SCATTERING WAVES BY A CYLINDRICAL CONDUCTIVE SURFACE

REFİK KORAY YAYLAK

DECEMBER 2022

ÇANKAYA UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF ELECTRONIC AND COMMUNICATION ENGINEERING

M.Sc. Thesis in

ELECTRONIC AND COMMUNICATION ENGINEERING

SCATTERING WAVES BY A CYLINDRICAL CONDUCTIVE SURFACE

REFİK KORAY YAYLAK

DECEMBER 2022

ABSTRACT

SCATTERING WAVES BY A CYLINDRICAL CONDUCTIVE SURFACE

YAYLAK, REFİK KORAY
M.Sc. in Electronic and Communication Engineering

Supervisor: Assist. Prof. Dr. Göker Şener
December 2022, 41 pages

Scattering of electromagnetic waves is the process by which electromagnetic waves change direction or frequency when interacting with a surface. This scattering event occurs as a result of the surface reflected or diffracted electromagnetic waves. In this thesis, the scattering of electromagnetic waves on a conductive cylindrical surface is discussed. The scattering of electromagnetic waves is usually analyzed using Maxwell's equations. The Maxwell's equations set up relations between the electric field and magnetic field. It provides mathematical modeling of phenomena such as electromagnetic wave propagation and scattering. First, these electromagnetic waves are modeled based on Maxwell's equations. Appropriate equations expressing the electric field and magnetic field at the surface of the cylinder are used. Then, the far-field approach is used. However, the far-field approach is valid in cases where the wavelength ratio of the distance to the observation point of the scattered electromagnetic waves is large enough. The far-field approach; It usually analyzes the parameters of the scattering event, such as scattering cross section, propagation angle, and scattered wave density. These parameters are used to determine the properties of the scattered wave. In this thesis, the behavior of the scattered fields from a cylindrical surface is analyzed in theory, and simulation with MATLAB. Then, the results are compared with Ansys HFSS software.

Keywords: Cylindrical Surface, Far-Field, Physical Optics

ÖZET

İLETKEN SİLİNDİRİK BİR YÜZEYDEN DALGALARIN SAÇILMASI

YAYLAK, REFİK KORAY
Elektronik ve Haberleşme Mühendisliği Yüksek Lisans

Danışman: Dr. Öğr. Üyesi Göker Şener
Aralık 2022, 41 sayfa

Elektromanyetik dalgaların saçılması, elektromanyetik dalgaların bir yüzeyle etkileşime girdiğinde yön veya frekans değiştirmesi sürecidir. Bu saçılma olayı, yüzeyin elektromanyetik dalgaları yansıtması ve kırması sonucu gerçekleşir. Bu tezde, iletken bir silindirik yüzey üzerinde elektromanyetik dalgaların saçılması ele alınmıştır. Elektromanyetik dalgaların saçılması genellikle Maxwell denklemleri kullanılarak analiz edilir. Maxwell denklemleri, elektrik alanı ve manyetik alanı birbirine bağlar. Elektromanyetik dalga yayılımı ve saçılması gibi olayların, matematiksel modellenmesini sağlar. İlk olarak, bu elektromanyetik dalgalar Maxwell denklemlerine dayanan bir şekilde modellenir. Silindirin yüzeyindeki elektrik alanı ve manyetik alanı ifade eden uygun denklemler kullanılır. Daha sonra, Uzak alan yaklaşımı kullanılır. Ancak, uzak alan yaklaşımı saçılan elektromanyetik dalgaların gözlem noktasına olan mesafenin dalga boyu oranının yeterince büyük olduğu durumlarda geçerlidir. Uzak alan yaklaşımı; saçılma olayının genellikle, saçılma kesitini, yayılım açısını, saçılan dalga yoğunluğu gibi parametreleri analiz eder. Bu parametreler, saçılan dalganın özelliklerini belirlemek için kullanılır, ve bununla birlikte saçılma olayı analiz edilir. Saçılan alanların davranışı teoride ve MATLAB ile simülasyonda analiz edilir, daha sonra sonuçlar Ansys HFSS yazılımı ile karşılaştırılır.

Anahtar Kelimeler: Silindirik Yüzey, Uzak Alan, Fiziksel Optik

ACKNOWLEDGEMENT

I want to sincerely thank Dr. Göker Şener for overseeing, providing particular direction, making comments, and encouraging me while I developed my thesis. It gives me great pleasure to specifically thank my family for their invaluable assistance.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF ABBREVIATIONS

PO	: Physical Optics
GO	: Geometric Optics
PTD	: Physical Theory of Diffraction
GTD	: Geometrical Theory of Diffraction
UTD	: Uniform Theory of Diffraction
MTPO	: Modified Theory of Physical Optics
E_i^S	: Incident Electrical Fields of Scattering Waves
E_r^S	: Reflected Electrical Fields of Scattering Waves
E_T	: Total Electric Fields Surface
H_T	: Total Magnetic Fields Surface
H_i	: Incident Fields of Magnetic Surface
H_r	: Reflected Fields of Magnetic Surface
J_{ms}	: Magnetic Flux Intensity
J_{es}	: Electric Surface Density
μ	: Permeability of Surface
μ_0	: Permeability of Free-Space
ε	: Permittivity of Surface
ε_0	: Permittivity of Free-Space

CHAPTER I

INTRODUCTION

.

1.1 REVIEW AND OBJECTIVES

The aim of this thesis is to investigate the scattering of electromagnetic waves by a cylindrical cap with conductive boundary conditions. The theory is verified by the simulation results. As far as the author's knowledge, no study has been found investigating this problem with an analytical approach as well as the computer simulation.

In the literature, Fresnel explained diffraction with wave theory. Developing the Huygens principle, Fresnel introduced a new principle by considering the overlapping interference (Superposition) of waves (Fresnel 1812). According to this principle, called the Huygens-Fresnel principle, when the incident wave is refracted by an obstacle, its wavelength gets shorter than the dimensions of the obstacle and the incident wave is considered as the primary wave. Secondly, each point of the advancing wave is approached as the source point of a new wave and these points are considered as (isotropic) sources that generate waves in all directions.

When Maxwell's electromagnetic theory was established, it has been understood that light has both particle and wave structure and the studies have progressed in this direction (Maxwell 1865). Later, Kirchoff developed the mathematical basis of Diffraction Theory with the Helmholtz equations and revealed that the diffraction field in the traditional Helmholtz-Kirchhoff approximation can be obtained by calculating the effect of waves propagating from all points within the aperture area (Kirchhoff 1883). Thereupon, Maggi developed Kirchhoff's Helmholtz equation based on it has integral solution. Maggi and Rubinowicz showed that the Kirchoff diffraction integral can be decomposed into boundary diffraction waves and geometric waves (Maggi 1888).

Before Fresnel, Young stated that diffraction is caused by the combination of a uniformly propagating wave and some waves reflected from the aperture surface (Young 1802). Rubinowicz then examined Young's ideas of diffraction (Rubinowicz 1917). Based on this, Miyamoto generalized Rubinowicz's studies (Miyamoto 1962). Since the method of Miyamoto and Wolf did not give results similar to Young's model for diffraction waves, Rubinowicz called these solutions Miyamoto-Wolf diffraction (Rubinowicz 1965).

In the following period, electromagnetic waves emitted from surfaces are observed with the method of Geometric Optics. However, in this method, discontinuous diffraction is not observed in the light-dark border crossing regions. First, Geometric Optical waves propagate unaffected by obstacles and have the same structure as incident fields. Second, the fields emanating from the edge of the obstacle are diffraction fields and Geometric Optics cannot be applied to them. For this reason, Sommerfeld, stating that the transmitted waves propagate from surfaces other than the real surface, found the edge diffraction coefficient by showing the scattered fields in terms of Geometric Optics and diffracted waves (Sommerfeld 1896). Sommerfeld's solution was reformulated by Wiener and Hopf to solve the integral equations named after them. Later, although Radlow analyzed the scattering problem with the Wiener-Hopf method, he could not fully express the diffracted waves (Radlow 1961). After that, Satterwhite developed a definitive solution (Satterwhite 1974). Moreover, Keller developed the Sommerfeld solution, one of the known solutions of simple shapes and made this theory, called the Geometric Theory of Diffraction, more multi-dimensional than the Kirchhoff wave theory. This method developed by Keller is a high-frequency approach (Keller 1962). The Geometric Theory of Diffraction for the impedance surface was formulated only after the work of Senior, Maliuzhinets, and Volakis (Senior 1952, Maliuzhinets 1958, Volakis 1986).

The asymptotic reduction approach used in Geometric Optics was first used by Michaeli (Michaeli 1984). This technique yielded the same results as the methods of Maggi and Rubinowicz. Scattering integral with Edge Point method evaluates asymptotic broadening of fully diffraction fields for high-frequency. The asymptotic diffraction field approaches infinity in the shadow boundary transition region. Regular equation expression, which is finite in this region, can be obtained by the Uniform Theory of Diffraction (UTD) (Kouyoumjian and Pathak 1974). The Geometrical Theory of Diffraction remains incapable in infinite fields at shadow boundaries, so the

Uniform Theory of Diffraction was developed to overcome this inadequacy. Also, James and Balanis are among the researchers who have conducted notable works in these fields (James 1986, Balanis 1987).

Physical Optics approach which is introduced by Macdonald is used as an efficient method to calculate electromagnetic scattering (Macdonald 1913). While Geometric Optics is a field-based method, Physical Optics is a current-based method. Physical Optics varies with frequency. The method of Physical Optics approximates the diffraction coefficients for the Geometric Theory of Diffraction. The most important reason why Physical Optics is preferred for calculating scattering fields from conductive surfaces is that it can be easily reduced to Geometric Optics equations at high frequencies. The solution expression is more general than Geometric Optics. The Physical Optics method determines the high frequency reflected fields, approximate surface current density and magnetic fields induced at the surface of a perfect conductor, but cannot evaluate edge diffraction fields. In this method, the solution of the Helmholtz equation for scattered fields is achieved with the help of the second scalar Green's theorem. Following, the reflected and refracted areas are expressed by applying the Stationary Phase method. Therefore, Umul developed an advanced theory of Physical Optics called The Modified Theory of Physical Optics (MTPO), which provides a definitive solution to the scattering problem (Umul 2004). Later, Umul defined the edge diffraction wave as a continuous field with no phase shift at the shadow or reflection boundary. In this way, the fields of Geometric Optics become an aperture wave (Umul 2008). Kirchhoff's theory of diffraction does not give edge diffractions; this problem also occurs in Physical Optics. Additional currents, called fringe currents, are added to obtain precise diffraction. In the next period, Ufimtsev introduced the Physical Theory of Diffraction (PTD) to eliminate this problem. In other words, Physical Theory of Diffraction was developed to improve the Physical Optical surface approach (Ufimtsev 1971). However, the theory has flaws; the apparent directions of the fringe currents cannot be evaluated. Nonetheless, the Modified Theory of Physical Optics has also removed the flaws of the Physical Theory of Diffraction. In this thesis, the corresponding boundary conditional surface current is obtained. First, the Physical Optics equation is written for the cylindrical surface. Transmitted and reflected waves are evaluated by the Far-Field method. The behavior of the evaluated areas is analyzed numerically.

1.2 ORGANIZATION OF THE THESIS

Chapter 1 presents the background on the subject. Chapter 2 gives information on the methods and approaches used in the study. In chapter 3, the scattered fields of a source originating from a cylindrical surface are examined by using the Physical Optics scattering equation. Finally, including the surface width and the position angle of the source, total scattered, reflected and diffracted fields are plotted for some problem parameters.

1.3 BACKGROUNDS

In this section, brief information is given about the functions used in the equations.

1.3.1 Conductive Surface

Surfaces that allow heat or electric current to pass through are called conductive surfaces. The conductive surface, here, is a type of scattering that maintains a magnetic surface density. This character is defined as the electromagnetic coupling of the resistive surface.

$$\Gamma = \frac{\sin\emptyset_0}{\sin\emptyset_0 + \sin\theta_n} \tag{1.1}$$

and

$$T = \frac{\sin\theta_n}{\sin\emptyset_0 + \sin\theta_n} \tag{1.2}$$

respectively. T and Γ are the transmission and reflection coefficients of a conductive surface. $\sin\theta_n$ is the angle of incidence and equal to $2R_m Z_0$. Z_0 is the free-space impedance. The electromagnetic fields satisfy the boundary conditions given in respectively,

$$n\ x\ (E_+ - E_-)|_s = -J_{ms} \tag{1.3}$$

$$n\ x\ H = n.\ R_m\ x\ J_{ms} \tag{1.4}$$

$$n\ x\ (H_+ - H_-)|_s = 0 \tag{1.5}$$

"+" and "–" refers to upper and lower parts of the conductive surface. n is the unit normal vector. d refers to width of dielectric surface. μ and μ_0 are the permeability of surface and free-space in respectively. The parameter R_m known as the surface conductivity, models the conductive surface. The surface conductivity is denoted by R_m which is equal to -j/[ωd(μ-μ_0)].

1.3.2 Bessel Function

The Bessel function, a special function used in mathematics, physics and engineering, was introduced by Friedrich Bessel. Being used to explain the distribution and reflection of electromagnetic waves, Bessel functions are utilized to model the reflection of planar electromagnetic waves from a circular conductor, in particular. The Bessel function is defined by,

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos\cos\,(nt - x\sin t)\;\; dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(nt - x\sin t)}\, dt \tag{1.6}$$

and

$$J_{-n}(x) = (-1)J_n(x) \tag{1.7}$$

Bessel functions represent the root component of a function in cylindrical coordinates, that is, they are used to solve wave equations in cylindrical coordinates.

1.3.3 Hankel Function

Hankel function was introduced by Hermann Hankel. Being closely related to the Bessel functions, Hankel function is a special type of function used in mathematics physics and engineering. There are two types of Hankel functions; denotes the inward and outward propagation values of the cylindrical wave functions, respectively, and are used to model the dispersion and reflection of an electromagnetic wave. The Hankel function is defined by the equation,

$$H_a^{(1)}(x) = \frac{1}{i\pi}\int_{-\infty}^{\infty + i\pi} e^{x\sin ht - at}\, dt \tag{1.8}$$

$$H_a^{(2)}(x) = -\frac{1}{i\pi}\int_{-\infty}^{\infty - i\pi} e^{x\sin ht\; - at}\, dt \tag{1.9}$$

and

$$H_{-a}^{(1)}(x) = e^{i\pi a} H_a^{(1)}(x) \tag{1.10}$$

$$H_{-a}^{(2)}(x) = e^{-i\pi a} H_a^{(2)}(x) \tag{1.11}$$

and

$$H_a^{(1)}(x) = \frac{J_{-a}(x) - e^{-i\pi a} J_a(x)}{i\sin\pi a} \tag{1.12}$$

$$H_a^{(2)}(x) = \frac{J_{-a}(x) - e^{-i\pi a} J_a(x)}{-i\sin\pi a} \tag{1.13}$$

That is, Hankel functions are used to solve wave equations in cylindrical coordinates and are defined in a similar way to Bessel functions, so they are also called Bessel functions of the third kind.

1.3.4 Fresnel Function

Introduced by Augustin Fresnel, The Fresnel Function is widely used in optics, radio communications, microwave engineering, surface science and other fields. It is applied in calculating electromagnetic field intensity in a medium where light is diffracting around objects. The Fresnel function is defined by the equation,

$$\mathrm{F}|\mathrm{x}| = \frac{e^{j\pi/4}}{\sqrt{\pi}} \int_x^{\infty} e^{-jt^2} dt \tag{1.14}$$

That is, in electromagnetic wave theory, solving problems such as surface propagation, reflection, and diffraction.

1.3.5 Helmholtz Equation

The Helmholtz equation, a wave equation used in mathematics, physics, and engineering, was introduced by Hermann Helmholtz. Also known as the reduced wave equation, the Helmholtz equation is used in the mathematical solution of electromagnetic wave problems, acoustic wave problems, hydrodynamic problems, and many other physical problems. Finding separable solutions to Helmholtz equations can be solved in cylindrical or spherical coordinates to produce Bessel functions. Substituting V(r,t) = V(r) = $e^{j\omega t}$ into the wave equation leads to a differential equation for the complex amplitude V(r);

$$\nabla^2\mathrm{V} + k^2\mathrm{V} = 0 \tag{1.15}$$

which is known as the Helmholtz equation, where,

$$c = 1/\sqrt{\mu\varepsilon} \tag{1.16}$$

$$k = \omega\sqrt{\mu\varepsilon} \tag{1.17}$$

$$k = \omega/c \tag{1.18}$$

k refers to as the wavelength. c is the speed of light. μ is the permeability, ε is the permittivity. Different solutions are obtained from other boundary conditions, defined by the equation on cylindrical coordinates.

$$R\frac{\partial}{\partial R}(R\frac{\partial G}{\partial R}) + \frac{\partial^2 G}{d\beta^2} + k^2R^2G = 0 \tag{1.19}$$

Where R refers to the vector, G refers to the Green's function in three-dimensional space. The Helmholtz equation demonstrates the propagation of electromagnetic waves in an environment and the field distribution in the environment excited by electromagnetic waves.

1.3.6 Green Function

Green's function was introduced by George Green. Green's functions, a type of function used in mathematics, physics, and engineering, facilitate the solution of inhomogeneous differential equations under certain boundary conditions. A Green's function can be expressed as the integral of a source function, and this function is used to solve the differential equation. That is, any linear differential operator can be modeled as a particular Green's function based on the dirac-delta function. The Green's function is defined by,

$$\mathrm{G}(\vec{p}, \vec{p}') = \frac{e^{-jkR}}{R} \tag{1.20}$$

and

$$R = \|\vec{R}\| = \|\vec{p} - \vec{p}'\| \tag{1.21}$$

Green's function, which is used for the propagation and dispersion of electromagnetic waves in electromagnetic wave problems, includes the phase and shows the distance from the source.

CHAPTER II

CURRENT BASED TECHNIQUES

2.1 FUNDAMENTAL OF TECHNIQUES

In this thesis, "Scattering from Conductive Cylindrical Surface" of electromagnetic waves emitted from a linear current source is investigated. Several methods have been developed so far to solve these problems. With these methods, calculations are made for various geometries and surfaces. Due to the deficiencies in each process, new techniques have been continued to be designed. In the thesis, the theory of Physical Optics is explained. In the following stages, the fields scattered from the infinitely long conductive cylindrical surface in the region of the linear current source are examined.

2.2 FAR-FIELD METHOD

The far-field approximation is a numerical method used to analyze the scattering of reflected and transmitted waves from cylindrical surfaces, especially considering the radiation pattern of the scattered wave and at the far-field region. It assumes that the observation point is far enough away that the wave front can be approached as a plane wave. In the context of other numerical methods, the far-field approach is typically applied when assessing diffuse fields at distant observation points. When using other numerical methods, the computational space is typically finite and computations are performed within a limited region. To obtain the far-field scattering pattern, the scattering field must be evaluated at observation points far enough from the scattering object. The far-field approach involves determining the angular spectrum of the scattered field, which characterizes the directional distribution of the wave in the far-field region. Various techniques such as Fourier transform, spherical wave broadening or plane wave decomposition can be used to extract far-field information from near-field results obtained in the computational field. Since the wave can be characterized by its direction and intensity in the far-field region, the far-field approximation allows for simplifications in the analysis and calculation of the scattering problem. Using the far-field approach, the radiation pattern of a conductive cylindrical surface, and scattering properties such as directionality and polarization can be analyzed at distant observation points. This knowledge is valuable for applications where the behavior of the scattered wave in the far-field region is of primary concern, such as antenna design, radar systems, and wireless communications.

2.3 PHYSICAL OPTICS

Physical Optics studies the propagation, reflection and diffraction of electromagnetic waves. That is, it is a method used to calculate the distribution of electromagnetic waves on surfaces. In this method, the surface is exposed to electromagnetic waves and the distribution of the surface is obtained by calculating the currents dispersed from these waves. The method also regards the features of electromagnetic waves such as wavelength, frequency, and polarization. Physical Optics approximates the field scattered from the target object, assuming the total area at each point on the target surface is the field that would be there if the target were flat, with the assumption that the target is far-field. When an electromagnetic wave illuminates a surface, the current is induced on the surface. These induced currents act as a second source, emitting electromagnetic fields in all directions. According to the Physical Optics approach, it is assumed that there are surface currents in the illuminated parts of the target and the current drops directly to zero in the shaded parts. Due to this sharp transition, the current values calculated away from the regular reflection regions and at the shadow boundaries deviate from the values they should be. Corner diffraction, multiple reflections, and surface waves are scattering processes that are not considered by the Physical Optics approach.

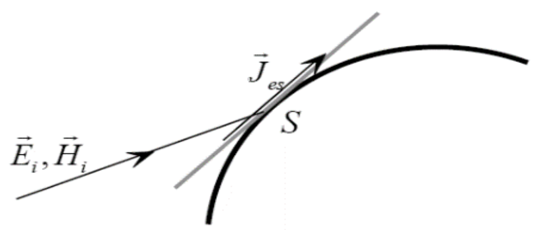

Figure 1: Induced surface by incident wave

J is the surface current induced by the incident wave can be seen in Figure 1. The boundary conditions are defined by the surfaces,

$$J_{es} = \begin{cases} \vec{n}\ x\ \vec{H}_t\ , Radiated \\ 0, \qquad Shadow \end{cases} \tag{2.1}$$

$$J_{ms} = \begin{cases} -\vec{n}\ x\ \vec{E}_t\ , Radiated \\ 0\ , \qquad Shadow \end{cases} \tag{2.2}$$

$$\vec{E}_i = \vec{E}_r \tag{2.3}$$

$$2\vec{E}_i = \vec{E}_t \tag{2.4}$$

$$\vec{H}_i = \vec{H}_r \tag{2.5}$$

$$2\vec{H}_i = \vec{H}_t \tag{2.6}$$

J_{es} is the surface current density and J_{ms} is the magnetic surface intensity. The total field on the surface is twofold the incident field in the Physical Optics. (the reflected fields as Geometric Optics fields). The total electric and magnetic field is equal to the sum of the scattered and incident fields.

$$\vec{E}_t = \vec{E}_i + \vec{E}_s \tag{2.7}$$

$$\vec{H}_t = \vec{H}_i + \vec{H}_s \tag{2.8}$$

Calculations of scattering fields from similar scattering objects such as spheres and cylinders are also frequently used in practice. According to this approach, the current density of a perfectly conductive scattering surface is twice the current density of the source it is in. With the solution of the Helmholtz equation of the Hertz vector obtained for the scattered field from this surface, the scattered field expressions of a local surface are obtained through the help of the second scalar Green's theorem. In addition, it is generally the preferred high-frequency approximation method since it gives quite accurate results when the observation point is the regular reflection of the far-field. Surface current density and magnetic field intensity are defined by,

$$J_{ms}(\vec{p}') = -\,(\vec{n} \times \vec{E}_t)|_s \tag{2.9}$$

$$J_{es}(\vec{p}') = (\vec{n} \times \vec{H}_t)|_s \tag{2.10}$$

The hertz equation is defined to be used in the scattering integral. 17. and 18. equations are written in the equation,

$$G(\vec{p}, \vec{p}') = \frac{e^{-jkR}}{R} \tag{2.11}$$

where

$$R = \|\vec{R}\| = \|\vec{p} - \vec{p}'\| \tag{2.12}$$

The required relations for electric vector potential and magnetic vector potential are given to establish the scattering integral as,

$$A = \frac{\mu_0}{4\pi} \iint J_{es}(\vec{p}') \frac{e^{-jkR}}{R} dS' \tag{2.13}$$

$$F = \frac{\varepsilon_0}{4\pi} \iint J_{ms}(\vec{p}') \frac{e^{-jkR}}{R} dS' \tag{2.14}$$

where A refers to electric vector potential and F refers to magnetic vector potential. Rotational equations of electric and magnetic vector potential equation for electric and magnetic field are given in respectively.

$$E = -jwA - \frac{1}{\varepsilon_0} \nabla x F \tag{2.15}$$

$$H = -jwF - \frac{1}{\mu_0} \nabla x A \tag{2.16}$$

where E refers to electrical fields and H refers to magnetic fields. The electric and magnetic fields equations are found in as,

$$E = \frac{jw\mu_0}{4\pi} \iint J_{es}(p') \frac{e^{-jkR}}{R} dS' \tag{2.17}$$

$$H = -\frac{jw\varepsilon_0}{4\pi} \iint J_{ms}(\vec{p}') \frac{e^{-jkR}}{R} dS' \tag{2.18}$$

The following formulas are used to convert the electrical and magnetic fields equations to each other.

$$E = \frac{1}{jw\varepsilon_0} \nabla x H \tag{2.19}$$

$$H = - \frac{1}{jw\mu_0} \nabla x E \tag{2.20}$$

The solution of the Helmholtz equation of the hertz vector for the scattered field from the surface is obtained with the help of the second scalar Green's theorem. The Hertz equation is defined by,

$$\pi \quad = - \frac{j\omega\varepsilon_0}{4\pi} \iint_{s'} (\vec{n}\ x\ \vec{H}_t)|_s \ \mathrm{G}(\vec{p}, \vec{p}')ds' \tag{2.21}$$

Shadow boundaries by a source can be seen in Figure 2

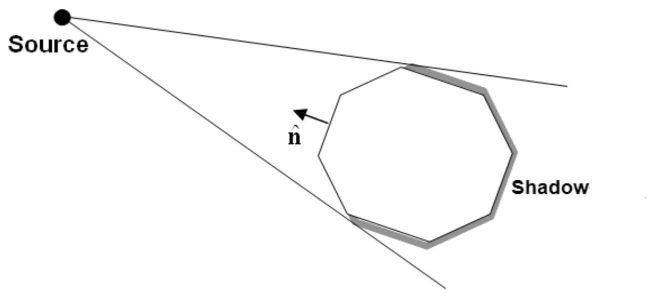

Figure 2: Shadow boundaries by the source

However, for electrically large targets, it is often the preferred high-frequency approximation method because it gives quite accurate results when the point of observation is in the far-field regular reflection zone.

2.4 DEBYE'S ASYMPTOTIC EXPANSION

The Debye Asymptotic is an asymptotic expansion for the Hankel Function of the first type. The technique, which is frequently used in microwave technology, the design of electromagnetically compatible devices, and applications such as radar systems, is also applied in electromagnetic wave problems, where the wavelength is short and the distribution is observed over long distances. This method provides asymptotic analysis of integral expressions used to calculate the dispersion of an electromagnetic wave.

$$H_0^{(1)}(kp) \approx \sqrt{\frac{2}{\pi}}\,\frac{e^{-jkp+j\pi/4}}{\sqrt{kp}} \tag{2.22}$$

and

$$H_0^{(2)}(kp_2) \approx \sqrt{\frac{2}{\pi}}\,\frac{e^{-jkp_2+j\pi/4}}{\sqrt{kp_2}} \tag{2.23}$$

are defined respectively. Debye Asymptotic expresses the solution of the wave scattering problem as a series; that is known as Debye functions. Debye functions vary depending on the wavelength and the angle of scattering and approach zero as the angle of scattering increases. It is used in scattering integral analysis.

CHAPTER III

SCATTERING FROM A CYLINDRICAL CONDUCTIVE SURFACE

.

3.1 APPROACH TO THE PROBLEM

An electromagnetic wave emitted from a linear source of electricity is scattered when it strikes a conductive cylindrical surface. To analyze this scattering event, some basic information and methods are considered. First, the electromagnetic wave emitted from the linear source of electricity can be regarded as a plane wave. This wave has properties such as wavelength (λ), frequency (f), wave number (k), polarization and propagation speed. To analyze scattering from a conductive cylindrical surface, a four-step approach is usually used.

3.1.1 Calculation of Inside and Outside Areas

The electromagnetic wave propagates inside and outside the conducting cylinder. The interior and exterior areas are calculated using Maxwell's equations and appropriate boundary conditions. At this stage, the wave field is expressed inside and outside the cylinder.

3.1.2 Calculation of Surface Flux Density

The surface flux density describes the interaction between the scattered electromagnetic wave and a conductive cylindrical surface. This intensity depends on the wave field integrated around the surface and the scattering angle. Methods such as Physical Optics are used to calculate this density.

3.1.3 Separation of Surface Flux Density into Reflection and Transmission Regions

The scattered electromagnetic wave is split into surface flux density components. These components are reflection and transmission components. This is called the Transition zone. The reflection component refers to the wave field reflected back from a conductive cylindrical surface, while the transition component refers to the transmitted wave field from the surface.

3.1.4 Integration of Surface Flux Density and Calculation of Surface Emitted Area

By integrating the reflection and transition components of the surface flux density, the radiated area on the surface is calculated. This calculation provides the magnitude, direction and distribution of the electromagnetic wave field scattered from a conductive cylindrical surface. It can be complex and difficult to fully analyze electromagnetic wave scattering emitted from a linear electric source from a conductive cylindrical surface. Therefore, numerical methods and simulations are often used. These methods are used to model and analyze electromagnetic wave scattering in detail.

3.2 GEOMETRY OF THE PROBLEM

This section explains the reflection from the cylindrical surface. TM mode planar wave is incident and R is the reflected ray, ρ is the direct ray for O observation point. S is the reflected point. $\emptyset$ *and* $\emptyset'$ represent the angle of the incident and direct ray respectively as shown in Figure 3.

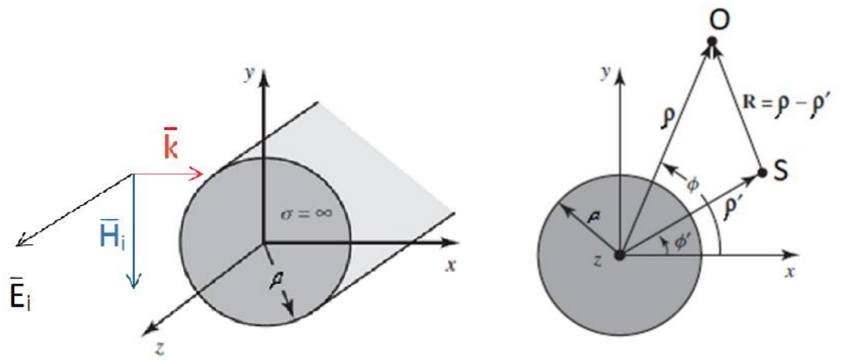

Figure 3: Geometry of a cylindrical surface (Incident TM wave)

3.3 CONSTRUCTING OF SCATTERING EQUATION

Plane wave is normally incident upon a conductive cylindrical surface of radius a, as shown in Figure 3. The electric field can be written as,

$$E_i = \mathrm{e}_z E_0 \mathrm{e}^{-jk\rho\cos\emptyset} \qquad (3.1)$$

The Green's function and surface current density is,

$$J^{PO} = 2n \; x \; H_i \tag{3.2}$$

$$\mathrm{G(O,S)} = \frac{e^{-jkR}}{R} \tag{3.3}$$

using the geometry of the surface given in figure, for the vector in three-dimensional space,

$$\mathrm{R} = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\emptyset - \emptyset')} \tag{3.4}$$

the Debye's asymptotic expansion for $kp \rightarrow \infty$ of the second-order Hankel function of the zeroth order;

$$H_0^{(2)}(k\rho \quad) \approx \sqrt{\frac{2}{\pi}} \frac{e^{-jk\rho \quad +j\pi/4}}{\sqrt{k\rho}} \tag{3.5}$$

which, according to the transformation can also be expressed as,

$$E_i = \mathrm{e}_z E_0 \sum_{-\infty}^{\infty} J^{-\mathrm{n}} J(k\rho)\mathrm{e}^{jn\emptyset} = \mathrm{e}_z E_0 \sum_0^{\infty} (-J)^{\mathrm{n}} \, \varepsilon_0 J^{\mathrm{n}}(k\rho) \cos(n\emptyset) \tag{3.6}$$

The equation obtained using Maxwell's Faraday equation and reduced for the corresponding magnetic field components,

$$H_i = \frac{-1}{j\mu\omega} \nabla \mathrm{x} E_i = \frac{-1}{j\mu\omega} (\mathrm{e}_\rho \frac{1}{\rho} \frac{\partial E_i}{\partial \emptyset} - \mathrm{e}_\emptyset \frac{\partial E_i}{\partial \rho}) \tag{3.7}$$

$$H_i^p = \frac{-1}{j\mu\omega} \frac{1}{\rho} \frac{\partial E_i}{\partial \emptyset} = = \frac{\varepsilon_o}{j\mu\omega} \frac{1}{\rho} \sum_{-\infty}^{\infty} \mathrm{n} j^{-\mathrm{n}+1} J_{\mathrm{n}}(k\rho)\mathrm{e}^{jn\emptyset} \tag{3.8}$$

$$H_i^\emptyset = \frac{1}{j\mu\omega} \frac{\partial E_i}{\partial \rho} = - \frac{\varepsilon_o k}{j\mu\omega} \sum_{-\infty}^{\infty} j^{-\mathrm{n}} J_{\mathrm{n}}^{\frac{\partial}{\partial (k\rho)}}(k\rho)\mathrm{e}^{jn\emptyset} \tag{3.9}$$

The total field is equal to,

$$E_T = E_i + E_s \tag{3.10}$$

As scattered fields move outward, should be represented by cylindrically moving wave functions,

$$E_s = \ \mathrm{e}_z E_0 \sum_{-\infty}^{\infty} h(Q)\, \mathrm{H}_\mathrm{n}^2(k\rho) \tag{3.11}$$

where $h(Q)$ represents the unknown amplitude coefficients and it can be found by applying the boundary conditions,

$$E_T = \sum_{-\infty}^{\infty}[J^{-\mathrm{n}} J_\mathrm{n}(k\rho)\mathrm{e}^{jn\emptyset} + h(Q)\mathrm{H}_\mathrm{n}^{(2)}] = 0 \tag{3.12}$$

and

$$h(Q) = \text{-}J^{-\mathrm{n}} \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \mathrm{e}^{jn\emptyset} \tag{3.13}$$

the scattered field reduces to,

$$E_s = \ -E_0 \sum_{-\infty}^{\infty} J^{-\mathrm{n}} \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \mathrm{H}_\mathrm{n}^{(2)}(k\rho)\ \mathrm{e}^{jn\emptyset} =$$

$$-E_0 \sum_{-\infty}^{\infty} \varepsilon_0 (-J)^\mathrm{n} \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \mathrm{H}_\mathrm{n}^{(2)}(k\rho)\cos(n\emptyset) \tag{3.14}$$

Using Maxwell's equations, the corresponding scattered magnetic field components are obtained,

$$H_s^p = \ \frac{-1}{j\mu\omega}\frac{1}{\rho}\frac{\partial E_s}{\partial\emptyset} = \frac{\varepsilon_o}{j\mu\omega}\frac{1}{\rho}\sum_{-\infty}^{\infty} \mathrm{n}j^{-\mathrm{n}+1} \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \mathrm{H}_\mathrm{n}^{(2)}(k\rho)\ \mathrm{e}^{jn\emptyset} \tag{3.15}$$

$$H_s^\emptyset = \frac{1}{j\mu\omega}\frac{\partial E_s}{\partial\rho} = \text{-}\ \frac{E_o k}{j\mu\omega}\sum_{-\infty}^{\infty} j^{-\mathrm{n}} \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \mathrm{H}_\mathrm{n}^{2\prime}(k\rho)\ \mathrm{e}^{jn\emptyset} \tag{3.16}$$

The total electric and magnetic field components are written as,

$$E_T^p = E_T^\emptyset = H_T = 0 \tag{3.17}$$

$$E_T = \ E_0 \sum_{-\infty}^{\infty} j^{-\mathrm{n}}\, [J_\mathrm{n}(k\rho) - \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \mathrm{H}_\mathrm{n}^{(2)}(k\rho)]\ \mathrm{e}^{jn\emptyset} \tag{3.18}$$

$$H_T^{p} = -\frac{E_o}{j\mu\omega}\frac{1}{\rho}\sum_{-\infty}^{\infty} \mathrm{n} j^{-\mathrm{n}+1}\,[J_\mathrm{n}(k\rho) - \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}\mathrm{H}_\mathrm{n}^{(2)}(k\rho)]\,\mathrm{e}^{jn\emptyset} \qquad (3.19)$$

$$H_T^{\emptyset} = \frac{E_o k}{j\mu\omega}\sum_{-\infty}^{\infty} j^{-\mathrm{n}}\,[J_\mathrm{n}{}'(k\rho) - \frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}\mathrm{H}_\mathrm{n}^{(2)\prime}(k\rho)]\,\mathrm{e}^{jn\emptyset} \qquad (3.20)$$

Bessel functions are converted with Wronskians equations,

$$\mathrm{Y}_\mathrm{n}{}'(k\rho)J_\mathrm{n}(k\rho) - J_\mathrm{n}{}'(k\rho)\mathrm{Y}_\mathrm{n}(k\rho) = 2/\pi k\rho \qquad (3.21)$$

$$J_s = n\ x\ H_T \qquad (3.22)$$

$$H_T^{\emptyset} = \frac{E_o k}{j\mu\omega}\sum_{-\infty}^{\infty} j^{-\mathrm{n}}\,[\frac{J_\mathrm{n}(k\rho)\mathrm{Y}_\mathrm{n}'(k\rho) - J_\mathrm{n}'(k\rho)\mathrm{Y}_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}]\mathrm{e}^{jn\emptyset} =$$

$$\mathrm{e}_z\frac{2E_o k}{\pi\rho\mu\omega}\sum_{-\infty}^{\infty} j^{-\mathrm{n}}\,\frac{\mathrm{e}^{jn\emptyset}}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)} \qquad (3.23)$$

Far-Field for $k\rho$->L

$$\mathrm{H}_\mathrm{n}^{(2)}(k\rho) = \sqrt{\frac{2j}{\pi k\rho}}\,j^{\mathrm{n}}\mathrm{e}^{-jk\rho} \qquad (3.24)$$

the modified expression is substituted into the scattering equation,

$$\frac{|E_s|}{|E_i|} = \frac{|-E_0\sqrt{\frac{2j}{\pi k}}\frac{\mathrm{e}^{-jk\rho}}{\sqrt{\rho}}\sum_{-\infty}^{\infty}\frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}\mathrm{e}^{jn\emptyset}|}{|E_0\mathrm{e}^{-jkx}|} = \sqrt{\frac{2}{\pi k\rho}}|\sum_{-\infty}^{\infty}\frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}\,\mathrm{e}^{jn\emptyset}| \qquad (3.25)$$

where

$$E_s = -E_0\sqrt{\frac{2j}{\pi k}}\,\frac{\mathrm{e}^{-jk\rho}}{\sqrt{\rho}}\sum_{-\infty}^{\infty}\frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}\,\mathrm{e}^{jn\emptyset} \qquad (3.26)$$

Conductivity for 2D,

$$\lim_{\rho\to\infty}\left(2\pi\rho\,\frac{|E_s|^2}{|E_i|^2}\right) = \frac{4}{\rho}|\sum_{-\infty}^{\infty}\frac{J_\mathrm{n}(k\rho)}{\mathrm{H}_\mathrm{n}^{(2)}(k\rho)}\,\mathrm{e}^{jn\emptyset}|^2 =$$

$$\frac{2\lambda}{\pi}\left|\sum_{-\infty}^{\infty} \varepsilon_0 \frac{J_{\mathrm{n}}(k\rho)}{\mathrm{H}_{\mathrm{n}}^{(2)}(k\rho)} \cos(n\emptyset)\right| \left|\sum_{-\infty}^{\infty} \varepsilon_0 \frac{J_{\mathrm{n}}(k\rho)}{\mathrm{H}_{\mathrm{n}}^{(2)}(k\rho)} \cos(n\emptyset)\right| \tag{3.27}$$

ρ<<λ for small radiuses,

$$\frac{J_0(k\rho)}{\mathrm{H}_{\mathrm{n}}^{(2)}(k\rho)} = \frac{1}{-\frac{2j}{\pi}\ln(0.89k\rho)} = \frac{\pi j}{2ln\,(0.89k\rho)} \tag{3.28}$$

Conductivity for 2D,

$$\frac{2\lambda}{\pi}\left(\frac{\pi^2}{4}\right)\left|\frac{1}{\ln(0.89k\rho)}\right|^2 = \frac{2\lambda}{\pi}\left|\frac{1}{\ln(0.89k\rho)}\right|^2 \tag{3.29}$$

(Balanis 2012)

CHAPTER IV

RESULTS

In the analysis of the results, the scattering wave expression in equation (3.26) is plotted in Matlab. This analytic result is compared with the Ansys HFSS simulation. Fig. 4a shows the HFSS simulation setup where a cylindrical perfectly conducting rod with various radius lengths is placed in front of a planar wave coming from far-field. The wave is TM polarized with respect to the conducting cylindrical surface as it was analyzed in the theory. The frequency is 10 GHz and the simulation is conducted in free space. The radiation patterns obtained from the HFSS simulation are sampled and transferred into Matlab. Later, the pattern obtained by the scattering electric field expression in (3.26) is compared with the HFSS results at different cylinder radiuses. The normalized electric field radiation pattern results are shown in Figure 6 and Figure 7.

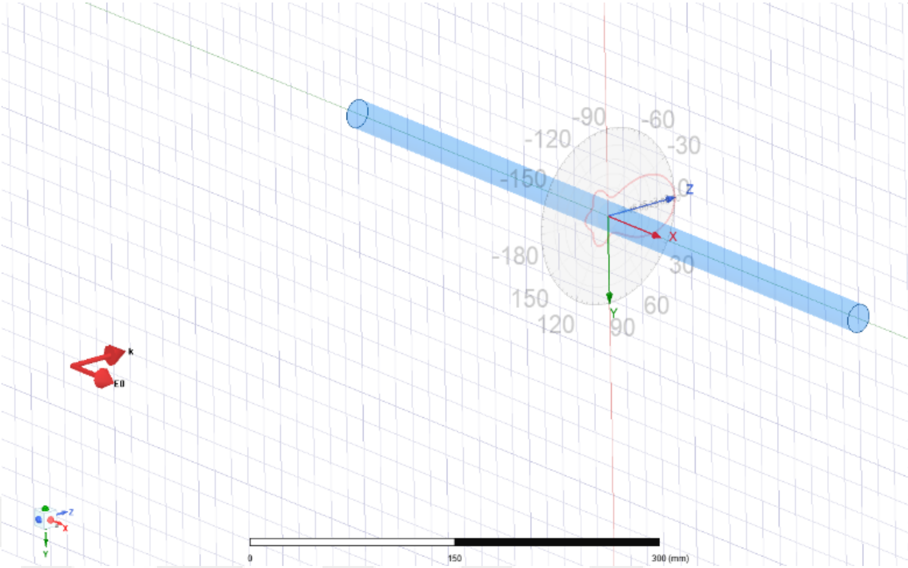

Figure 4: HFSS simulation setup

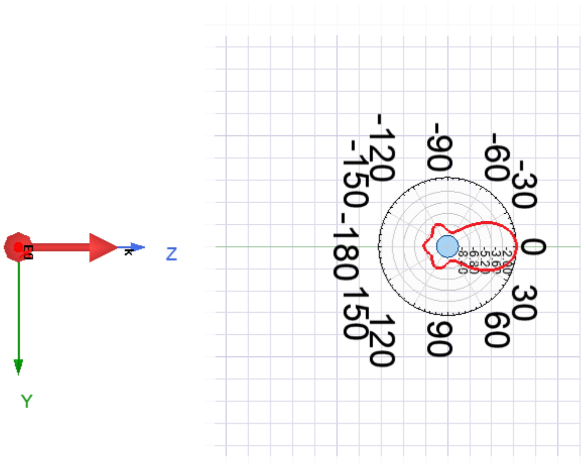

Figure 5: HFSS simulation setup side view

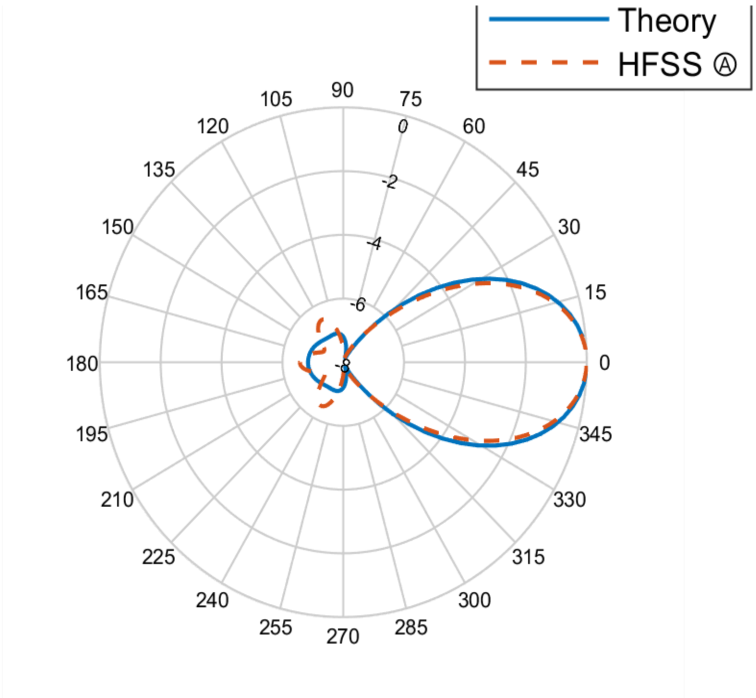

Figure 6: Radiation pattern in dB, cylinder radius = 10mm

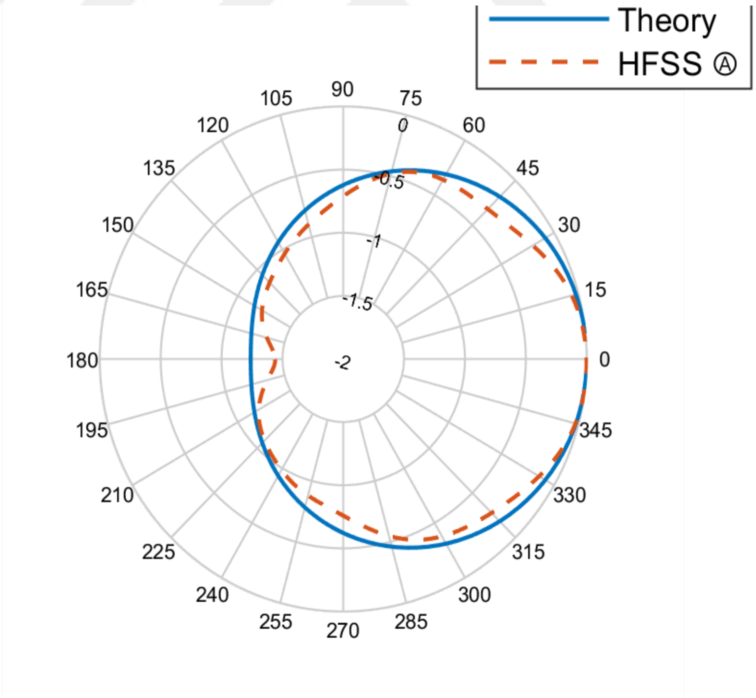

Figure 7: Radiation pattern in dB, cylinder radius = 1mm

In all simulations, the summation in (3.26) is truncated to n=[-100, 100]. However, even for n=[-10,10] truncation resulted in a very accurate plots. The Matlab code for plotting the scattered electric far-field pattern is shown in Figure 7.

```
% Scattering from a cylindrical conductor surface
clear all;
freq=10*1e9;
c=3e8;
lambda=c/freq;
Bt=2*pi/lambda;
rho=200;
E0=10;
phi=0.1:0.05:2*pi;
Et=0;
a=0.001; % cylinder radius of 1mm.
for n = -100:1:100
    Ea=((besselj(n,Bt*a)./besselh(n,2,Bt*a))).*exp(1i*n.*phi);
    Et=Et+Ea;
end
Es=-E0*sqrt((2*1i)/(pi*Bt))*(exp(-1i*Bt*rho)/sqrt(rho))*Et;
% polar(phi,abs(Es));
% theta=0;
% polarpattern(phi*180/pi,20*log10(abs(Es)));
polarpattern(phi*180/pi,20*log10(abs(Es)./max(abs(Es))),'LineStyle','-'); hold on;
```

Figure 8: Matlab simulation

The results show that equation (3.26) is a good approximation to scattered fields from a conducting cylindrical surface when the incident field is a TM wave. The HFSS scattered fields are slightly different than the theoretical patterns due to the fact that edge diffractions are not accounted in Physical Optics.

CHAPTER V

CONCLUSION

The total field is the summation of the scattered and incident waves. For the total field, an alternate representation can be referred to by the following equation,

$$E_i^S + E_r^S = E_T \tag{5.1}$$

The Physical Optics (PO) equation is derived for the conductive surface. The method of Physical Optics (PO) is used to obtain scattered fields. The results are compared with the Ansys HFSS simulation software, and it is concluded that Physical optics method yields good results for the scattering of electromagnetic waves from conducting cylindrical surfaces. In addition, edge diffraction algorithms can be employed with the Physical optics method in order to obtain more accurate results.

REFERENCES

Balanis C.A. (1982), *Antenna Theory: Analysis and Design*, Harper and Row Publishers, New York.

Balanis C. A. (2012), *Advanced Engineering Electromagnetics*, Second Edition, John Wiley & Sons, USA.

Born M. and Wolf E. (1980), *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, Seventh Edition, Cambridge University Press, United Kingdom.

Dinç G. (2014), *Line Integral Representation of Fringe Waves For Perfectly Conducting and Impedance Surfaces, Physical Optics Image* (Master's Thesis), Çankaya Universiy Graduate School of Natural and Applied Sciences, Ankara.

Fresnel A. J (1812), *Mémoire sur la Diffraction de la Lumiére*, Annales de Chimie des Physique, Paris.

James G. L. (1979), *Geometrical Theory of Diffraction for Electromagnetic Waves*, Third Edition, Peter Peregrinus on behalf of IEEE, United Kingdom.

Kranzer H. C. and Radlow J. (1965), "An Asymptotic Method for Solving Perturbed Wiener-Hopf Problems", *Journal of Mathematics and Mechanics*, Vol. 14, pp. 41-59.

Keller J. B. (1962), "Geometrical Theory of Diffraction", *Journal of the Optical Society of America*, Vol. 52, pp. 116-130.

Kirchhoff, G. (1883), *Vorlesungen Uber Mechanik*, Teubner, Leipzig.

Kouyoumjian R. G. and Pathak P. H. (1974), "A Uniform Theory of Diffraction for Edge in a Perfectly Conducting Surface", *IEEE*, Vol. 62, No. 11, pp. 1448-1461.

Macdonald H. M. (1913), "The Diffraction of Light by an Opaque Prism", *Proceedings of the London Mathematical Society*, Vol. s2-12, Issue 1. pp. 430-432.

Maggi G. A. (1888), "Sulla Propagazione Libera e Perturbata Delle Onde Luminose in un Mezzo Izotropo", *Annali di Matematica Pura ed Applicata*, Vol. 16, pp. 21-48.

Maliuzhinets G. D. (1958), "Excitation, Reflection and Emission of Surface Waves from a Wedge with Given Face Impedances", *Soviet Physics Doklady*, Vol. 3, pp. 752-755.

Maxwell J. C. (1865), "A Dynamical Theory of the Electromagnetic Field", *Philosophical Transactions of Royal Society of London*, Vol. 155, pp. 459-512.

Michaeli A. (1984), "Equivalent Edge Currents for Arbitrary Aspects of Observation", *IEEE Transactions on Antennas and Propagation*, Vol. 32, Issue 3, pp. 252-258.

Michaeli A. (1995), "Incremental Diffraction Coefficients for the Extended Physical Theory of Diffraction" *IEEE Transactions on Antennas and Propagation*, Vol. 43, Issue 7, pp. 732-734.

Miyamoto K. and Wolf E. (1962), "Generalization of the Maggi-Rubinowicz Theory of the Boundary Diffraction Wave: Part I-II", *Journal of the Optical Society of America*, Vol. 52, pp. 615-636.

Radlow J. (1961), "Diffraction by a Quarter-plane", *The Archive for Rational Mechanics and Analysis*, Vol. 8, pp. 139-158.

Rubinowicz A. (1917), "Die Beugung Welle in der Kirchoffschen Theorie der Beugungsercheinungen", *Annalen der Physik*, Vol. 4, pp. 257-278.

Satterwhite R. (1974), "Diffraction by a Quarter Plane, the Exact Solution, and Some Numerical Results", *IEEE Transactions on Antennas and Propagation*, Vol. 22, pp. 500-503.

Senior T. B. A. and Volakis J. L. (1995), *Approximate Boundary Conditions in Electromagnetics*, IEEE, London.

Sommerfeld A. (1954), *Optics*, Academic Press, New York

Sommerfeld A. (1896), "Mathematische Theorie der Diffraction", *Mathematische Annalen*, Vol. 47, pp. 317-374.

Ufimtsev P. Y. (2007), *Fundamentals of The Physical Theory of Diffraction*, Wiley, New Jersey.

Umul Y. Z. (2004), "Modified Theory of Physical Optics", *Optics Express*, Vol. 12, pp. 4959-4972.

Umul Y. Z. (2008), "MTPO Based Potential Function of the Boundary Diffraction Wave Theory", *Optics and Laser Technology*, Vol. 40, pp. 769-774.

Umul Y. Z. (2008), "Scattering of a Line Source by a Cylindrical Parabolic Impedance Surface", *Journal of the Optical Society of America,* Vol. 25, pp. 1652–1659.

Umul Y. Z. (2008), "Uniform Theory of The Boundary Diffraction Wave", *Optics and Laser Technology*, Vol. 41, pp. 285-288.

Umul Y. Z. (2008), "Uniform Version of The Modified Theory of Physical Optics Based Boundary Diffraction Wave Theory", *Journal of Modern Optics*, Vol. 55, pp. 2797-2804.

Volakis J. L. (1986), "A Uniform Geometrical Theory of Diffraction for an Imperfectly Conducting Half-plane", *IEEE Transactions on Antennas and Propagation*, Vol. 34, Issue 2, pp. 172-180.

Young T. (1802), "On the Theory of Light and Colours", *Philosophical Transactions Royal Society*, Vol. 92, pp. 12-48.

APPENDICES

Appendix 1: Other Articles and Studies to help with this topic

Kara M. (2016) “Scattering of a Plane Wave by a Cylindrical Parabolic Perfectly Electric Conducting Reflector”, *Optik,* Vol. 127, pp. 4531–4535.

Stutzman W. L. and Thiele G. A. (1998), *Antenna Theory and Design*, Wiley, New York.

Umul Y. Z. (2005), “Modified Theory of Physical Optics Approach to Wedge Diffraction Problems”, *Optics Express,* Vol. 13, pp. 216-224.

Umul Y. Z. (2011), “Physical Optics Theory for The Diffraction of Waves by Impedance Surfaces”, *Journal of the Optical Society of America*, Vol. 28, pp. 255-262.

Umul Y. Z. (2012), “Physical Optics Theory of Resistive Surfaces”, *Optics and Laser Technology,* Vol. 44, pp. 357–361.

Umul Y. Z. (2013), “The MTPO/Malyughinetz Hybrid Method for the Scattering Analysis of Parabolic Impedance Reflectors”, *Optik,* Vol. 124, pp. 5577–5584.

Umul Y. Z. (2016), “Three-dimensional Modified Theory of Physical Optics” *Optik*, Vol. 127, pp. 819–824.

Umul Y. Z. (2017), “Diffraction of Waves by a Conductive Half-plane”, *Optik,* Vol. 131, pp. 29-35.

Umul Y. Z. (2019), “Interaction of Electromagnetic Waves by a Cylindrical Parabolic Mesh Reflector”, *Optik*, Vol. 179, pp. 135–144.

Umul Y. Z. (2020), “The Factorization Process in the Modified Theory of Physical Optics” *Optik*, Vol. 205, No.164249.

Umul Y. Z. (2021), “Diffraction by an Interface Between Perfect Electromagnetic Conductor and Conductive Surfaces”, *Optik*, Vol. 241, No.166982.

Yalçın U. (2007), “Scattering from a Cylindrical Reflector: Modified Theory of Physical Optics Solution”, *Journal of the Optical Society of America*, Vol. 24, pp. 502-506.

Yalçın U. and Sarnik M. (2013), “Uniform Diffracted Fields from a Cylindrical Reflector with MTPO”, The Scientific World Journal, Vol. 2013, pp. 1-6