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Numerical solutions of the Wolbachia invasive model using Levenberg-Marquardt backpropagation neural network technique

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ABSTRACT

The current study presents the numerical solutions of the Wolbachia invasive model (WIM) using the neural network Levenberg-Marquardt (NN-LM) backpropagation technique. The dynamics of the Wolbachia model is categorized into four classes, namely Wolbachia-uninfected aquatic mosquitoes (A_n^*), Wolbachia-uninfected adult female mosquitoes (F_n^*), Wolbachia-infected aquatic mosquitoes (A_w^*), and Wolbachia-infected adult female mosquitoes (F_w^*). A reference dataset for the proposed NN-LM technique is created by solving the Wolbachia model using the Runge-Kutta (RK) numerical method. The reference dataset is used for validation, training, and testing of the proposed NN-LM technique for three different cases. The obtained numerical results from the proposed neural network technique are compared with the results obtained from the RK method for accuracy, correctness, and efficiency of the designed methodology. The validation of the proposed solution methodology is checked through the mean square error (MSE), error histograms, error plots, regression plots, and fitness plots.

Introduction

Diseases transmitted by vectors, including yellow fever, dengue fever, zika virus, chikungunya virus, and others, pose a threat to public health. For instance, there are 390 million new dengue infections each year, and dengue has a widespread global distribution, while it is estimated that 3.9 billion people are at high risk [1]. Mosquitoes belonging to the species Aedes aegypti and Aedes albopictus are the most common carriers of viruses that cause dengue fever, Zika, chikungunya, and yellow fever. Significant effort has been placed on restricting mosquito longevity by introducing genetically engineered mosquitoes or by infecting mosquitoes with different strains of Wolbachia in order to prevent the transmission of these diseases [2].

Wolbachia is a bacterium that is extremely common and is naturally present in 50% of insect species, including certain mosquitoes, dragonflies, and butterflies. Independent risk assessments reveal that the release of mosquitoes with Wolbachia virus does not pose a health risk to people or the environment [3]. Insect cells are where Wolbachia originates and is carried from one generation to the next via the eggs of insects.

The tools and ideas of mathematics are used in epidemiology, engineering, social science research systems, and medicine. Mathematical techniques are used not just in medicine, engineering, and science as well as in languages. A mathematical system can assist in identifying and investigating the effects of various devices and forecast their behavior [4]. A mathematical model may be measured in various ways, involving game theoretic models, statistical networks, and dynamical networks [5]. Mathematical categories are frequently used to describe logical

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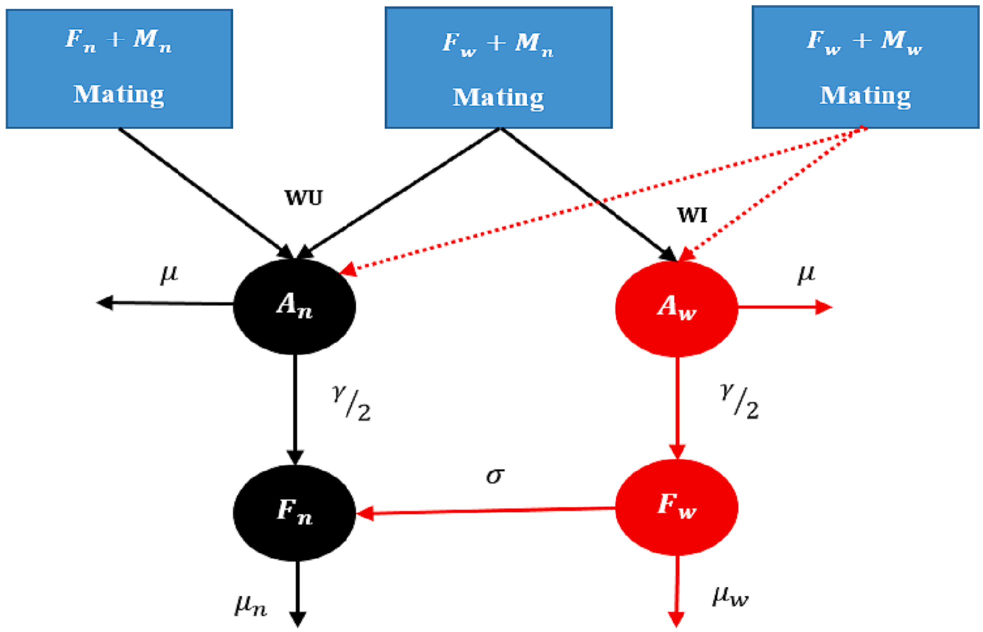


Fig. 1. Schematic diagram of the model (1). The subscripts n and w denote the population of the Wolbachia-uninfected (WU) and Wolbachia-infected (WI) mosquitoes, respectively.

Table 1
Description of the parameters used in model (1).

Parameter	Description	Unit	Values	Reference
ρ_1	The proportion of WU eggs that are produced when Wolbachia-infected female mates with WI male mosquitoes	Dimensionless	[0.05–0.1]	[10]
ρ_2	The proportion of WU eggs that are produced when WI female mates with WU male	Dimensionless	[0.05–0.1]	[10]
K	Carrying capacity of the aquatic stage	———	[10^4-10^8]	Assumed
σ	Loss rate of Wolbachia infection	Per day	0.04	Assumed
γ	Maturation rate	Per day	0.11	[10,11]
μ	Aquatic death rate	Per day	0.02	[8]
μ_w	Death rate of WI mosquitoes	Per day	0.68	[2,10]
μ_n	Death rate of WU mosquitoes	Per day	0.68	[2,8]
ϕ_n	Reproduction rate for WU mosquitoes	Eggs per day	2	[8,9]
ϕ_w	Reproduction rate for WI mosquitoes	Eggs per day	2	[10,11]

ideas. Several studies rely on scientific integrity areas to establish the mathematical system's validity, which creates a highly theoretical breakthrough to accept the outcomes of repeated exploration [6,7]. In epidemiology, mathematical modeling has emerged as a basic, significant, and essential tool for understanding infectious disease dynamics and infection control recovery processes [8,9]. An epidemic model is regarded as an excellent and innovative design that may forecast any potential outbreak of the infection and be successful in controlling its transmission or propagation [10].

Biological neural networks are the premise of the computational model known as the Neural Network (NN). In order to create artificial neural networks, artificial neurons, or "nodes" are coupled. The NN's architecture is crucial for performing computations. Some neurons have been designed so they can take in information from their surroundings. These neurons are situated in a layer known as the Input layer. Each neuron in the input layer generates a result that is passed into the next layer. An output is produced by the Artificial Neuron by combining one or more inputs. The sum of each node is frequently weighted, and the total is then compiled by using an activation function. With the use of the neural network, the input–output mapping feature becomes stronger [11].

Neural networks (NNs) are utilized in stochastic numerical computing techniques related to the neural network to assess the efficiency of linear or non-linear differential equations in formulating distinct issue models. A feed-forward neural network is utilized by Umar, M., et al. [12] to analyze an HIV infection disease model. Many other researchers employed the feed-forward backpropagation neural network, to tackle a wide range of challenging issues [13–14]. While there is a significant amount of research available online that employs computing concepts such as neural networks, deep learning, and learning algorithms. Umar, M., et al. [15] studied an SIR nonlinear model for dengue fever to predict the number of infections efficiently and proficiently using neural network technique. The neural network technique, which is becoming more popular throughout the globe, is used by Botmart, T., et al. [16] to solve the SEIR-NDC model. Numerous stochastic-based techniques have been used to examine different swarming and evolving systems [17]. Similar to this, Sabir, Z., et al., [18] utilized fractional Mayer wavelet neural networks to investigate multi-singular Lane-Emden model, SITR models are examined by utilizing a neural-network technique for the coronavirus [19], Guirao, J.

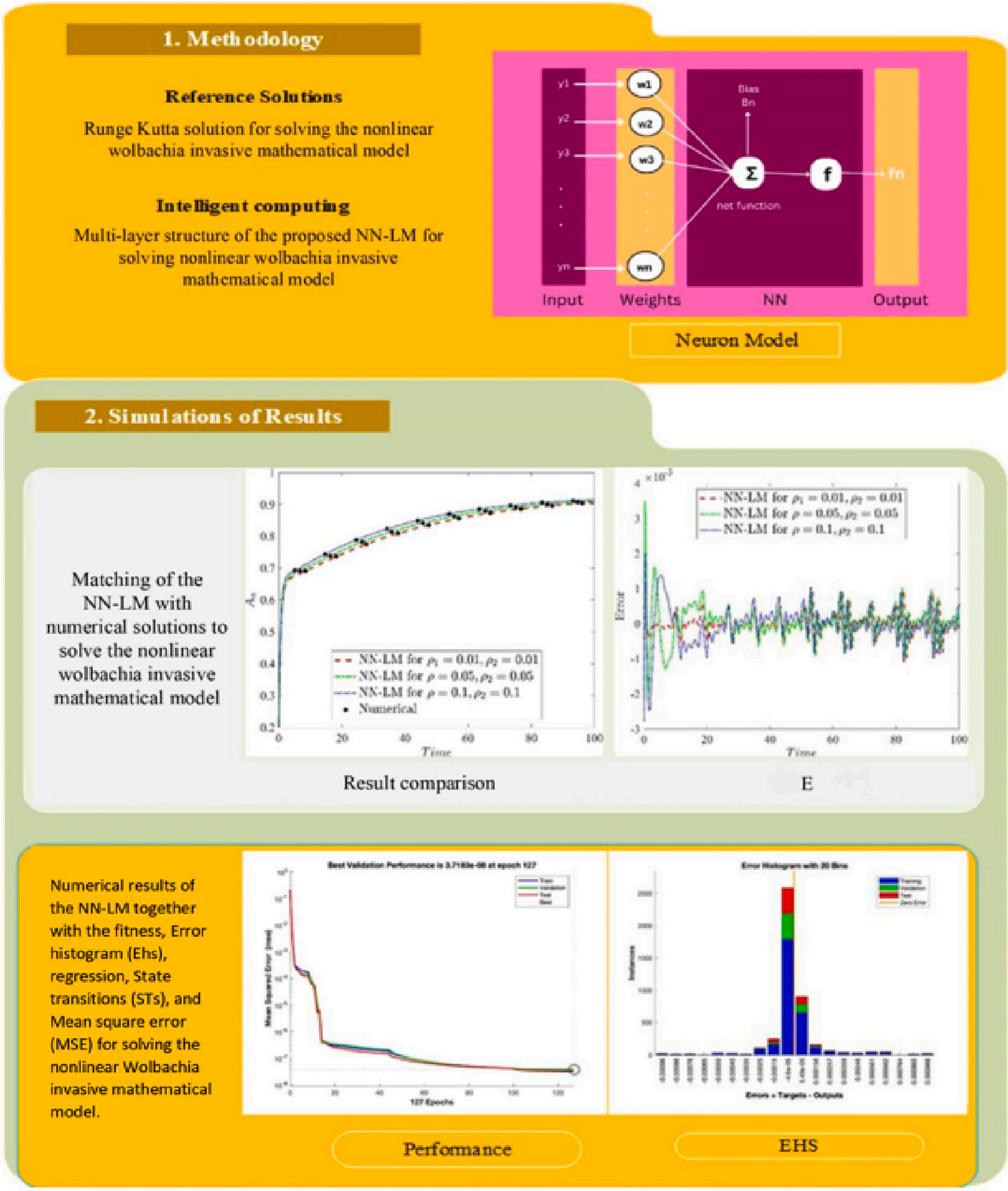


Fig. 2. Process diagrams of the proposed NN-LM technique for solving the Wolbachia invasive model (WIM) system.

L., et al., [20] used NN process to work on the functional order singular model. The proposed NN-LM is utilized to solve fractional, fluid, and Lonngren-wave models [21–26]. Through the use of the NN-LM, Sabir, Z. et al. [27] analyzed a sixth-order singular nonlinear pantograph model using the neural network technique. Using the Levenberg-Marquardt Neural Network (NN-LM) approach, Shoaib, M. et al. [28] analyzed heat transfer impact on Maxwell nanofluid over the vertical stretching surface. Further, the Emden-Fowler nonlinear system is studied by Sabir, Z., et al. [29] using the neuro-swarming technique.

In this research work, we develop an advanced neural network (NN) technique for solving a mathematical model that consists of both Wolbachia-uninfected (WU) and Wolbachia-infected (WI) mosquitoes using the novel features of the Levenberg-Marquardt (LM) method. Wolbachia-infected mosquitoes significantly reduce the transmission of dengue virus to humans. Using Wolbachia-infected mosquitoes for dengue control has several advantages over traditional mosquito control methods such as insecticide spraying. Firstly, it is a sustainable approach as Wolbachia-infected mosquitoes can reproduce and continue to reduce the mosquito population over time. Secondly, it is targeted specifically at reducing the transmission of dengue and other mosquito-borne viruses. Finally, it is less harmful to the environment and non-target species compared to traditional insecticide spraying. Overall, using Wolbachia-infected mosquitoes for dengue control is a promising strategy that has shown success in several countries, including Australia, Indonesia, Brazil, and Vietnam. To the best of the authors' knowledge, this technique has not been implemented before for solving the Wolbachia invasive model (WIM). The numerical results obtained from the proposed neural network technique are compared with the traditional RK4 method, and these results show a good agreement.

Mathematical model

We develop a mosquito population model in which the mosquito's population is divided into two groups: Wolbachia-uninfected (WU)

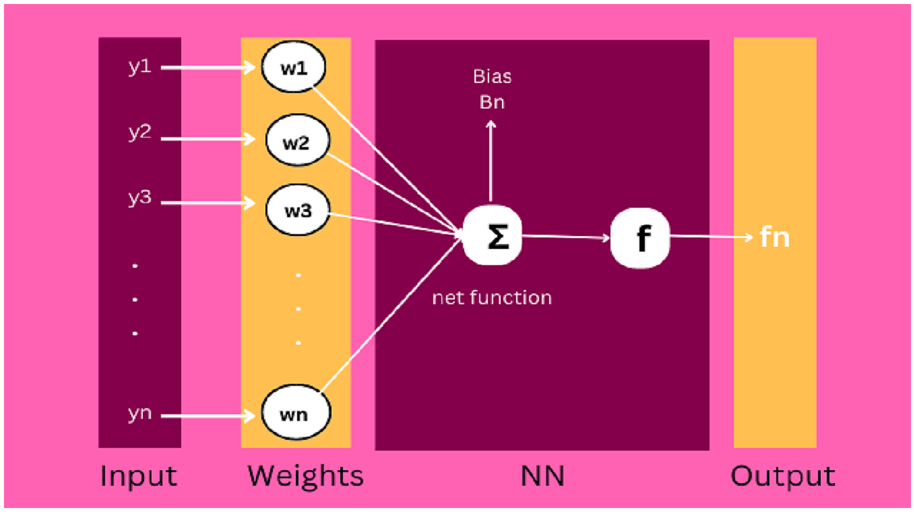


Fig. 3. Proposed structure for the neuron.

Table 2
Pseudocode of neural network Levenberg-Marquardt (NN-LM):

NN-LM Construction: Using RK method in MATLAB input dataset and reference dataset is created.
Selection of dataset: Selected datasets must be in matrices or vectors form
Start: Distribution of datasets for the training, testing and validation purpose and selection of neurons

- 70% data is used for training
- 15% data is used for testing
- 15% data is used for validation
- 10 number of hidden neurons are used

Construction: Each input is assigned a weight by the algorithm. A transfer function of the input is created by adding together all of the input weights and bias.
Stopping criteria: The following requirements must be satisfied for the procedure to be stopped.

- Mu has its maximum value
- Maximum iteration has been achieved
- The minimum value of the performance matric is achieved
- The value of the gradient becomes smaller than the minimum specified gradient

If the required outcomes are obtained, then save the results; else, the network should be retrained.
Retrain network: Train the network again by changing the parameters and saving outputs: Ending of NN-LM
Save results: At the end save graphical results
Ending of NN-LM

mosquitoes and Wolbachia-infected (WI) mosquitoes. Further, the population of WU and WI mosquitoes is divided into two stages: the aquatic stage which includes three immature phases (eggs, larva, and pupa) of mosquitoes, and the adult stage which includes adult male and female mosquitoes. The number of Wolbachia-uninfected aquatic and adult female mosquitoes are denoted by A_n and F_n, respectively. Similarly, A_w and F_w denote the WI aquatic and adult female mosquitoes, respectively.

Wolbachia-carrying females have a reproductive advantage due to cytoplasmic incompatibility (CI). Wolbachia-carrying females can effectively produce offspring when mating with either Wolbachia-uninfected or Wolbachia-infected males, but Wolbachia-uninfected females can only successfully produce offspring when mating with Wolbachia-uninfected males. This is because of the cytoplasmic incompatibility condition. When Wolbachia-uninfected females mate with Wolbachia-infected males, an embryo is formed, but the embryo is not healthy and dies, thus preventing reproduction. Therefore, CI causes complicated dynamics for the mosquito population in the presence of Wolbachia, which also has an impact on the dynamics of disease transmission [30].

To model the growth rate, we consider the following cases when a female mosquito mates randomly with male mosquitoes and produce offspring.

- When a WU female mates with WU male mosquitoes, WU offspring are produced only.

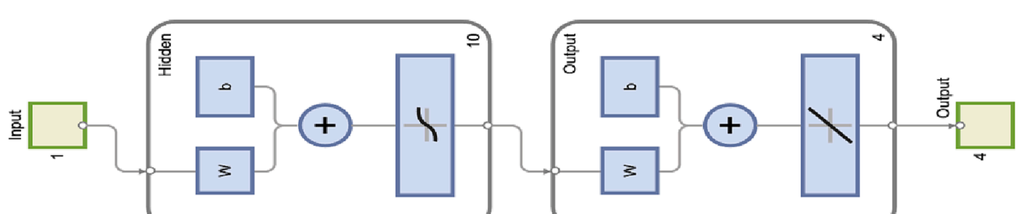
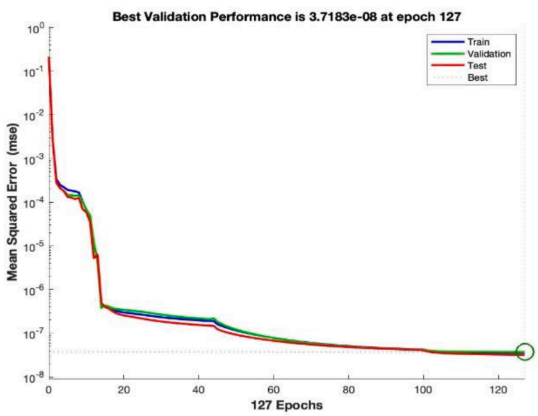


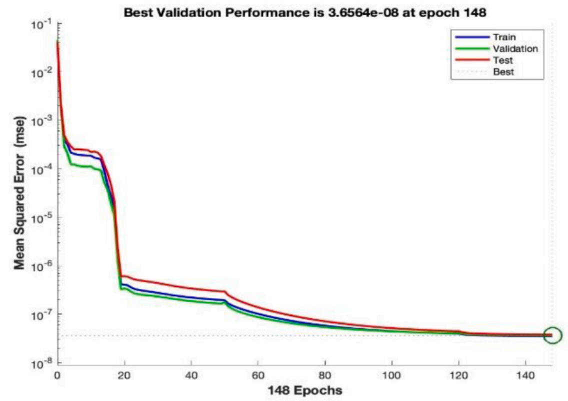
Fig. 4. Proposed NN-LM to resolve the nonlinear Wolbachia invasive model (WIM).

Table 3
Wolbachia invasive model (WIM) solved using NNs-LM.

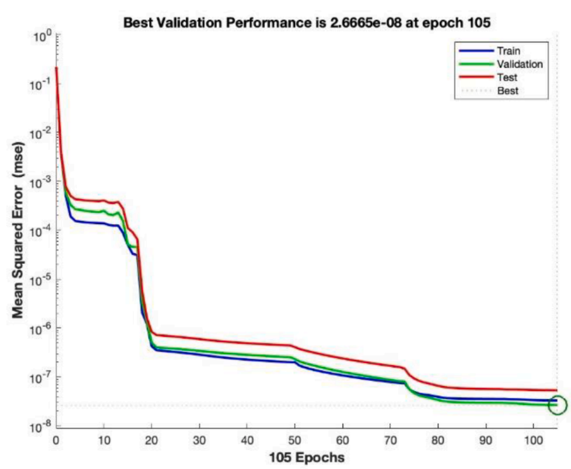
Case	MSE			Gradient	Performance	Epoch	Mu
	Training	Testing	Validation				
1	3.45×10^{-08}	4.08×10^{-08}	3.71×10^{-08}	9.47×10^{-08}	3.45×10^{-08}	127	1×10^{-10}
2	3.41×10^{-08}	3.92×10^{-08}	3.65×10^{-08}	9.91×10^{-08}	3.41×10^{-08}	148	1×10^{-10}
3	4.98×10^{-08}	5.51×10^{-08}	2.66×10^{-08}	8.37×10^{-08}	4.99×10^{-08}	105	1×10^{-10}



(a) **Case I:** MSE for Wolbachia invasive model (WIM)



(b) **Case II:** MSE for Wolbachia invasive model (WIM)



(c) **Case III:** MSE for Wolbachia invasive model (WIM)

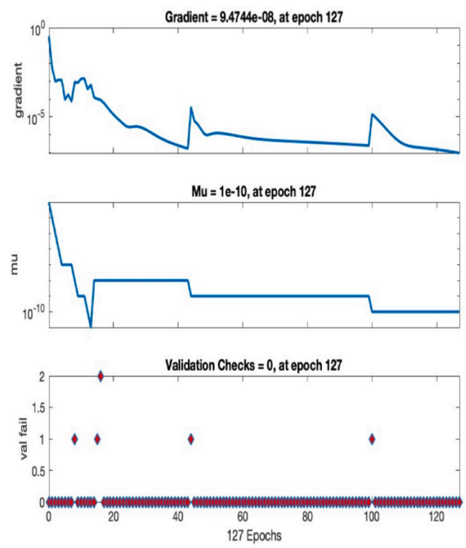
Fig. 5. The nonlinear Wolbachia invasive model (WIM) is solved utilizing the presented NN-LM, according to MSE performances.

- Due to cytoplasmic incompatibility (CI), no offspring are produced when a Wolbachia-uninfected (WU) female mates with WI male mosquitoes.
- Due to imperfect maternal transmission, WI and WU offspring are produced in a certain proportion when a WI female mosquito mates with either WI or WU male mosquitoes.

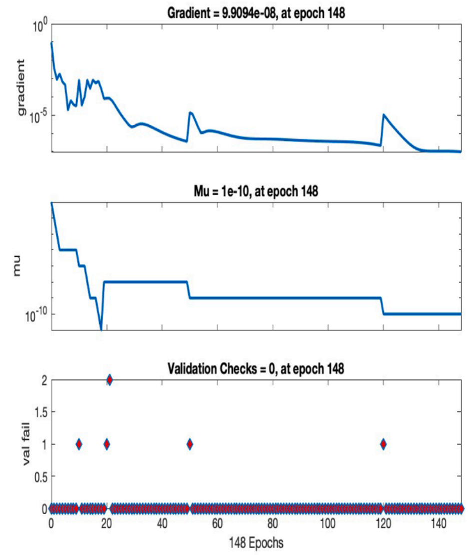
The growth rates of Wolbachia-uninfected (WU) and Wolbachia-infected (WI) females are denoted by φ_n and φ_w, respectively. Both, WU and WI aquatic stage mosquitoes mature with rate γ and die with rate μ. The death rate of the WU adult female mosquitoes is shown by μ_n whereas the death rate of WI adult female is denoted by μ_w. A proportion τ of immature mosquitoes are female and $1-\tau$ are male mosquitoes. However, in this model, we assume female-to-male ratio to be 1:1, so we take $\tau = 1/2$. K represents the carrying capacity of mosquitoes in the aquatic stage. The schematic diagram of the model is shown in Fig. 1 along with the state variables. In the model, the subscript n stands for Wolbachia-uninfected mosquitoes and w for Wolbachia-infected mosquitoes. All the parameters that appear in model (1) are defined in Table 1.

The resulting model consists of a system of ordinary differential equations (ODEs) and is given below:

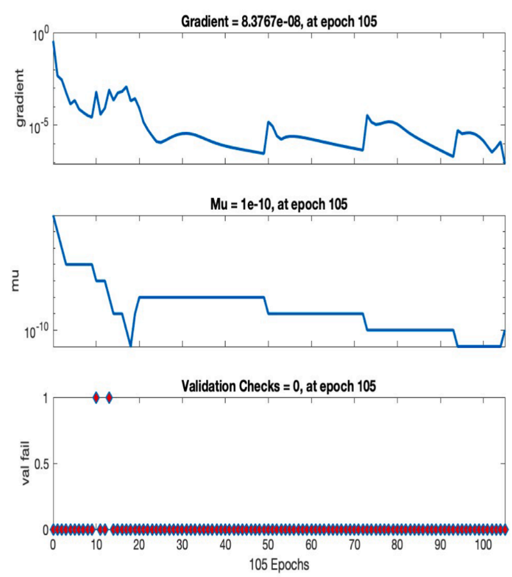
$$
\begin{cases}
\dfrac{dA_n^*}{dt} = \left[\dfrac{\phi_n F_n^{*2} + \rho_1\phi_w F_w^{*2} + \rho_2\phi_w F_n^* F_w^*}{F_n^* + F_w^*}\right]\left(1 - \dfrac{A_n^* + A_w^*}{K}\right) - (\mu+\gamma)A_n^*,\\
\dfrac{dA_w^*}{dt} = \left[\dfrac{(1-\rho_1)\phi_w F_w^{*2} + (1-\rho_2)\phi_w F_n^* F_w^*}{F_n^{*2} + F_w^{*2}}\right]\left(1 - \dfrac{A_n^* + A_w^*}{K}\right) - (\mu+\gamma)A_w^*,\\
\dfrac{dF_n^*}{dt} = \dfrac{\gamma}{2}A_n^* + \sigma F_w^* - \mu_n F_n^*,\\
\dfrac{dF_w^*}{dt} = \dfrac{\gamma}{2}A_w^* - \sigma F_w^* - \mu_w F_w^*.
\end{cases}
\tag{1}
$$



(a) **Case I:** State Transitions for Wolbachia invasive model (WIM)



(b) **Case II:** State Transitions for Wolbachia invasive model (WIM)



(c) **Case III:** State Transitions for Wolbachia invasive model (WIM)

Fig. 6. State Transitions performances using the NN-LM for solving Wolbachia invasive model (WIM).

The * above state variables denote that the variables are in dimensional form. To make the model (1) in dimensionless form, we take $A_n = \frac{A_n^*}{K}$, $A_w = \frac{A_w^*}{K}$, $F_w = \frac{F_w^*}{K}$, and $A_n = \frac{F_n^*}{K}$. Model (1) in dimensionless form takes the following form,

$$\begin{cases} \dfrac{dA_n}{dt} = \left[\dfrac{\phi_n F_n^2 + \rho_1 \phi_w F_w^2 + \rho_2 \phi_w F_n F_w}{F_n + F_w}\right]\left(1 - (A_n + A_w)\right) - (\mu + \gamma) A_n, \\ \dfrac{dA_w}{dt} = \left[\dfrac{(1-\rho_1)\phi_w F_w^2 + (1-\rho_2)\phi_w F_n F_w}{F_n + F_w}\right]\left(1 - (A_n + A_w)\right) - (\mu + \gamma) A_w, \\ \dfrac{dF_n}{dt} = \dfrac{\gamma}{2} A_n + \sigma F_w - \mu_n F_n, \\ \dfrac{dF_w}{dt} = \dfrac{\gamma}{2} A_w - \sigma F_w - \mu_w F_w. \end{cases} \tag{2}$$

Methodology: NN-LM

For a better comprehension of the approach, the suggested NN-LM to solve the Wolbachia invasive model (WIM) that is described with four compartments (A_n, F_n, A_w, and F_w) based on system of ordinary differential equations, and a neural network model developed with the LM approach is proposed and utilized. The innovation of the proposed research is described as follows:

- NN-LM with a novel methodology or design is created for the mathematical model incorporating both Wolbachia-infected and Wolbachia-uninfected mosquitoes.
- To acquire a better-approximated solution for the Wolbachia invasive model (WIM), an innovative application of NN-LM technique is applied.
- Obtained the dataset using the Runge-Kutta numerical method. The neural network procedure is based on stochastic numerical

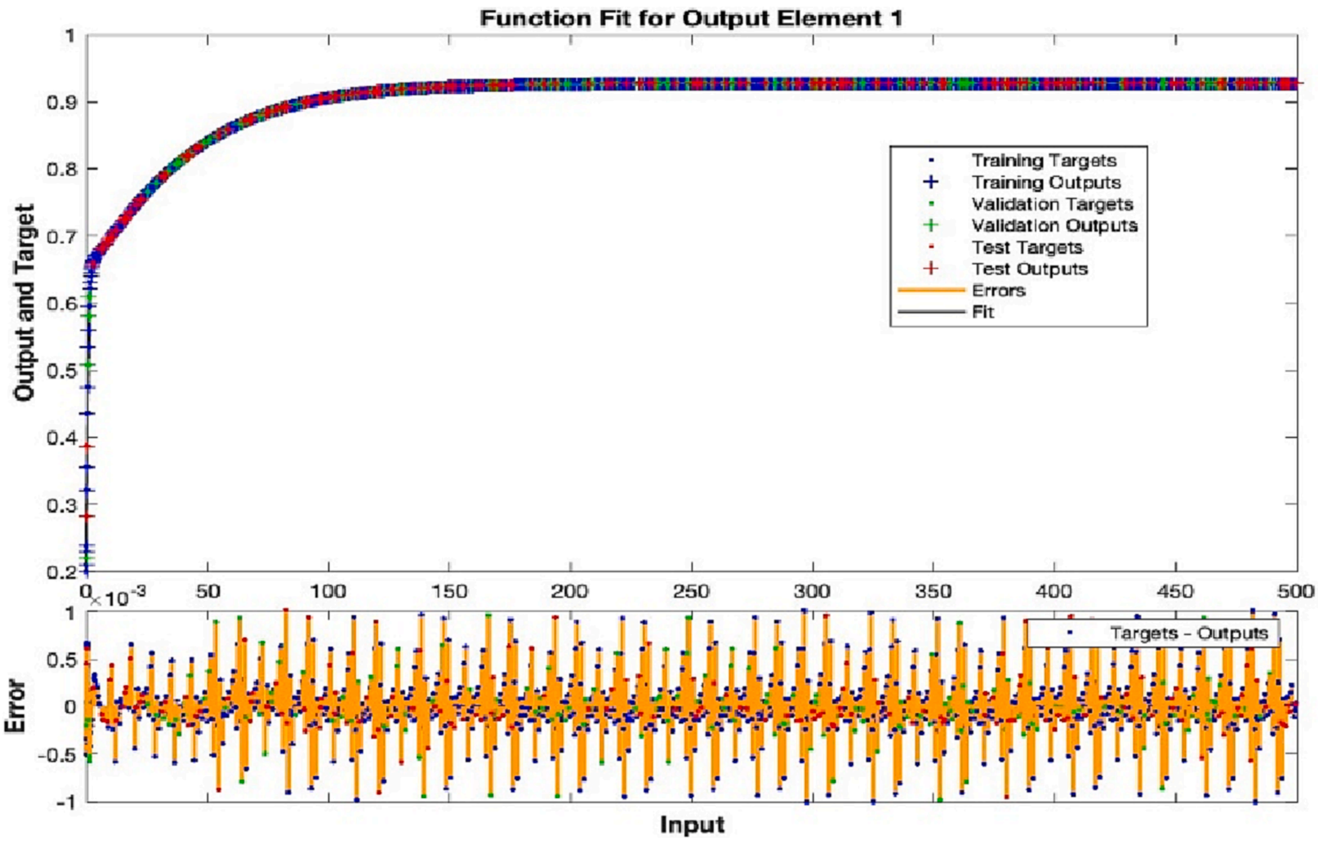


Fig. 7. Case 1: Comparison of the outcomes through NN-LM for solving Wolbachia invasive model (WIM).

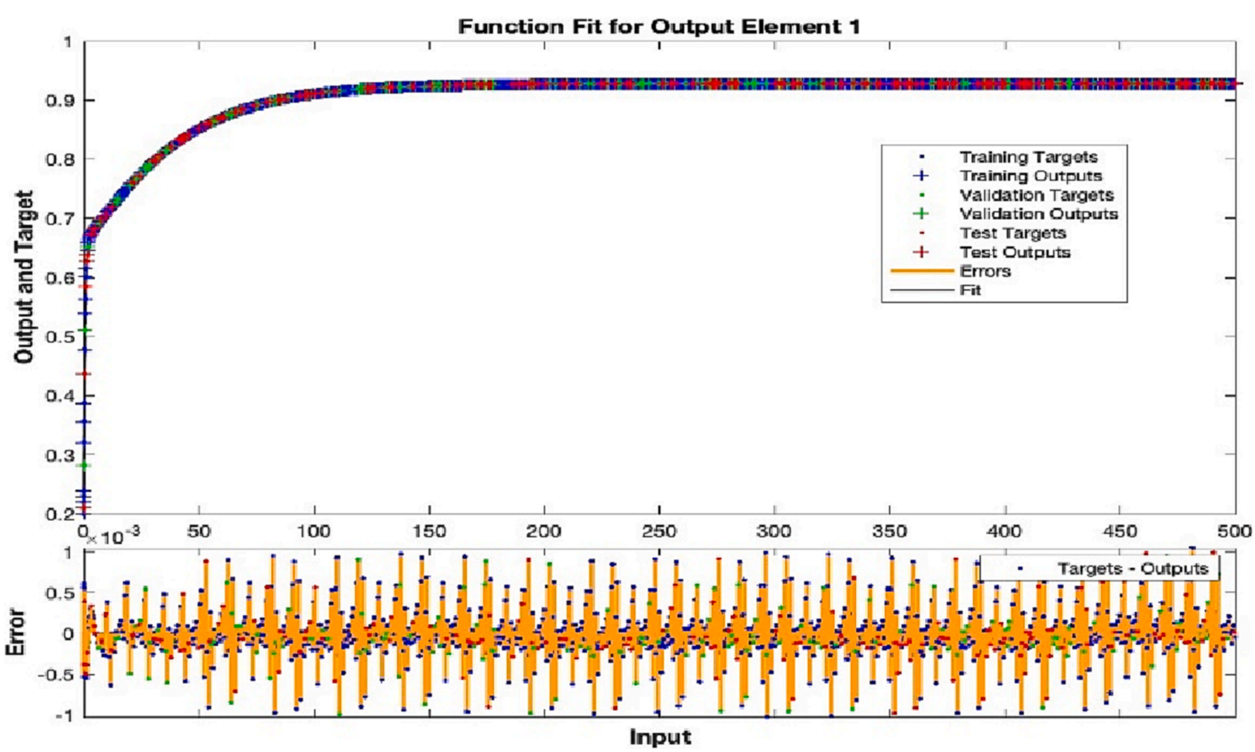


Fig. 8. Case 2: Comparison of the outcomes through NN-LM for solving Wolbachia invasive model (WIM).

computation for the approximate solution; and compares it with the reference solution via training (70%), testing (15%), and validation (15%).

- The results obtained from the NN-LM technique and the numerical results are shown graphically, and they show a good agreement.

- Statistical analysis shows that the outcomes from the proposed NN-LM procedure are accurate and trustworthy.
- To maintain the precision of the proposed NN-LM solution, the training, validation, and testing outputs of the Wolbachia invasive model (WIM) for three different cases.
- The MSE graphs display the model's convergence or stability.
- The graphical plots for regression analysis (RA), mean square error (MSE), fitness curves, and error histograms (EH) show the effective performance of the proposed NN-LM technique (see Fig. 2, Fig. 3 and Table 2).

Results and discussion

In this section, we present the numerical solutions of the Wolbachia model using the proposed NN-LM technique for three different cases. The mathematical structure for each case is presented below:

Case- 1: Consider the Wolbachia invasive model (WIM) system (2) with $\rho_1 = 0.01$, $\rho_2 = 0.01$, $\sigma = 0.04$, $\beta = 0.5$, $\gamma = 0.11$, $\mu = 0.02$, $\mu_w = 0.68$, $\mu_n = 0.68$, $\phi_w = 2$, $\phi_n = 2$, is given as:

$$\begin{cases} \dfrac{dA_n}{dt} = \left[\dfrac{2F_n^2 + 0.01(2)F_w^2 + 0.01(2)F_nF_w}{F_{\overline{w}} + F_w}\right]\left(1-(A_n+A_w)\right) - (0.02+0.11)A_n, \\ \dfrac{dA_w}{dt} = \left[\dfrac{(1-0.01)(2)F_w^2 + (1-0.01)(2)F_nF_w}{F_n + F_w}\right]\left(1-(A_n+A_w)\right) - (0.02+0.11)A_w, \\ \dfrac{dF_n}{dt} = \dfrac{0.11}{2}A_n + 0.04F_w - 0.68F_n, \\ \dfrac{dF_w}{dt} = \dfrac{0.11}{2}A_w - 0.04F_w - 0.68F_w. \end{cases} \tag{3}$$

Case- 2: Consider the Wolbachia invasive model (WIM) system with $\rho_1 = 0.05$, $\rho_2 = 0.05$, $\sigma = 0.04$, $\beta = 0.5$, $\gamma = 0.11$, $\mu = 0.02$, $\mu_w = 0.68$, $\mu_n = 0.68$, $\phi_w = 2$, $\phi_n = 2$, is given as:

$$\begin{cases} \dfrac{dA_n}{dt} = \left[\dfrac{2F_n^2 + 0.05(2)F_w^2 + 0.05(2)F_nF_w}{F_n + F_w}\right]\left(1-(A_n+A_w)\right) - (0.02+0.11)A_n, \\ \dfrac{dA_w}{dt} = \left[\dfrac{(1-0.05)(2)F_w^2 + (1-0.05)(2)F_nF_w}{F_n + F_w}\right]\left(1-(A_n+A_w)\right) - (0.02+0.11)A_w, \\ \dfrac{dF_n}{dt} = \dfrac{0.11}{2}A_n + 0.04F_w - 0.68F_n, \\ \dfrac{dF_w}{dt} = \dfrac{0.11}{2}A_w - 0.04F_w - 0.68F_w \end{cases} \tag{4}$$

Case- 3: Consider Wolbachia invasive model (WIM) system with $\rho_1 = 0.1$, $\rho_2 = 0.1$, $\sigma = 0.04$, $\beta = 0.5$, $\gamma = 0.11$, $\mu = 0.02$, $\mu_w = 0.68$, $\mu_n = 0.68$, $\phi_w = 2$, $\phi_n = 2$, is given as:

$$\begin{cases} \dfrac{dA_n}{dt} = \left[\dfrac{2F_n^2 + 0.1(2)F_w^2 + 0.1(2)F_nF_w}{F_n + F_w}\right]\left(1-(A_n+A_w)\right) - (0.02+0.11)A_n, \\ \dfrac{dA_w}{dt} = \left[\dfrac{(1-0.1)(2)F_w^2 + (1-0.1)(2)F_nF_w}{F_n + F_w}\right]\left(1-(A_n+A_w)\right) - (0.02+0.11)A_w, \\ \dfrac{dF_n}{dt} = \dfrac{0.11}{2}A_n + 0.04F_w - 0.68F_n, \\ \dfrac{dF_w}{dt} = \dfrac{0.11}{2}A_w - 0.04F_w - 0.68F_w. \end{cases} \tag{5}$$

The numerical solutions by utilizing the NN-LM to solve the Wolbachia mathematical model were achieved using the MATLAB 'nftool' command in combination with 10 neurons, 70% of the data used for training and 15% used for validation and testing, respectively. Fig. 4 shows the structure of the neural network for the nonlinear Wolbachia model.

The MSE convergence for training, validation, epochs, testing, and complexity investigations provided in Table 3 is accomplished to solve the Wolbachia invasive model (WIM).

The graphic illustrations through the NN-LM to solve the Wolbachia invasive model (WIM) are shown in Figs. 5-14. The appropriate

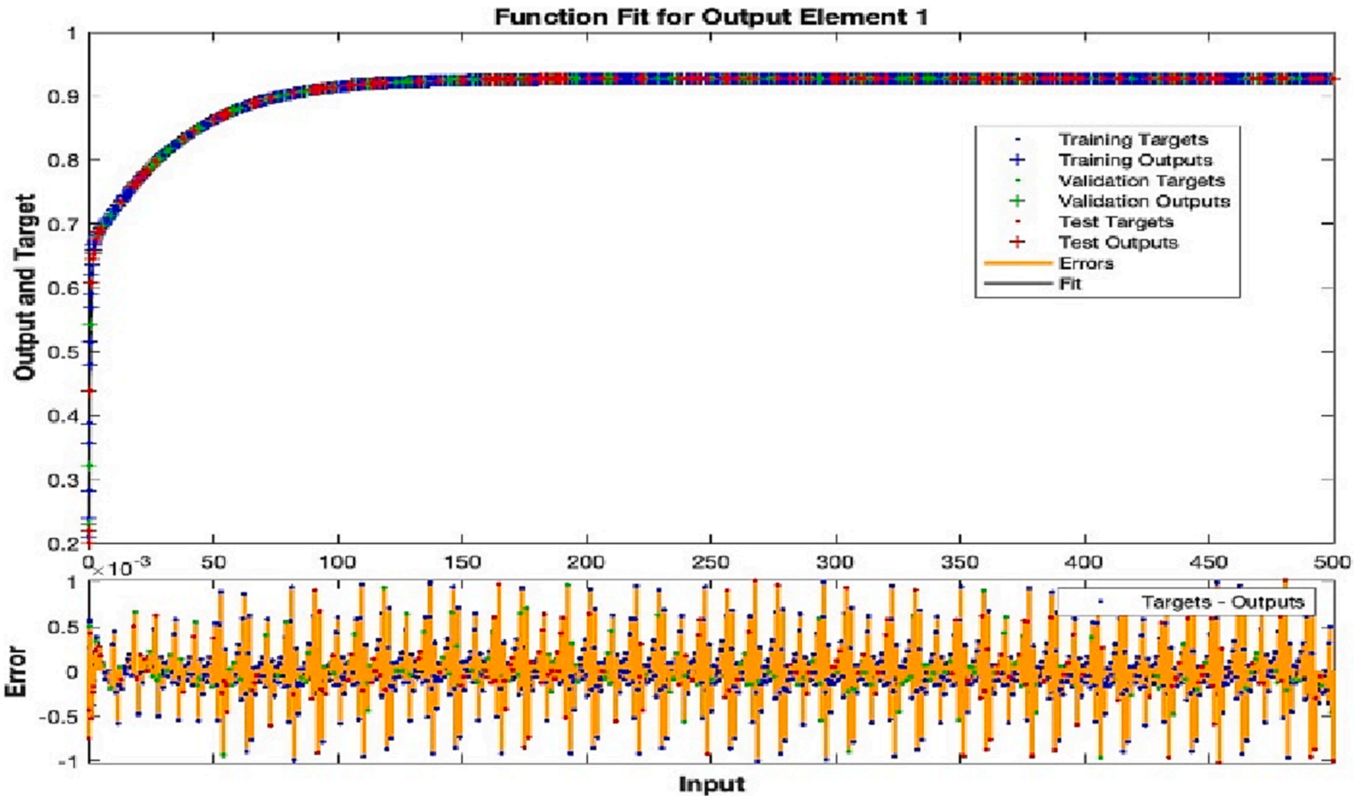


Fig. 9. Case 3: Comparison of the outcomes through NN-LM for solving Wolbachia invasive model (WIM).

numerical schemes of each scenario of the Wolbachia invasive model (WIM) are obtained in Fig. 5 utilizing the performances. The proposed technique has the finest validation performance in Fig. 5. Since the error is minimized after a few training epochs, but it may grow on the validation data set if the network starts to over fit the training data. The performance from the epoch with the lowest validation error is chosen as the best. The best performance values in three different cases for the Wolbachia invasive mathematical model produces its best outputs during epochs 127, 148 and 105 that are nearby 3.71×10^{-08}, 3.65×10^{-08} and 2.66×10^{-08}, respectively.

Fig. 6 presents the gradient, Mu, and validation performances of the proposed NN-LM to solve the Wolbachia invasive mathematical model. The values of gradient, Mu, and validation performances for Case I are 9.4744×10^{-08}, 1×10^{-10}, 3.71×10^{-08}, for Case II the values ranges 9.9094×10^{-08}, 1×10^{-10}, 3.65×10^{-08}, and for Case III these values are 8.3767×10^{-08}, 1×10^{-10}, 2.66×10^{-08}. These illustrations indicate exactness, convergence, and accuracy of the proposed NN-LM to simulate the Wolbachia invasive mathematical model. The fitting curve values for each variation of the Wolbachia invasive model (WIM) are shown in Figs. 7-9 which show a comparison between the solution of NN-LM and reference solutions.

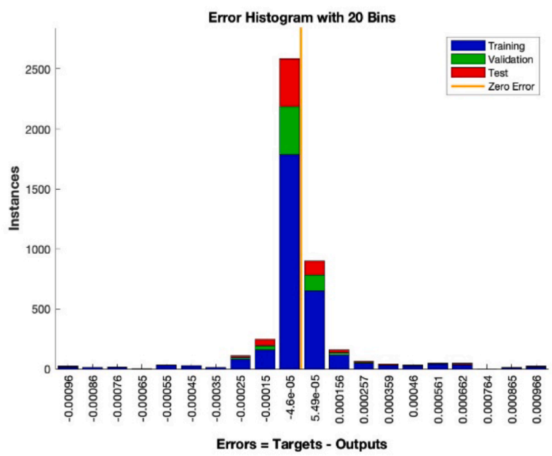
The error histograms (Ehs) plots values between targets (reference solution values) and outputs using the validation, training, and testing of data based on the NN-LM to solve the Wolbachia invasive model (WIM) are presented in Fig. 10 (a-f) for two classes: A_n and F_n. Most of the error bars lie near the zero-error line, showing that the error is very small, which demonstrate that our proposed technique is accurate. The error values for all cases of class A_n range from $10^{-4}to10^{-5}$ and similarly for all cases of class F_n the error values are in between $10^{-4}to10^{-6}$.

The regression plots are shown in Figs. 11-13. The regression plots represent the network output with respect to the target for the training, validation, and test datasets. One can observe that the correlation is around 1 for all cases when the Wolbachia invasive model (WIM) is solved using the proposed NN-LM technique. The testing, validation, and training plots entitle the correctness of the designed NN-LM to solve the nonlinear Wolbachia invasive model (WIM).

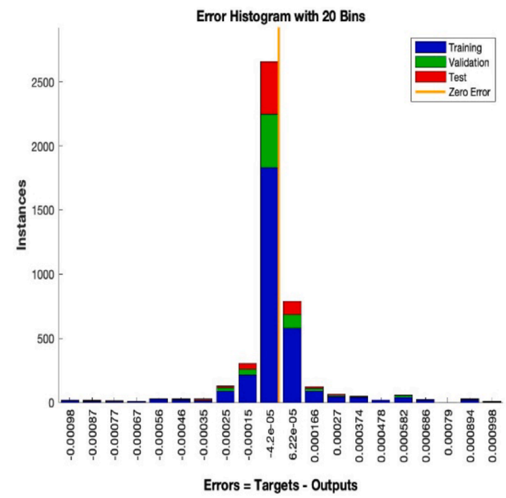
The comparison of the numerical solutions obtained from the proposed NN-LM technique with numerical solutions obtained using the RK method is shown in Fig. 14 along with the error plots. Comparison of the numerical results obtained from the NN-LM technique and using the RK method for three different cases are shown in Fig. 14 (a) for the class A_n. The error plots in Fig. 14 (b) for A_n class shows that the overall error is less than 10^{-3}, which shows a good agreement between the numerical results and results obtained from the proposed NN-LM technique. Similarly, the plots presented in Fig. 14 (c-h) for A_w, F_n, and F_w classes show the accuracy of the proposed method for solving the Wolbachia invasive model.

Conclusion

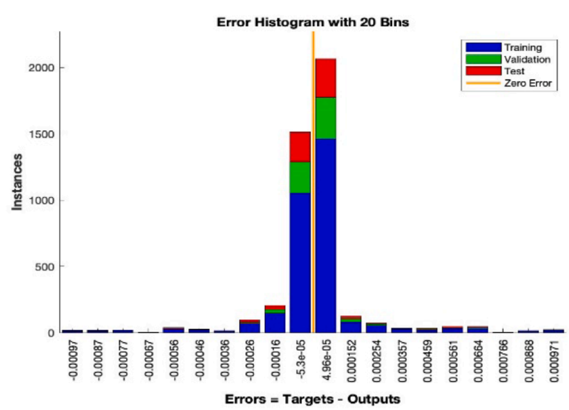
This research aimed to develop an advanced neural network (NN) technique for solving a mathematical model that consists of both Wolbachia-uninfected (WU) and Wolbachia-infected (WI) mosquitoes using the novel features of the Levenberg-Marquardt (NN-LM) method. Artificial intelligence based on Levenberg-Marquardt neural network approach has never been presented before for the numerical solutions of the Wolbachia invasive model (WIM). Three different cases have been discussed to show the performance of the proposed Levenberg-Marquardt neural network approach using training (70%), testing (15%), and validation (15%) data sets along with 10 hidden neurons. The numerical results obtained from the trained neural network model have been compared with the numerical results obtained from the RK method, and the results show a good agreement. The plots for regression analysis (RA), mean square error (MSE), fitness curves, and error histograms (EH) are also presented to show the effective performance of the proposed NN-LM technique.



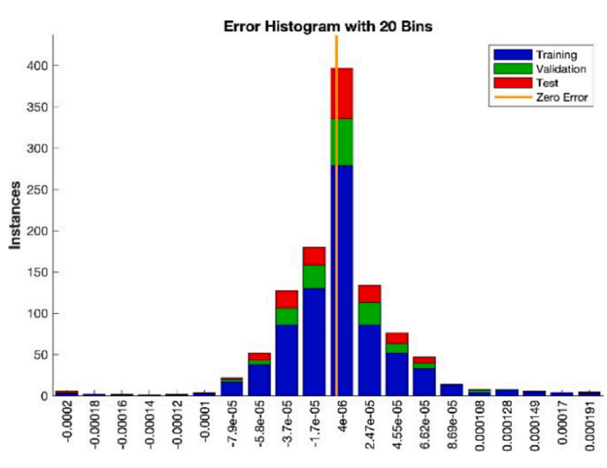
(a) **Case I**: Error Histogram class A_n

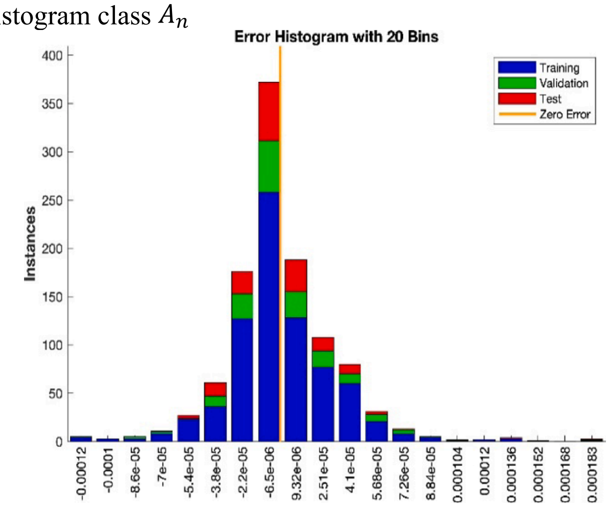


(b) **Case II**: Error Histogram class A_n



(c) **Case III**: Error Histogram class A_n

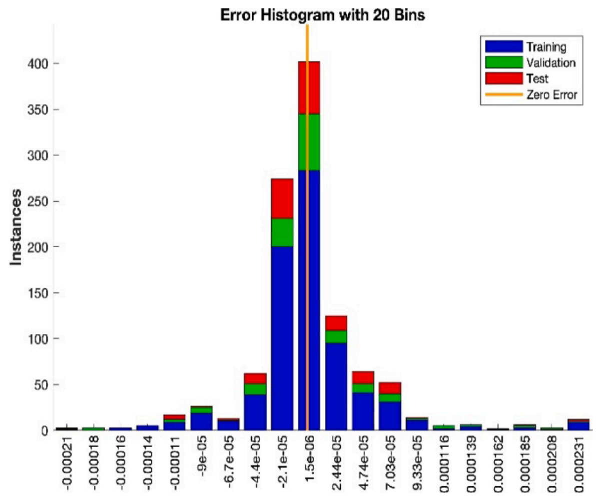




(d) **Case I**: Error Histogram class F_n

(e) **Case II**: Error Histogram class F_n

Fig. 10. Error Histograms through the proposed NN-LM.



(f) **Case III**: Error Histogram class F_n

Fig. 10. (*continued*).

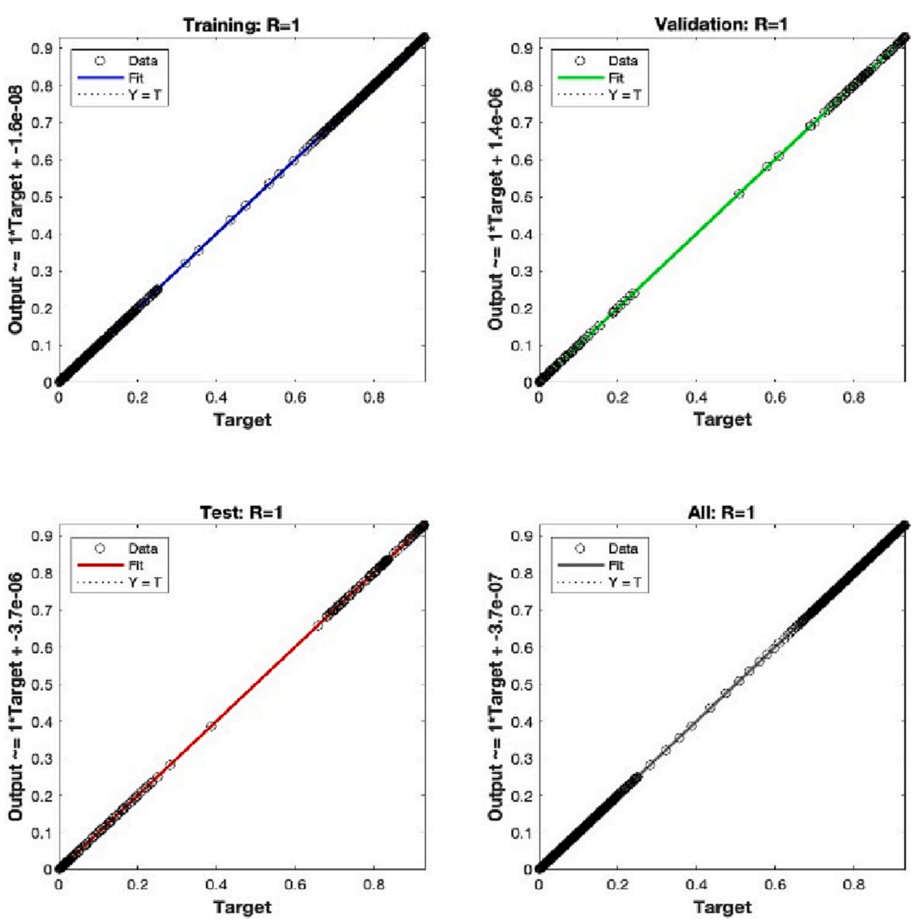


Fig. 11. **Case I**: Regression plots utilizing the NN-LM solving the nonlinear Wolbachia invasive model (WIM).

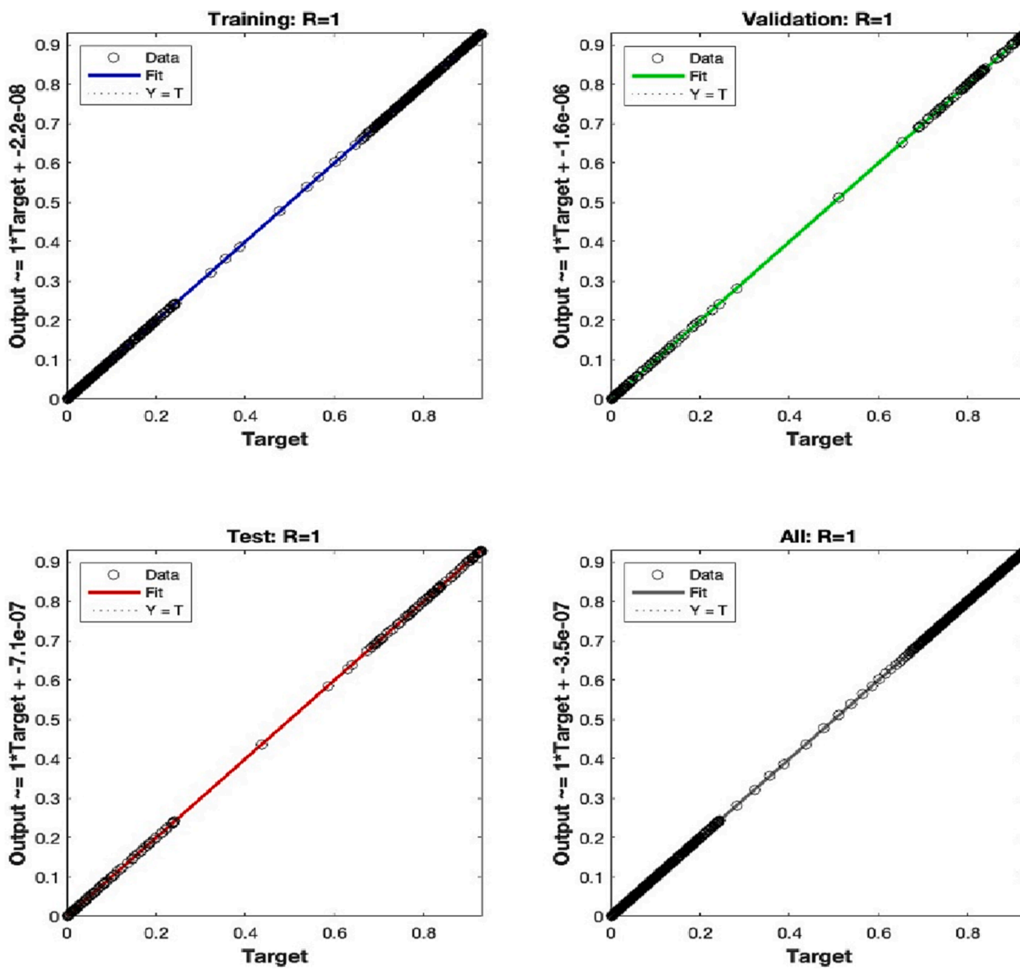


Fig. 12. Case II: Regression plots utilizing the NN-LM solving the nonlinear Wolbachia invasive model (WIM).

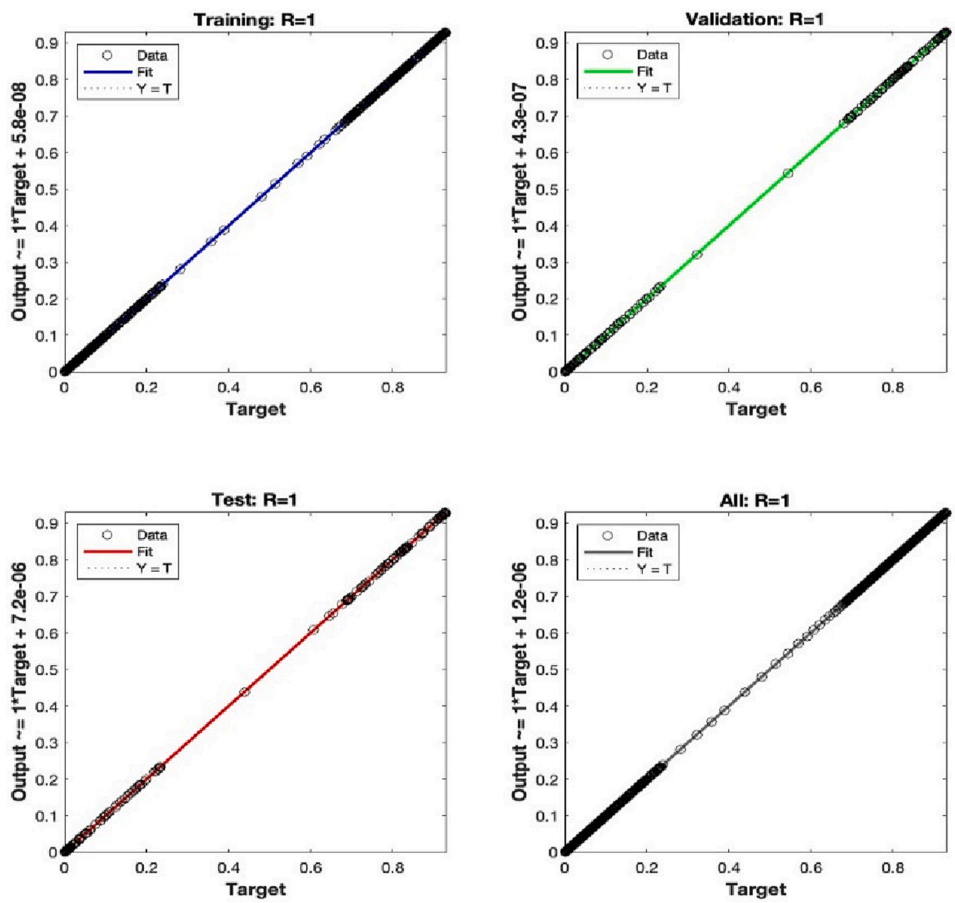
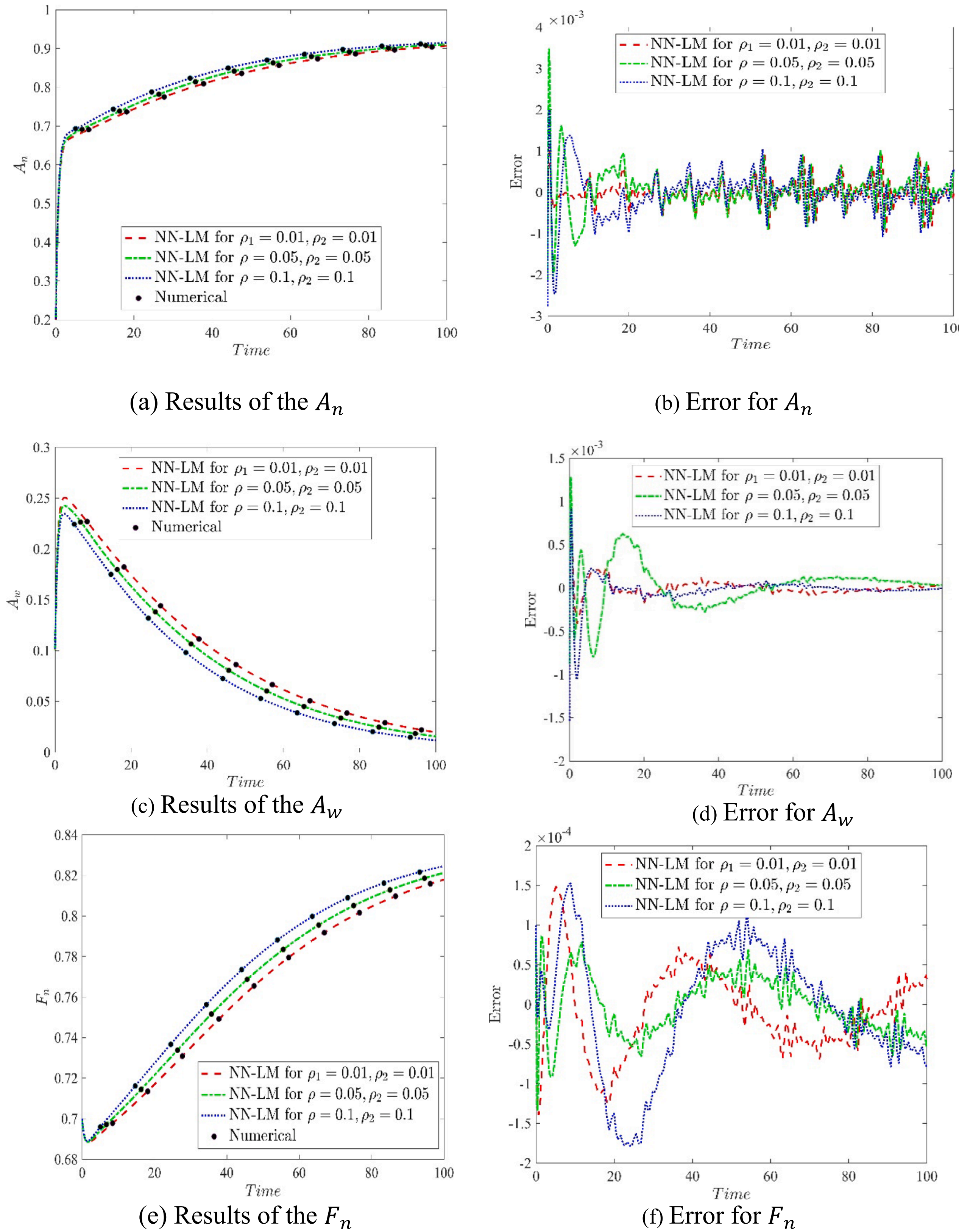


Fig. 13. Case III: Regression plots utilizing the NN-LM solving the nonlinear Wolbachia invasive model (WIM).



(a) Results of the A_n

(b) Error for A_n

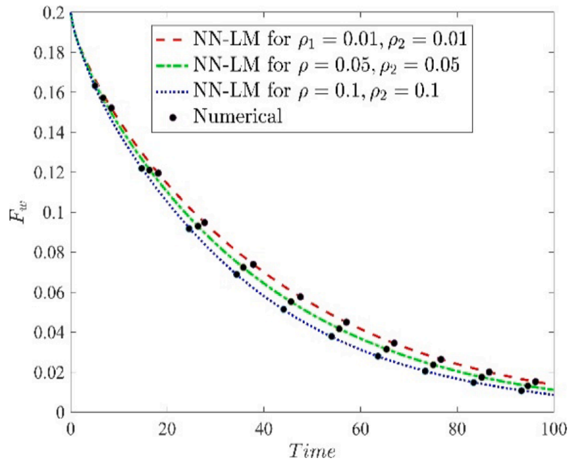
(c) Results of the A_w

(d) Error for A_w

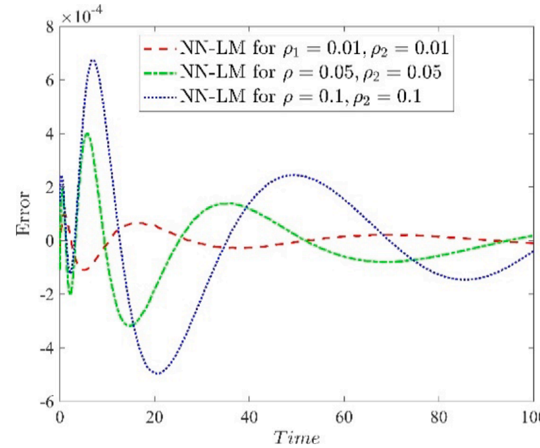
(e) Results of the F_n

(f) Error for F_n

Fig. 14. Comparison and error values of the presented NN-LM and the reference solutions for solving the nonlinear Wolbachia invasive model (WIM).



(g) Results of the Fw



(h) Error for F_w

Fig. 14. (*continued*).

CRediT authorship contribution statement

Zeshan Faiz: Conceptualization, Investigation, Writing – review & editing, Supervision. **Shumaila Javeed:** Conceptualization, Investigation, Writing – review & editing, Supervision. **Iftikhar Ahmed:** Conceptualization, Investigation, Writing – review & editing, Supervision. **Dumitru Baleanu:** Data curation, Methodology, Project administration, Writing – review & editing, Software, Visualization. **Muhammad Bilal Riaz:** Data curation, Methodology, Project administration, Writing – review & editing, Software, Visualization. **Zulqurnain Sabir:** Data curation, Methodology, Project administration, Writing – review & editing, Software, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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