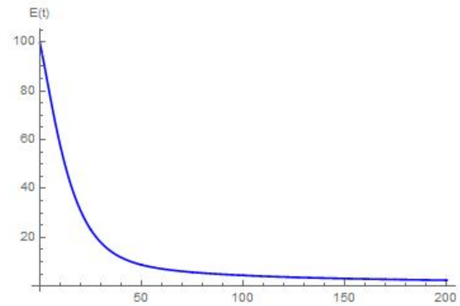
(a) Susceptible (b) Exposed (c) Infected (d) Quarantined (e) Recovered (f) Vaccinated

Figure 3: The behaviour of the solution when the vaccine's inefficacy is very low and the value of the fractional order is $\nu = 0.7$

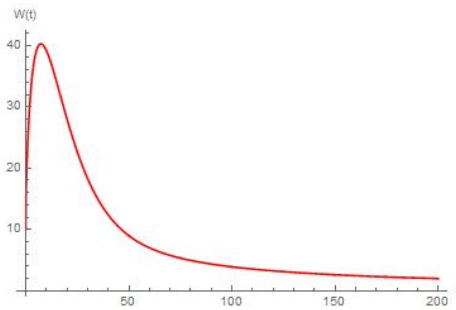
The above figure shows the behaviour of the susceptible, exposed, infected, quarantined, recovered, and the vaccinated group of individuals along with the time when the inefficacy of the vaccine is very low and the value of the fractional order derivative is $\nu = 0.7$. We can see that if the vaccine is working good against the omicron infection, then the exposed, infected, and the quarantined group of individuals decreases and the recovered group increases.



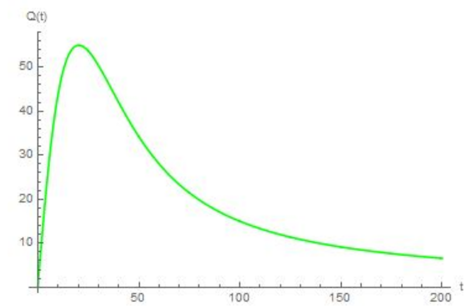
(a) Susceptible



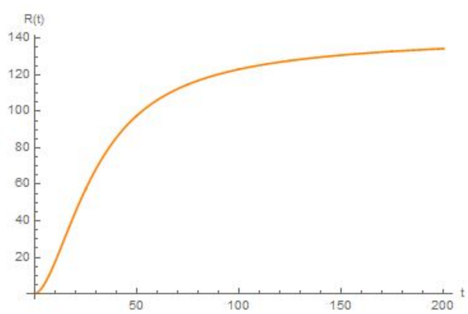
(b) Exposed



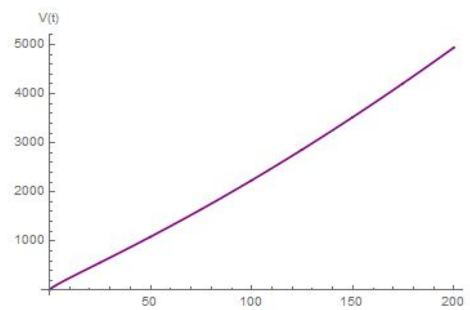
(c) Infected



(d) Quarantined



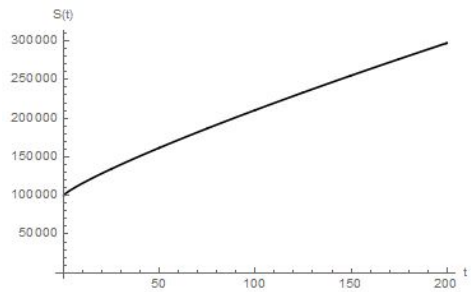
(e) Recovered



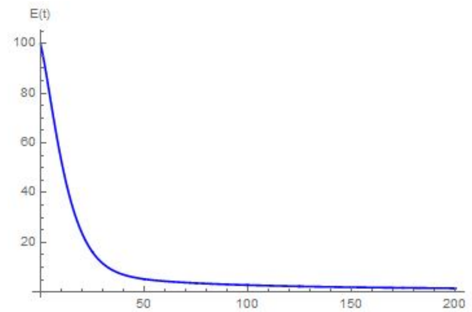
(f) Vaccinated

Figure 4: The behaviour of the solution when the vaccine's inefficacy is very low and the value of the fractional order is $\nu = 0.81$

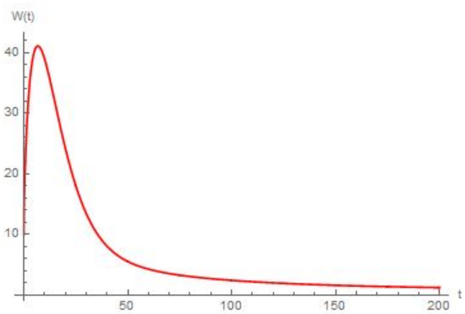
The above figure shows the behaviour of the susceptible, exposed, infected, quarantined, recovered, and the vaccinated group of individuals along with the time when the inefficacy of the vaccine is very low and the value of the fractional order derivative is $\nu = 0.81$. We can see that if the vaccine is working good against the omicron infection, then the exposed, infected, and the quarantined group of individuals decreases and the recovered group increases.



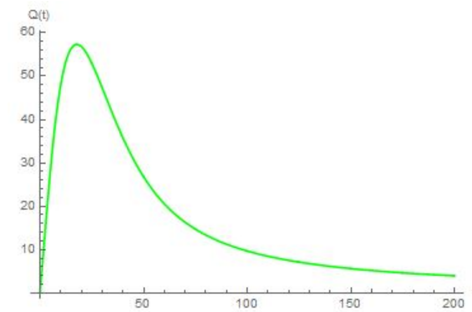
(a) Susceptible



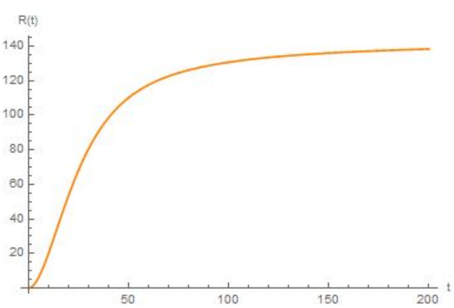
(b) Exposed



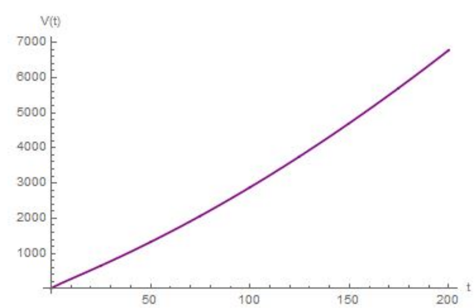
(c) Infected



(d) Quarantined



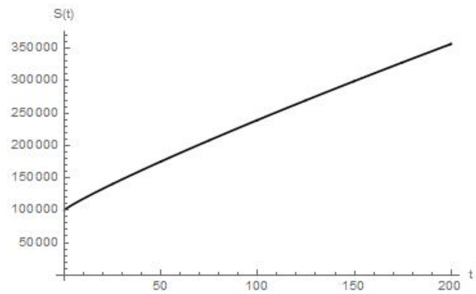
(e) Recovered



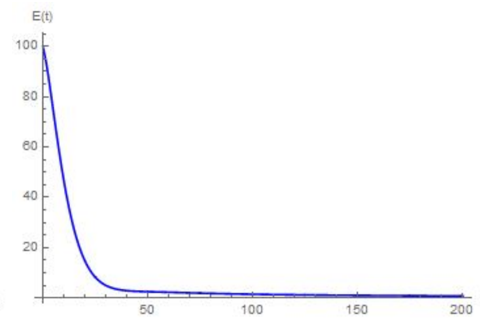
(f) Vaccinated

Figure 5: The behaviour of the solution when the vaccine's inefficacy is very low and the value of the fractional order is $\nu = 0.85$

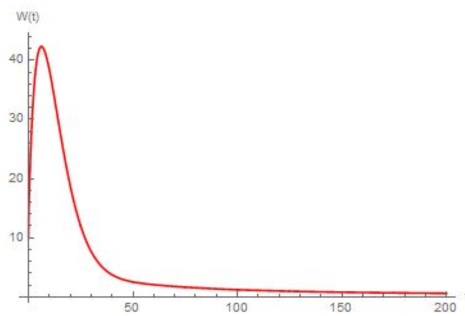
The above figure shows the behaviour of the susceptible, exposed, infected, quarantined, recovered, and the vaccinated group of individuals along with the time when the inefficacy of the vaccine is very low and the value of the fractional order derivative is $\nu = 0.85$. We can see that if the vaccine is working good against the omicron infection, then the exposed, infected, and the quarantined group of individuals decreases and the recovered group increases.



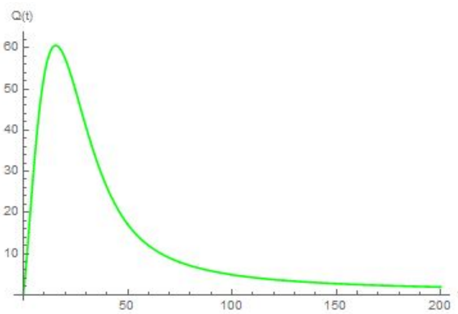
(a) Susceptible



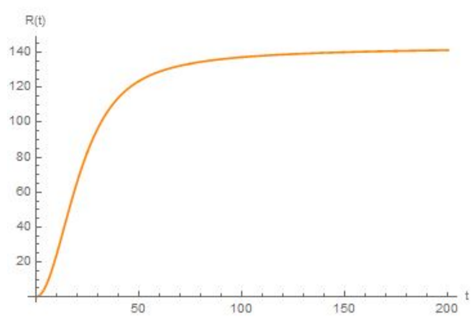
(b) Exposed



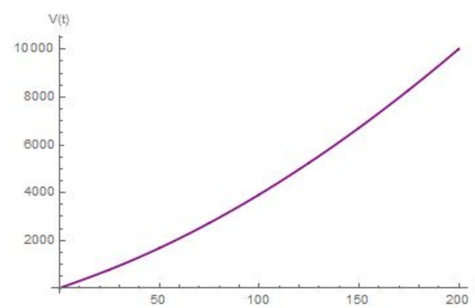
(c) Infected



(d) Quarantined



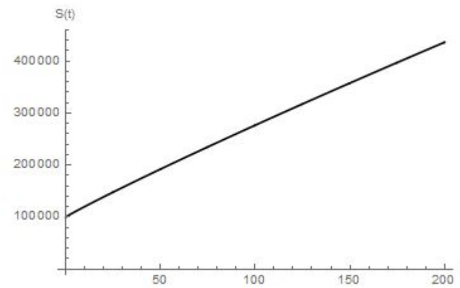
(e) Recovered



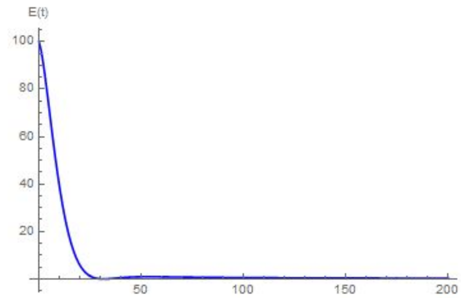
(f) Vaccinated

Figure 6: The behaviour of the solution when the vaccine's inefficacy is very low and the value of the fractional order is $\nu = 0.899$.

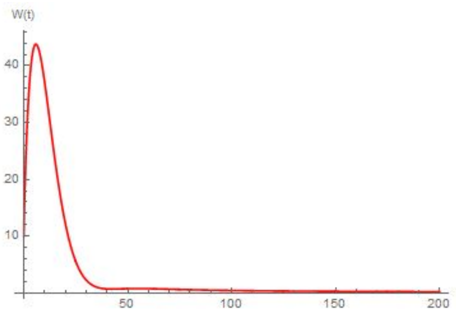
The above figure shows the behaviour of the susceptible, exposed, infected, quarantined, recovered, and the vaccinated group of individuals along with the time when the inefficacy of the vaccine is very low and the value of the fractional order derivative is $\nu = 0.899$. We can see here that if the vaccine is effective against the omicron variant then the exposed and infected group of individuals tend to zero and thus showing that the infection due to Omicron is eradicating.



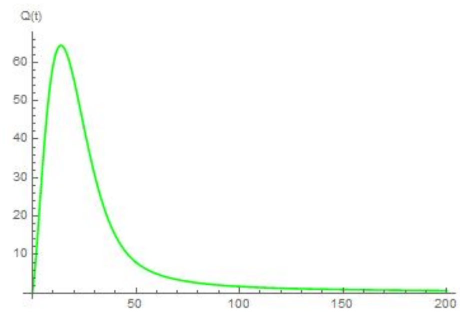
(a) Susceptible



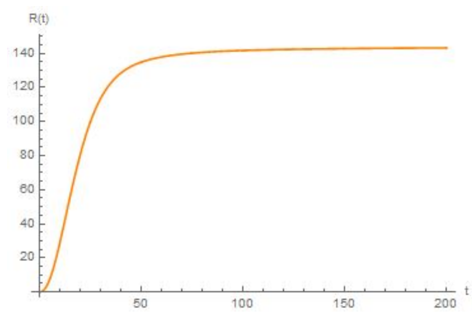
(b) Exposed



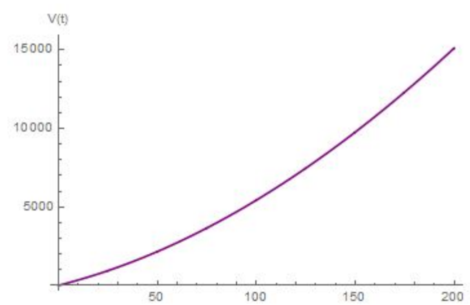
(c) Infected



(d) Quarantined



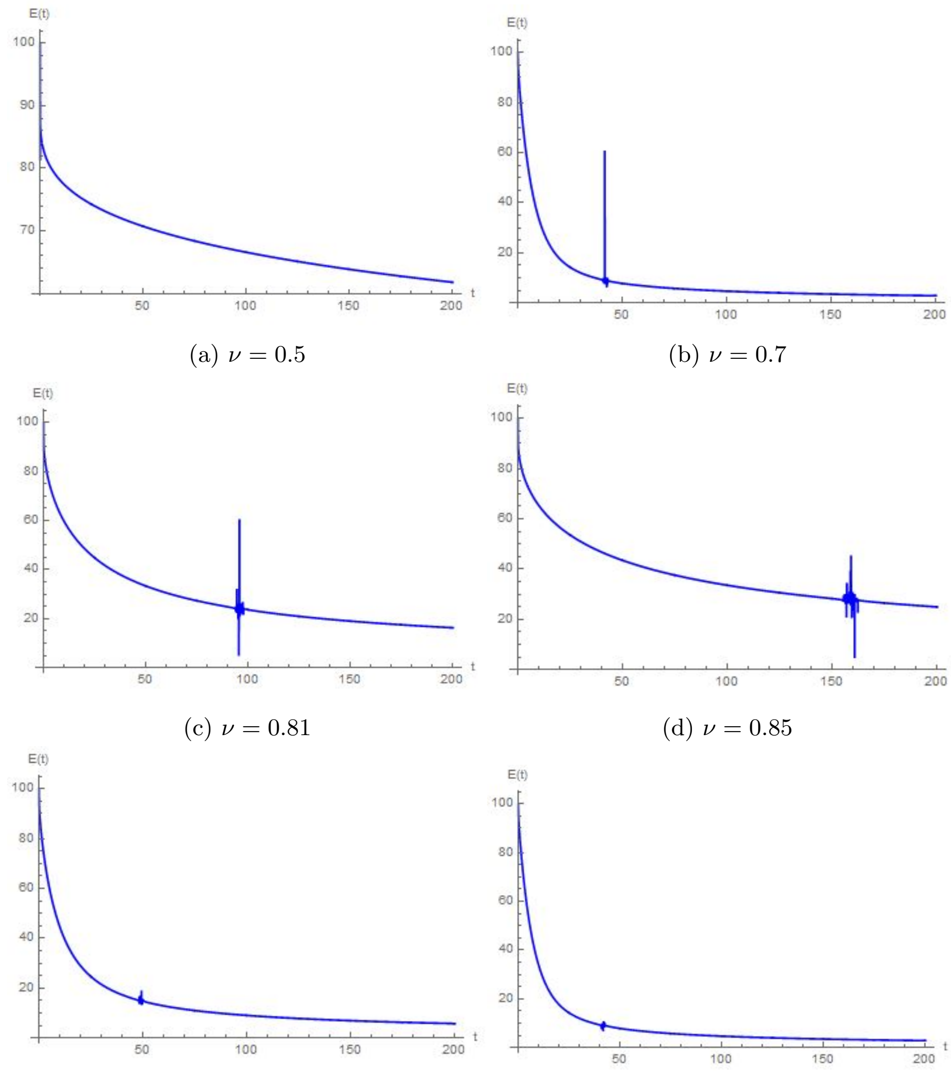
(e) Recovered



(f) Vaccinated

Figure 7: The behaviour of the solution when the vaccine's inefficacy is very low and the value of the fractional order is $\nu = 0.95$.

The above figure shows the behaviour of the susceptible, exposed, infected, quarantined, recovered, and the vaccinated group of individuals along with the time when the inefficacy of the vaccine is very low and the value of the fractional order derivative is $\nu = 0.95$. We can see here that if the vaccine is effective against the omicron variant then the exposed and infected group of individuals tend to zero and thus showing that the infection due to Omicron is eradicating.



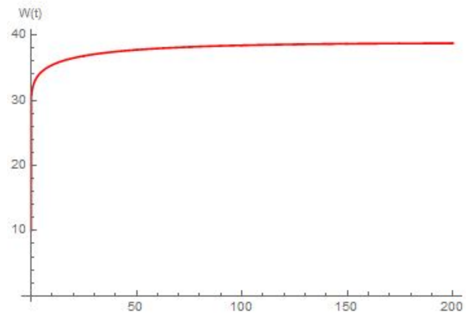
(a) $\nu = 0.5$ (b) $\nu = 0.7$

(c) $\nu = 0.81$ (d) $\nu = 0.85$

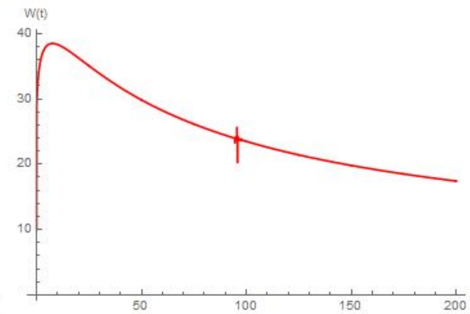
(e) $\nu = 0.899$ (f) $\nu = 0.95$

Figure 8: The behaviour of the exposed class when the efficiency of the vaccine is not much at different fractional order.

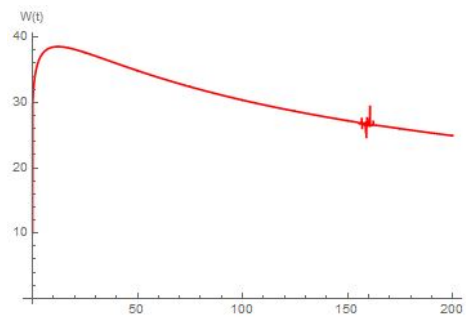
The above figure shows the behaviour of the exposed class when the vaccine is not efficient against the Omicron variant for different fractional order derivatives viz $\nu = 0.5, \nu = 0.7, \nu = 0.81, \nu = 0.85, \nu = 0.899, \nu = 0.95$. Also we can see a steady up and down randomly, this means that if the vaccine is inefficient against the Omicron variant then the number of exposed individuals will increase rapidly creating a havoc. Also in this case the exposed group of individuals will never be zero and the infection will continue to spread.



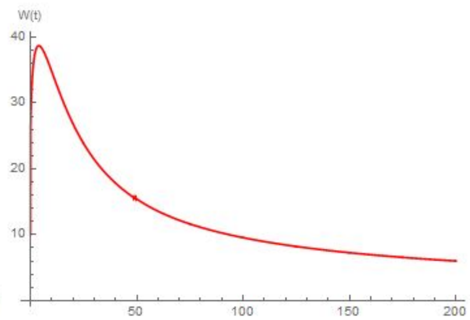
(a) $\nu = 0.5$



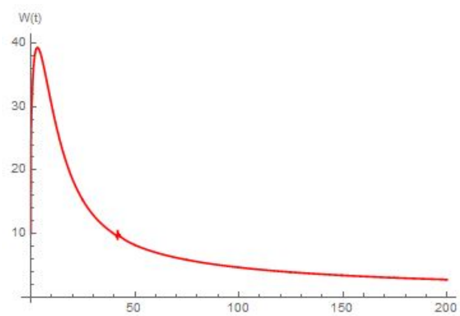
(b) $\nu = 0.7$



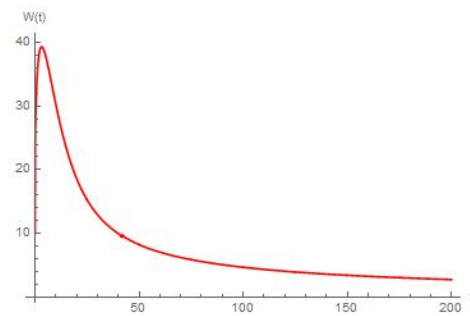
(c) $\nu = 0.81$



(d) $\nu = 0.85$



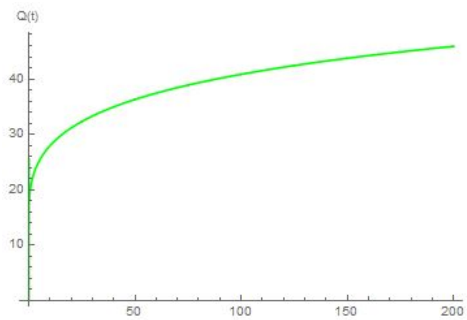
(e) $\nu = 0.899$



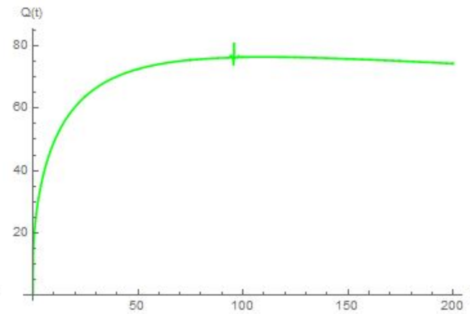
(f) $\nu = 0.95$

Figure 9: The behaviour of the infected class when the efficiency of the vaccine is not much at different fractional order.

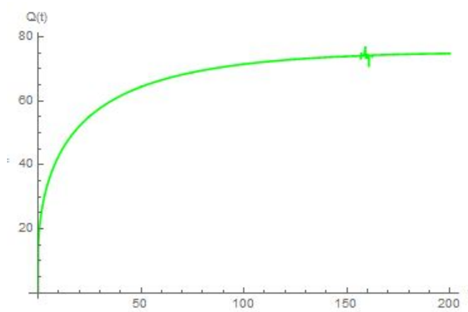
The above figure shows the behaviour of the infected class when the vaccine is not efficient against the Omicron variant for different fractional order derivatives viz $\nu = 0.5, \nu = 0.7, \nu = 0.81, \nu = 0.85, \nu = 0.899, \nu = 0.95$. Also we can see a steady up and down randomly, this means that if the vaccine is inefficient against the Omicron variant then the number of infected individuals will increase rapidly creating a havoc. Also in this case the infected group of individuals will never be zero and the infection will continue to spread.



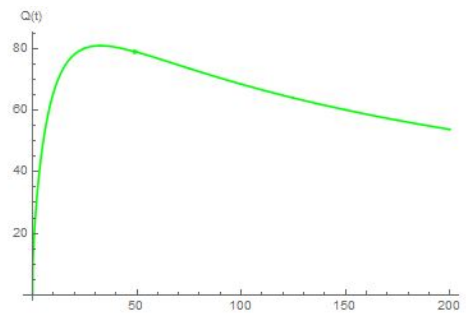
(a) $\nu = 0.5$



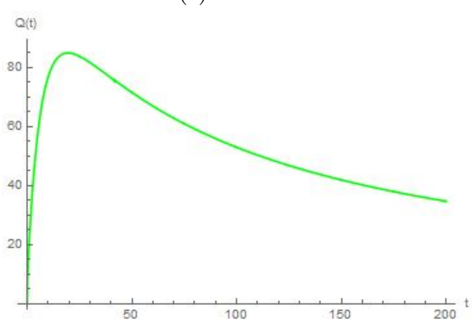
(b) $\nu = 0.7$



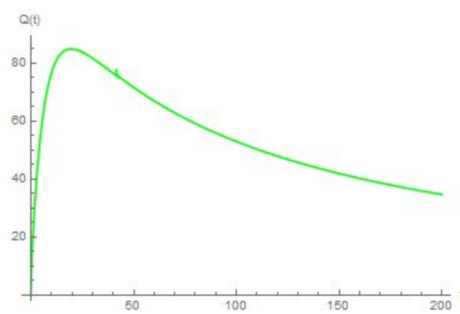
(c) $\nu = 0.81$



(d) $\nu = 0.85$



(e) $\nu = 0.899$



(f) $\nu = 0.95$

Figure 10: The behaviour of the quarantined class when the efficiency of the vaccine is not much at different fractional order.

The above figure shows the behaviour of the quarantined class when the vaccine is not efficient against the Omicron variant for different fractional order derivatives viz $\nu = 0.5, \nu = 0.7, \nu = 0.81, \nu = 0.85, \nu = 0.899, \nu = 0.95$. Also we can see a steady up and down randomly, this means that if the vaccine is inefficient against the Omicron variant then the number of quarantined individuals will increase rapidly creating a havoc. Also in this case the quarantined group of individuals will never be zero.

7 Conclusion

In this paper, we have developed an extended SER mathematical model for the spread of Omicron variant. The model is then extended to the fractional mathematical model in frame of AB fractional derivative. After proving the existence and the uniqueness of the solution, we had found the numerical solution using the Predictor-Corrector method. Using this numerical scheme, we have presented the solutions graphically. From figure-2-7, We see that if the inefficacy of the vaccine is low that means the vaccines are working good against the Omicron variant then exposed and infected individuals decreases well and the recovery rate is increasing. We have also analysed that in the case when the inefficacy of the vaccine is low, the exposed, infected, quarantined class of the individuals decreases as the value of the fractional order increases, and the the recovered and the vaccinated class of individuals increases as the value of the fractional order increases. From figure-6, we see that when the value of the fractional order is 0.95, the exposed, the infected, and the quarantined class of individuals become zero and the number of recovered and the vaccinated individuals is very high for $\nu = 0.95$.
The behaviour of the solution when the inefficacy of the vaccine is high this means that the vaccines are not efficiently working against the omicron variant is given in figure 8, 9, and 10. In figure-8, 9, and 10, we have shown the behaviour for the exposed, infected, quarantined class of individuals respectively at different fractional orders when the inefficacy of the vaccine is high, and we see that there is a peak in the exposed, infected, quarantined of individuals and how this peak varies at different fractional orders. In this case, we see that the exposed and the infected class of individuals can never become zero and the quarantined number of individuals is increasing. This means that vaccination against the Omicron is the only possible way to get rid of this infection.

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References

[1] Mayergoyz ID. Mathematical Models of Hysteresis and their Applications. Academic Press. 2003.

[2] Miller KS, Ross B. An introduction to the fractional calculus and fractional differential equations. Wiley-Intersci. Publ.1993

[3] Podlubny I. Fractional differential equations. An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. Math. Sci. Eng. 1999.

[4] Mohammed WW, Alshammari M, Cesarano C, et al. Brownian motion effects on the stabilization of stochastic solutions to fractional diffusion equations with polynomials. Mathematics. 2022; 10: 1-9.

[5] Cesarano C. Generalized special functions in the description of fractional diffusive equations. Commun. Appl. Ind. Math. 2019; 10: 31-40.

[6] Mohammed WW, Alshammari M, Cesarano C, et al. Brownian motion effects on the stabilization of stochastic solutions to fractional diffusion equations with polynomials. Mathematics. 2022; 10: 1-9.

[7] Baleanu D, Abadi MH, Jajarmi A, et al. A new comparative study on the general fractional model of COVID-19 with isolation and quarantine effects. Alexandria Eng. J. 2022; 61:6; 4779-4791.

[8] Ndairou F, Area I, Nieto JJ, et al. Fractional model of Covid-19 applied to Galicia, Spain and Portugal. Chaos, Solitons Fractals. 2021; 144: 110652.

[9] Kumar A, Prakash A, Baskonus HM. The epidemic Covid-19 model via Caputo-Fabrizio fractional operator. Waves in Random and Complex Media. 2022.

[10] Veeresha P, Gao W, Prakasha DG, Malagi NS, et al. New dynamical behaviour of the coronavirus (2019-nCoV) infection system with non-local operator from reservoirs to people. Inf. Sci. Lett. 2021; 10:2; 205-212.

[11] Nabi KN, Kumar P, Erturk VS. Projections and fractional dynamics of COVID-19 with optimal control strategies. Chaos, Sol. Frac. 2021; 145: 110689.

[12] Atangana A, Araz SI. A novel COVID-19 model with fractional differential operators with singular and non-singular kernels: Analysis and numerical scheme based on Newton polynomial. Alexandria Eng. J. 2021; 60:4: 3781-3806.

[13] Srinivasa K, Baskonus HM, Sanchez YG. Numerical solutions of the mathematical models on the digestive system and Covid-19 pandemic by Hermite Wavelet Technique. Symmetry. 2021; 13:12.

[14] Kumar P, Erturk VS, Arcila MM, et al. A case study of 2019-nCOV cases in Argentina with the real data based on daily cases from March 03, 2020 to March 29, 2021 using classical and fractional derivatives. Adv. Difference Equ. 2021.

[15] Atangana E, Atangana A. Facemasks simple but poweful weapons to protect against COVID-19 spread: Can they have sides effects? Results Phys. 2020; 19:103425.

[16] Mohammadi H, Rezapour S, Jajarmi A. On the fractional SIRD mathematical model and control for the transmission of COVID-19: The first and the second waves of the disease in Iran and Japan. ISA Trans. 2021.

[17] Baleanu D, Abadid MH, Jajarmi A, et al. A new comparative study on the general fractional model of COVID-19 with isolation and quarantine effects. *Alexandria Eng. J.* 2021

[18] Altaf KM, Atangana A. Dynamics of Ebola Disease in the framework of different fractional derivatives. Entropy. 2019.

[19] Atangana A, Araz SI. Modeling and forecasting the spread of COVID-19 with stochastic and deterministic approaches: Africa and Europe. Adv. Difference Equ. 2021; 1:57.

[20] Rezapour S, Mohammadi H, Jajarmi A. A new mathematical model for Zika virus transmission. Adv. Difference Equ. 2020.

[21] Baleanu D, Jajarmi A, Mohammadid H, et al. A new study on the mathematical modeling of human liver with Caputo–Fabrizio fractional derivative. Chaos Sol. Frac. 2020

[22] Alkahtani BST, Alzaid SS. A theoretical analysis of a SEAIJR model of Spanish flu with fractional derivative. *Results Phys.* (2021)

[23] Kucharski AJ, Klepac P, Conlan AJK, et al.. Effectiveness of isolation, testing, contact tracing, and physical distancing on reducing transmission of SARS-CoV-2 in different settings: a mathematical modelling study. Lancet Infect Dis. 2020; 1151-1160.

[24] Ngonghala CN, Iboi E, Eikenberry S, et al. Mathematical assessment of the impact of non-pharmaceutical interventions on curtailing the 2019 novel Coronavirus. Math. Biosci. 2020.

[25] Nabi KN. Forecasting COVID-19 pandemic: a data-driven analysis. Chaos Solitons Fractals 2020.

[26] Ghostine R, Gharamti M, Hassrouny S, et al. An extended SEIR model model with vaccination for forecasting the COVID-19 pandemic in Saudi Arabia using an ensemble kalman filter. Mathematics. 2021.

[27] Barros LC, Lopes MM, Pedro FS, et al.. The memory effect on fractional calculus: an application in the spread of COVID-19. Comput. Appl. Math. 2021; 40:72.

[28] Aguilar JFG, Atangana A. New chaotic attractors: Application of fractal-fractional differentiation and integration.Math. Methods Appl. Sci. 2020; 44:4: 3036-3065

[29] Area I, Fernandez FJ, Nieto JJ, Tojo FA. Concept and solution of digital twin based on a stieltjes differential equation. Math. Methods Appl. Sci. 2022; 45:772; 7451-7465.

[30] Alsalami OM, Sahoo SK, Tariq M, et al. Some new fractional integral inequalities pertaining to generalized fractional integral operator. Symmetry. 2022; 14: 1-9.

[31] Jassim HK, Ahmad H, Shamaoon A, et al. An efficient hybrid technique for the solution of fractional-order partial differential equations. Carpathian J. Math. 2021; 13: 790-804.

[32] Ahmet OA, Deniz E., Yuksel E. On some integral inequalities via conformable fractional integrals. Appl. math. nonlinear sci. 2020: 6, 489-498.

[33] Gurbuz M, Yildiz C. Some new inequalities for convex functions via Riemann-Liouville fractional integrals. Appl. math. nonlinear sci.2020; 6(1).

[34] Aghili A. Complete solution for the time fractional diffusion problem with mixed boundary conditions by operational method. Appl. math. nonlinear sci. 2020; 6:1, 9-20.

[35] Raja MM, Vijayakumar V, Shukla A, et al. On the approximate controllability results for fractional integrodifferential systems of order $1 < r < 2$ with sectorial operators. J. Comput. Appl. Math. 2022; 415.

[36] Gao J, Alotaibi FS, Ismail RI. The model of sugar metabolism and exercise energy expenditure based on fractional linear regression equation. Appl. math. nonlinear sci. 2021; 7(1): 123-132.

[37] Dubey RS, Goswami P, Baskonus HM, et al. On the existence and uniqueness analysis of fractional blood glucose-insulin minimal model. Int. J. Model. Simul. 2022.

[38] Liu C. Precision algorithms in second-order fractional differential equations. Appl. math. nonlinear sci. 2021; 7(1): 155-164.

[39] Wang Y, Veeresha P, Prakasha DG, Baskonus HM, et al. Regarding deeper properties of the fractional order Kundu-Eckhaus equation and massive thirring model. Comput. Model. Eng. Sci. 2022.

[40] Chen S, Ren Y. Small amplitude periodic solution of Hopf bifurcation theorem for fractional differential equations of balance point in group competitive martial arts. Appl. math. nonlinear sci. 2021; 7(1): 207-214.

[41] Alkahtani BST, Alzaid SS. A theoretical analysis of a SEAIJR model of Spanish flu with fractional derivative. Results Phys. 2021.

[42] Atangana A, Baleanu D. New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model. Therm. Sci. 2016; 20:2; 763-769.

[43] Koca I. Modeling the spread of Ebola virus with Atangana-Baleanu fractional operators. Eur. Phys. J. Plus. 2018; 133:100.

[44] Kumar P, Erturk VS, Arcila MM. A new fractional mathematical modelling of COVID-19 with the availability of vaccine.Results Phys. 2021

[45] Baleanu D, Jajarmi A, Hajipour M. On the nonlinear dynamical systems within the generalized fractional derivatives with Mittag-Leffler Kernel. Nonlinear Dyn. 2018; 397-414.