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Quantum dot transition rate modifying by coupling to lattice plasmon

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Abstract

In this study, a plasmonic system coupled to a quantum dot is defined to generate the entanglement between two non-simultaneous emitted output modes. The quantum dot with three energy levels creates two different transition rates by which non-simultaneous photons are emitted. Thus, it seems that the entanglement between two emitted modes is forbidden. However, the simulation results show the entanglement between the output modes. It is because the original transition rates of the quantum dot are modified due to the lattice plasmon coupling effect. It means that the effective transition rate affected by the lattice plasmon plays a key role. The lattice plasmon coupling to quantum dot at some locations leads to a simultaneous transition by which the entanglement between output modes is established. The entangled output modes refer to the entangled photons with a specific frequency (e.g., the emission frequency). This unique behavior is theoretically discussed and the results show that using the lattice plasmon can change the transition rates by which the two emitted modes become entangled.

Keywords Plasmonic · Entanglement · Quantum dot · Lattice plasmon · Simultaneous decay rate

1 Introduction

In recent years, the effects of quantum systems' features such as superposition of states, uncertainty values of observables, and non-local non-trivial correlation have been widely investigated in the field of quantum sciences, image processing, and communication (Ekert 1991; Braunstein and Loock 2004; Shih 2007; Phillips et al. 2016). In these cases, entanglement (Simon 2000; Salmanogli and Gecim 2020; Laurat et al. 2005; Ge et al. 2015; Deesuwan 2010) is one of the most important quantum features utilized in different applications such as quantum illumination systems (Salmanogli et al. 2020), quantum communication (Ekert 1991; Braunstein and Loock 2004), and quantum sensory applications (Salmanogli 2021). Entanglement is basically generated due to the nonlinear properties of

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the material at which the momentum and frequency should be conserved (Kim et al. 2002; Salmanogli et al. 2019; Salmanogli 2201, 2204). Therefore, nonlinearity is a key factor to produce entangled states. Because of that fact, it has been recently shown that plasmonic nanoparticles (NPs) have an ability to induce the non-classicality correlation (Salmanogli and Geçim 2018a, 2018b). In contrast to photons, Plasmon resonance has a unique ability, such that it can be localized at a nanoscale. The latter important property has been deeply investigated, indicating that the plasmonic mode can be squeezed into volume far below the diffraction limit (Tame et al. 2013) and the related quantum state can be determined through the method as the quantum state tomography (Dominguez et al. 2013). Nonetheless, the plasmonic modes tend to decay so fast, a dramatic factor that is hard to control (Salmanogli 2019). Thus, the decay rate of the plasmonic NPs needs to be decreased. One of the methods for this purpose is using the retardation field effect, lattice plasmon, by which the decay rate of the plasmonic system is strongly changed.

Lattice plasmon is generated by an array of plasmonic NPs and shows much more efficient coupling to quantum dot (QD) than the sole NPs (Salmanogli and Geçim 2018a, 2018b). This type of plasmon is strongly contributed to the retarded field effect of a chain of NPs (Ross et al. 2016; Zou et al. 2004) in which NPs near field interact with the photonic mode at the far field. The photonic modes (optical diffraction) associated with the array periodicity are effectively coupled to the plasmon modes of each particle, leading to a combined mode. From the optic point of view, light scattering from each particle can be coherently re-scattered several times which means that scattering light is confined for a long time in an array (Ross et al. 2016).

In this work, the effect of the lattice plasmon coupled to QD is investigated on the entanglement between modes emitting by the QD. It has been shown that lattice-plasmon induced by the retarded field effect can strongly influence two-mode entanglement (Salmanogli and Geçim 2018b). However, there is a critical point by which the entanglement between modes can be destroyed. That problem is issued from the emitting photons with different decay rates, which are non-simultaneously emitting from the QD's with varying energy levels. Nevertheless, we think that lattice plasmon coupled with QD can affect the QD decaying rates at some locations, so the entanglement between modes is partially established. Thus, the study focuses on this point and theoretically proves that using lattice plasmon affects the effective decay rate of the QD at some locations where the lattice plasmon is created, causes to generate entanglement.

2 Theoretical and backgrounds

The schematic of the system containing an array of the plasmonic NPs and a QD is depicted in Fig. 1. In this system, 20×20 plasmonic NPs are periodically arranged with an inter-distance *d*. A QD is considered at a typical point in space with coordinates x, y, and z with $z >= 2R_{NPs}$, where R_{NPs} is the radius of NPs. By exciting the array of NPs with a wave, a high-intensity near field so close to each NP is generated. This field couples to the photonic mode (diffraction light) whereby the lattice-plasmon is produced, which effectively interacts with the QD. The QD is supposed to have three energy levels excited with two different plasmon modes ω_1 and ω_2. It is assumed that energy level 2 is far away from the ground level (Gerry and Eberly 1990), then it can be supposed as a two-energy level QD. The coupling lattice plasmon dramatically changes QD's density matrix behavior due to its effective coupling with QD, leading to the non-local

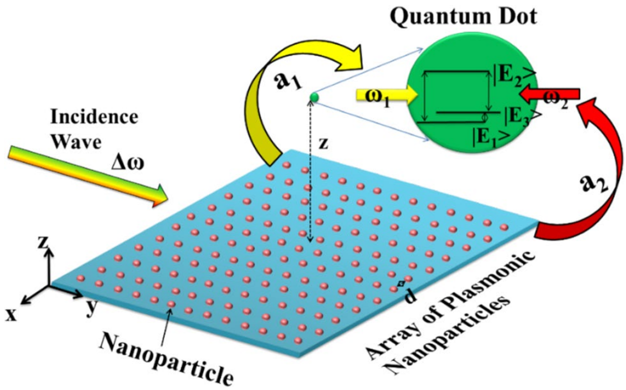

Fig. 1 Schematic of the plasmonic system; the array of the NPs coupling to a QD at distance z with three energy levels

manipulation of the output modes. This result can be explained by the fact that the retarded field effect is an important key to constructing the lattice plasmon in the array, and this phenomenon induces entanglement in output modes.

However, as demonstrated in Fig. 1, the energy difference between E_2 and E_1 (ω_1) is different from the energy difference between E_2 and E_3 (ω_2). This means that two emitting photons as the output are not simultaneous. At the first look, it seems that they cannot become entangled. Nonetheless, it will theoretically show that at some space where the QD can be placed, the lattice plasmon effect can create an effective decay rate by which the entanglement between two modes becomes established.

Thus, we want to show how the coupled lattice plasmon can modify the QD's transition rates to establish the entanglement between output modes.

The effective interaction Hamiltonian between the array of NPs and QD has been derived (Salmanogli and Geçim 2018a, 2018b) as:

$$H_{I,eff} = g(\sigma_+ a_1 a_2^+ + \sigma_- a_2 a_1^+), \quad g = (-2\hbar g_{12} g_{20} / \Delta) \tag{1}$$

where Δ, a_i and a_i^+ (i = 1, 2) are the detuning factor between energy levels 1 and 3, and the Bosonic operators (lattice-plasmon modes). Also, $\sigma_+ = \sigma_{13}$ and $\sigma_- = \sigma_{31}$ are the effective atomic raising and lowering operators between levels 1 and 3. Moreover, g_{20} and g_{12} are the NPs-QD coupling strength for mode I, and mode II, respectively. Based on the QD energy level illustrated in Fig. 1 and considering the energy conservation, one can deduce that $\hbar\Delta >> E_3 – E_1$. Based upon this supposition, level 2 is far off resonance and will be adiabatically removed. That is why the factor $E_2\sigma_{22}$ was ignored.

To investigate the effect of the lattice plasmon on effective coupling factor (g), the extended form of the interaction Hamiltonian is derived and presented as:

$$H_I = \hbar\left\{g_{20}(r).\left[a_1^+\sigma_{20} + \sigma_{02}a_1\right] + g_{21}(r).\left[a_2^+\sigma_{21} + \sigma_{12}a_2\right]\right\} \tag{2}$$

where NP-QD coupling strengths are given by Waks and Sridharan (2010); Salmanogli 2016):

$$g_{20}(r) = \left(-2\mu_{20}/\hbar\right)\left(\mathrm{R_{NPs}}/\mathrm{r}\right)^3 \sqrt{\hbar\omega_p/\left(2\varepsilon_0 \mathrm{V_m}\right)}$$
$$g_{21}(r) = \left(-2\mu_{21}/\hbar\right)\left(\mathrm{R_{NPs}}/\mathrm{r}\right)^3 \sqrt{\hbar\omega_p/\left(2\varepsilon_0 \mathrm{V_m}\right)} \quad (3)$$

where μ_{20}, μ_{21} are the dipole momentum of transition rates for considered energy levels. Also, R_{NPs}, r, ħ, ε_0, ω_p, and V_m are the NP's radius, QD location, Plank's constant, free space dielectric constant, plasmon frequency, and volume, respectively.

The main aim is to calculate the effect of the plasmonic field a_1 and a_2 on the transition times γ_{20} and γ_{21}. In fact, γ_{20} and γ_{21} are the natural life-time of related energy levels and determine the dipole momentum of transition rates ($\mu = \sqrt{(3\pi\hbar\varepsilon_0 c^3\gamma/\omega^3)}$). Therefore, using the total Hamiltonian and the related dynamics equation of motions (Salmanogli and Geçim 2018a, 2018b) and by substituting $a_1 = (\varepsilon\sqrt{\kappa_1} - ig_{20}(r)\sigma_{-1})/(i\Delta_{20} + 0.5\kappa_1)$, $a_2 = (\varepsilon\sqrt{\kappa_2} - ig_{21}(r)\sigma_{-2})/(i\Delta_{21} + 0.5\kappa_2)$ where $\Delta_{20} = \omega_p - \omega_{20}$, $\Delta_{21} = \omega_p - \omega_{21}$. in Eq. (4):

$$\dot{\sigma}_{-1} = -(i\omega_{20} + \gamma_{20})\sigma_{-1} + ig_{20}(r).\,a_1.\,\sigma_z - \Omega\sigma_z$$
$$\dot{\sigma}_{-2} = -(i\omega_{21} + \gamma_{21})\sigma_{-2} + ig_{21}(r).\,a_2.\,\sigma_z - \Omega\sigma_z \quad (4)$$

The related decay rates are concluded as:

$$\gamma_{20\mathrm{m}} = \gamma_{20} + \mathrm{g}_{20}(\mathrm{r})^2/\left(\mathrm{i}\Delta_{20} + 0.5\kappa_1\right)$$
$$\gamma_{21\mathrm{m}} = \gamma_{21} + \mathrm{g}_{21}(\mathrm{r})^2/\left(\mathrm{i}\Delta_{21} + 0.5\kappa_2\right) \quad (5)$$

where γ_{20m} and γ_{21m} are modified natural life-times regarding to coupling to the lattice plasmon modes. It is known from the energy levels in Fig. 1 that $\omega_{20} > \omega_{21}$, so $\Delta_{20} < \Delta_{21}$. From Eq. (5), one can conclude that the transition rates of the different states will be changed due to the lattice plasmon coupling. It is worth mentioning that after coupling the lattice plasmon to QD, it enforces deals with the modified version of the transition rates γ_{20m} and γ_{21m} rather than γ_{20} and $\gamma_{21.}$ The manipulation of the energy level's lifetime or transition rate means that it has a possibility to have a simultaneous transition rate which strongly depends on the lattice plasmon coupling field. Here, one can return back to the main question of the article which was: is it possible to create an entanglement between states with two different decay rates? Clearly, it is impossible to have entanglement between output modes (entangled photons with a specific frequency) if the original transition rates γ_{20} and γ_{21} is considered. It is because the original transition rates $\gamma_{20} \neq \gamma_{21}$ ($\gamma_{20} < \gamma_{21}$), which means that the two modes cannot be entangled. But what about the modified transition rates? It is shown that the contributed transition rates will be changed through the coupling to lattice plasmon. Indeed, we want to show that it has a possibility to have the same transition rates after coupling, which means that the entanglement between two lattice plasmon coupling modes can be established.

3 Results and discussions

Here in this section, the question asked above is answered using the simulations. Initially, the far-field area is considered. From Eq. (3), it is clearly understood that NPs-QD coupling strength is strongly decreased by increasing the NPs-QD distance. It means that at the far-field region, one can assume $g_{20}(r) \sim g_{21}(r) = g(r)$. Moreover, in this article, it is supposed that $\kappa_1 = \kappa_2 = \kappa$, then $g(r)^2/(i\Delta_{20} + 0.5\kappa) > g(r)^2/(i\Delta_{21} + 0.5\kappa)$ suggesting that it has a possibility to have $\gamma_{20m} = \gamma_{21m}$. It is because of $\Delta_{20} < \Delta_{21}$.

Now consider a near-field area where the lattice plasmon is coupled to QD. It is obvious from Eq. (3) that $g_{20}(r)$ and $g_{21}(r)$ straightforwardly depend on the dipole momentum of the transition which is defined as $\mu = \sqrt{(3\pi\hbar\varepsilon_0 c^3\gamma/\omega^3)}$. Therefore, by considering $\omega_{20} > \omega_{21}$ and $\gamma_{20} < \gamma_{21}$, it can be found that $\mu_{21} > \mu_{20}$, so, $g_{21}(r) > g_{20}(r)$; this point is illustrated in Fig. 2a and b. In Fig. 2, it is depicted that NPs-QD coupling of mode II is greater than mode I. That is attributed to the fact that QD's dipole momentum coupled with mode II is greater than with mode I.

Finally, Eq. (5) shows that there is a chance to have $\gamma_{20m} = \gamma_{21m}$ at some specific locations not everywhere. This means that it is the lattice plasmon that causes it to have a simultaneous transition rate. For this reason, a simulation is carried out and the results are illustrated in Fig. 3. It is shown that the modified transition rates (or modified natural life-time) can simultaneously occur where the lattice plasmon resonance is fully established. These locations are indicated with the black dashed circles on the figures. Moreover, the places where the modified transition rates are not equal are indicated with the red dashed circle. Thus, one can compare the results illustrated in Fig. 3a and b with Fig. 3c. Therefore, it is better to focus on the locations where γ_{20m} and γ_{21m} are equal, indicated with black dashed circle and follow the black dashed arrow. The black dashed arrow shows the locations where 2η (criterion of entanglement between two output modes (Simon 2000; Salmanogli and Gecim 2020; Laurat et al. 2005; Ge et al. 2015)) becomes less than 1, which means that the two modes become entangled at the mentioned location. It means that in the location addressed, the entangled photons with a specific frequency (e.g., $\sim \omega_1 \pm \omega_2$) are created. Now, consider the red dashed circle followed by the red arrows indicating locations where 2η is greater than 1, which means that two modes become separable at those locations. From Fig. 3, one can deduce that lattice plasmon can affect the modified decay rates and change them in such a way that the emitted photons become entangled. It is attributed to the fact that by coupling the

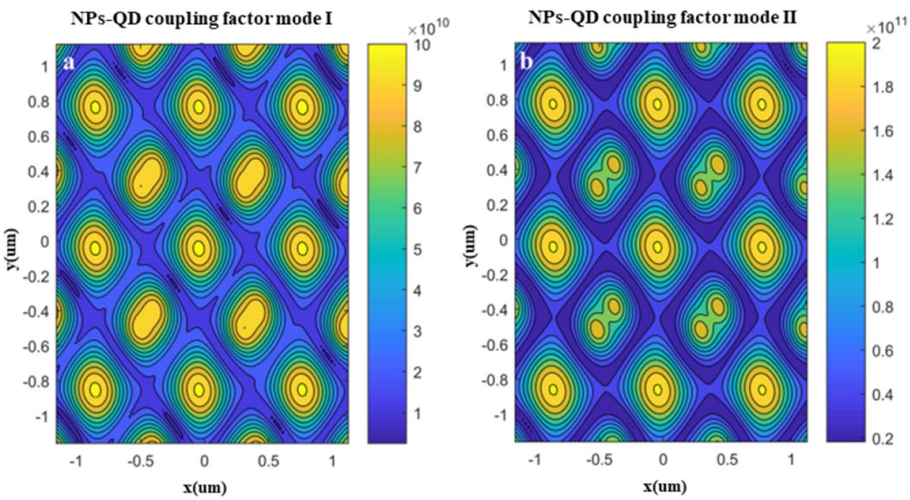

Fig. 2 **a** $g_{20}(r)$ versus x (um) and y(um), **b** $g_{21}(r)$ versus x (um) and y(um)

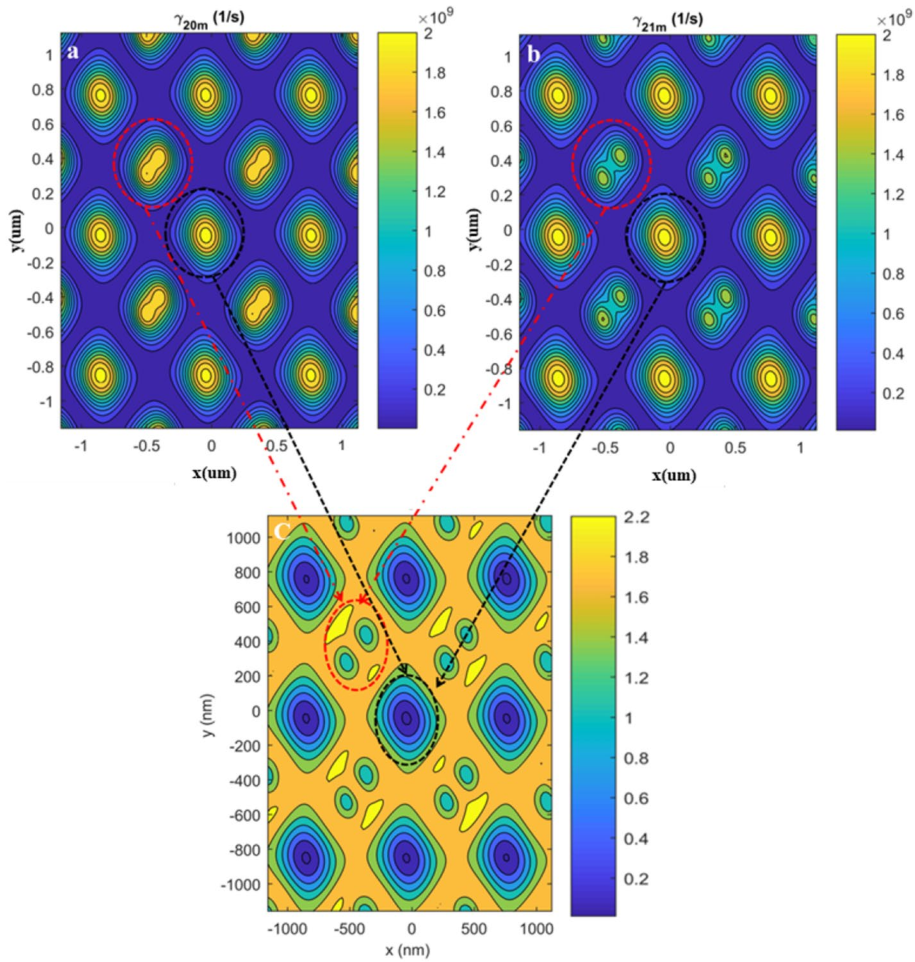

Fig. 3 **a** the modified transition rates at mode I γ_{20m} versus x (nm) and y(nm), **b** the modified transition rates at mode I γ_{21m} versus x (nm) and y(nm), **c** Two-mode entanglement (2η) for coherent initial state at z = 130 nm with array specifications: d = 808 nm, N_p = 400, R_{NPs} = 35 nm; λ_{inc} = 808 nm

QD to a lattice plasmon, the transition rates of the QD can be modified, resulting in the entanglement between the output modes. In principle, by carefully engineering the interaction between the QD and the lattice plasmon, it is possible to manipulate the emission properties and generate entangled photons with desired characteristics, including at specific emission frequencies. Indeed, it is the emission frequency that determines at which frequency the entangled photons can be generated. However, it is worth mentioning that lattice plasmon is created due to the interaction of the plasmonic mode with the photonic mode; consequently, the interference phenomenon plays a critical role in making the lattice plasmon. That is why in some places the lattice plasmon cannot be completely created, so the effect of the lattice plasmon is subsided. As a result, the emitted modes become separable.

4 Conclusion

In this study, the main aim was to study the effect of the lattice plasmon on the entanglement between emitted photons of a three-level quantum dot. Initially, it was shown that the natural lifetime (decay rate) of the emitted atoms is not simultaneous. So, it seemed that it was impossible to create entanglement between modes. For further investigation, a plasmonic system coupled to a QD was defined in which the generated lattice plasmon could affect the QD's original decay rate. It was theoretically and by simulations shown that although the natural lifetime of the emitted photons was not equal, lattice plasmon could manipulate the contributed lifetimes. In other words, by coupling the QD to a lattice plasmon, the transition rates of the QD were modified, so that the entanglement between the output modes was created. Thus, it was shown that it is possible to manipulate the emission properties and generate entangled photons by engineering the interaction between the QD and the lattice plasmon at specific emission frequencies.

Author contributions Sude Hatam contributed to the study's conception and manuscript preparation; Ahmad Salmanogli contributed to the design, simulation, and results analysis; H. Selcuk Gecim contributed to the editing and manuscript preparation.

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Data and materials availability The data generated during and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval Not applicable. There are no ethical concerns associated with this work.

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