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Some more bounded and singular pulses of a generalized scale-invariant analogue of the Korteweg–de Vries equation

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ABSTRACT

We investigate a generalized scale-invariant analogue of the Korteweg–de Vries (KdV) equation, establishing a connection with the recently discovered short-wave intermediate dispersive variable (SIdV) equation. To conduct a comprehensive analysis, we employ the Generalized Kudryashov Technique (KT), Modified KT, and the sine–cosine method. Through the application of these advanced methods, a diverse range of traveling wave solutions is derived, encompassing both bounded and singular types. Among these solutions are dark and bell-shaped waves, as well as periodic waves. Significantly, our investigation reveals novel solutions that have not been previously documented in existing literature. These findings present novel contributions to the field and offer potential applications in various physical phenomena, enhancing our understanding of nonlinear wave equations.

Introduction

From past many years, integrable systems have been considered as hot research area due to their several applications in science and engineering [1,2]. The most important and active research on integrable system is studying solitons or solitary waves of integrable systems via different approaches [3–7]. Specifically, KdV equations and its modified versions were analyzed by using various techniques [8–10]. The classical KdV equation is expressed by:

$$\mathcal{U}_t + 6\mathcal{U}\mathcal{U}_x + \mathcal{U}_{xxx} = 0. \tag{1}$$

The KdV equation finds significance in various contexts. It serves as the governing equation for the string in the Fermi-Pasta–Ulam-Tsingou problem when considering the continuum limit. Furthermore, it accurately portrays the behavior of long waves in shallow water and internal waves in a density-stratified ocean. Moreover, it study both acoustic waves on a crystal lattice and ion acoustic waves in a plasma. In literature, it was studied that the standard KdV equation admit solitary waves solutions. The single soliton solution of the Eq. (1) is expressed as:

$$\mathcal{U}(x,t) = \frac{\alpha}{2} sech^2\left[\frac{\sqrt{\alpha}}{2}(x - \alpha t - x_0)\right]. \tag{2}$$

The SIdV equation is an intriguing recent discovery in the field of nonlinear partial differential equations. It represents a generalization of the widely applied Korteweg–de Vries (KdV) equation, which is renowned for its ability to describe various physical phenomena, such as shallow water waves. The SIdV equation is characterized by its unique combination of short-wave and intermediate dispersive properties, bridging the gap between traditional short-wave equations and the KdV equation. The SIdV is expressed as [11]:

$$\mathcal{U}_t + \left(\frac{2\mathcal{U}_{xx}}{\mathcal{U}}\right)\mathcal{U}_x = \mathcal{U}_{xxx}. \tag{3}$$

The SIdV's applicability to physics holds significant potential in various domains. In the realm of nonlinear optics, where light waves interact

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with materials, the SIdV equation may offer valuable insights into the behavior of certain types of waves with intermediate dispersive effects. Similarly, in plasma physics, a field encompassing waves with a wide range of scales, the SIdV equation could be instrumental in understanding wave dynamics that exhibit intermediate dispersive characteristics. In fluid dynamics, the SIdV equation might find relevance in studying waves within fluid systems that display dispersive effects at intermediate scales. Even in quantum mechanics, where wave-like behavior is fundamental, the SIdV equation could shed light on wave packet dynamics with intermediate dispersive features.

Obtaining the SIdV equation from a physical system involves a delicate interplay of mathematical analysis and physical insight. Researchers start by formulating the governing equations that describe the behavior of the specific physical system under consideration. These equations are typically derived from the fundamental principles and laws of physics relevant to the phenomena being studied. Next, the dynamics of the system are analyzed to identify characteristic scales, such as spatial dimensions or time scales, that play a crucial role.

By employing asymptotic methods, such as the method of multiple scales or perturbation theory, researchers can derive simplified models that capture the dominant behaviors at different scales. During this process, certain patterns or terms may be recognized, leading to the generalization of the KdV equation and the emergence of the SIdV equation. This may entail introducing additional parameters or functions that encompass intermediate dispersive behaviors.

Once derived, the SIdV equation needs validation against numerical simulations or experimental data from the original physical system to ensure its accuracy and relevance. The equation's successful application to various physical phenomena opens up new avenues for understanding and modeling wave dynamics with intermediate dispersive properties, and it holds promise for advancing our comprehension of nonlinear wave phenomena across different scientific disciplines.

The SIdV equation encompasses higher-order dispersive effects and gives a solid explanation of wave prorogation in various physical systems. On contrast, the KdV equation is popular for its ability to study long waves in different media, the SIdV equation generalizes its applicability to short-wave phenomena. It denotes the behavior of waves with higher frequencies and shorter wavelengths, allowing for a more detail understanding of wave propagation in some contexts. The authors in [12] proposed a generalized SIdV equation in the form:

$$\mathcal{U}_t + \left(3(1-\varrho)\mathcal{U} + (1+\varrho)\frac{\mathcal{U}_{xx}}{\mathcal{U}}\right)\mathcal{U}_x - \gamma\,\mathcal{U}_{xxx} = 0. \tag{4}$$

Various methods have been introduced to extract exact solutions of nonlinear PDEs. For example, sine–cosine method [13], tan-cot method [14], F-expansion method [15], and many more [16–19]. Besides these techniques some more techniques like bifurcation analysis and neural networks have also been presented in the literature [20–23]. Here, we use three analytical methods such as the Generalized Kudryashov Technique (KT), Modified KT, and the sine–cosine method to analyze new exact solutions of the considered model. These solutions have not been studied before.

Generalized Kudryashov technique

The generalized Kudryashov (GK) method holds significant importance and utility when it comes to identifying the analytical soliton solutions to the nonlinear PDEs. To obtain a range of precise solutions for the proposed model, we outline the general procedure of the GK technique in this section. The solution's general form is determined using the GK technique as follows: Consider the following general nonlinear PDE

$$\mathcal{P}(\mathcal{A}, \mathcal{A}_x, \mathcal{A}_t, \mathcal{A}_{xx}, \mathcal{A}_{xt}, \dots) = 0, \tag{5}$$

here $\mathcal{A} = \mathcal{A}(x,t)$. Take start with transformation

$$\eta = \beta x - \alpha t. \tag{6}$$

Substituting Eq. (6) into Eq. (5), one can obtain the following nonlinear ODE:

$$\mathcal{U}(\mathcal{A}, \mathcal{A}', \mathcal{A}'', \mathcal{A}''', \dots) = 0, \tag{7}$$

here "′" stands for ordinary derivative with respect to η.

Then use the following form of the solution of the ODE under study.

$$\mathcal{U}(x,t) = \frac{\Omega_0 + \sum_{\gamma=1}^{\varsigma} \Omega_\gamma \mathcal{K}^\gamma(\zeta)}{\varrho_0 + \sum_{\gamma=1}^{\sigma} \varrho_\gamma \mathcal{K}^\gamma(\zeta)}, \tag{8}$$

where ς and $\sigma \in Z^+$, $\Omega_\gamma(\gamma = 1,2,3,\dots,\varsigma)$ and $\varrho_\gamma(\gamma = 1,2,3,\dots,\sigma)$ are unknown coefficients that are to be found later and η is defined in Eq. (6). Moreover we have

$$\mathcal{K}(\eta) = \frac{1}{1 + \mathcal{B}exp(\eta)}, \tag{9}$$

here $\mathcal{B}$ is the constant of integration and $\mathcal{K}(\eta)$ is general solution of Riccati equation as follows

$$\mathcal{K}'(\eta) = \mathcal{K}^2(\eta) - \mathcal{K}(\eta), \tag{10}$$

here "′" stands for ordinary derivative with respect to ζ. Using homogeneous balance principle, by comparing the highest power of nonlinear term with the highest order derivative in the resultant ODE after integrating Eq. (7) as much times as possible, one can obtain the values of ς and σ. Then substituting solution (8) and Eq. (9) into resultant ODE a polynomial in various powers of $\mathcal{K}(\eta)$ can be achieved. Furthermore, by equating the powers of $\mathcal{K}(\eta)$ to zero, an algebraic system can be obtained. Solving this system allows one to determine the values of Ω_γ, ϱ_γ, and other parameters, enabling the derivation of exact solutions.

Modified Kudryashov technique

In this part, we provide the general algorithm of the modified KT method. In this technique one has to find the ODE presented in Eq. (7), then the following expansion can be used

$$\mathcal{U}(\eta) = \sum_{\kappa=0}^{\vartheta} \frac{C_\kappa}{(1 + exp(\eta))^\kappa}, \tag{11}$$

where $C_0, C_1, C_2, \dots, C_\vartheta$ are constants to be calculated from Eq. (7). Using the homogeneous balance principle on the resultant ODE after integrating Eq. (7) multiple times, one can determine the values of ϑ. By substituting the Eq. (11) into the obtained ODE, a polynomial in various powers of $\exp(\eta)$ can be obtained. Equating the coefficients of the powers of $\exp(\eta)$ to zero yields an algebraic system. After the solution of this system, one can find C_κ and other parameters, resulting in the derivation of exact solutions.

Applications

In this part we present the applications of the proposed methods to the suggested model and calculate some novel soliton solutions. Therefore to do so, consider the following transformation

$$\zeta = \beta x - \alpha t. \tag{12}$$

Substituting Eq. (12) into the considered Eq. (4), we obtained the following ODE

$$\mathcal{U}(\zeta)\left(-\beta^3 - \gamma\,\mathcal{U}(\zeta)^3 - \alpha\,\mathcal{U}'(\zeta) + \beta\,\mathcal{U}'(\zeta)\left(\frac{\beta^2(\varrho+1)\mathcal{U}''(\zeta)}{\mathcal{U}(\zeta)} + 3(1-\varrho)\mathcal{U}(\zeta)\right)\right) = 0. \tag{13}$$

One integration Eq. (13) gives

$$\begin{aligned} &-2\beta^3\gamma\,\mathcal{U}(\zeta)\mathcal{U}''(\zeta) + \beta^3(\gamma+\varrho+1)\mathcal{U}'(\zeta)^2 \\ &-\alpha\,\mathcal{U}(\zeta)^2 - 2\beta(\varrho-1)\mathcal{U}(\zeta)^3 = 0\,, \end{aligned} \tag{14}$$

$$
\begin{aligned}
& -2(\varrho-1)\beta\Omega_0^3\rho_0-\alpha\Omega_0^2\rho_0^2=0\\
& \Omega_0\left(\beta^3\gamma\rho_0(\Omega_1\rho_0-\Omega_0\rho_1)+(\varrho-1)\beta\Omega_0(\Omega_0\rho_1+3\Omega_1\rho_0)+\alpha\rho_0(\Omega_0\rho_1+\Omega_1\rho_0)\right)=0\\
& \beta^3\left((\varrho+1)(\Omega_1\rho_0-\Omega_0\rho_1)^2-\gamma\left(\Omega_0^2\rho_1(6\rho_0+\rho_1)-2\Omega_0\rho_0(\Omega_1(3\rho_0+\rho_1)-4\rho_0(\Omega_2+\Omega_3))+\Omega_1^2\rho_0^2\right)\right)\\
& -6(\varrho-1)\beta\Omega_0\left(\Omega_0\Omega_1\rho_1+\Omega_0\rho_0(\Omega_2+\Omega_3)+\Omega_1^2\rho_0\right)-\alpha\left(\Omega_0^2\rho_1^2+\rho_0^2\left(2\Omega_0(\Omega_2+\Omega_3)+\Omega_1^2\right)\right.\\
& \left.+4\Omega_0\Omega_1\rho_0\rho_1\right)=0\\
& \beta^3\left((\varrho+1)(\Omega_1\rho_0-\Omega_0\rho_1)^2-\gamma\left(\Omega_0^2\rho_1(6\rho_0+\rho_1)-2\Omega_0\rho_0(\Omega_1(3\rho_0+\rho_1)-4\rho_0(\Omega_2+\Omega_3))+\Omega_1^2\rho_0^2\right)\right)\\
& -6(\varrho-1)\beta\Omega_0\left(\Omega_0\Omega_1\rho_1+\Omega_0\rho_0(\Omega_2+\Omega_3)+\Omega_1^2\rho_0\right)-\alpha\left(\Omega_0^2\rho_1^2+\rho_0^2\left(2\Omega_0(\Omega_2+\Omega_3)+\Omega_1^2\right)+4\Omega_0\Omega_1\rho_0\rho_1\right)=0\\
& 2\beta^3\left(\gamma\left(2\Omega_0^2\rho_0\rho_1+\Omega_0\left(2\rho_0(\Omega_2+\Omega_3)(5\rho_0-2\rho_1)-\Omega_1\left(2\rho_0^2+2\rho_0\rho_1+\rho_1^2\right)\right)+\Omega_1\rho_0(\Omega_1(2\rho_0+\rho_1)\right.\right.\\
& \left.\left.-3\rho_0(\Omega_2+\Omega_3))\right)-(\varrho+1)(\Omega_1\rho_0-\Omega_0\rho_1)(-\Omega_0\rho_1+\Omega_1\rho_0-2\rho_0(\Omega_2+\Omega_3))\right)-2(\varrho-1)\beta\left(\Omega_1\rho_0\left(6\Omega_0\times\right.\right.\\
& \left.\left.(\Omega_2+\Omega_3)+\Omega_1^2\right)+3\Omega_0\rho_1\left(\Omega_0(\Omega_2+\Omega_3)+\Omega_1^2\right)\right)-2\alpha\left(\rho_0\rho_1\left(2\Omega_0(\Omega_2+\Omega_3)+\Omega_1^2\right)+\Omega_0\Omega_1\rho_1^2\right.\\
& \left.+\Omega_1\rho_0^2(\Omega_2+\Omega_3)\right)=0\\
& \beta^3\left(2\gamma\Omega_1\left(-\rho_0\rho_1(-\Omega_0+\Omega_2+\Omega_3)+\Omega_0\rho_1^2+9\rho_0^2(\Omega_2+\Omega_3)\right)-2(\varrho+1)\Omega_1\rho_0(\Omega_0\rho_1+(\Omega_2+\Omega_3)(4\rho_0-\rho_1))\right.\\
& +(\varrho+1)\left(8\Omega_0\rho_0\rho_1(\Omega_2+\Omega_3)+\Omega_0\rho_1^2(\Omega_0-2(\Omega_2+\Omega_3))+4\rho_0^2(\Omega_2+\Omega_3)^2\right)-\gamma\left(4\rho_0^2(\Omega_2+\Omega_3)(3\Omega_0\right.\\
& \left.\left.+\Omega_2+\Omega_3)-20\Omega_0\rho_0\rho_1(\Omega_2+\Omega_3)+\Omega_0\rho_1^2(6(\Omega_2+\Omega_3)-\Omega_0)\right)+\Omega_1^2\rho_0(\varrho\rho_0-3\gamma\rho_0+\rho_0-2\gamma\rho_1)\right)\\
& -2(\varrho-1)\beta\left(3\rho_0(\Omega_2+\Omega_3)\left(\Omega_0(\Omega_2+\Omega_3)+\Omega_1^2\right)+\Omega_1\rho_1\left(6\Omega_0(\Omega_2+\Omega_3)\right.\right.\\
& \left.\left.+\Omega_1^2\right)\right)-\alpha\left(\rho_1^2\left(2\Omega_0(\Omega_2+\Omega_3)+\Omega_1^2\right)+4\Omega_1\rho_0\rho_1(\Omega_2+\Omega_3)+\rho_0^2(\Omega_2+\Omega_3)^2\right)=0
\end{aligned}
$$

Box I.

balance highest power of nonlinearity and highest order derivative in Eq. (14), we can express

$$\varsigma=\sigma+2, \tag{15}$$

$\sigma\neq 0$ is free parameter.

Application of GK method

Here we present the application of the GK method to the proposed model. In Eq. (15), when $\sigma=1$, then one can obtain from Eq. (15), that $\varsigma=3$. So the general solution of model (14), will be of the form

$$\mathcal{U}(x,t)=\mathcal{U}(\zeta)=\frac{\Omega_0+\Omega_1\mathcal{K}(\zeta)+\Omega_2\mathcal{K}^2(\zeta)+\Omega_3\mathcal{K}^3(\zeta)}{\rho_0+\rho_1\mathcal{K}(\zeta)}. \tag{16}$$

Substituting Eq. (16) into Eq. (14), we obtain

$$
\begin{aligned}
& -2\beta^3\gamma((\mathcal{K}(\zeta))-1)\,(\mathcal{K}(\zeta))\mathcal{K}(\zeta)^2(\Omega_2+\Omega_3)+(\Omega_1)\\
& +(\Omega_0)\left(\mathcal{K}(\zeta)\left(\mathcal{K}(\zeta)(\Omega_2+\Omega_3)(\rho_1\mathcal{K}(\zeta))(2\rho_1\mathcal{K}(\zeta))\right.\right.\\
& +(6\rho_0-\rho_1)+3\rho_0((2\rho_0-\rho_1))+\rho_0\rho_1(\Omega_1-2\Omega_0)\\
& \left.-\Omega_0\rho_1^2+2\rho_0^2(\Omega_1-2(\Omega_2+\Omega_3))\right)\\
& \left.+\rho_0(\Omega_0\rho_1-\Omega_1\rho_0)\right)\\
& +\beta^3((\mathcal{K}(\zeta))-1)^2(\mathcal{K}(\zeta))^2(\gamma+\varrho+1)(\mathcal{K}(\zeta))\\
& \times\,(\Omega_2+\Omega_3)((\rho_1\mathcal{K}(\zeta))+2\rho_0)-\Omega_0\rho_1+(\Omega_1\rho_0)^2\\
& -2\beta(\varrho-1)((\rho_1\mathcal{K}(\zeta))+\rho_0)\times\mathcal{K}(\zeta)^2(\Omega_2+\Omega_3)\\
& +(\Omega_1)+(\Omega_0)^3-\alpha((\rho_1\mathcal{K}(\zeta))+\rho_0)^2\mathcal{K}(\zeta)^2\\
& \times\,(\Omega_2+\Omega_3)+(\Omega_1)+(\Omega_0)^2=0,
\end{aligned} \tag{17}
$$

comparing various powers of $\mathscr{K}(\zeta)$, we get the equations see Boxes I and II. Solving the algebraic system presented above, we obtain the following sets of parameters values

$$Set\ I:\ \Omega_0=-\frac{\beta^2\gamma\rho_0}{\varrho-1},\Omega_2=-\Omega_3,$$
$$\rho_1=\frac{\Omega_1-\varrho\Omega_1}{\beta^2\gamma},\alpha=2\beta^3\gamma$$
$$Set\ II:\ \Omega_0=-\frac{\beta^2\gamma\rho_0}{\varrho-1},\Omega_1=\frac{\beta^2\gamma\rho_0}{\varrho-1},$$
$$\Omega_2=-\Omega_3,\rho_1=-\rho_0,\alpha=2\beta^3\gamma$$
$$Set\ III:\ \Omega_0=0,\Omega_1=-\frac{2\beta^2\rho_0(\varrho-2\gamma+1)}{\varrho-1},$$
$$\Omega_2=\frac{2\varrho\beta^2\rho_0-4\beta^2\gamma\rho_0+2\beta^2\rho_0-\varrho\Omega_3+\Omega_3}{\varrho-1},$$
$$\rho_1=0,\alpha=\beta^3(\varrho-\gamma+1).$$

Now substituting above sets of parameters into the Eq. (16) and using Eq. (12), we obtain the following exact solutions

$$S_1=\frac{\frac{\Omega_3}{\left(Be^{\beta x-2\beta^3\gamma t}+1\right)^3}-\frac{\Omega_3}{\left(Be^{\beta x-2\beta^3\gamma t}+1\right)^2}-\frac{\beta^2\gamma\rho_0}{\varrho-1}+\frac{\beta^2\gamma\rho_0}{(\varrho-1)\left(Be^{\beta x-2\beta^3\gamma t}+1\right)}}{\rho_0-\frac{\rho_0}{Be^{\beta x-2\beta^3\gamma t}+1}}. \tag{18}$$

$$S_2=\frac{\frac{\beta^2\gamma\rho_0}{(\varrho-1)(Be^{\beta x-\alpha t}+1)}-\frac{\beta^2\gamma\rho_0}{\varrho-1}-\frac{\Omega_3}{(Be^{\beta x-\alpha t}+1)^2}+\frac{\Omega_3}{(Be^{\beta x-\alpha t}+1)^3}}{\rho_0-\frac{\rho_0}{Be^{\beta x-\alpha t}+1}}. \tag{19}$$

See Eq. (20) given in Box III.

Application of MK method

Here we present the application of the MK technique. From Eq. (11), using the homogeneous balance principle we get that $\vartheta=2$. Therefore from Eq. (11), we have

$$\mathcal{U}(x,t)=\mathcal{U}(\zeta)=C_0+\frac{C_1}{1+exp(\eta)}+\frac{C_2}{(1+exp(\eta))^2}. \tag{21}$$

Now substituting the Eq. (21) into Eq. (13), we obtain the following

$$
\begin{aligned}
& -2\beta^3\gamma e^{\beta x+\alpha t}\left(C_1\left(e^{2\beta x+\alpha t}-1\right)+2C_2\left(2e^{\beta x+\alpha t}-1\right)\right)\\
& \left(C_0\left(e^{\beta x+\alpha t}+1\right)^2+C_1e^{\beta x+\alpha t}+C_1+C_2\right)\\
& +2(1-\varrho)\beta\left(C_0\left(e^{\beta x+\alpha t}+1\right)^2+C_1e^{\beta x+\alpha t}\right.\\
& \left.+C_1+C_2\right)^3-\left(e^{\beta x+\alpha t}+1\right)^2\varrho\left(C_0\left(e^{\beta x+\alpha t}+1\right)^2\right.\\
& \left.+C_1e^{\beta x+\alpha t}+C_1+C_2\right)^2+\beta^3e^{2\beta x+\alpha t}(\alpha+\gamma+1)\\
& \left(C_1e^{\beta x+\alpha t}+C_1+2C_2\right)^2=0.
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
&(\Omega_2+\Omega_3)\left(\beta^3\left(\gamma\left(\rho_0\rho_1(6\Omega_0-6\Omega_1+\Omega_2+\Omega_3)+\rho_1^2(\Omega_1-6\Omega_0)+6\rho_0^2(\Omega_1-\Omega_2-\Omega_3)\right)-2(\varrho+1)\left(\rho_0\rho_1\times\right.\right.\right.\\
&\left.\left.(-\Omega_0-\Omega_1+\Omega_2+\Omega_3)\ +\Omega_0\rho_1^2+\rho_0^2(\Omega_1-2(\Omega_2+\Omega_3))\right)\right)+3(\varrho-1)\beta\left(\Omega_0\rho_1(\Omega_2+\Omega_3)+\Omega_1^2\rho_1+\Omega_1\rho_0\times\right.\\
&\left.\left.(\Omega_2+\Omega_3)\right)+\alpha\rho_1(\Omega_1\rho_1+\rho_0(\Omega_2+\Omega_3))\right)=0\\
&(\Omega_2+\Omega_3)\left(\beta^3\left((\varrho+1)\left(\rho_1^2(-2\Omega_0+\Omega_2+\Omega_3)+2\rho_0\rho_1(\Omega_1-4(\Omega_2+\Omega_3))+4\rho_0^2(\Omega_2+\Omega_3)\right)-\gamma\left(\rho_1^2(6\Omega_0\right.\right.\right.\\
&\left.\left.-\ 6\Omega_1+\Omega_2+\Omega_3)-10\rho_0\rho_1(-\Omega_1+\Omega_2+\Omega_3)+8\rho_0^2(\Omega_2+\Omega_3)\right)\right)-2(\varrho-1)\beta(\Omega_2+\Omega_3)(3\Omega_1\rho_1+\rho_0(\Omega_2\\
&\left.+\ \Omega_3))-\alpha\rho_1^2(\Omega_2+\Omega_3)\right)=0\\
&-4\beta^3\gamma\Omega_1\rho_1^2(\Omega_2+\Omega_3)+4\beta^3\rho_0\rho_1(\varrho+\gamma+1)(\Omega_2+\Omega_3)^2-12\beta^3\gamma\rho_0\rho_1(\Omega_2+\Omega_3)^2-2\beta^3\rho_1^2\times\\
&(\varrho+\gamma+1)(\Omega_2+\Omega_3)^2+6\beta^3\gamma\rho_1^2(\Omega_2+\Omega_3)^2-2(\varrho-1)\beta\rho_1(\Omega_2+\Omega_3)^3=0\\
&\beta^3\rho_1^2(\varrho+\gamma+1)(\Omega_2+\Omega_3)^2-4\beta^3\gamma\rho_1^2(\Omega_2+\Omega_3)^2=0.
\end{aligned}
$$

Box II.

$$
S_3=\frac{\frac{\Omega_3}{\left(\mathcal{B}e^{\beta x-\beta^3 t(\varrho-\gamma+1)}+1\right)^3}+\frac{2\varrho\beta^2\rho_0-4\beta^2\gamma\rho_0+2\beta^2\rho_0-\varrho\Omega_3+\Omega_3}{(\varrho-1)\left(\mathcal{B}e^{\beta x-\beta^3 t(\varrho-\gamma+1)}+1\right)^2}-\frac{2\beta^2\rho_0(\varrho-2\gamma+1)}{(\varrho-1)\left(\mathcal{B}e^{\beta x-\beta^3 t(\varrho-\gamma+1)}+1\right)}}{\rho_0}. \tag{20}
$$

Box III.

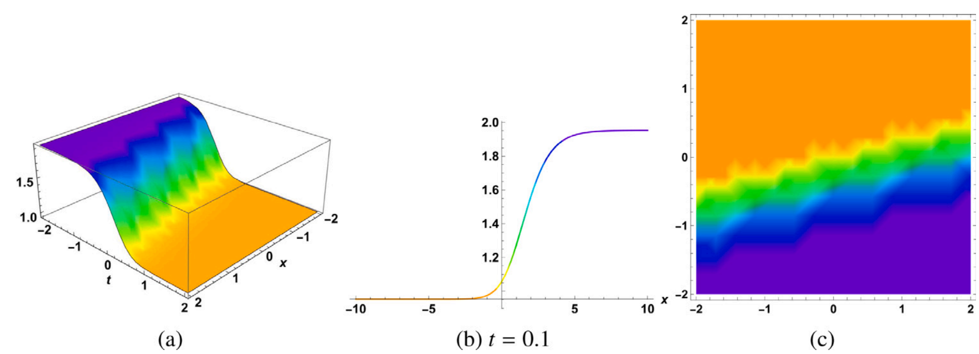

Fig. 1. Picture of S_1 with parameters $\varrho=-1.1$, $\gamma=2$, $\mathcal{B}=2$, $\beta=1$, $\Omega_3=1$, $\rho_0=1$.

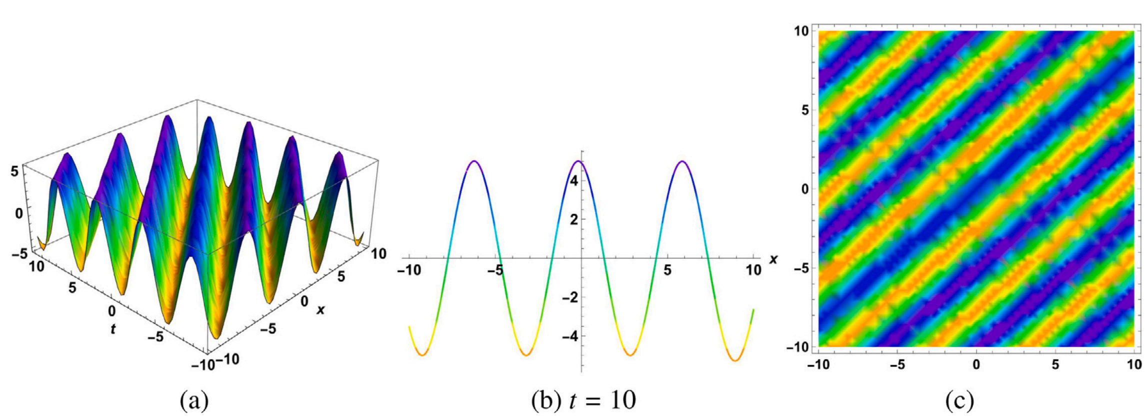

Fig. 2. Picture of S_2 with parameters $\varrho=0.1$, $\gamma=1$, $\mathcal{B}=1$, $\beta=0.1$, $\Omega_3=1$, $\rho_0=0.2$.

$$
\begin{aligned}
&exp(\zeta)^0: \ -2\varrho C_0^3\beta + 2C_0^3\beta - 6\varrho C_0^2C_1\beta + 6C_0^2C_1\beta - 6\varrho C_0^2C_2\beta + 6C_0^2C_2\beta - C_0^2\alpha - 6\varrho C_0C_1^2\beta + 6C_0C_1^2\beta\\
&-12\varrho C_0C_1C_2\beta + 12C_0C_1C_2\beta - 2C_0C_1\alpha - 6\varrho C_0C_2^2\beta + 6C_0C_2^2\beta - 2C_0C_2\alpha - 2\varrho C_1^3\beta + 2C_1^3\beta\\
&-6\varrho C_1^2C_2\beta + 6C_1^2C_2\beta - C_1^2\alpha - 6\varrho C_1C_2^2\beta + 6C_1C_2^2\beta - 2C_1C_2\alpha - 2\varrho C_2^3\beta + 2C_2^3\beta - C_2^2\alpha = 0\\
&exp(\zeta)^1: \ -2\varrho C_0^3\beta + 2C_0^3\beta - C_0^2\alpha = 0\\
&exp(\zeta)^2: \ -12\varrho C_0^3\beta + 12C_0^3\beta - 6\varrho C_0^2C_1\beta + 6C_0^2C_1\beta - 6C_0^2\alpha - 2C_0C_1\beta^3\gamma - 2C_0C_1\alpha = 0\\
&exp(\zeta)^3: \ -30\varrho C_0^3\beta + 30C_0^3\beta - 30\varrho C_0^2C_1\beta + 30C_0^2C_1\beta - 6\varrho C_0^2C_2\beta + 6C_0^2C_2\beta - 15C_0^2\alpha - 6\varrho C_0C_1^2\beta\\
&+6C_0C_1^2\beta - 4C_0C_1\beta^3\gamma - 10C_0C_1\alpha - 8C_0C_2\beta^3\gamma - 2C_0C_2\alpha + \varrho C_1^2\beta^3 - C_1^2\beta^3\gamma + C_1^2\beta^3 - C_1^2\alpha = 0\\
&exp(\zeta)^4: \ -40\varrho C_0^3\beta + 40C_0^3\beta - 60\varrho C_0^2C_1\beta + 60C_0^2C_1\beta - 24\varrho C_0^2C_2\beta + 24C_0^2C_2\beta - 20C_0^2\alpha - 24\varrho C_0C_1^2\beta\\
&+24C_0C_1^2\beta - 12\varrho C_0C_1C_2\beta + 12C_0C_1C_2\beta - 20C_0C_1\alpha - 12C_0C_2\beta^3\gamma - 8C_0C_2\alpha - 2\varrho C_1^3\beta + 2C_1^3\beta\\
&+2\varrho C_1^2\beta^3 + 2C_1^2\beta^3 - 4C_1^2\alpha + 4\varrho C_1C_2\beta^3 - 6C_1C_2\beta^3\gamma + 4C_1C_2\beta^3 - 2C_1C_2\alpha = 0\\
&exp(\zeta)^5: \ -12\varrho C_0^3\beta + 12C_0^3\beta - 30\varrho C_0^2C_1\beta + 30C_0^2C_1\beta - 24\varrho C_0^2C_2\beta + 24C_0^2C_2\beta - 6C_0^2\alpha - 24\varrho C_0C_1^2\beta\\
&+24C_0C_1^2\beta - 36\varrho C_0C_1C_2\beta + 36C_0C_1C_2\beta + 2C_0C_1\beta^3\gamma - 10C_0C_1\alpha - 12\varrho C_0C_2^2\beta + 12C_0C_2^2\beta\\
&+4C_0C_2\beta^3\gamma - 8C_0C_2\alpha - 6\varrho C_1^3\beta + 6C_1^3\beta - 12\varrho C_1^2C_2\beta + 12C_1^2C_2\beta + 2C_1^2\beta^3\gamma - 4C_1^2\alpha\\
&-6\varrho C_1C_2^2\beta + 6C_1C_2^2\beta + 6C_1C_2\beta^3\gamma - 6C_1C_2\alpha + 4C_2^2\beta^3\gamma - 2C_2^2\alpha = 0\\
&exp(\zeta)^6: \ -30\varrho C_0^3\beta + 30C_0^3\beta - 60\varrho C_0^2C_1\beta + 60C_0^2C_1\beta - 36\varrho C_0^2C_2\beta + 36C_0^2C_2\beta - 15C_0^2\alpha - 36\varrho C_0C_1^2\beta\\
&+36C_0C_1^2\beta - 36\varrho C_0C_1C_2\beta + 36C_0C_1C_2\beta + 4C_0C_1\beta^3\gamma - 20C_0C_1\alpha - 6\varrho C_0C_2^2\beta + 6C_0C_2^2\beta\\
&-12C_0C_2\alpha - 6\varrho C_1^3\beta + 6C_1^3\beta - 6\varrho C_1^2C_2\beta + 6C_1^2C_2\beta + \varrho C_1^2\beta^3 + 3C_1^2\beta^3\gamma + C_1^2\beta^3 - 6C_1^2\alpha\\
&+4\varrho C_1C_2\beta^3 + 4C_1C_2\beta^3 - 6C_1C_2\alpha + 4\varrho C_2^2\beta^3 - 4C_2^2\beta^3\gamma + 4C_2^2\beta^3 - C_2^2\alpha = 0.
\end{aligned} \tag{23}
$$

Box IV.

Comparing various powers of $exp(\zeta)$, $\zeta = 2\beta x + \alpha t$, we get the equations (see Box IV). Solving the system presented above, we obtain the following non trivial set of values

$$
\begin{aligned}
C_0 &= 0, C_1 = -\frac{2\beta^2(\varrho - 2\gamma + 1)}{\varrho - 1},\\
C_2 &= \frac{2\left(\varrho\beta^2 - 2\beta^2\gamma + \beta^2\right)}{\varrho - 1}, \alpha = \beta^3(\varrho - \gamma + 1),
\end{aligned} \tag{24}
$$

substituting above values into Eq. (21), we get the following solution

$$
\begin{aligned}
S_4 = &\frac{2\left(\varrho\beta^2 - 2\beta^2\gamma + \beta^2\right)}{(\varrho - 1)\left(e^{\beta x - \beta^3 t(\varrho - \gamma + 1)} + 1\right)^2}\\
&- \frac{2\beta^2(\varrho - 2\gamma + 1)}{(\varrho - 1)\left(e^{\beta x - \beta^3 t(\varrho - \gamma + 1)} + 1\right)}.
\end{aligned} \tag{25}
$$

Application of sine–cosine method

In this part, we use the sine–cosine technique to calculate some novel exact solutions of the suggested equation. In this technique we use the following Sine expansion:

$$
\mathcal{U}(x,t) = \mathcal{U}(\zeta) = \Upsilon sin(\omega\zeta)^{\mathrm{r}}, \tag{26}
$$

where

$$
\mathcal{U}(\zeta)'' = \mathrm{r}(\mathrm{r}-1)\Upsilon\omega^2 sin(\omega\zeta)^{\mathrm{r}-2} - \mathrm{r}^2\Upsilon\omega^2 sin(\omega\zeta)^{\mathrm{r}}, \tag{27}
$$

substituting Eqs. (26) and (27) into Eq. (13), we obtain the following

$$
\begin{aligned}
&-\Upsilon^2\alpha(sin(\omega\zeta))^{2\mathrm{r}} - 2\beta(-1+\alpha)\Upsilon^3(sin(\omega\zeta))^{3\mathrm{r}}\\
&+(\beta)^3\mathrm{r}^2(1+\alpha+\gamma)\Upsilon^2\ (\omega)^2\\
&(1-(sin(\omega\zeta))^2)(sin(\omega\zeta))^{-2+2\mathrm{r}} - 2\beta^3\gamma\Upsilon(sin(\omega\zeta))^{\mathrm{r}}\\
&((-1+\mathrm{r})\mathrm{r}\Upsilon\omega^2\\
&(1-(sin(\omega\zeta))^2)(sin(\omega\zeta))^{-2+\mathrm{r}}\\
&-\mathrm{r}\Upsilon(\omega)^2(sin(\omega\zeta))^{\mathrm{r}}) = 0.
\end{aligned} \tag{28}
$$

Now there is one possible case:

$$
\begin{aligned}
&\mathrm{r} - 2 \neq 0\\
&3\mathrm{r} + 2 - 2\mathrm{r} = 0\\
&\alpha\mathrm{r}^2\beta^3\Upsilon^2\omega^2 - \mathrm{r}^2\beta^3\gamma\Upsilon^2\omega^2 + \mathrm{r}^2\beta^3\Upsilon^2\omega^2\\
&+2\mathrm{r}\beta^3\gamma\Upsilon^2\omega^2 - 2\varrho\beta\Upsilon^3 + 2\beta\Upsilon^3 = 0\\
&-\varrho\mathrm{r}^2\beta^3\Upsilon^2\omega^2 + \mathrm{r}^2\beta^3\gamma\Upsilon^2\omega^2 - \beta^3\mathrm{r}^2\Upsilon^2\omega^2 - \Upsilon^2\nu = 0,
\end{aligned} \tag{29}
$$

solving above system gives the following values

$$
\begin{aligned}
&\mathrm{r} = -2, \omega = -\frac{\sqrt{\omega}}{2\sqrt{-\varrho\beta^3 + \beta^3\gamma - \beta^3}},\\
&\Upsilon = \frac{-\varrho\omega + 2\gamma\omega - \omega}{2(\varrho - 1)\beta(\varrho - \gamma + 1)},\\
&\mathrm{r} = -2, \omega = \frac{\sqrt{\omega}}{2\sqrt{-\varrho\beta^3 + \beta^3\gamma - \beta^3}},\\
&\Upsilon = \frac{-\varrho\omega + 2\gamma\omega - \omega}{2(\varrho - 1)\beta(\varrho - \gamma + 1)},
\end{aligned} \tag{30}
$$

substituting above values into Eq. (26), we obtained the following solutions

$$
S_5 = \frac{(-\varrho\omega + 2\gamma\omega - \omega)\csc^2\left(\frac{\sqrt{\omega}(\beta x - \alpha t)}{2\sqrt{-\varrho\beta^3 + \beta^3\gamma - \beta^3}}\right)}{2(\varrho - 1)\beta(\varrho - \gamma + 1)}. \tag{31}
$$

$$
S_6 = \frac{(-\varrho\omega + 2\gamma\omega - \omega)\csc^2\left(\frac{\sqrt{\omega}(\beta x - \alpha t)}{2\sqrt{-\varrho\beta^3 + \beta^3\gamma - \beta^3}}\right)}{2(\varrho - 1)\beta(\varrho - \gamma + 1)}. \tag{32}
$$

Now consider the following Cosine expansion:

$$
\mathcal{U}(x,t) = \mathcal{U}(\zeta) = \Upsilon cos(\omega\zeta)^{\mathrm{r}}, \tag{33}
$$

where

$$
\mathcal{U}(\zeta)'' = \mathrm{r}(\mathrm{r}-1)\Upsilon\omega^2 cos(\omega\zeta)^{\mathrm{r}-2} - \mathrm{r}^2\Upsilon\omega^2 cos(\omega\zeta)^{\mathrm{r}}, \tag{34}
$$

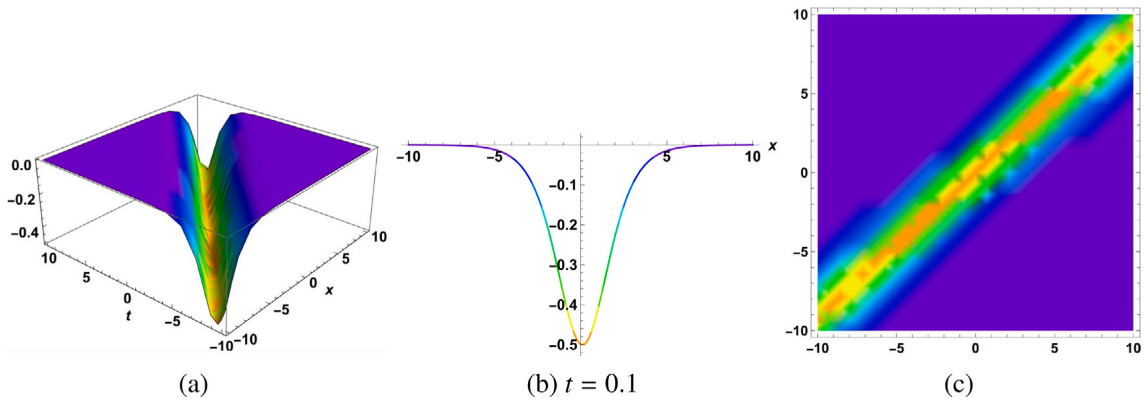

Fig. 3. Picture of S_4 with parameters $\varrho = 1,\ \gamma = 1,\ \beta = 1$.

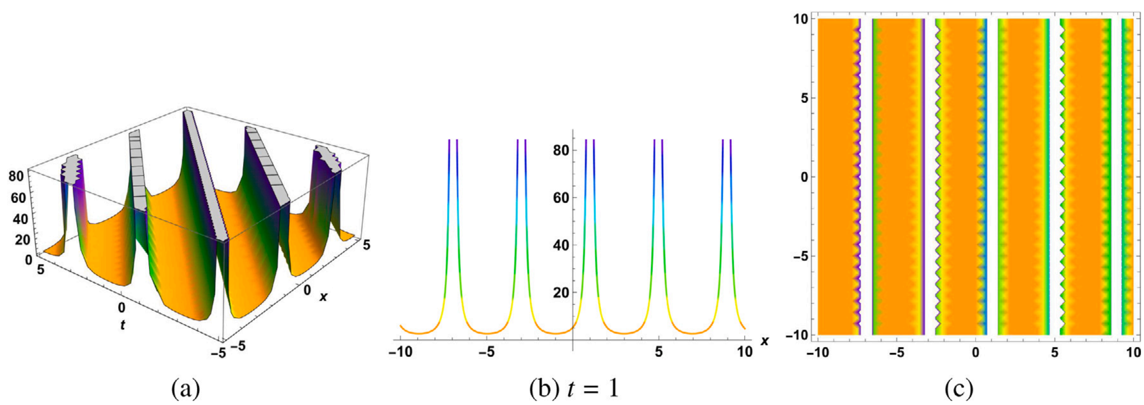

Fig. 4. Picture of S_5 with parameters $\varrho = 0.1,\ \gamma = 1.5,\ \alpha = 1,\ \beta = 1$.

substituting Eqs. (33) and (34) into Eq. (13), we obtain the following

$$\begin{aligned}
&-Y^2\alpha(cos(\omega\zeta))^{(2\mathrm{r})} - 2\beta(-1+\alpha)Y^3(cos(\omega\zeta))^{3\mathrm{r}}\\
&+\beta^3\mathrm{r}^2(1+\alpha+\gamma)Y^2\,\omega^2\\
&(1-(cos(\omega\zeta))^2)(cos(\omega\zeta))^{-2+2\mathrm{r}} - 2\beta^3\gamma Y(cos(\omega\zeta))^{\mathrm{r}}\\
&((-1+\mathrm{r})\mathrm{r}Y\omega^2\\
&(1-(cos(\omega\zeta))^2)(cos(\omega\zeta))^{-2+\mathrm{r}}\\
&-\mathrm{r}Y\omega^2(cos(\omega\zeta))^{\mathrm{r}}) = 0.
\end{aligned} \tag{35}$$

Now here is also one possible case:

$$\begin{aligned}
&\mathrm{r}-2 \neq 0\\
&3\mathrm{r}+2-2\mathrm{r} = 0\\
&\alpha\mathrm{r}^2\beta^3Y^2\omega^2 - \mathrm{r}^2\beta^3\gamma Y^2\omega^2 + \mathrm{r}^2\beta^3Y^2\omega^2\\
&+2\mathrm{r}\beta^3\gamma Y^2\omega^2 - 2\varrho\beta Y^3 + 2\beta Y^3 = 0\\
&-\varrho\mathrm{r}^2\beta^3Y^2\omega^2 + \mathrm{r}^2\beta^3\gamma Y^2\omega^2 - \beta^3\mathrm{r}^2Y^2\omega^2 - Y^2\nu = 0,
\end{aligned} \tag{36}$$

solving above system gives the following values

$$\begin{aligned}
&\mathrm{r} = -2, \omega = -\frac{\sqrt{\omega}}{2\sqrt{-\varrho\beta^3+\beta^3\gamma-\beta^3}},\\
&Y = \frac{-\varrho\omega+2\gamma\omega-\omega}{2(\varrho-1)\beta(\varrho-\gamma+1)},\\
&\mathrm{r} = -2, \omega = \frac{\sqrt{\omega}}{2\sqrt{-\varrho\beta^3+\beta^3\gamma-\beta^3}},\\
&Y = \frac{-\varrho\omega+2\gamma\omega-\omega}{2(\varrho-1)\beta(\varrho-\gamma+1)},
\end{aligned} \tag{37}$$

substituting above values into Eq. (33), we get the following solutions

$$S_7 = \frac{(-\varrho\alpha+2\gamma\alpha-\alpha)\sec^2\left(\frac{\sqrt{\alpha}(\beta x-\alpha t)}{2\sqrt{-\varrho\beta^3+\beta^3\gamma-\beta^3}}\right)}{2(\varrho-1)\beta(\varrho-\gamma+1)} \tag{38}$$

$$S_8 = \frac{(-\varrho\alpha+2\gamma\alpha-\alpha)\sec^2\left(\frac{\sqrt{\alpha}(\beta x-\alpha t)}{2\sqrt{-\varrho\beta^3+\beta^3\gamma-\beta^3}}\right)}{2(\varrho-1)\beta(\varrho-\gamma+1)} \tag{39}$$

Simulations and discussion

In this study, the geometric behavior of some obtained solutions is described, and their physical interpretations are presented through graphical representations in 2D, 3D, and density plots. Fig. 1 illustrates the dynamics of solution S_1, showcasing the kink solitary wave behavior in both 3D-space and 2D-plane. This kink solitary wave corresponds to a localized wave profile with a sharp transition from one amplitude to another.

Fig. 2 portrays the physical interpretation of the exact solution S_2, revealing periodic solitonic behavior. Solitons are solitary waves that maintain their shape and speed during propagation, and the periodic solitonic behavior observed here demonstrates stable, periodic waveforms. Moving on, Fig. 3 displays the evolution of the solution S_4 in 3D and 2D plots, representing a dark soliton structure. Dark solitons in the SIdV equation arise from the delicate balance between nonlinear and dispersive effects. The nonlinear term tends to compress the wave, while the dispersive term leads to wave spreading or dispersion. This interplay allows for the formation of a localized, dark region within the wave profile. Dark solitons have been observed in various physical systems and can play significant roles in the dynamics of nonlinear waves.

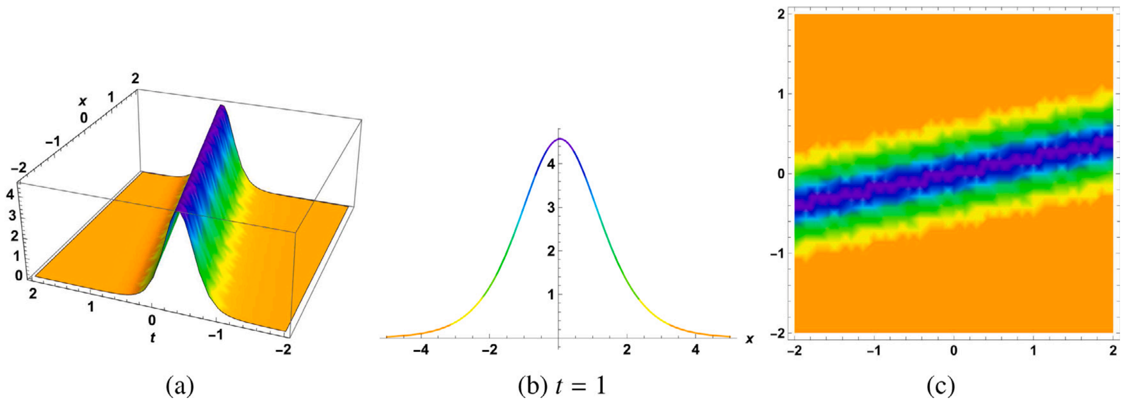

Fig. 5. Picture of S_6 with parameters $\rho = 0.1,\ \gamma = -2,\ \alpha = 5,\ \beta = 1$.

Fig. 4 presents the graphical representation of the solution S_5, showcasing a localized wave with sharp peaks and periodicity. This type of solution is relevant in scenarios where localized wave structures are observed, and the sharp peaks indicate a well-defined wave packet. Lastly, Fig. 5 illustrates the geometric behavior of the solution S_6, which exhibits a localized wave solution with a bell-shaped profile. This bell-shaped profile is characteristic of certain physical phenomena, and the solution's localization suggests a well-defined region of wave concentration. The physical interpretations of these solutions demonstrate the richness of behaviors that can arise from the generalized SIdV equation. The derived solutions provide valuable insights into the complex dynamics of dispersive waves in various physical contexts, ranging from kink solitary waves to periodic solitonic behaviors and dark soliton structures. Understanding and analyzing these geometric features is crucial for comprehending the intricate interplay between nonlinear and dispersive effects in different physical systems.

Further investigations could explore the stability and interactions of these solutions, as well as their applicability to specific physical scenarios. Analyzing the physical meanings of these solutions in different contexts could deepen our understanding of wave phenomena and inspire new applications in various branches of physics and engineering. Overall, the study of the geometric behavior of these solutions contributes significantly to the broader field of nonlinear wave dynamics and dispersive wave equations.

Conclusion

In this study, we have investigated a generalized Short-Wave Intermediate Dispersive Variable (SIdV) equation, establishing its connection with the well-known Korteweg–de Vries (KdV) equation and the recently discovered SIdV equation. Notably, both the KdV equation and the generalized SIdV equation share a common one-soliton solution. Through the application of advanced analytical techniques, including the Generalized Kudryashov Technique (KT), Modified KT, and the sine–cosine method, we have successfully derived a diverse array of traveling wave solutions. These solutions encompass both bounded and singular types, such as dark and bell-shaped waves, as well as periodic waves. Remarkably, our findings have unveiled novel solutions that were previously unreported in existing literature, and we have provided explicit closed-form expressions for these solutions.

Future work

The implications of the generalized SIdV equation and the newly discovered solutions are promising for various areas of physics and engineering. One potential avenue for future work lies in the application of the generalized SIdV equation to model plasma dynamics, particularly in regions where dispersive and nonlinear effects play a crucial role. This could be highly relevant in studying plasma instabilities and wave propagation in plasmas, as the generalized SIdV equation might offer new insights into the complex behaviors of plasma waves. Further investigations can also explore the broader applicability of the derived solutions in other physical systems and engineering problems where intermediate dispersive behaviors are encountered. Additionally, examining the stability and robustness of these novel solutions in practical scenarios would be beneficial for assessing their reliability and feasibility in real-world applications. Besides this the suggested model can also be studied using fractional operators in future [24–26]. Overall, the continued exploration of the generalized SIdV equation and its solutions holds great promise for advancing our understanding of nonlinear wave phenomena and their applications in diverse scientific disciplines.

CRediT authorship contribution statement

Sayed Saifullah: Conceptualization, Methodology, Writing – original draft. **M.M. Alqarni:** Validation, Writing – review & editing, Revision. **Shabir Ahmad:** Conceptualization, Methodology, Writing – original draft. **Dumitru Baleanu:** Validation, Formal analysis, Investigation. **Meraj Ali Khan:** Writing – review & editing, Revision. **Emad E. Mahmoud:** Writing – review & editing, Revision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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References

[1] Kivshar Yuri S, Malomed Boris A. Dynamics of solitons in nearly integrable systems. Rev Modern Phys 1989;61(4):763.

[2] Kartashov Yaroslav V, Malomed Boris A, Torner Lluis. Solitons in nonlinear lattices. Rev Modern Phys 2011;83(1):247.

[3] Nasreen N, Seadawy AR, Lu D, Arshad M. Optical fibers to model pulses of ultra-short via gernalized third-order nonlinear schrodinger equation by using extended and modified rational expansion method. J Nonlinear Opt Phys Mater 2023.
[4] Nasreen N, Lu D, Zhang Z, Akgul A, Younas U, Nasreen S, et al. Propagation of optical pulses in fiber optics modelled by coupled space–time fractional dynamical system. Alex Eng J 2023;73:173–87.
[5] Ismaelal HF, Younas U, Sulaiman TA, Nasreen N, Shah N, Ali MR. Non classical interaction aspects to a nonlinear physical model. Results Phys 2023;49:106520.
[6] Nasreen N, Younas U, Sulaiman TA, Zhang Z, Lu D. A variety of M-truncated optical solitons to a nonlinear extended classical dynamical model. Results Phys 2023;51:106722.
[7] Nasreen N, Younas U, Lu D, Zhang Z, Rezazadeh H, Hosseinzadeh MA. Propagation of solitary and periodic waves to conformable ion sound and langmuir waves dynamical system. Opt Quantum Electron 2023;55:868.
[8] Ma Wen Xiu. Complexiton solutions to the Korteweg–de Vries equation. Phys Lett A 2002;301(1–2):35–44.
[9] Wazwaz Abdul-Majid. The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations. Appl Math Comput 2007;184(2):1002–14.
[10] Wazwaz Abdul-Majid. Two-mode fifth-order KdV equations: necessary conditions for multiple-soliton solutions to exist. Nonlinear Dynam 2017;87:1685–91.
[11] Sen Abhijit, Ahalpara Dilip P, Thyagaraja Anantanarayanan, Krishnaswami Govind S. A KdV-like advection–dispersion equation with some remarkable properties. Commun Nonlinear Sci Numer Simul 2012;17(11):4115–24.
[12] Alzaleq Lewa, Manoranjan Valipuram, Alzalg Baha. Exact traveling waves of a generalized scale-invariant analogue of the Korteweg–de Vries equation. Mathematics 2022;10(3):414.
[13] Wazwaz A-M. The tanh and the sine-cosine methods for the complex modified K dV and the generalized K dV equations. Comput Math Appl 2005;49(7–8):1101–12.
[14] Naowarat Surapol, Saifullah Sayed, Ahmad Shabir, la Sen Manuel De. Periodic, singular and dark solitons of a generalized geophysical KdV equation by using the Tanh-Coth method. Symmetry 2023;15(1):135.
[15] Zhao Yun-Mei. F-expansion method and its application for finding new exact solutions to the Kudryashov-Sinelshchikov equation. J Appl Math 2013;2013.
[16] Ren Bo. Interaction solutions for mKP equation with nonlocal symmetry reductions and CTE method. Phys Scr 2015;90(6):065206.
[17] Ren Bo, Cheng Xue-Ping, Lin Ji. The (2+ 1)(2+ 1)-dimensional Konopelchenko–Dubrovsky equation: nonlocal symmetries and interaction solutions. Nonlinear Dynam 2016;86:1855–62.
[18] Ahmad Israr, Jalil Abdul, Ullah Aman, Ahmad Shabir, la Sen Manuel De. Some new exact solutions of (4+ 1)-dimensional Davey–Stewartson-Kadomtsev–Petviashvili equation. Results Phys 2023;45:106240.
[19] Ahmad Shabir, Saifullah Sayed, Khan Arshad, Inc Mustafa. New local and nonlocal soliton solutions of a nonlocal reverse space–time mKdV equation using improved Hirota bilinear method. Phys Lett A 2022;450:128393.
[20] Xu Changjin, Mu Dan, Liu Zixin, Pang Yicheng, Liao Maoxin, Li Peiluan. Bifurcation dynamics and control mechanism of a fractional-order delayed Brusselator chemical reaction model. MATCH Commun Math Comput Chem 2023;89(1):73–106.
[21] Xu CJ, Cui XH, Li PL, Yan JL, Yao LY. Exploration on dynamics in a discrete predator–prey competitive model involving time delays and feedback controls. J Biol Dyn 2023;17(1):2220349.
[22] Li PL, Lu YJ, Xu CJ, Ren J. Insight into hopf bifurcation and control methods in fractional order BAM neural networks incorporating symmetric structure and delay. Cogn Comput 2023. http://dx.doi.org/10.1007/s12559-023-10155-2.
[23] Mu Dan, Xu Changjin, Liu Zixin, Pang Yicheng. Further insight into bifurcation and hybrid control tactics of a chlorine dioxide-iodine-malonic acid chemical reaction model incorporating delays. MATCH Commun Math Comput Chem 2023;89(3):529–66.
[24] Ou Wei, Xu Changjin, Cui Qingyi, Liu Zixin, Pang Yicheng, Farman Muhammad, et al. Mathematical study on bifurcation dynamics and control mechanism of tri-neuron BAM neural networks including delay. Math Methods Appl Sci 2023. http://dx.doi.org/10.1002/mma.9347.
[25] Xu Changjin, Cui Qingyi, Liu Zixin, Pan Yuanlu, Cui Xiaohan, Ou Wei, et al. Extended hybrid controller design of bifurcation in a delayed chemostat model. MATCH Commun Math Comput Chem 2023;90(3):609–48.
[26] Xu Changjin, Mu Dan, Pan Yuanlu, Aouiti Chaouki, Yao Lingyun. Exploring bifurcation in a fractional-order predator–prey system with mixed delays. J Appl Anal Comput 2023;13(3):1119–36. http://dx.doi.org/10.11948/20210313.