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ORIGINAL PAPER

Nonautonomous lump-periodic and analytical solutions to the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili equation

Marwan Alquran · Tukur Abdulkadir Sulaiman · Abdullahi Yusuf · Ali S. Alshomrani · Dumitru Baleanu

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Abstract This work establishes the lump periodic and exact traveling wave solutions for the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili equation. We use the Hirota bilinear method, as well as the robust integration techniques tanh–coth expansion and rational sine–cosine, to provide such innovative solutions. In order to explain specific physical difficulties, innovative lump periodic and analytical solutions have been investigated. These discoveries have been proven to be useful in the transmission of long-wave and high-power communications networks. It is important to highlight that the results given in this work depict new features and reflect previously unknown physical dynamics for the governing model.

M. Alquran (✉)
Department of Mathematics and Statistics, Jordan University of Science and Technology, Irbid, Jordan
e-mail: marwan04@just.edu.jo

T. A. Sulaiman · A. Yusuf
Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon

T. A. Sulaiman · A. Yusuf
Department of Computer Engineering, Biruni University, Istanbul, Turkey

A. S. Alshomrani
Department of Mathematics, King Abdul Aziz University, Jeddah, Saudi Arabia

D. Baleanu
Department of Mathematics, Cankaya University, Ankara, Turkey

D. Baleanu
Institute of Space Sciences, Magurele, Bucharest, Romania

D. Baleanu
Lebanese American University, 11022801 Beirut, Lebanon

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1 Introduction

Two or more variables are involved in a partial differential equation (PDE) [1]. During the twentieth century, there was considerable advancement in the study of differential equations. The main cause for this is the growing number of mathematical applications in fields such as medicine, engineering, computer science, mathematical biology, and aerodynamics. Nonlinear PDEs are used to mathematically characterize various physical phenomena [2–4].

The ability to get analytical solutions to traveling waves traced by nonlinear differential equations is a significant addition in nonlinear sciences since it depicts heterogeneous natural events such as solitons, vibrations, and speed distribution [5–7].

Solitons are nonlinear diffusive PDE solutions that describe any physical system. A solitary wave, also known as a soliton in mathematics and physics, is a spiral wave feature that maintains its structure as it spreads at an unchanging velocity [8].

PDEs are used to explain all real-world processes, resulting in a complex collection of problems that can-not be solved perfectly but may be addressed analytically by considering their actual scenarios and theoretical potential. There have various ways to employ PDEs and apply them in real-word problems such as heat unsteady flow of a micropolar fluid over a curved stretched surface [9–12]. Moreover, to examine analytic solutions of PDEs, several effective strategies have been presented in the literature. Exploring the dynamic behavior of tangible actual systems has long been an important study subject for mathematicians throughout history. During the twentieth century, researchers began to investigate the in-depth analysis of nonlinear systems and their centripetal compositions, and much attention was paid to Chaos theory, which states that PDEs and ODEs can exhibit astonishingly diverse behavior, allowing around settled schemes to be exponentially episodic for flaring time. Soliton theory allows mathematicians to attest to the quasi-linear behavior of nonlinear PDEs (systems) [8,13–20].

It is generally established that nonlinear interaction solutions can be interpreted by lump solutions [21]. Many researchers have studied lump solutions for integrable equations during the last few decades [22–25]. Furthermore, other investigations demonstrate the occurrence of a collision between lumps and other kinds of exact solutions to nonlinear aspects. It could be noticed, however, that several methodologies have been used to generate interaction phenomena for PDEs. To guarantee that solutions exist, values are sometimes set to these constant coefficients. It is important to remember that research on how to deal with these interaction events is still lacking. Owing to this, we got the motivation to consider the lump and exact solutions for the (3+1)-dimensional generalized Kadomtsev–Petviashvili equation provided by [26–29]

$$\tau_{xt}+\alpha\tau_{xxxx}+\beta(\tau\tau_{xx}+\tau_x^2)+\delta(\tau_{yy}+\tau_{zz})=0, \quad (1)$$

where τ is the wave-amplitude function of x, y, z and t. The parameter β is the coefficient of the nonlinear terms, and α is the coefficient of the dispersion term. The parameter δ is the coefficient of the dispersionless terms. With the change in the output is not equal to the change in the input, nonlinearity results. Waves of various wavelengths spread at various phase velocities when there is dispersion. The speed at which a wave moves across a medium is referred to as its phase velocity. This is the speed at which any particular frequency portion of the wave moves in phase. Findings have proven that wave pulses can sustain their form and speed in the form of solitons during the transmission process when the scattering effect and nonlinear impact of the medium attain a stable equilibrium [26].

2 Lump-periodic solution

Here, the lump-periodic solutions to the variable coefficients form of Eq. (1) are presented.

The variable coefficients form of Eq. (1) is regulated as

$$\begin{aligned}&\tau_{xt}+\alpha(t)\tau_{xxxx}+\beta(t)(\tau\tau_{xx}\\&\quad+\tau_x^2)+\delta(t)(\tau_{yy}+\tau_{zz})=0.\end{aligned}\quad (2)$$

Plugging the transformation parameter

$$\tau(x,y,z,t)=12(\ln\psi(x,y,z,t))_{xx} \quad (3)$$

in (1), by taking $\beta(t)=\alpha(t)$, we reach

$$\begin{aligned}&\psi_y^2-\delta(t)\psi_z^2-\psi(\psi_{zz}+\psi_{yy})+3\alpha(t)\psi_{xx}^2\\&\quad-\psi_x(\psi_t+4\alpha(t)\psi_{xxx})\\&\quad+\psi(\psi_{xt}+\alpha(t)\psi_{xxxx})=0.\end{aligned}\quad (4)$$

Using the following as a result of Eq. (4)

$$\begin{aligned}\psi(x,y,z,t)&=\varpi_1(t)\cosh(\zeta_1)\\&\quad+\varpi_2(t)\cos(\zeta_2)+\varpi_3(t)\cosh(\zeta_3),\end{aligned}\quad (5)$$

where $\zeta_1=d_3(t)+d_1y+d_2z+x$, $\zeta_2=d_6(t)+d_4y+d_5z+x$, $\zeta_3=d_9(t)+d_7y+d_8z+x$.

Substituting Eq. (5) into (4) yields an equation in powers of $\cos(\cdot)$, $\cosh(\cdot)$. Performing some symbolic computations, we reach the following solutions:

(I): With

$$\begin{aligned}&d_5=d_2-(d_1-d_4),\\&d_3(t)=\int\left(d_1^2(-\delta(t)-d_2^2\delta(t)+2\alpha(t)\right)dt+K_1,\\&d_6(t)=\int\left(d_1^2\delta(t)-2d_4d_1\delta(t)-d_2^2\delta(t)\right.\\&\qquad\left.+2\sqrt{d_2^2\,(d_1-d_4)^2\left(-\delta(t)^2\right)}-2\alpha(t)\right)dt\\&\qquad+K_2,\ \varpi_2(t)=i\varpi_1(t),\ \varpi_3(t)=0,\end{aligned}$$

we have

$$\psi^{I}(x,y,z,t)=i\varpi_1(t)\cos\Bigg(\int\Big(d_1^2\delta(t)-2d_4d_1\delta(t)+2\sqrt{-d_2^2(d_1-d_4)^2\delta(t)^2}-d_2^2\delta(t)-2\alpha(t)\Big)dt+\frac{z\left(d_2^2\delta(t)-\sqrt{-d_2^2\,(d_1-d_4)^2\delta(t)^2}\right)}{d_2\delta(t)}+d_4y+K_2+x\Bigg)+\varpi_1(t)\cosh\Bigg(\int\Big(d_1^2(-\delta(t))-d_2^2\delta(t)+2\alpha(t)\Big)dt+d_1y+d_2z+K_1+x\Bigg). \tag{6}$$

Thus,

$$\tau^{I}(x,y,z,t)=12\Big((\cosh(\Theta_2)\varpi_1(t)-i\cos(\Theta_1)\varpi_1(t))(\cosh(\Theta_2)\varpi_1(t)+i\cos(\Theta_1)\varpi_1(t))-(\sinh(\Theta_2)\varpi_1(t)-i\sin(\Theta_1)\varpi_1(t))^2\Big)\Big/(\cosh(\Theta_2)\varpi_1(t)+i\cos(\Theta_1)\varpi_1(t))^2, \tag{7}$$

where $\Theta_1=\int(d_1^2\delta(t)-2d_4d_1\delta(t)+2\sqrt{-d_2^2(d_1-d_4)^2\delta(t)^2}-d_2^2\delta(t)-2\alpha(t))dt+K_2+\frac{z\left(d_2^2\delta(t)-\sqrt{-d_2^2(d_1-d_4)^2\delta(t)^2}\right)}{d_2\delta(t)}+d_4y+x$, $\Theta_2=\int\left(d_1^2(-\delta(t))-d_2^2\delta(t)+2\alpha(t)\right)dt+K_1+d_1y+d_2z+x$.

(II): With

$$d_8=\frac{d_5^2\delta(t)-\sqrt{d_5^2\,(d_4-d_7)^2\left(-\delta(t)^2\right)}}{d_5\delta(t)},$$

$$d_6(t)=\int\left(d_4^2(-\delta(t))-d_5^2\delta(t)-2\alpha(t)\right)dt+K_3,$$

$$d_9(t)=\int\left(d_4^2\delta(t)-2d_7d_4\delta(t)-d_5^2\delta(t)+2\sqrt{d_5^2\,(d_4-d_7)^2\left(-\delta(t)^2\right)}+2\alpha(t)\right)dt+K_4,\quad \varpi_1(t)=0,\quad \varpi_2(t)=i\varpi_3(t),$$

we get

$$\psi^{II}(x,y,z,t)=\varpi_3(t)\cosh\Bigg(\int\Big(d_4^2\delta(t)+-2d_7d_4\delta(t)2\sqrt{-d_5^2\,(d_4-d_7)^2\delta(t)^2}+-d_5^2\delta(t)+2\alpha(t)\Big)dt+K_4+\frac{z\left(d_5^2\delta(t)-\sqrt{-d_5^2\,(d_4-d_7)^2\delta(t)^2}\right)}{d_5\delta(t)}+d_7y+x\Bigg)+i\varpi_3(t)\cos\Bigg(\int\Big(d_4^2(-\delta(t))-d_5^2\delta(t)-2\alpha(t)\Big)dt+K_3+d_4y+d_5z+x\Bigg). \tag{8}$$

Thus,

$$\tau^{II}(x,y,z,t)=12\alpha(t)\Big((\cosh(\Theta_3)\varpi_3(t)-i\cos(\Theta_4)\varpi_3(t))(\cosh(\Theta_3)\varpi_3(t)+i\cos(\Theta_4)\varpi_3(t))-(\sinh(\Theta_3)\varpi_3(t)-i\sin(\Theta_4)\varpi_3(t))^2\Big)\Big/\beta(t)(\cosh(\Theta_3)\varpi_3(t)+i\cos(\Theta_4)\varpi_3(t))^2, \tag{9}$$

where $\Theta_3=\int\left(d_4^2\delta(t)-2d_7d_4\delta(t)+2\sqrt{-d_5^2\,(d_4-d_7)^2\delta(t)^2}-d_5^2\delta(t)+2\alpha(t)\right)dt+K_4+\frac{z\left(d_5^2\delta(t)-\sqrt{-d_5^2(d_4-d_7)^2\delta(t)^2}\right)}{d_5\delta(t)}+d_7y+x$, $\Theta_4=\int\left(d_4^2(-\delta(t))-d_5^2\delta(t)-2\alpha(t)\right)dt+K_3+d_4y+d_5z+x$.

Figures 1 and 2 represent the propagation of the obtained solutions (7) and (9), respectively.

3 Analytical solutions

In this section, we construct more new exact solitons for the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili by means of two modified schemes, the extended tanh–coth method, and the rational sine–cosine method. First, we recall the proposed KP model which is

$$\tau_{xt}+\alpha\tau_{xxxx}+\beta(\tau\tau_{xx}+\tau_x^2)+\delta(\tau_{yy}+\tau_{zz})=0. \tag{10}$$

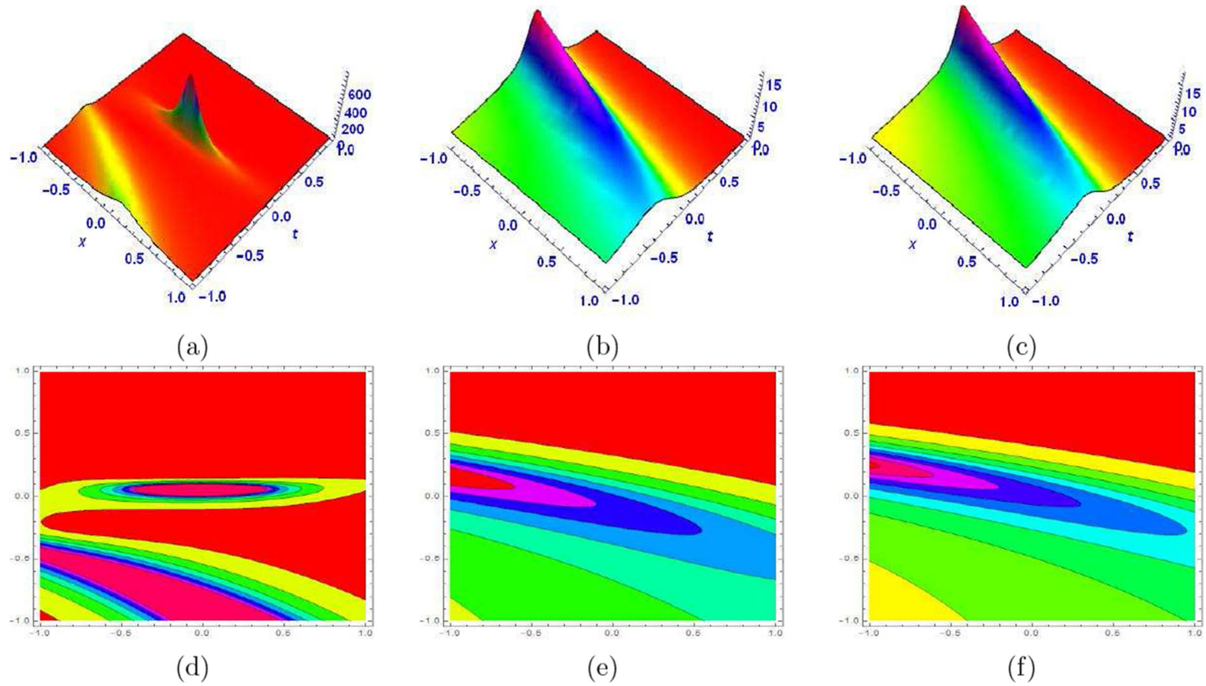

Fig. 1 The 3D and contour profiles of solution (7) under $\alpha(t) = e^t$, $\delta(t) = \tanh(t)$

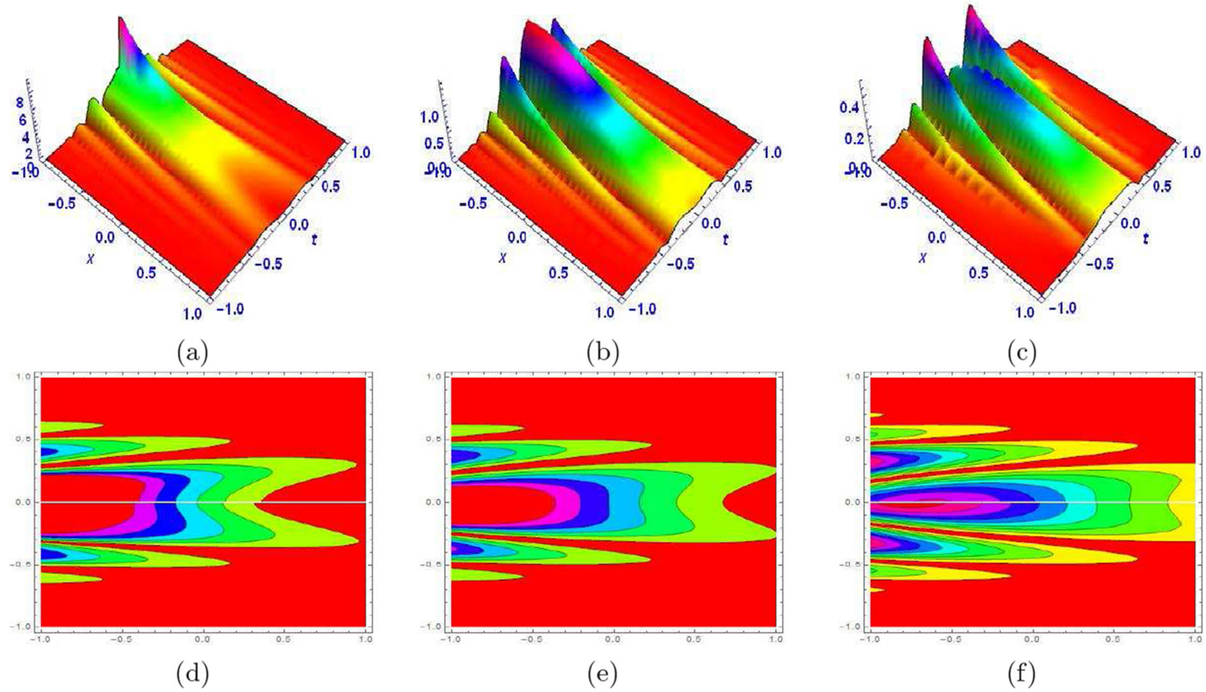

Fig. 2 The 3D and contour profiles of solution (9) under $\alpha(t) = \varpi_3(t) = \delta(t) = t$

Then, we convert the PDE (10) into an ODE via the new independent variable $\zeta = x + ay + bz - ct$. The resulting ODE is

$$\left(\delta\left(a^2+b^2\right)-c\right)U(\zeta)+\frac{1}{2}\beta U^2(\zeta)+\alpha U''(\zeta)=0, \tag{11}$$

where $U(\zeta) = \tau(x, y, z, t)$. Next, we solve (11) by the suggested schemes.

3.1 Tanh–coth expansion method

The extended tanh–coth expansion scheme [30–33] provides the solution of (11) in the following form

$$U(\zeta) = A_0 + A_1 Y + A_2 Y^2 + \frac{B_1}{Y} + \frac{B_2}{Y^2}, \tag{12}$$

for $Y = Y(\zeta)$ is the solution of the auxiliary differential equations $Y' = \mu(1 - Y^2)$ with solution $Y = \tanh(\zeta)$ or $Y = \coth(\zeta)$. Differentiating (12) implicitly twice, we reach

$$U''(\zeta) = \frac{2\mu^2\left(Y^2-1\right)\left(A_1Y^5+A_2\left(3Y^2-1\right)Y^4+B_2Y^2-B_1Y-3B_2\right)}{Y^4}. \tag{13}$$

By substitution of (12) and (13) in (11), and considering each coefficient of $Y^j : j = -4, -2, \ldots, 4$ to zero, will provide a nonlinear algebraic system with the unknowns A_0, A_1, A_2, B_1, B_2 as well as the other parameters a, b, c and μ.

$$\begin{aligned}
&0=B_2\left(12\alpha\mu^2+\beta B_2\right), \quad 0=2B_1\left(2\alpha\mu^2+\beta B_2\right),\\
&0=2B_2\left(a^2\delta-8\alpha\mu^2+A_0\beta+b^2\delta-c\right)+\beta B_1^2,\\
&0=2\left(B_1\left(a^2\delta-2\alpha\mu^2+A_0\beta+b^2\delta-c\right)+A_1\beta B_2\right),\\
&0=A_0\left(2\delta\left(a^2+b^2\right)-2c\right)+A_0^2\beta\\
&\quad+2\left(A_2\left(2\alpha\mu^2+\beta B_2\right)+A_1\beta B_1+2\alpha B_2\mu^2\right),\\
&0=2\left(A_1\left(a^2\delta-2\alpha\mu^2+A_0\beta+b^2\delta-c\right)+A_2\beta B_1\right),\\
&0=2A_2\left(a^2\delta-8\alpha\mu^2+A_0\beta+b^2\delta-c\right)+A_1^2\beta,\\
&0=2A_1\left(2\alpha\mu^2+A_2\beta\right),\\
&0=A_2\left(12\alpha\mu^2+A_2\beta\right).
\end{aligned} \tag{14}$$

In the above system, we have the following exact solutions

$$\begin{aligned}
\tau_1(x,y,z,t)=&\frac{4\alpha\mu^2}{\beta}(1-3\tanh^2(\mu(-t(a^2\delta-4\alpha\mu^2\\
&+b^2\delta)+ay+bz+x))),\\
\tau_2(x,y,z,t)=&\frac{12\alpha\mu^2}{\beta}\operatorname{sech}^2(\mu(-t(a^2\delta+4\alpha\mu^2+b^2\delta)\\
&+ay+bz+x)),\\
\tau_3(x,y,z,t)=&-\frac{12\alpha\mu^2}{\beta}\tanh^2(\mu(-t(a^2\delta-16\alpha\mu^2\\
&+b^2\delta)+ay+bz+x))-\frac{12\alpha\mu^2}{\beta}\coth^2\\
&(\mu(-t(a^2\delta-16\alpha\mu^2+b^2\delta)\\
&+ay+bz+x))-\frac{8\alpha\mu^2}{\beta},
\end{aligned}$$

and

$$\begin{aligned}
\tau_4(x,y,z,t)=&-\frac{48\alpha\mu^2}{\beta}\operatorname{csch}^2(2\mu(-t(a^2\delta\\
&+16\alpha\mu^2+b^2\delta)+ay+bz+x)),\\
\tau_5(x,y,z,t)=&-\frac{12\alpha\mu^2}{\beta}\operatorname{csch}^2(\mu(-t(a^2\delta\\
&+4\alpha\mu^2+b^2\delta)+ay+bz+x)),\\
\tau_6(x,y,z,t)=&\frac{4\alpha\mu^2}{\beta}(1-3\coth^2(\mu(-t(a^2\delta\\
&-4\alpha\mu^2+b^2\delta)+ay+bz+x))).
\end{aligned} \tag{15}$$

In Fig. 3, we present the propagations of the obtained solutions τ_1 and τ_2. Both are categorized as the motion of periodic waves. Moreover, the other solutions τ_3–τ_4 are periodic moving waves.

3.2 Rational sine–cosine method

The enhanced rational sine–cosine approach provides the following solution for (11) [34–38]:

$$U = U(\zeta) = \frac{1 + A\sin(\mu\zeta)}{B + F\cos(\mu\zeta)}, \tag{16}$$

for A, B, F are constants to be reached later. Now, we insert (16) in (11) and collect the coefficients of the functions $\sin^i(\zeta) : i = 0, 1, 2, 3$, $\cos(\zeta)$ and

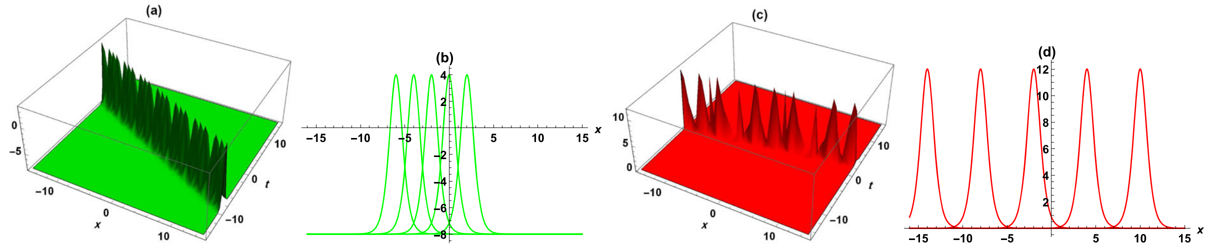

Fig. 3 **a** 3D plot of τ_1. **b** 2D plot of τ_1. **c** 3D plot of τ_2. **d** 2D plot of τ_2

$\sin(\zeta)\cos(\zeta)$. We reach the system

$$\begin{aligned}
0 &= 2a^2B^2\delta + 2a^2\delta F^2 + 2b^2B^2\delta + 2b^2\delta F^2 \\
&\quad - 2B^2c + \beta B - 2cF^2 + 2\alpha F^2\mu^2, \\
0 &= F\left(B\left(4a^2\delta + 2\alpha\mu^2 + 4b^2\delta - 4c\right) + \beta\right), \\
0 &= 2A\left(B^2\left(a^2\delta - \alpha\mu^2 + b^2\delta - c\right)\right. \\
&\quad \left. + F^2\left(a^2\delta + 2\alpha\mu^2 + b^2\delta - c\right) + \beta B\right), \\
0 &= AF\left(B\left(2a^2\delta + \alpha\mu^2 + 2b^2\delta - 2c\right) + \beta\right), \\
0 &= -2F^2\left(a^2\delta - \alpha\mu^2 + b^2\delta\right) + A^2\beta B + 2cF^2, \\
0 &= A^2\beta F, \quad 0 = 2AF^2\left(c - \delta\left(a^2 + b^2\right)\right).
\end{aligned} \tag{17}$$

Computing the equations above, we reach the following new two solutions

$$\tau_7(x, y, z, t) = \frac{1}{F\cos\left(\frac{\sqrt{\beta}\left(-\delta t\left(a^2+b^2\right)+ay+bz+\frac{\beta t}{6F}+x\right)}{\sqrt{6}\sqrt{\alpha}\sqrt{F}}\right) - F},$$

$$\tau_8(x, y, z, t) = \frac{1}{F\cosh\left(\frac{\sqrt{\beta}\left(-t\left(\delta\left(a^2+b^2\right)+\frac{\beta}{6F}\right)+ay+bz+x\right)}{\sqrt{6}\sqrt{\alpha}\sqrt{F}}\right) + F}. \tag{18}$$

Figure 4 shows the physical implication of both τ_7 and τ_8. It can be observed that they are periodic and bell-shaped, respectively.

3.3 Rational cosine–sine method

Here, we solve (11) by considering the following rational form

$$U(\zeta) = \frac{1 + A\cos(\zeta)}{B + \sin(\zeta)}. \tag{19}$$

Applying the same steps as the preceding section, we provide the following solutions:

$$\tau_9(x, y, z, t) = \frac{-1}{F \mp F\sin\left(\frac{\sqrt{\beta}\left(-\delta t\left(a^2+b^2\right)+ay+bz+\frac{\beta t}{6F}+x\right)}{\sqrt{6}\sqrt{\alpha}\sqrt{F}}\right)},$$

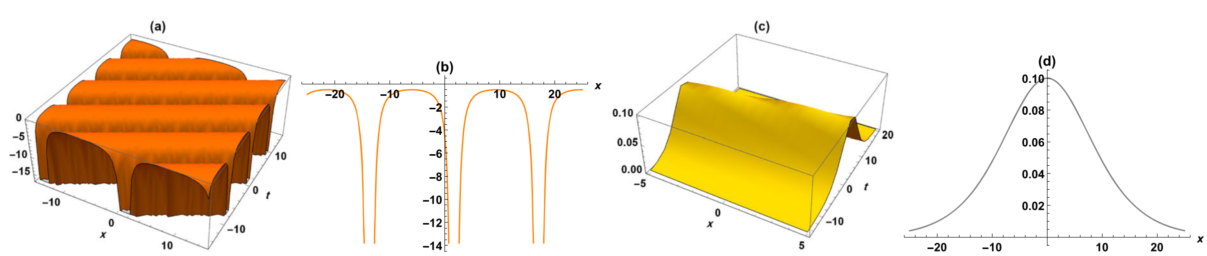

Fig. 4 **a** 3D plot of τ_7. **b** 2D plot of τ_7. **c** 3D plot of τ_8. **d** 2D plot of τ_8

$$\tau_{10}(x,y,z,t) = \frac{1}{F \mp i\,F \sinh\left(\frac{\sqrt{\beta}\left(-t\left(\delta(a^2+b^2)+\frac{\beta}{6F}\right)+ay+bz+x\right)}{\sqrt{6}\sqrt{\alpha}\sqrt{F}}\right)}. \quad (20)$$

We point here that the physical structures of τ_9 and τ_{10} are similar to τ_7 and τ_8.

4 Conclusion

In this work, the lump periodic and exact traveling wave solutions for the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili problem were determined using a robust integration method. The Hirota bilinear methodology, as well as the effective tanh–coth expansion and rational sine–cosine procedures, were used to develop such innovative solutions. The above-mentioned approach was utilized to effectively extract the obtained solutions. Among the discovered solutions are lump periodic, trigonometric, rational, and hyperbolic functions, which form a fascinating pattern of waves in nonlinear physical phenomena. The derived solutions' interesting physical patterns have been presented in 3D and 2D as well as the contour surfaces. Furthermore, all of the offered solutions satisfied the original equation. The reported nonautonomous lump-periodic and analytical solutions in this study could be helpful in explaining the physical meaning of different nonlinear models.

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Data Availability Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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