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New soliton solutions of the mZK equation and the Gerdjikov-Ivanov equation by employing the double $(G'/G, 1/G)$-expansion method

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ABSTRACT

In the electrical transmission lines, the processing of cable signals distribution, computer networks, high-speed computer databases and discrete networks can be investigated by the modified Zakharov-Kuznetsov (mZK) equation as a data link propagation control model in the study of nonlinear Schrödinger type equations as well as in the analysis of the generalized stationary Gardner equation. The proposed Gerdjikov–Ivanov model can be used in the field of nonlinear optics, weakly nonlinear dispersion water waves, quantum field theory etc. In this work, we developed complete traveling wave solutions with specific t-type, kink type, bell-type, singular solutions, and periodic singular solutions to the proposed mZK equation and the Gerdjikov-Ivanov equation with the aid of the double $(G'/G, 1/G)$- expansion method. These settled solutions are very reliable, durable, and authentic which can measure the fluid velocity and fluid density in the electrically conductive fluid and be able to analysis of the flow of current and voltage of long-distance electrical transmission lines too. These traveling wave solutions are available in a closed format and make them easy to use. The proposed method is consistent with the abstraction of traveling wave solutions.

Introduction

It is well acquainted that the nonlinear evolution equations (NLEEs) are used to explain the various occurrences in the fields of plasma physics, solid state physics, relativistic physics, optics fibers, chemical physics, chemical kinematics, fluid mechanics, propagation of shallow water waves, flat wave propagation, river mobility, electromagnetic and so on [1–5]. The concept of soliton is involved in the propagation of a large-scale type of wave and most of the solutions of NLEEs are soliton types [6,7]. Hence in the field of science and technology, the exploration of soliton solutions of NLEEs keeps a vital role, especially in system analysis, nonlinear transmission lines, electric control themes, mechanical engineering, chemical engineering, signal processing, gas dynamics, optical telecommunication, electromagnetism, ocean engineering, biomedical problems, nuclear physics, nanofiber technology, etc. Variant kinds of solutions of NLEEs can be recognized via the several types of the analytical process such as periodic waves, singular solutions, breather waves, rational wave solutions, optical solutions, rogue waves, and soliton types of solutions [8,9]. Lately, numerous potential and workable techniques have been investigated for getting constructive solutions for NLEEs. These techniques comprehend the Rieman- Hilbert method [10,11], the Jacobi elliptic function method [12,13], the Lie symmetric analysis [14,15], the auxiliary equation method [16,17], the Sine-Gordon expansion method [18,19], the tan-cot function method [20], the Sardar-subequation technique [21], the simple equation method [22], the modified simple equation method [23], the first

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integral method [24,25], the Hirota's bilinear method [26,27], the homogeneous balance method [28,29], the Darboux-Like transformation method [30] etc. Recently Ibrahim E. Inan et al. proposed the $\exp(-\varphi(\xi))$-expansion method for some exact solutions of (2 + 1) and (3 + 1) dimensional constant coefficients KdV equations [31]. Furthermore, a few numbers of scientists have searched for another better technique that is more feasible and active than any other former techniques such as Mia et al. executed the further investigations to extract abundant new exact traveling wave solutions of some NLEEs by employing the generalized (G'/G)-expansion method [32]. Furthermore, many investigators have used this method to get the proper solutions for NLEEs [33–38]. Lately, Iqbal et al. and Miah et al. initiated the Study on the Date-Jimbo-Kashiwara-Miwa equation with Conformable Derivative Dependent on time parameter to find the exact dynamic wave solutions and the fifth order Caudrey-Dodd-Gibbon equation for exact travelling wave solutions respectively by applying the double $(G'/G, 1/G)$-expansion method [39,40]. After all many researchers [41–45] are applying this technique to find the exact travelling wave solutions of NLEEs. In this research work, we present and apply the double $(G'/G, 1/G)$-expansion method to investigate the mZK equation and the Gerdjikov-Ivanov equation respectively which are given bellow,

The mZK equation is written in the following form:

$$v_t + Avv_x + Bv^2v_x + Mv_{xxx} + Nv_{xyy} = 0 \tag{1.1}$$

where A, B, M and N are constants.

And the Gerdjikov-Ivanov equation has the following form:

$$iq_t + q_{xx} - iq^2q_x^* + \frac{1}{2}|q|^4q = 0 \tag{1.2}$$

where q is a complex function of x and t and q^* its complex conjugate.

Eq. (1.1) plays a crucial role in illuminating the internal workings of concrete composite phenomena in the fields of deep-ocean wave behavior, plasma physics, two-dimensional discrete electrical lattice, and nonlinear optics. This equation is derived for the first time by Munro and Parkes [46] to explain how weakly non-linear ion-acoustic waves behave in a plasma of hot isothermal electrons and cold ions when a consistent magnetic field is present.

While Eq. (2.2) has an essential purpose in non-linear fiber optics. Moreover, photonic crystal fibers have numerous significant uses for it. Xu and He [47] found the rogue wave and the breather solution for this equation by the two-fold DT from a periodic "seed" with a constant amplitude. For the first time, the non-balanced Riccati-Bernoulli Sub-ODE and the balanced modified extended tanh-function methods are utilized in [48] to get the new optical solitons of this equation.

The double $(G'/G, 1/G)$-expansion method is a general simple analytical method that can be used to investigate a variety of solutions with different geometrical structures for different NLEEs with constant or variable coefficients in different topics of science. Al-Shawba et al. [49] used an extension of this method to discuss the solutions behavior for the nonlinear time fractional clannish random Walker's parabolic (CRWP) equation, nonlinear time fractional SharmaTassoOlver (STO) equation, and nonlinear space–time fractional KleinGordon equation. Further, Demiray et al. [33] investigated new solutions for Boussinesq type equations with the aid of the double $(G'/G, 1/G)$-expansion and (1/G')-expansion methods.

This paper is organized as follows: In section 1, a short introduction has been given. The methodology of the double $(G'/G, 1/G)$-expansion method has been given in section 2. The mZK equation has been proposed and explored through the double $(G'/G, 1/G)$- expansion method in section 3. In section 4, the Gerdjikov-Ivanov equation has been analyzed. We give the picturesque manifestation and discussion of the solutions in section 5 and finally, we give the conclusion in section 6.

Exploration of the double $(G'/G, 1/G)$-expansion method

In this part, we succinctly recapitulate the premier keys to the double $(G'/G, 1/G)$-expansion method for proposing the constructive wave solutions of the above alluded NLEEs. Now, we consider a supplementary first-degree ordinary differential equation (ODE) having parameter coefficient as follows,

$$\frac{d^2G}{d\rho^2} + \lambda G = kG = G(\rho) \tag{2.1}$$

and considering the two new variables in the following model,

$$T = G'/G, W = 1/G \tag{2.2}$$

Thus, we can assign in terms of T and W in the following,

$$T' = -T^2 + kW - \lambda, W' = -TW \tag{2.3}$$

The solution of Eq. (2.1) relies upon λ and according to its sign, we give three calcifications which are given below,

Class I. For $\lambda > 0$, we get a complete solution of Eq. (2.1) as mentioned below,

$$G(\rho) = D\sin\left(\sqrt{\lambda}\rho\right) + E\cos\left(\sqrt{\lambda}\rho\right) + \frac{k}{\lambda} \tag{2.4}$$

where the coefficients C and D are arbitrary constants. As a result, it allows

$$W^2 = \frac{\lambda(T^2 - 2kW + \lambda)}{\lambda^2\alpha - k^2} \tag{2.5}$$

where $\alpha = D^2 + E^2$.

Class II. For $\lambda < 0$, we get another general solution of Eq. (2.1) as mentioned below,

$$G(\rho) = D\sinh\left(\sqrt{-\lambda}\rho\right) + E\cosh\left(\sqrt{-\lambda}\rho\right) + \frac{k}{\lambda} \tag{2.6}$$

and consequently,

$$W^2 = -\frac{\lambda(T^2 - 2kW + \lambda)}{\lambda^2\beta + k^2} \tag{2.7}$$

where $\beta = D^2 - E^2$.

Class III. For $\lambda = 0$, we get a rational function solution of Eq. (2.1) as mentioned below,

$$G(\rho) = \frac{k}{2}\rho^2 + D\rho + E \tag{2.8}$$

and hence

$$W^2 = \frac{(T^2 - 2kW)}{D^2 - 2kE} \tag{2.9}$$

Now, we consider a nonlinear evolution equation in terms of three independent variables in the polynomial Z and its partial derivative, which is given below,

$$Z\left(v, v_x, v_y, v_t, v_{xx}, v_{yy}, v_{tt}, v_{xy}, v_{xt}, v_{yt}\cdots\right) = 0 \tag{2.10}$$

Now, we allow the premier actions to the double $(G'/G, 1/G)$-expansion method stage-wise as follows,

Stage I. Associating the arguments x, y and t into a single new variable,

$v(x, y, t) = v(\rho)$; $\rho = x + y - ct$, (2.11)

where c is constants. Here, the resembling ordinary differential equation of Eq. (2.10) is mentioned below,

$$U(v, -cv', v', v', v'', v'', \cdots) = 0 \tag{2.12}$$

where U be a polynomial in $v(\rho)$ and the prime denotes the ordinary derivative regarding ρ.

Stage II. Let us express the solution of Eq. (2.12) in terms of $T(\rho)$ and $W(\rho)$ as mentioned bellow,

$$v(\rho) = a_0 + \sum_{l=1}^{S}\left(a_l T^l + b_l T^{l-1} W\right) \tag{2.13}$$

Here $a_l(l = 1, 2, \cdots, S)$, $b_l(l = 1, 2, \cdots, T)$, c, λ and k will be defined later and the value of S can be obtained with a homogeneous equilibrium.

Stage III. Enter the value of S in Eq (2.13) and replace the modified equation into Eq. (2.12). By exercising Eq. (2.3), Eq. (2.5), Eq. (2.7) and Eq. (2.9), the left of Eq. (2.12) will be converted to a polynomial of $T(\rho)$ and $W(\rho)$. Comparing the same indices of the polynomial to zero generates a group of equations in $a_l(l = 1, 2, \cdots, S)$, $b_l(l = 1, 2, \cdots, S)$, $c, \lambda(\lambda > 0), k, C$ and D.

Stage IV. With the help of a computer program like Mathematica, the algebraic equations obtained in stage 3 gives the solutions in a_l, b_l, c, $\lambda(\lambda > 0), k, C$ and D. Setting $a_l, b_l, c, \lambda(\lambda > 0)$ and k into the resolved Eq. (2.13), we can gain a travelling wave solution developed across the trigonometric functions of Eq. (2.12). After setting the wave conversion in Eq. (2.11) into Eq. (2.13), we attain the required solutions to the NEEs.

Stage V. Likewise Stage III and Stage IV, we can attain two more solutions of Eq. (2.12) which are hyperbolic (for $\lambda < 0$) and rational (for $\lambda = 0$) function solutions.

Exact solutions of the mZK equation

Here, we assign the double $(G'/G, 1/G)$-expansion method to look for the formative solutions of the above-mentioned equation. Now, we take the wave transformation,

$v(x, y, t) = v(\rho)$ and $\rho = x + y - ct$, (3.1).

where 'c' denotes the wave number. Employing this conversion, we alter the mZK equation demonstrated in Eq. (1.1) into an ordinary differential equation mention below,

$$-cv + A\frac{v^2}{2} + B\frac{v^3}{3} + (M + N)v'' = 0 \tag{3.2}$$

where the prime indicates the ordinary derivatives with respect to ρ. By employing the homogeneous balance rule in Eq. (3.2), we achieve the balance number $S = 1$. Putting this balance number in Eq. (2.13), we get the following form,

$$v(\rho) = a_0 + a_1 T(\rho) + b_1 W(\rho) \tag{3.3}$$

where the functions $T(\rho)$ and $W(\rho)$ are mentioned in Eq. (2.2) and Eq. (2.3). Relying on the notations of λ, we acquire three basic solutions of Eq. (3.2) which have been cited in case-1, case-2 and case-3 orderly.

Case-1. For, $\lambda > 0$.

Differentiating Eq. (3.3) two times then using Eq. (2.3) and Eq. (2.5), we alter the left part of the equation (3.3) in terms of T and W. Now equalizing the mentioned values into the obtained polynomials to zero, we get a set of equations in terms of a_0, a_1, b_1, λ and c. After closing this process and implicating the computer program Mathematica, we get one set of values of the constants mentioned below,

$$a_0 = 0, a_1 = 0, b_1 = -\frac{6(M+N)k}{A}, \alpha = \frac{k^2\{A^2 - 6B\lambda(M+N)\}}{A^2\lambda^2}, c = -\lambda(M+N) \tag{3.4}$$

Putting the values from Eq. (3.4) into Eq. (3.3), we get the solutions of Eq. (3.2) as follows,

$$v(\rho) = -\frac{6(M+N)k}{A}\frac{1}{\left[Dsin(\sqrt{\lambda}\rho) + Ecos(\sqrt{\lambda}\rho) + \frac{k}{\lambda}\right]} \tag{3.5}$$

Now, putting $D = 0$ but $k \neq 0$ and $E = \frac{k\sqrt{(A^2 - 6B\lambda(M+N))}}{A\lambda}$ then setting $\rho = (x + y - ct)$, we get the trigonometric function solution of Eq. (1.1) mention below,

$$v(x, y, t) = -\frac{6\lambda(M+N)}{\left(\sqrt{A^2 - 6B\lambda(M+N)}cos\left(\sqrt{\lambda}(x + y - ct)\right) + A\right.} \tag{3.6}$$

Now, putting $E = 0$ but $k \neq 0$ and $D = \frac{k\sqrt{(A^2 - 6B\lambda(M+N))}}{A\lambda}$ then setting $\rho = (x + y - ct)$, we get the following trigonometric function solution of Eq. (1.1) mention below,

$$v(x, y, t) = -\frac{6\lambda(M+N)}{\left(\sqrt{A^2 - 6B\lambda(M+N)}sin\left(\sqrt{\lambda}(x + y - ct)\right) + A\right.} \tag{3.7}$$

Case-2. For.$\lambda < 0$

Analogous to case-1, we get another set of solutions mention below,

$a_0 = -\frac{A}{2B}$, $a_1 = \pm\frac{\sqrt{-3(M+N)}}{\sqrt{2B}}$, $b_1 = \pm\frac{\sqrt{36B^2k^2(M+N)^2 + A^4(D^2 - E^2)}}{2AB}$,$.c = -\frac{A^2}{6B}$

$\lambda = \frac{A^2}{6B(M+N)}$; $B(M + N)\langle 0$. (3.8).

Setting the values from Eq. (3.8) into Eq. (3.3), we have the solution of Eq. (3.2) given below,

$$v(\rho) = -\frac{A}{2B} \pm \frac{\sqrt{3\lambda(M+N)}}{\sqrt{2B}}\frac{Dcosh\left(\sqrt{-\lambda}\rho\right) + Esinh\left(\sqrt{-\lambda}\rho\right)}{Dsinh\left(\sqrt{-\lambda}\rho\right) + Ecosh\left(\sqrt{-\lambda}\rho\right) + \frac{k}{\lambda}}$$
$$\pm\frac{\sqrt{36B^2k^2(M+N)^2 + A^4(D^2 - E^2)}}{2AB}\frac{1}{\left[Dsinh\left(\sqrt{-\lambda}\rho\right) + Ecosh\left(\sqrt{-\lambda}\rho\right) + \frac{k}{\lambda}\right]} \tag{3.9}$$

where $\lambda = \frac{A^2}{6B(M+N)}$; $B(M + N)\langle 0$.

Now, taking $E = 0$, and $k = 0$ but $D \neq 0$ and inserting $\rho = (x + y - ct)$, we have the hyperbolic function solution of Eq. (1.1) which is given below,

$$v(x, y, t) = -\frac{A}{2B}\left[1 \pm coth\left(\sqrt{-\lambda}(x + y - ct)\right)\right.$$
$$\left.\pm cosech\left(\sqrt{-\lambda}(x + y - ct)\right)\right] \tag{3.10}$$

where $= \frac{A^2}{6B(M+N)}$; $B(M + N)\langle 0$.

Case-3. For $\lambda = 0$.

Corresponding to case-1 and case-2, we have another set mentioned below,

$$a_0 = -\frac{3A}{4B}, a_1 = 0, b_1 = \frac{12k(M+N)}{A}, c = -\frac{3A^2}{16B}, E = \frac{24Bk^2(M+N) + A^2D^2}{2A^2k} \tag{3.11}$$

Engaging Eq. (3.11) into Eq. (3.3), we attain the solution of Eq. (3.2) given below,

$$v(\rho) = -\frac{3A}{4B} + \frac{12k(M+N)}{A}\frac{1}{\left(\frac{k}{2}\rho^2 + D\rho + E\right)} \tag{3.12}$$

Now, putting $D = 0$, $E = \frac{12Bk(M+N)}{A^2}$ and $k \neq 0$ then inserting $\rho = (x + y - ct)$, we acquire a rational function solution of Eq. (1.1) as the followings,

$$v(x, y, t) = -\frac{3A}{4B} + \frac{24A(M+N)}{A^2(x + y - ct)^2 + 24B(M+N)} \tag{3.13}$$

Investigation of the Gerdjikov-Ivanov equation

In this part, we also engage the double $(G'/G, 1/G)$-expansion method to look for the abstract solutions of the Gerdjikov-Ivanov equation. Consider the wave transformation,

$q(x, t) = \sqrt{h}e^{\frac{i}{p}\varphi(x,t)}$; $h = h(x, t)$; $p \neq 0$. (4.1).

where h implies the density of fluid. Now Eq. (1.2) leads to.

$h_t + \left(\frac{2}{p}uh - \frac{1}{2}h^2\right)_x = 0$; $u = \varphi_x$. (4.2).

With new wave conversion $\rho = x - ct$, we write Eq. (4.2) as the following form,

$$-ch' + \left(\frac{2}{p}uh - \frac{1}{2}h^2\right)' = 0 \quad (4.3)$$

By integrating, we have

$$u = \frac{p}{2}\left(c + \frac{1}{2}h + \frac{a}{h}\right) \quad (4.4)$$

where a is integrating constant. On this occasion, the above-mentioned equation can be explained as follows:

$$u_t + \frac{2}{p}uu_x = p(h^{-\frac{1}{2}}(h^{\frac{1}{2}})_{xx})_x + \left(hu + \frac{2}{p}h^2\right)_x \quad (4.5)$$

By facilitating and integrating, Eq. (4.5) can be written as follows,

$$q(x,t) = \sqrt{\frac{2\lambda}{c[mcos(\sqrt{\lambda}(x-ct)) + 1]}}e^{\frac{i}{2}[\left(1+\frac{a}{2\lambda}\right)cx + \frac{2\sqrt{\lambda}\tanh^{-1}\left\{\sqrt{\frac{(m-1)}{(m+1)}}\tan[\frac{1}{2}\sqrt{\lambda}(x-ct)]\right\}}{c\sqrt{m^2-1}} + \frac{acmsin(\sqrt{\lambda}(x-ct))}{2\lambda^{\frac{3}{2}}}]} \quad (4.14)$$

$$-uh_t + hu_t - 2uhh_x - h^2u_x - 2p^2h^2h_x - \frac{p}{2}h_{xxx} + 2rh_x + 2\left(\int u_t dx\right)_x = 0 \quad (4.6)$$

where r is a constant. Placing Eq. (4.4) into Eq. (4.6) leads to,

$$\left(\frac{p}{2}c^2 - \frac{p}{2}a + 2r\right)h' - \frac{3p}{2}chh' - \frac{11p}{4}h^2h' - \frac{p}{2}(h^3)'' = 0 \quad (4.7)$$

If we are setting the integrating constant $a = 0$ and integrating two times with refers to ρ, Eq. (4.7) will be of the following form,

$$(h')^2 = \left(c^2 + \frac{4}{k}r\right)h^2 - ch^3 - \frac{11}{12}h^4 \quad (4.8)$$

By engaging the homogeneous balance rule in Eq. (4.8), we find the balance number $T = 1$ and placing it in Eq. (2.13), the following form can be obtained,

$$h(\rho) = a_0 + a_1T(\rho) + b_1W(\rho) \quad (4.9)$$

where the functions $T(\rho)$ and $W(\rho)$ have been narrated in Eq. (2.2) and Eq. (2.3). Depending on the signs of λ, we attain three fundamental solutions of Eq. (4.8) which are given below,

Case-1. For $\lambda > 0$.

Differentiation twice of Eq. (4.9) and employing Eq. (2.3) and Eq. (2.5), we alter the left-hand part of the equation (4.9) in terms of T and W. Now equating the constants of coefficients into the attained expressions equal to zero, we have a group of algebraic equations in a_0, a_1, a_2, b_1, b_2, λ and c. After ending this process and engaging the computer software Mathematica, we get set the solution as follows,

$$a_0 = 0, a_1 = 0, b_1 = \pm\frac{2\lambda\sqrt{3(D^2+E^2)}}{\sqrt{3c^2-11\lambda}}, k = \pm\frac{c\lambda\sqrt{3(D^2+E^2)}}{\sqrt{3c^2-11\lambda}}, r = -\frac{p}{4}(c^2+\lambda) \quad (4.10)$$

By putting the above values into the Eq. (4.9), we have trigonometric function solutions of Eq. (4.8) mentioned below,

$$h(\rho) = \pm\frac{2\lambda\sqrt{3(D^2+E^2)}}{\sqrt{3c^2-11\lambda}}\frac{1}{[Dsin(\sqrt{\lambda}\rho) + Ecos(\sqrt{\lambda}\rho) + \frac{k}{\lambda}]} \quad (4.11)$$

By putting $D = 0$, $k = \pm\frac{c\lambda\sqrt{3}E}{\sqrt{3c^2-11\lambda}}$ but $E \neq 0$ then inserting $\rho = (x-ct)$ in Eq. (4.11), we have.

$h(x,t) = \frac{2\lambda}{c[mcos(\sqrt{\lambda}(x-ct))+1]}$. [By taking '+' sign] (4.12).

Now engaging Eq. (4.12) into Eq. (4.4) and using the relation $u = \varphi_x$ then integrating with respect to x, we have

$$\varphi(x,t) = \frac{p}{2}[\left(1+\frac{a}{2\lambda}\right)cx + \frac{2\sqrt{\lambda}\tanh^{-1}\left\{\sqrt{\frac{(m-1)}{(m+1)}}\tan[\frac{1}{2}\sqrt{\lambda}(x-ct)]\right\}}{c\sqrt{m^2-1}} + \frac{acmsin(\sqrt{\lambda}(x-ct))}{2\lambda^{\frac{3}{2}}}] \quad (4.13)$$

$$m = \frac{\sqrt{3c^2-11\lambda}}{c\sqrt{3}}$$

By using Eq. (4.12) and Eq. (4.13) in Eq. (4.1), we find a trigonometric function solution of Eq. (1.2) as follows,

$$m = \frac{\sqrt{3c^2-11\lambda}}{c\sqrt{3}}$$

Again putting $E = 0$, $k = \pm\frac{c\lambda\sqrt{3}D}{\sqrt{3c^2-11\lambda}}$ but $D \neq 0$ then inserting $\rho = (x-ct)$, we have.

$h(x,t) = \frac{2\lambda}{c[msin(\sqrt{\lambda}(x-ct))+1]}$. [By taking '+' sign] (4.15).

By using Eq. (4.15) and Eq. (4.4) in Eq. (4.1), we find a trigonometric function solution of Eq. (1.2) as follows,

$$q(x,t) = \sqrt{\frac{2\lambda}{c[msin(\sqrt{\lambda}(x-ct))+1]}}e^{\frac{i}{2}[\left(1+\frac{a}{2\lambda}\right)cx + \frac{2\sqrt{\lambda}\tanh^{-1}\left\{\frac{m+\tan[\frac{1}{2}\sqrt{\lambda}(x-ct)]}{\sqrt{1-m^2}}\right\}}{c\sqrt{1-m^2}} - \frac{acmcos(\sqrt{\lambda}(x-ct))}{2\lambda^{\frac{3}{2}}}]} \quad (4.16)$$

$$m = \frac{\sqrt{3c^2-11\lambda}}{c\sqrt{3}}$$

Case-2. For $\lambda < 0$

Alike to case-1, we get another set of solutions as follows,

$$a_0 = 0, a_1 = 0, b_1 = \pm\frac{2\lambda\sqrt{3(D^2-E^2)}}{\sqrt{11\lambda-3c^2)}}, k = \pm\frac{c\lambda\sqrt{3(D^2-E^2)}}{\sqrt{11\lambda-3c^2}}, r = -\frac{p}{4}(c^2+\lambda) \quad (4.17)$$

By putting the above values into the Eq. (4.9), we have hyperbolic function solutions of Eq. (4.8) mentioned below,

$$h(\rho) = \pm\frac{2\lambda\sqrt{3(D^2-E^2)}}{\sqrt{11\lambda-3c^2)}}\frac{1}{[Dsinh(\sqrt{-\lambda}\rho) + Ecosh(\sqrt{-\lambda}\rho) + \frac{k}{\lambda}]} \quad (4.18)$$

Placing $D = 0$, $k = \pm\frac{c\lambda\sqrt{3}E}{\sqrt{3c^2-11\lambda}}$ but $E \neq 0$ then inserting $\rho = (x-ct)$, we have.

$h(x,t) = -\frac{2\lambda}{c[mcosh(\sqrt{-\lambda}(x-ct))+1]}$. [By taking '-' sign] (4.19)

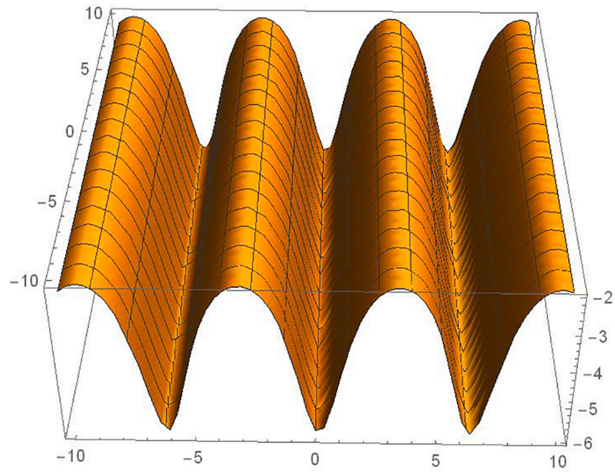

Fig. 1a. The 3D figure of $|\boldsymbol{q}(\boldsymbol{x},\boldsymbol{t})|$ implies the periodic soliton of the Eq. (3.6) within the range $(\boldsymbol{x},\boldsymbol{t}) \in [-10,10]$ for the parameters $|\boldsymbol{u}(\boldsymbol{x},\boldsymbol{t})|\boldsymbol{\lambda} = 1, \boldsymbol{M} = 1, \boldsymbol{N} = 1, \boldsymbol{A} = 4, \boldsymbol{B} = 1$, and $\boldsymbol{c} = -2$.

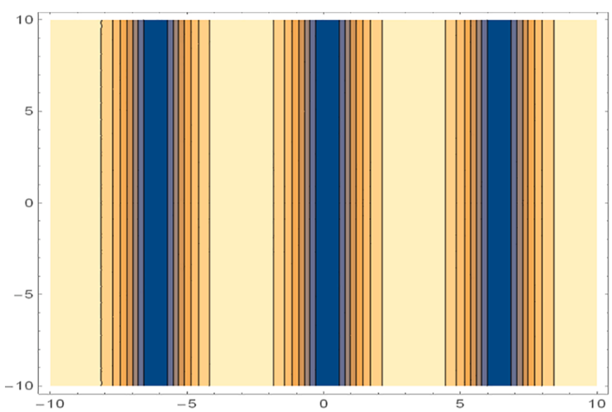

Fig. 1b. The figure of $|\boldsymbol{q}(\boldsymbol{x},\boldsymbol{t})|$ implies the contour shape of the Eq. (3.6) within the range $(\boldsymbol{x},\boldsymbol{t}) \in [-10,10]$ for the parameters $\boldsymbol{\lambda} = 1, \boldsymbol{M} = 1, \boldsymbol{N} = 1, \boldsymbol{A} = 4, \boldsymbol{B} = 1$, and $\boldsymbol{c} = -2$.

$$m = \frac{\sqrt{3c^2 - 11\lambda}}{c\sqrt{3}}$$

By using Eq. (4.19) and Eq. (4.4) in Eq. (4.1), we a get a hyperbolic function solution of Eq. (1.2) as follows,

$$q(x,t)=\sqrt{-\frac{2\lambda}{c\left[mcosh\left(\sqrt{-\lambda}(x-ct)\right)+1\right]}}e^{\frac{i}{2}\left[\left(1+\frac{a}{2\lambda}\right)cx+\frac{2\sqrt{-\lambda}tan^{-1}\left[\frac{\left[\left(m-1\right)\tanh\left[\frac{1}{2}\sqrt{-\lambda}(x-ct)\right]\right]}{\sqrt{-\left(1+m^2\right)}}\right]}{c\sqrt{(m^2-1)}}-\frac{acmsinh\left(\sqrt{-\lambda}(x-ct)\right)}{2(-\lambda)^{\frac{3}{2}}}\right]} \tag{4.20}$$

$$m = \frac{\sqrt{3c^2 - 11\lambda}}{c\sqrt{3}}$$

Case-3. For $\lambda = 0$.

Alike to case-1 and case-2, we get another set of solutions as follows,

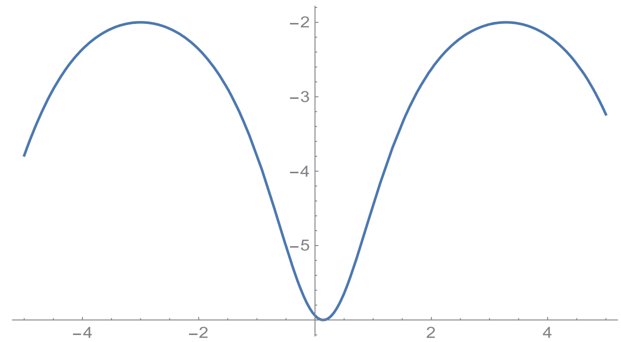

Fig. 1c. The 2D surface of $|\boldsymbol{q}(\boldsymbol{x},\boldsymbol{t})|$ implies the projection of 3D form of Eq. (3.6) within the range $\boldsymbol{x} \in [-5,5]$ for the parameters $\boldsymbol{\lambda} = 1, \boldsymbol{M} = 1, \boldsymbol{N} = 1, \boldsymbol{A} = 4, \boldsymbol{B} = 1, \boldsymbol{c} = -2$ and $\boldsymbol{t} = 1$.

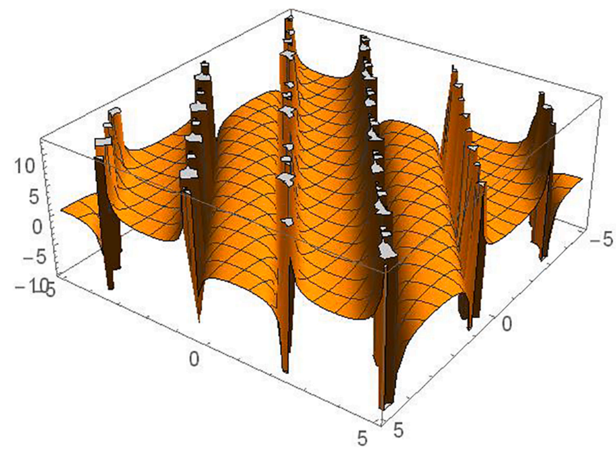

Fig. 2a. The 3D figure of $|\boldsymbol{q}(\boldsymbol{x},\boldsymbol{t})|$ implies the singular periodic soliton of the Eq. (3.10) within the range $(\boldsymbol{x},\boldsymbol{t}) \in [-5,5]$ for the parameters $\boldsymbol{\lambda} = -\frac{2}{3}, \boldsymbol{M} = 1, \boldsymbol{N} = 1, \boldsymbol{A} = 4, \boldsymbol{B} = -2$, and $\boldsymbol{c} = \frac{4}{3}$.

$$a_0 = 0,\ a_1 = 0,\ b_1 = \frac{2\left(-3cE \pm \sqrt{9c^2E^2 - 33D^2}\right)}{11},\ k = \frac{c}{11}\left(3Ec \pm \sqrt{9c^2E^2 - 33D^2}\right),\ r = -\frac{c^2p}{4} \tag{4.21}$$

By putting the above values into the Eq. (4.9), we have the solutions of Eq. (4.8) mention below,

$$h(\rho) = \frac{2\left(-3cE \pm \sqrt{9c^2E^2 - 33D^2}\right)}{11}\frac{1}{\left(\frac{k}{2}\rho^2 + D\rho + E\right)} \tag{4.22}$$

Taking $D = 0$ but $E \neq 0$ and $k = \frac{6Ec^2}{11}$ then inserting $\rho = (x - ct)$, we have

$$h(x,t) = -\frac{12c}{3c^2(x-ct)^2 + 11} \tag{4.23}$$

By using Eq. (4.23) and Eq. (4.4) in Eq. (4.1), we a get a rational function solution of Eq. (1.2) as follows,

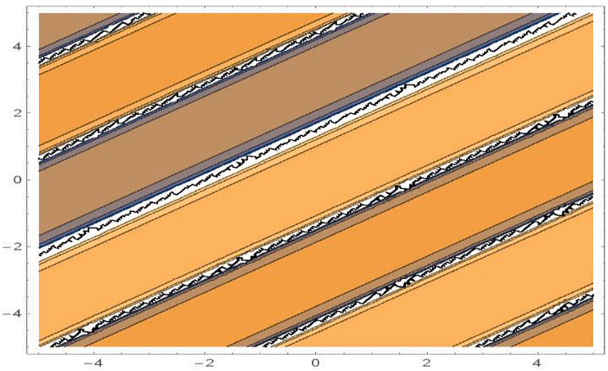

Fig. 2b. The figure of $|q(x,t)|$ implies the contour shape of the Eq. (3.10) within the range $(x,t) \in [-5,5]$ for the parameters $\lambda = -\frac{2}{3}, M = 1, N = 1, A = 4, B = -2$, and $c = \frac{4}{3}$.

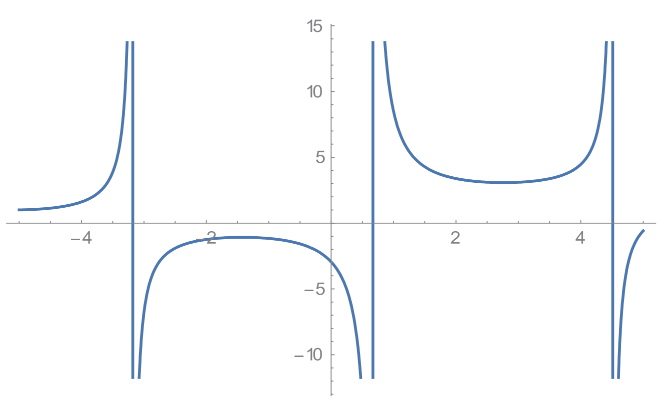

Fig. 2c. The 2D surface of $|q(x,t)|$ implies the projection of 3D form of Eq. (3.10) within the range $x \in [-5,5]$ for the parameters $\lambda = -\frac{2}{3}, M = 1, N = 1, A = 4, B = -2, c = \frac{4}{3}$ and $t = 2$.

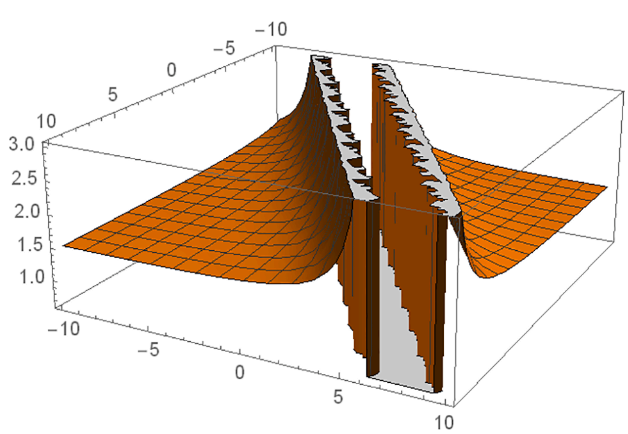

Fig. 3a. The 3D figure of $|q(x,t)|$ implies the singular kink shape soliton of the Eq. (3.13) within the range $(x,t) \in [-10,10]$ for the parameters $M = 1, N = 1, A = 4, B = -2$, and $c = \frac{3}{2}$.

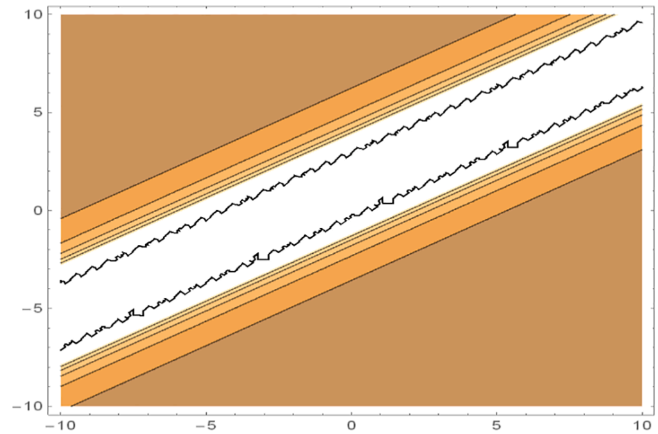

Fig. 3b. The figure of $|q(x,t)|$ implies the contour shape of the Eq. (3.13) within the range $(x,t) \in (x,t) \in [-10,10]$ for the parameters $M = 1, N = 1, A = 4, B = -2$, and $c = \frac{3}{2}$.

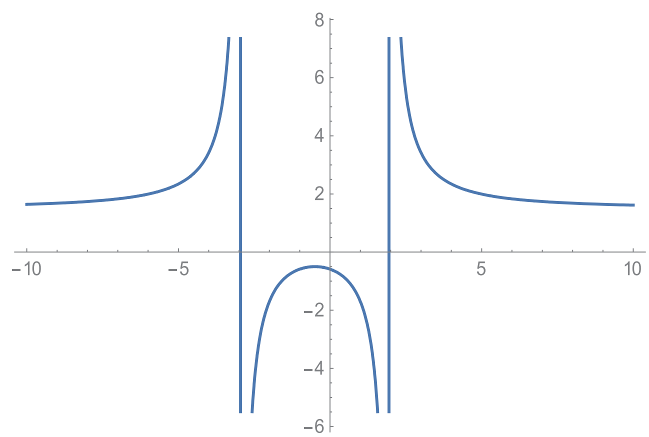

Fig. 3c. The 2D surface of $|q(x,t)|$ implies the projection of 3D form of Eq. (3.13) within the range $x \in [-10,10]$ for the parameters $M = 1, N = 1, A = 4, B = -2, c = \frac{3}{2}$ and $t = 1$.

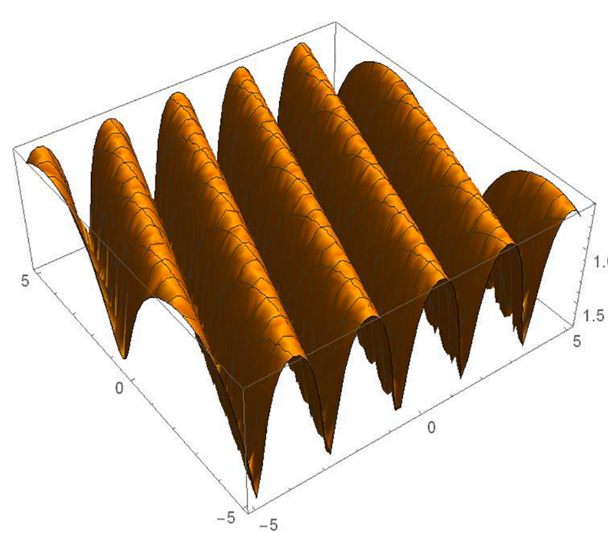

Fig. 4a. The 3D figure of $|q(x,t)|$ implies the periodic soliton of the Eq. (4.14) within the range $(x,t) \in [-5,5]$ for the parameters $c = 3, \lambda = 1$, and $m = \frac{4}{3\sqrt{3}}$.

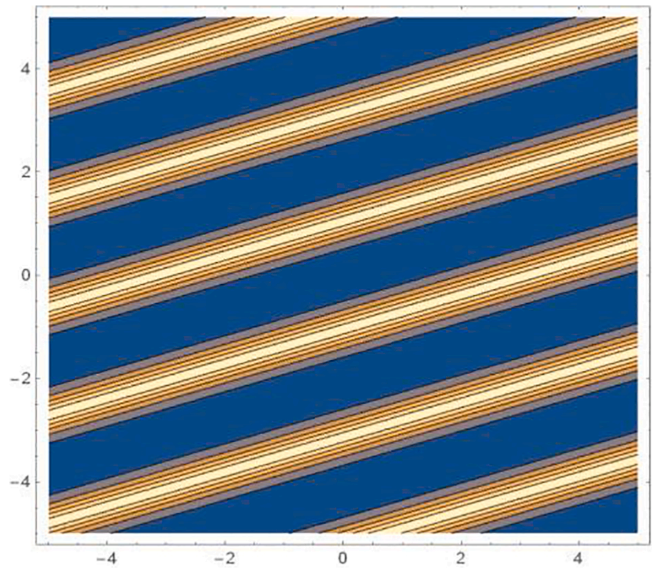

Fig. 4b. The figure of $|q(x,t)|$ implies the contour shape of the Eq. (4.14) within the range $(x,t) \in [-5,5]$ for the parameters $c = 3$, $\lambda = 1$, and $m = \frac{4}{3\sqrt{3}}$.

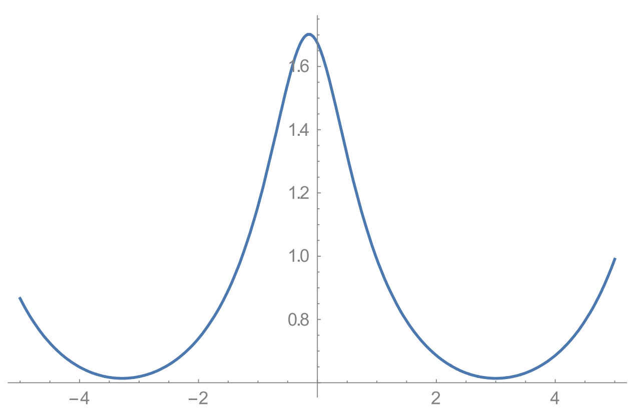

Fig. 4c. The 2D surface of $|q(x,t)|$ implies the projection of 3D form of Eq. (4.14) within the range $x \in [-5,5]$ for the parameters $c = 3$, $\lambda = 1$, $m = \frac{4}{3\sqrt{3}}$ and, $t = 1$.

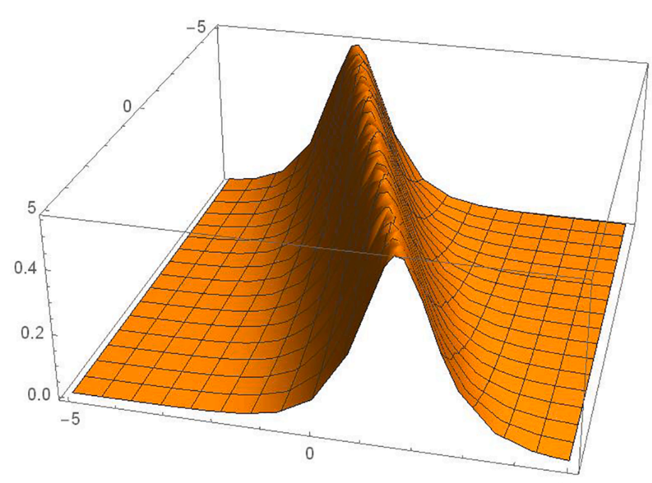

Fig. 5a. The 3D figure of $|q(x,t)|$ implies the bell shape soliton of the Eq. (4.20) within the range $(x,t) \in [-5,5]$ for the parameters $c = 3$, $\lambda = -1$, and $m = \frac{\sqrt{38}}{3\sqrt{3}}$.

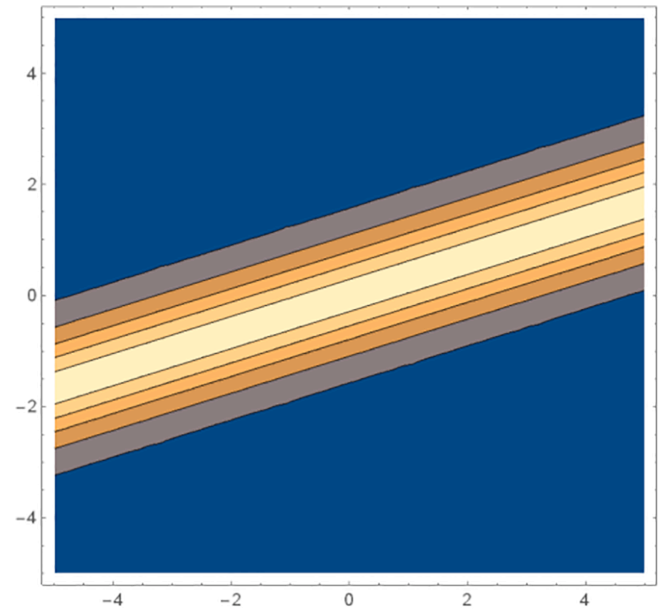

Fig. 5b. The figure of $|q(x,t)|$ implies the contour shape of the Eq. (4.20) within the range $(x,t) \in [-5,5]$ for the parameters $c = 3$, $\lambda = -1$, and $m = \frac{\sqrt{38}}{3\sqrt{3}}$.

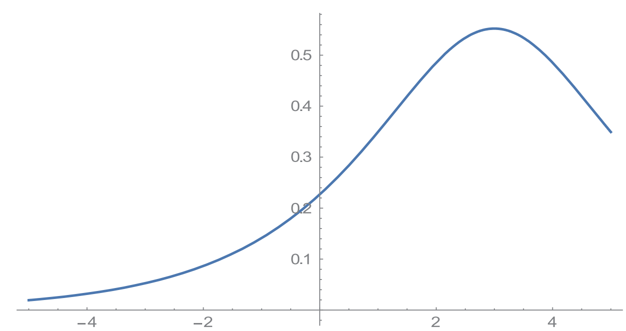

Fig. 5c. The 2D surface of $|q(x,t)|$ implies the projection of 3D form of Eq. (4.20) within the range $x \in [-5,5]$ for the parameters $c = 3$, $\lambda = -1$, $m = \frac{\sqrt{38}}{3\sqrt{3}}$ and $t = 1$.

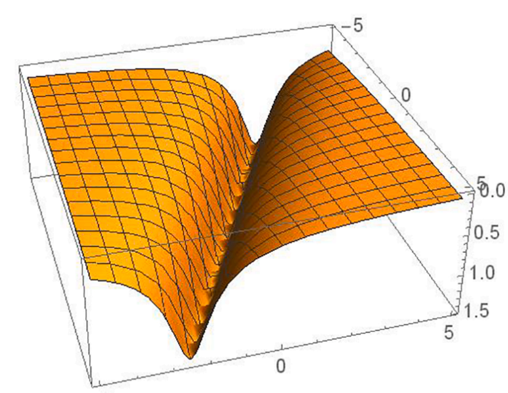

Fig. 6a. The 3D figure of $|q(x,t)|$ implies the anti-bell shape soliton of the Eq. (4.24) within the range $(x,t) \in [-5,5]$ for the parameter $c = -2$.

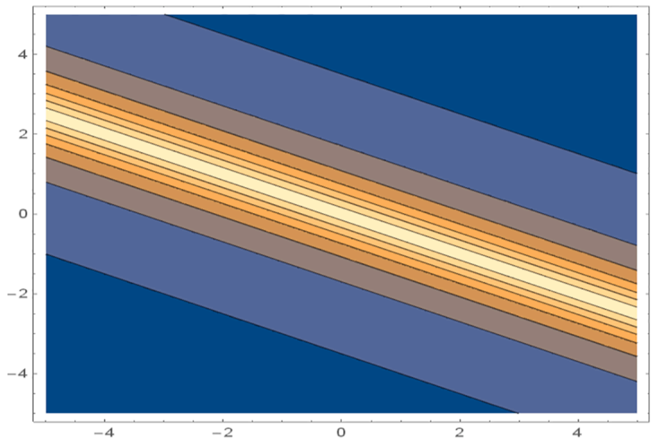

Fig. 6b. The figure of $|q(x,t)|$ implies the contour shape of the Eq. (4.24) within the range $(x,t) \in [-5,5]$ for the parameters $c = -2$.

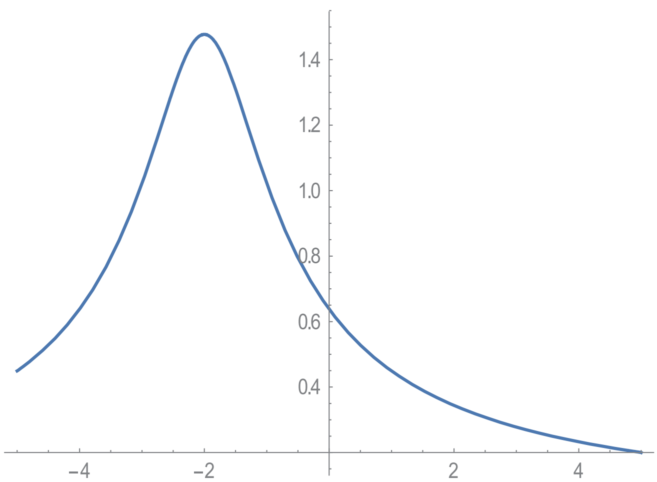

Fig. 6c. The 2D surface of $|q(x,t)|$ implies the projection of 3D form of Eq. (4.24) within the range $x \in [-5,5]$ for the parameters $c = -2$, and $t = 1$.

$$q(x,t) = \sqrt{-\frac{12c}{3c^2(x-ct)^2+11}}e^{\frac{i}{2}[(c-\frac{a\left(11+3c^4t^2\right)}{12c})x+\frac{1}{4}atc^2x^2-\frac{1}{12}acx^3-2\sqrt{\frac{3}{11}}\tan^{-1}[\sqrt{\frac{3}{11}}c(x-ct)]]} \tag{4.24}$$

where c is negative.

Graphical representation and discussion

Here, we delineate the graphical illustration as well as its physical significances of our gained traveling wave solutions to the proposed equations. As the characteristic of investigative solutions relies on the geometrical composition so we obvious several form of soliton solutions like periodic soliton, singular periodic soliton, anti-kink shape soliton, bell shape soliton, and anti-bell shape solitons are traced in this part with 3D, contour, and 2D form. Here, we have explained our six attained solutions. Firstly, we draw the figures of the of Eq. (3.6) in three formats which are 3D, contour and the projection of 3D i.e. 2D figure and the type of solution is periodic soliton within the range $(x,t) \in [-10,10]$ for the parameters $\lambda = 1, M = 1, N = 1, A = 4, B = 1$, and $c = -2$ which is given in Fig. 1(a). The contour shape of Eq. (3.6) has been displayed in Fig. 1(b) for the same range and same parameters. In Fig. 1(c) the 2D surface i.e. the projection of 3D form within the range $x \in [-5,5]$ for the parameters $\lambda = 1$, $M = 1$, $N = 1$, $A = 4$, $B = 1$, $c = -2$ and $t = 1$ has been shown. Now the Eq. (3.10) implies the singular periodic soliton within the range $(x,t) \in [-5,5]$ for the parameters $\lambda = -\frac{2}{3}, M = 1, N = 1, A = 4, B = -2$, and $c = \frac{4}{3}$ which is shown in Fig. 2(a) and the corresponding contour shape and 2D shape are given in Figs. 2(b) and 2(c) respectively. The Eq. (3.13) signifies the singular kink shape soliton within the range $(x,t) \in [-10,10]$ for the parameters $M = 1, N = 1, A = 4, B = -2$, and $c = \frac{3}{2}$ which is shown in Fig. 3(a) and its resembling contour shape and 2D shape are given in Figs. 3(b) and 3(c) respectively. The Eq. (4.14) implies the periodic soliton within the range $(x,t) \in [-5,5]$ for the parameters $c = 3, \lambda = 1$, and $m = \frac{4}{3\sqrt{3}}$ which is shown in Fig. 4(a) and its analogous contour shape and 2D shape are given in Figs. 4(b) and 4(c) respectively. The Eq. (4.20) indicates the bell shape soliton within the range $(x,t) \in [-5,5]$ for the parameters $c = 3, \lambda = -1$, and $m = \frac{\sqrt{38}}{3\sqrt{3}}$ which is shown in Fig. 5(a) and its corresponding contour shape and 2D shape are given in Figs. 5(b) and 5(c) respectively. Finally, Fig. 6(a) of the Eq. (4.24) implies the anti-bell shape soliton within the range $(x,t) \in [-5,5]$ for the parameter $c = -2$ and its corresponding contour shape and 2D shape are plotted in Figs. 6(b) and 6(c) respectively.

Conclusion

In this research work, we attain exact travelling wave solutions of the mZK equation and Gerdjikov-Ivanov equation. These solutions are individual which are trigonometric function, hyperbolic and rational function. In the various conditions and situations, these wave solutions are going to be very obligate. The obtained solutions are stable, adaptable, and capable to travel long dimensions. The attained solutions explain the flow of current and voltage which can be applied to sketch the electrical transmission lines. So, these solutions may be sharp to resolve the complex occurrence blooming in science and engineering and susceptible to amplifying the impression on further investigation. These investigations exhibit through the double $(G'/G, 1/G)$-expansion method, which is completely proficient and, well fitted to search the closed form wave solution for the difficulties deliberated in this paper. We believe that this method will be of great help in solving a variety of real and complex nonlinear evolution equations in the future. At the same time, we hope to have more innovative methods to provide new ideas for the two equations investigated in this work.

CRediT authorship contribution statement

M. Ashik Iqbal: Methodology, Formal analysis, Software, Writing – original draft. **Dumitru Baleanu:** Resources, Validation, Funding acquisition, Writing – review & editing. **M. Mamun Miah:** Project administration, Supervision, Writing – review & editing. **H.M. Shahadat Ali:** Conceptualization, Data curation, Visualization, Investigation. **Hashim M. Alshehri:** Conceptualization, Data curation, Visualization, Investigation. **M.S. Osman:** Resources, Validation, Funding acquisition, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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